



UNIVERSITATEA TEHNICĂ

DIN CLUJ-NAPOCA

Modeling and control of a quadcopter

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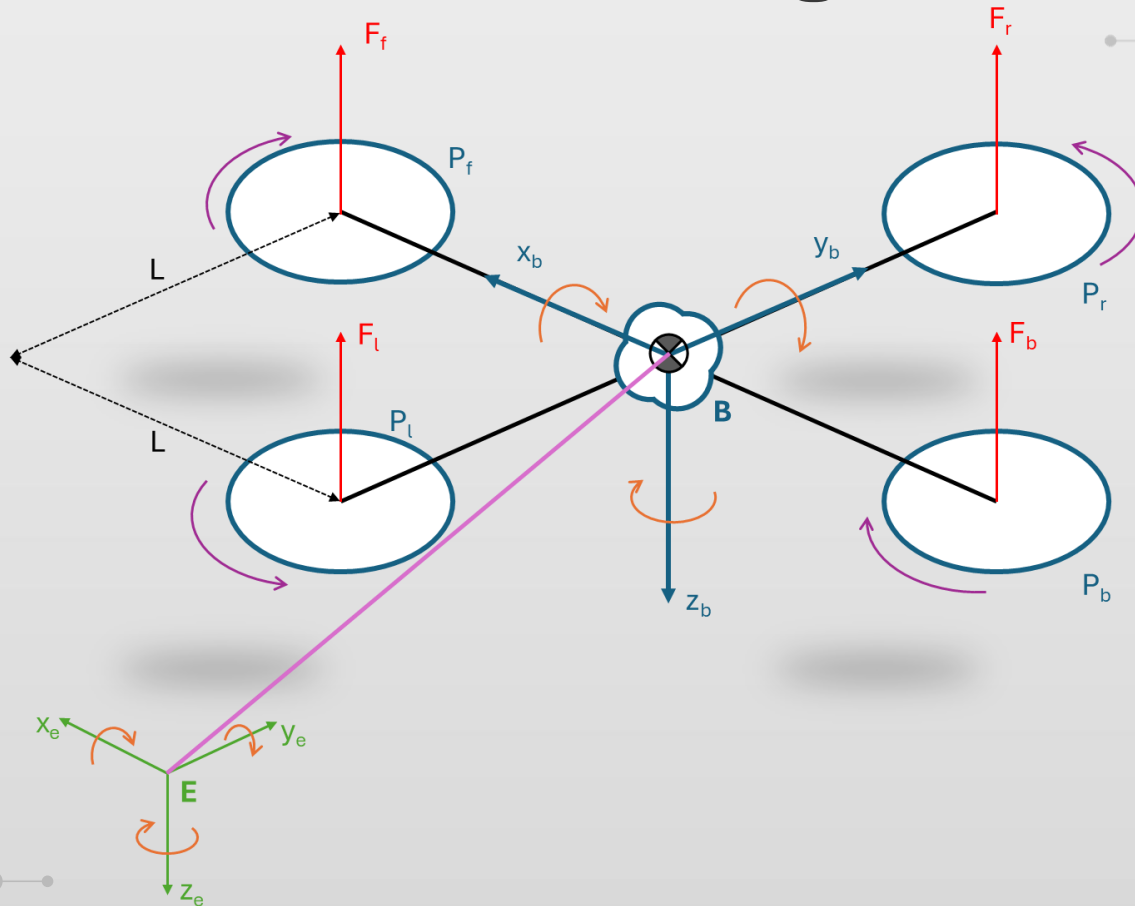
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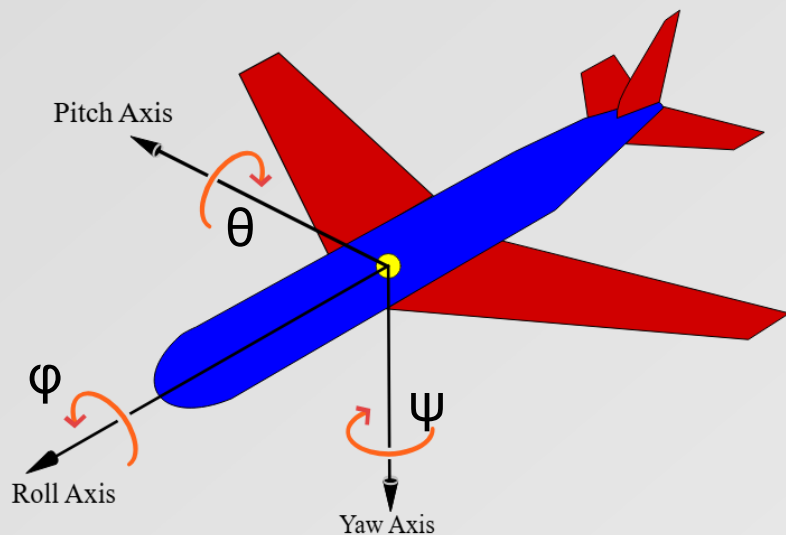
Introduction



