MINISTERUL EDUCAȚIEI
UNIVERSITATEA TEHNICĂ
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Nonlinear systems – analysis and control

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Agenda

Analysis

- Planar systems and phase portraits
- Limit cycles
- Lyapunov stability
- Nonlinear canonical forms

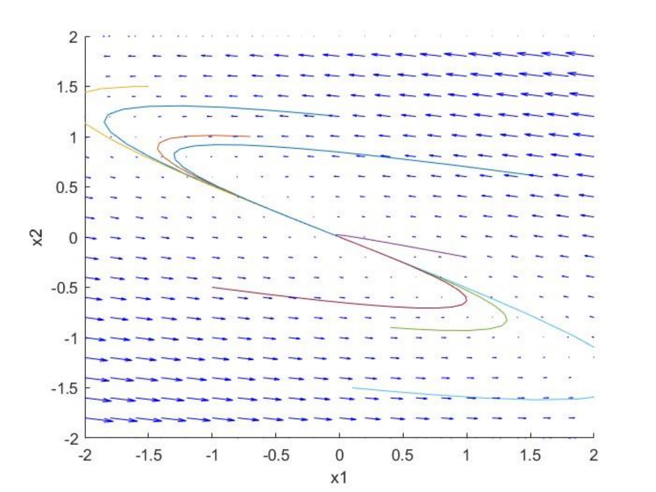
Control

- Feedback linearization
- Sliding mode control