Modeling



Euler angles



$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Mathematical models

Newton

$$\mathbf{E}: \left[\underbrace{x \ y \ z}_{\text{transl}} \ \underbrace{\phi \ \theta \ \psi}_{\text{rot}} \right]^T$$

$$\mathbf{B}: \left[\underbrace{u \ v \ w}_{\text{transl}} \ \underbrace{p \ q \ r}_{\text{rot}} \right]^T$$

$$M_1 = J_1 \dot{p} - (J_2 - J_3)qr$$

$$M_2 = J_2 \dot{q} - (J_3 - J_1)rp$$

$$M_3 = J_3 \dot{r} - (J_1 - J_2)pq$$

Lagrange

$$\mathbf{q} = [x \ y \ z \ \phi \ \theta \ \psi]^T$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\tau} \end{bmatrix}$$

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + C^{E/B} \begin{bmatrix} 0 \\ 0 \\ -F_{thrust} \end{bmatrix}$$

$$J(\eta)\ddot{\eta} + C(\eta, \dot{\eta})\dot{\eta} = \boldsymbol{\tau} = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

Hamilton

$$\mathbf{p} = \frac{\partial L}{\partial \dot{\mathbf{q}}}$$

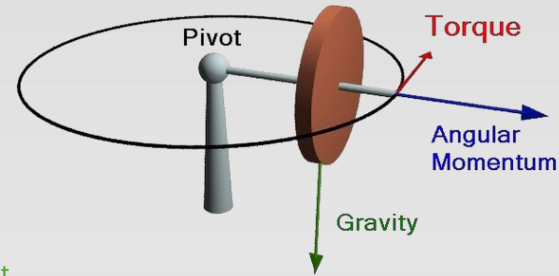
$$H(\mathbf{p}, \mathbf{q}, t) = \mathbf{p}\dot{\mathbf{q}} - L(\mathbf{q}, \dot{\mathbf{q}}, t)$$

$$\dot{\mathbf{q}} = \frac{\partial H}{\partial \mathbf{p}}$$

$$\dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{q}} + \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\tau} \end{bmatrix}$$

Additional phenomena

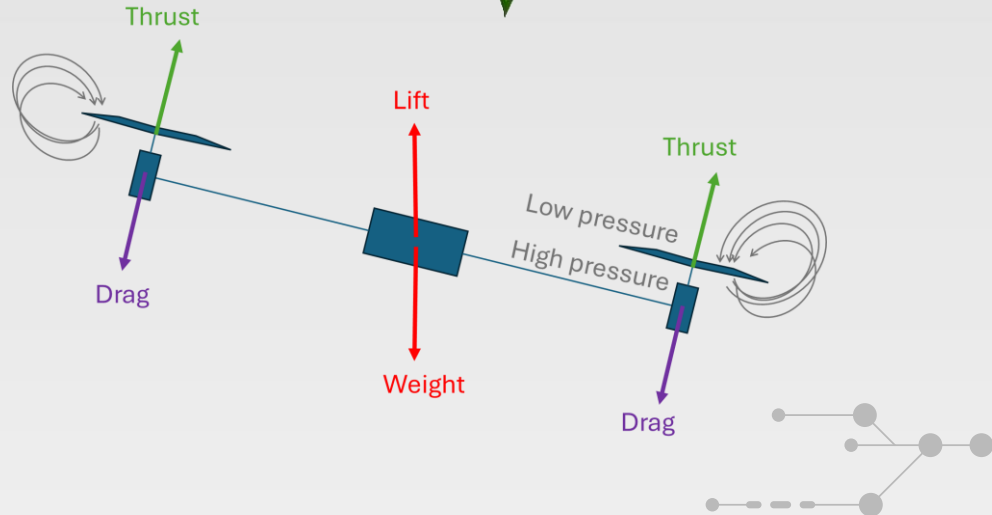
Gyroscopic effects



Aerodynamic resistance

Ground effects

Wind disturbances





Control

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u}$$

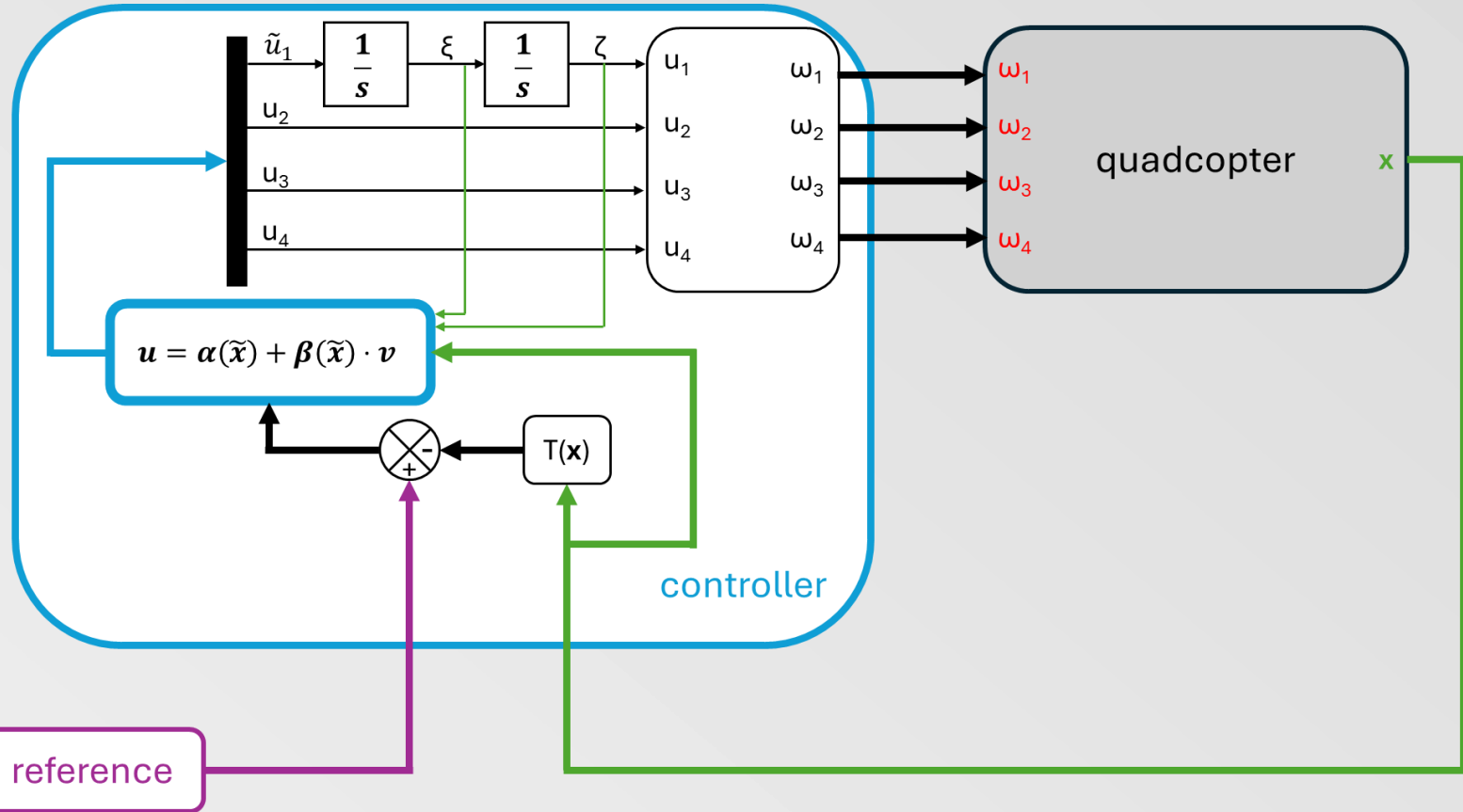
$$\mathbf{x} = [x \quad y \quad z \quad \phi \quad \theta \quad \psi \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad p \quad q \quad r]^T$$

$$\mathbf{u} = [F_{thrust} \quad M_1 \quad M_2 \quad M_3]^T := [u_1 \quad u_2 \quad u_3 \quad u_4]^T$$

$$f(\mathbf{x}) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ p + q(\sin \phi \tan \theta) + r(\cos \phi \tan \theta) \\ q(\cos \phi) - r(\sin \phi) \\ \frac{\sin \phi}{\cos \theta} q + \frac{\cos \phi}{\cos \theta} r \\ 0 \\ 0 \\ g \\ \frac{J_2 - J_3}{J_1} qr \\ \frac{J_3 - J_1}{J_2} rp \\ \frac{J_1 - J_2}{J_3} pq \end{bmatrix}$$

$$g(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{m} (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{m} (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{m} \cos \phi \cos \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{J_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{J_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{J_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Feedback linearization



SMC 2

$$\sigma_1(z) = c_z(z_d - z) + (\dot{z}_d - \dot{z})$$

$$\sigma_2(\psi) = c_\psi(\psi_d - \psi) + (\dot{\psi}_d - \dot{\psi})$$

$$\sigma_3(x, \theta) = c_1(\dot{x}_d - \dot{x}) + c_2(x_d - x) + c_3(\dot{\theta}_d - \dot{\theta}) + c_4(\theta_d - \theta)$$

$$\sigma_4(y, \phi) = c_5(\dot{y}_d - \dot{y}) + c_6(y_d - y) + c_7(\dot{\phi}_d - \dot{\phi}) + c_8(\phi_d - \phi)$$

Adaptive SMC

$$\mathbf{x} = [x \quad y \quad z \quad \phi \quad \theta \quad \psi]^T$$

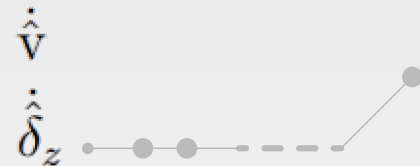
$$\ddot{\mathbf{x}} = f(\mathbf{x}) + G(\mathbf{x})U - \mathbf{v} + \delta_z(\mathbf{x})$$

$$G(\mathbf{x}) = \begin{bmatrix} -\frac{1}{m}(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) & 0 & 0 & 0 & 1 & 0 \\ -\frac{1}{m}(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) & 0 & 0 & 0 & 0 & 1 \\ -\frac{1}{m} \cos \phi \cos \theta & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{J_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{J_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{J_3} & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{g}_s}$$