- Backstepping

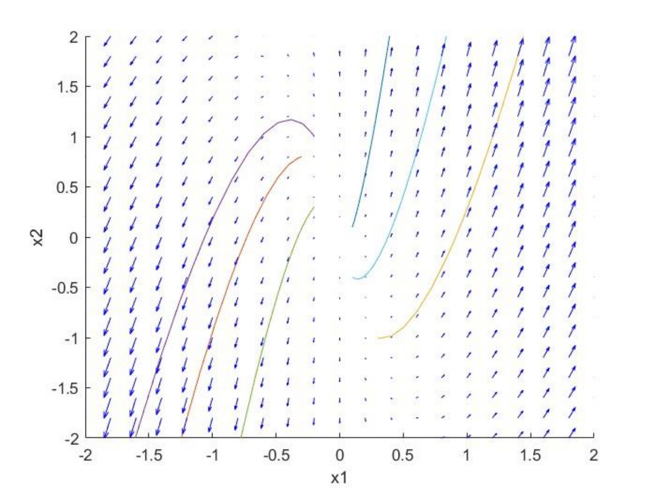
Analysis

Planar systems and phase portraits

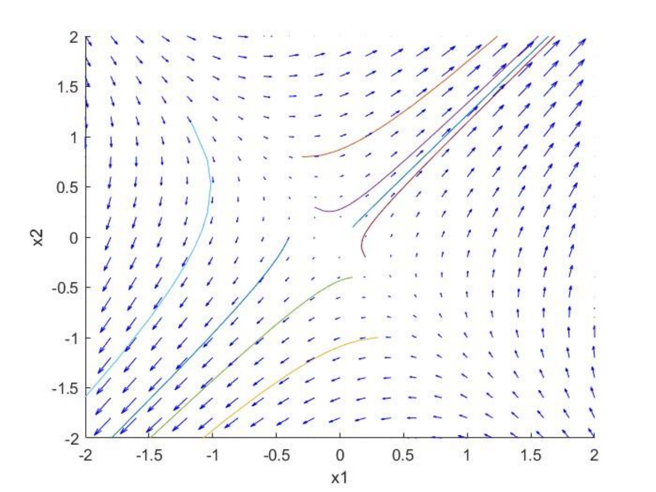
stable node

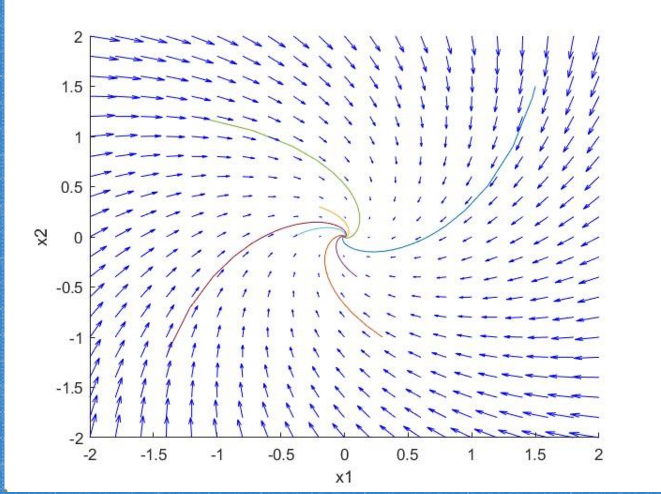


unstable node

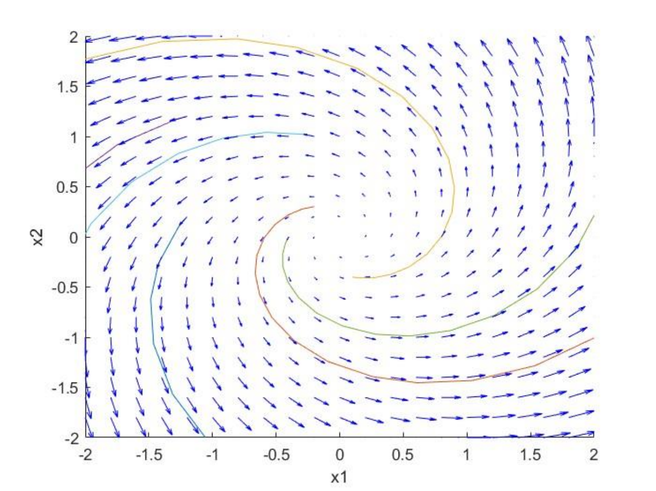


saddle point

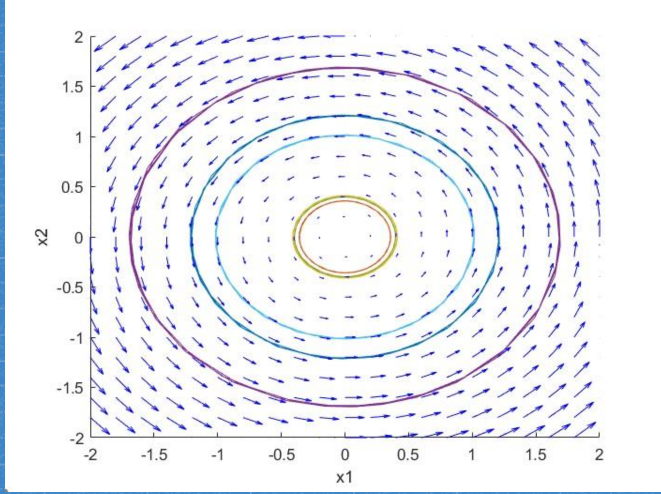




stable foci



unstable foci



center

Limit cycles

periodic solution:

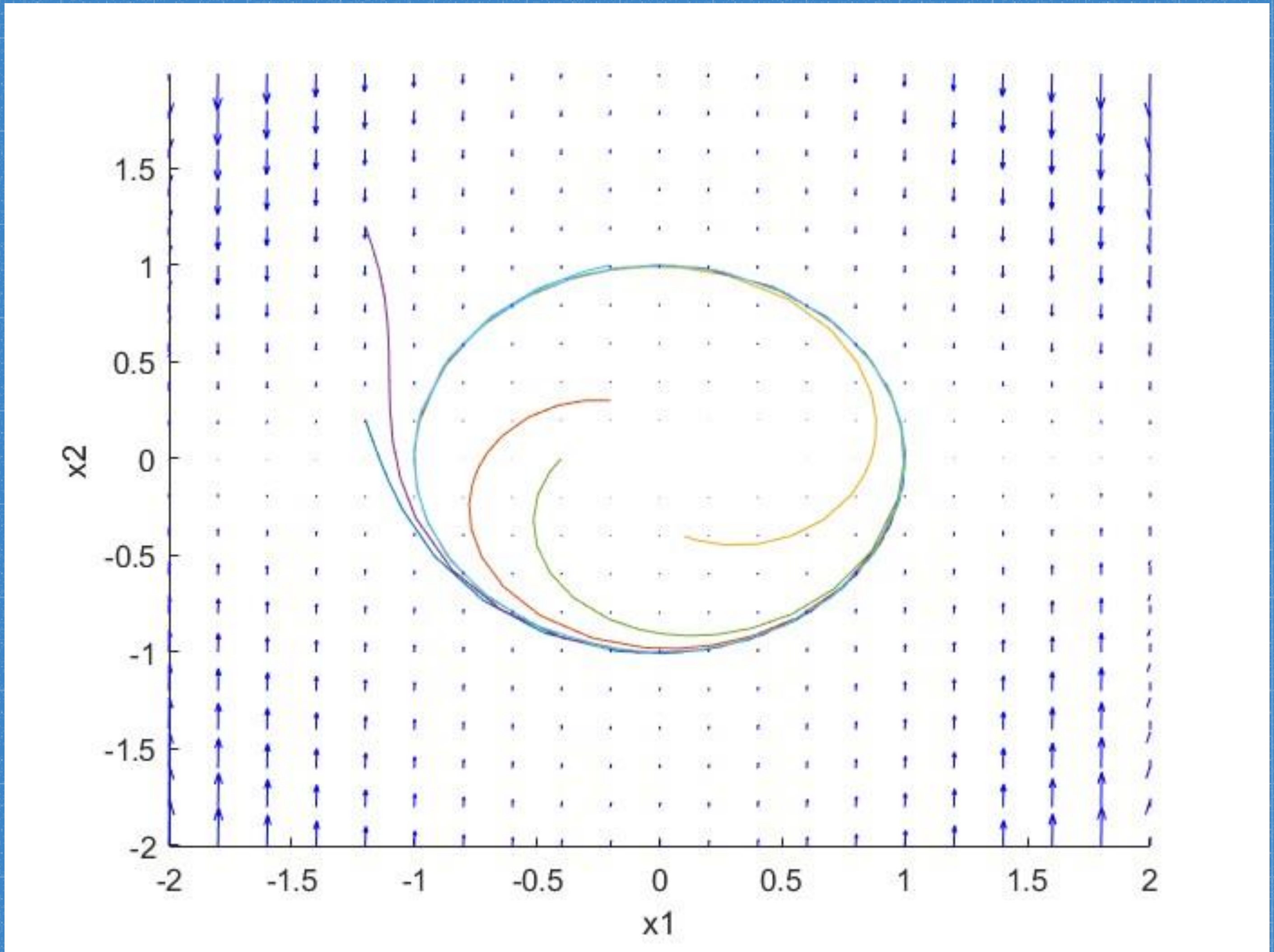
$x(t) = x(t+T), \forall t \geq 0$

min(T) > 0 -> period of the solution

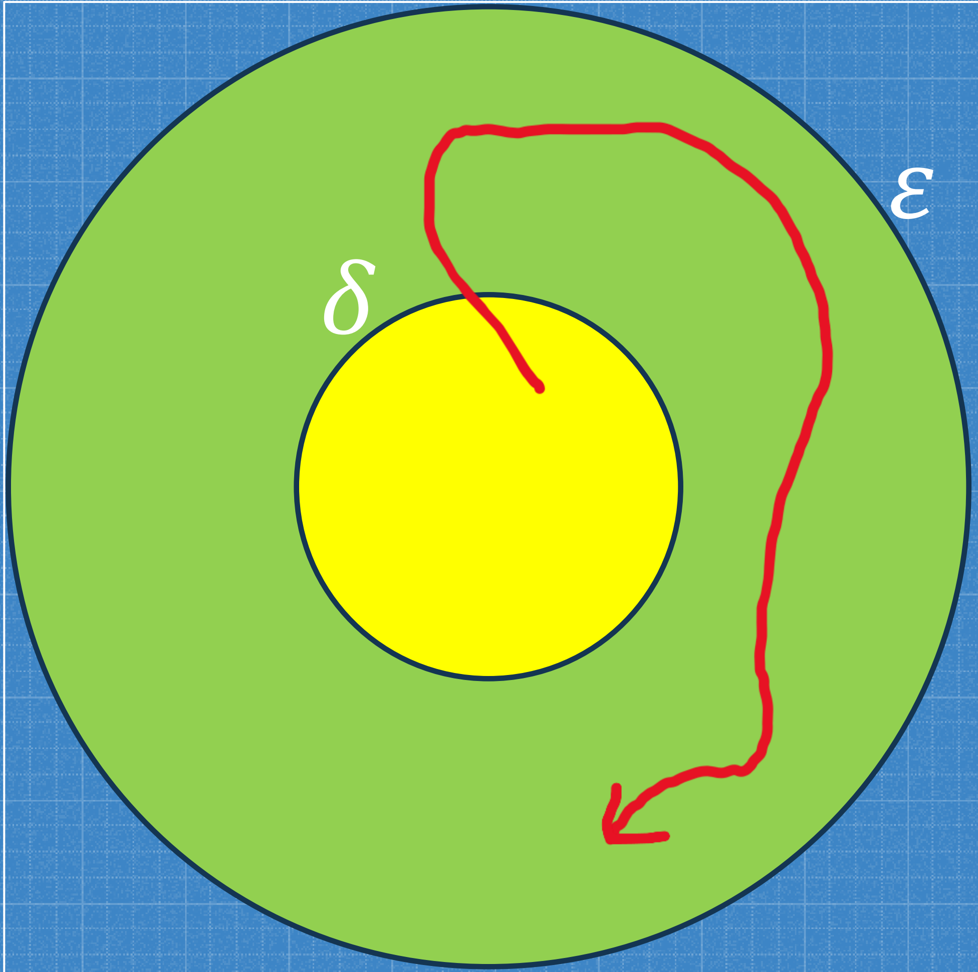
Limit cycles:

- stable
- unstabile
- semi-stable

Stable limit cycle



Lyapunov stability



- stability
- asymptotic stability
- exponential stanility

Lyapunov's Theorem

Let $V(x) > 0, \forall x \neq 0$ și $V(0) = 0$

$\dot{V}(\mathrm{x}) \leq 0, \forall x \neq 0$ implies stability

$\dot{V}(\mathrm{x}) < 0, \forall x \neq 0$ implies asymptotic stability

Forme canonice neliniare

- Normal canonical form

$$\dot{\eta} = f_0(\eta, \xi)$$
$$\dot{\xi} = A_c\xi + B_c\left[L_f^{\rho}h(x) + L_gL_f^{\rho-1}h(x)u\right]$$
$$\mathrm{y} = C_c\xi$$