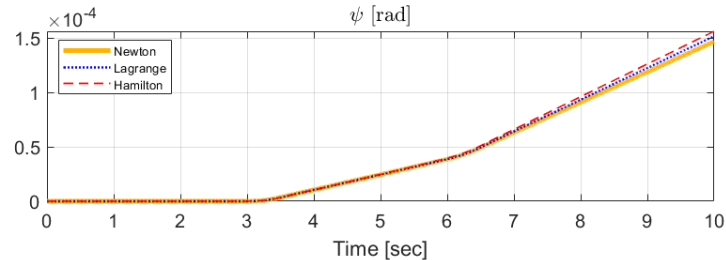
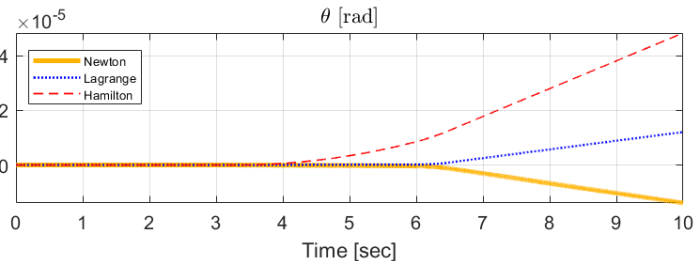
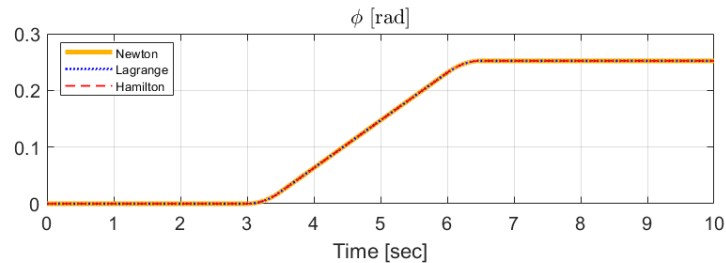
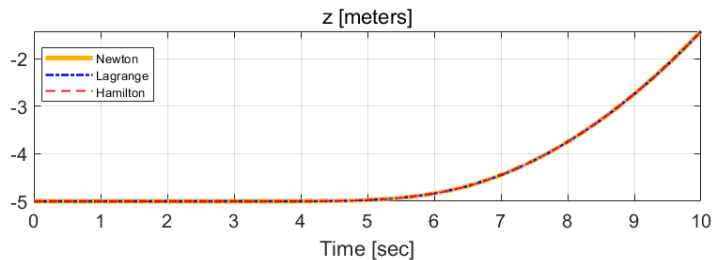
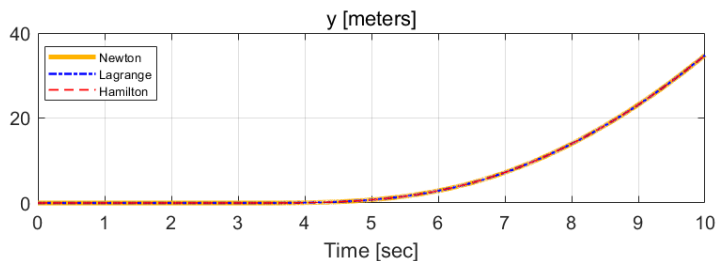
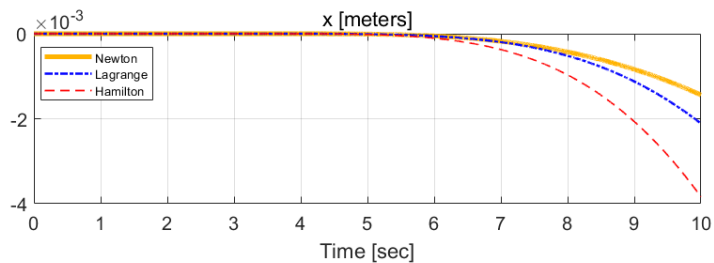
$$U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} \bigg\} \mathbf{u}_s$$

$$\mathbf{v} = \mathbf{g}_s \mathbf{u}_s$$

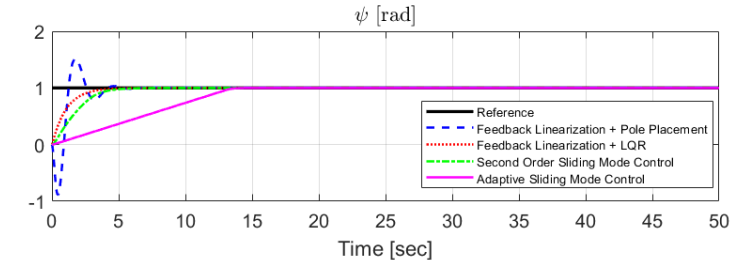
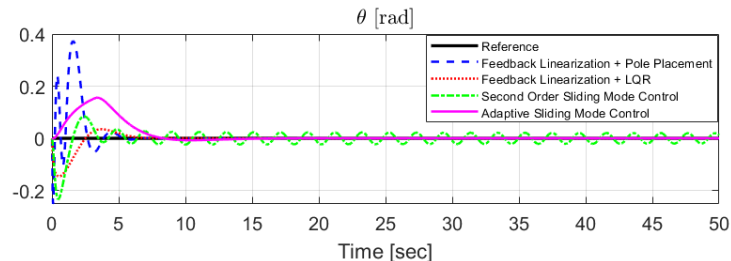
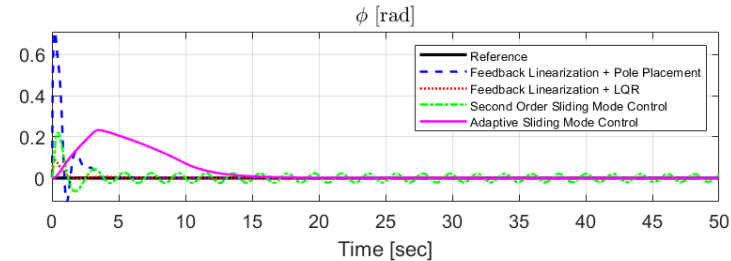
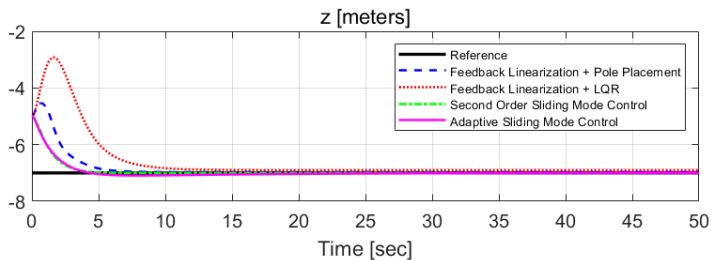
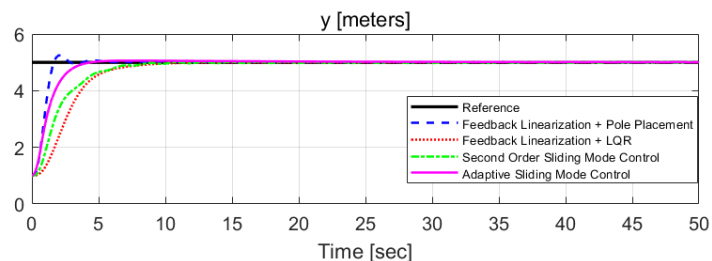
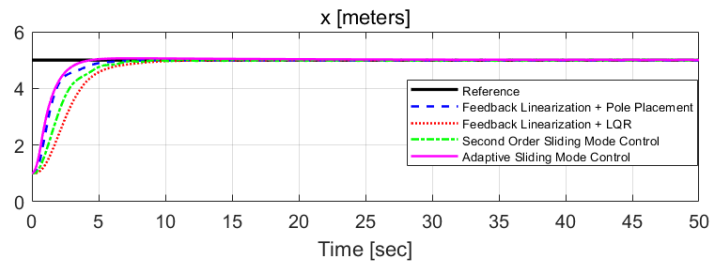
$$\sigma(\mathbf{x}) = \dot{\mathbf{x}} - \dot{\mathbf{x}}_d + K(\mathbf{x} - \mathbf{x}_d)$$



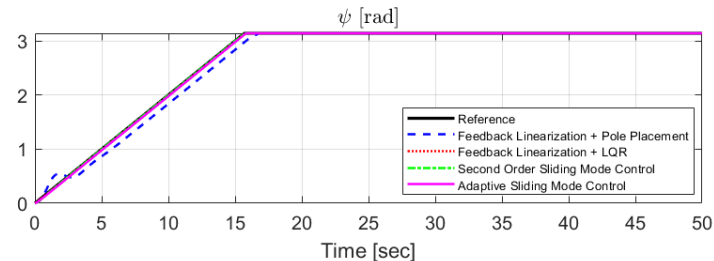
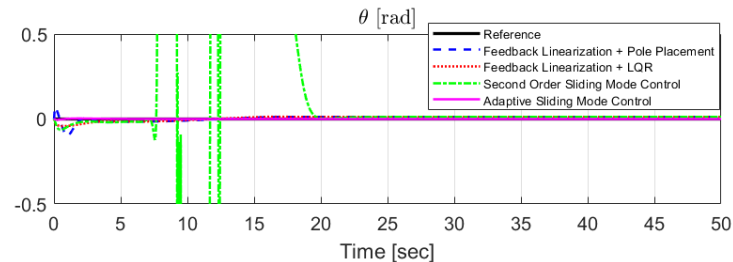
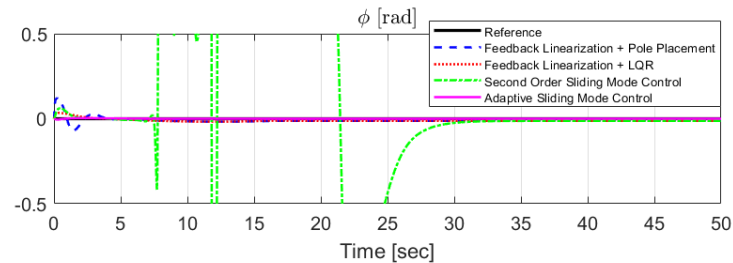
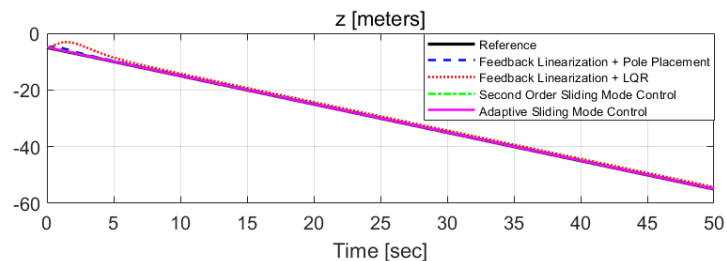
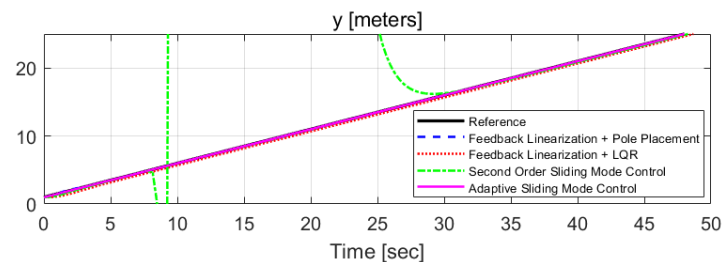
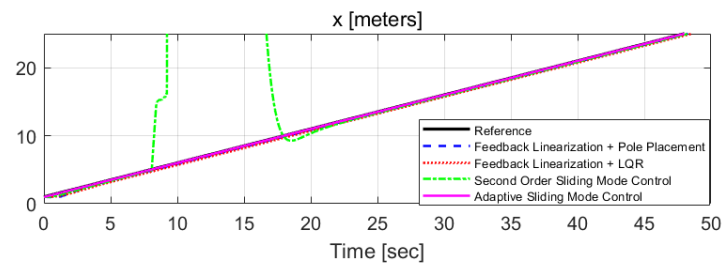
Roll angle stimulation (open loop)



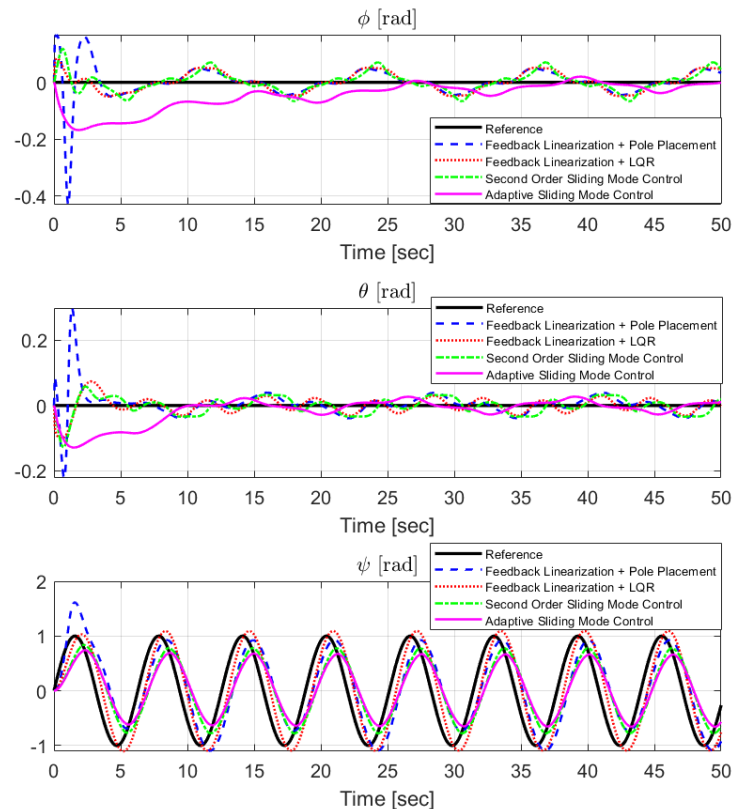
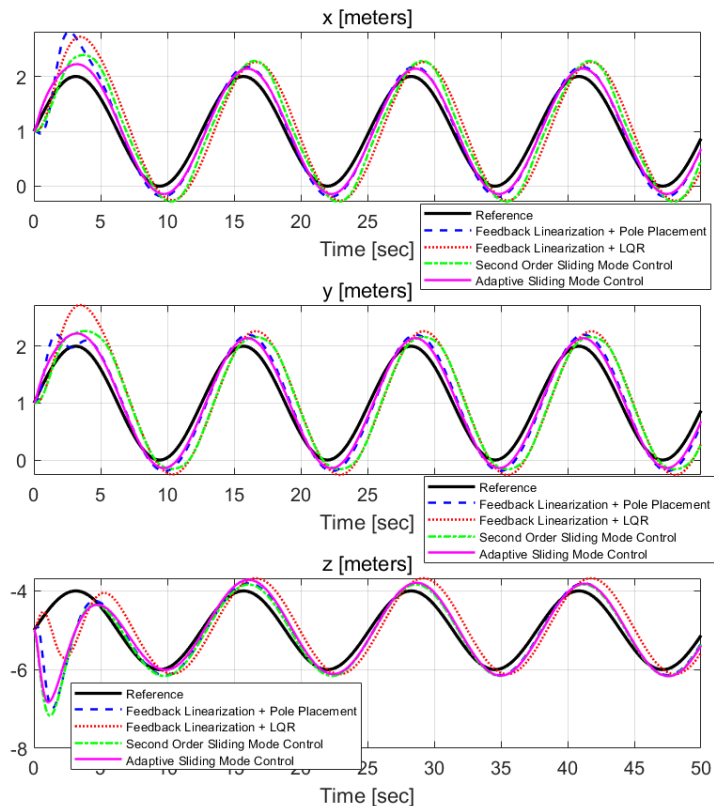
Trajectory tracking - step



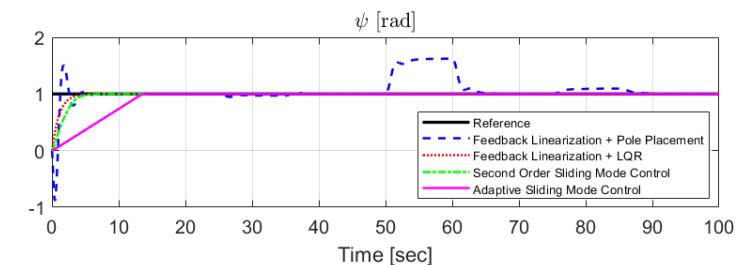
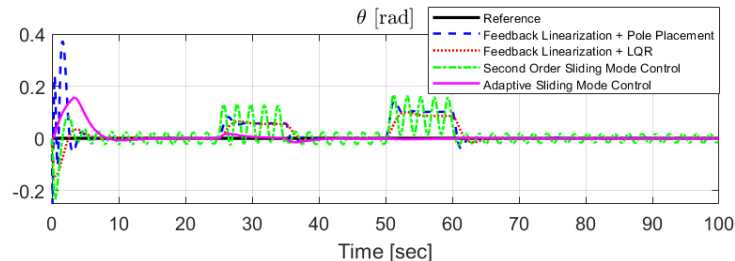
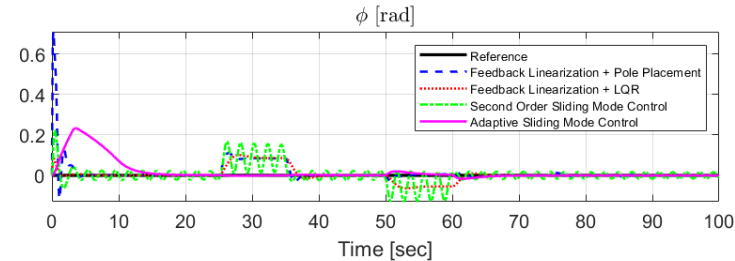
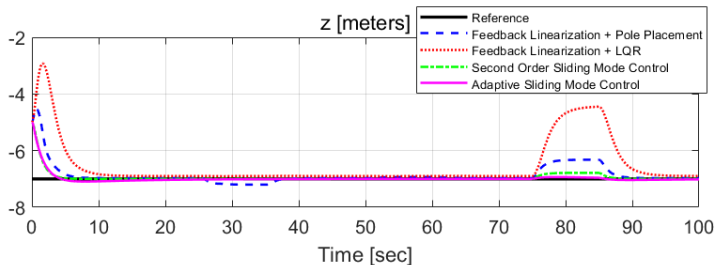
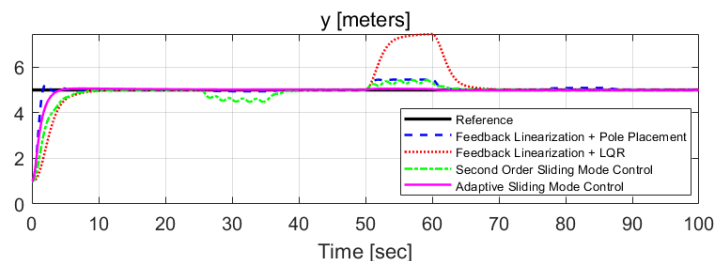
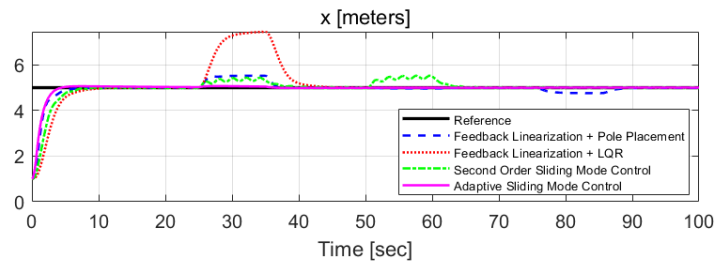
Trajectory tracking - ramp



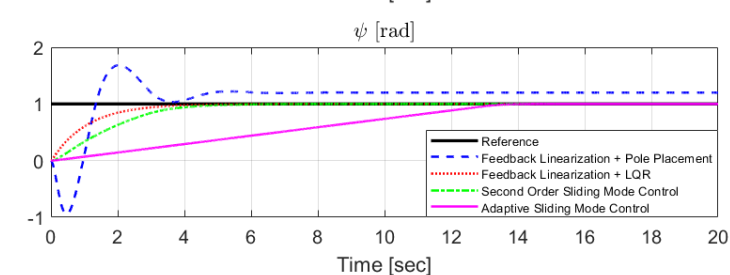
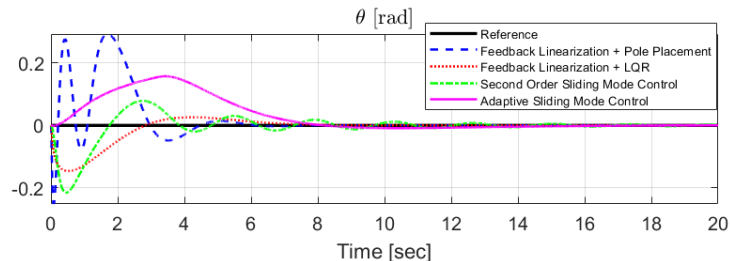
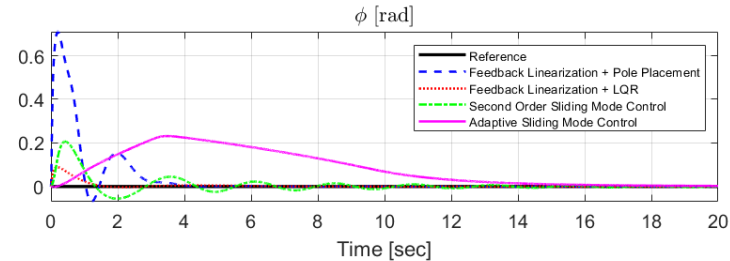
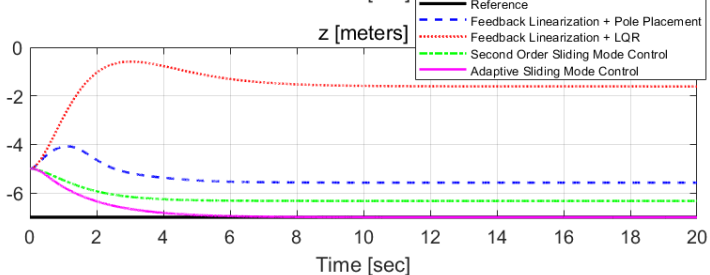
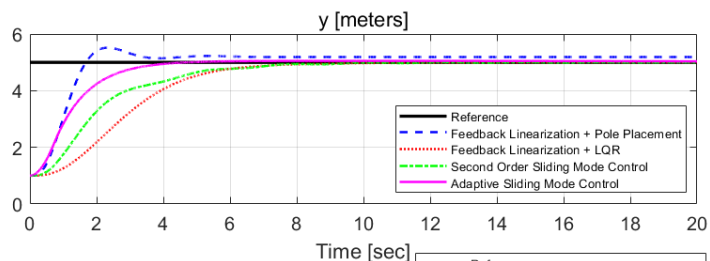
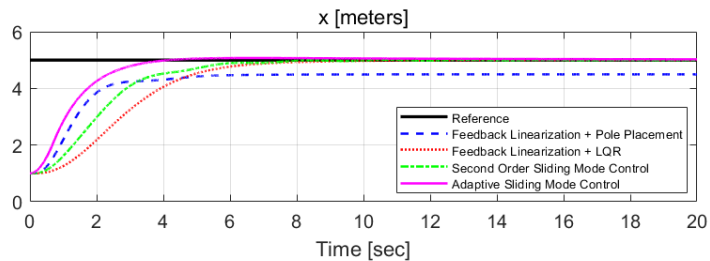
Trajectory tracking – sine waves



Step response during disturbances



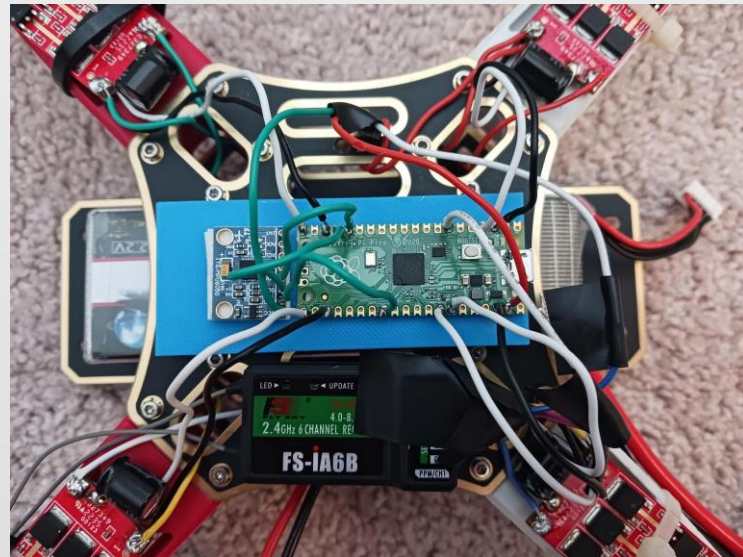
Step response during uncertainties





Conclusions

- Influence of state-space and approximations over control techniques and performance
- Motor Mixing Algorithm
- Undesired behavior in practice: overshoot and oscillations
- Future developments: state and disturbance observers, comparison with other techniques, discrete analysis, integration on computing systems



The background is a light gray with various abstract geometric elements. In the top left, there is a grid of dots. In the top right, there are complex line patterns, including a large circular structure with concentric rings and radial lines. In the bottom left, there are more line patterns and small circles. In the bottom right, there is a small cluster of connected dots.

Thanks!