- Controller canonical form

$$\dot{x} = Ax + B[\psi(x) + \gamma(x)u]$$

- Observer canonical form

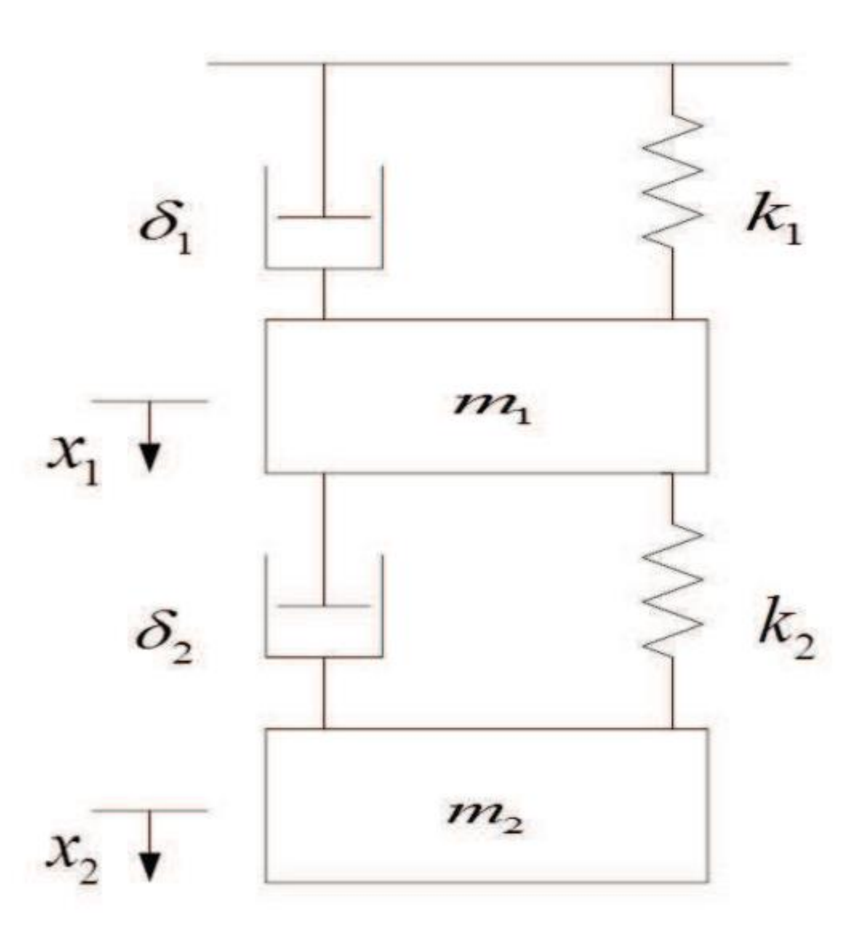
$$\dot{x} = Ax + \psi(u, y)$$
$$y = Cx$$

Control

Plant description

- mechanical system with 2 masses, 2 springs and 2 dampers

$$
\begin{cases}
m_1\ddot{x}_1 = -\delta_1\dot{x}_1 - k_1x_1 + \mu_1x_1^3 - k_2\left(x_1 - x_2\right) \\
\qquad + \mu_2(x_1 - x_2)^3 \\
m_2\ddot{x}_2 = -\delta_2\dot{x}_2 - k_2\left(x_2 - x_1\right) \\
\qquad + \mu_2(x_2 - x_1)^3 + u
\end{cases}
$$



Feedback linearization

Verifying whether a system is feedback linearizable

Theorem: The *system* $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}$ *is feedback linearizable in a neighbourhood* $x_0 \in D$ *if and only if there exists a domain* $D_x \subset D, x_0 \in D_x$*, such that:*

- *matrix* $G(x) = [g(x), ad_f g(x), \dots, ad_f^{n-1} g(x)]$ *has rank n* $\forall x \in D_x$
- *distribution D =* $span\{g, ad_f g, \dots, ad_f^{n-2} g\}$ *is involutive in* D_x

Implementation:

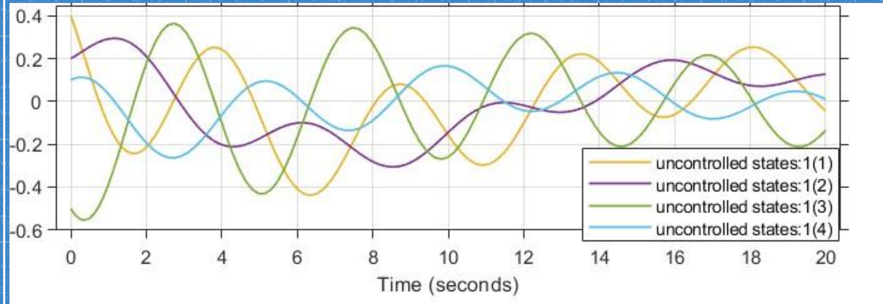
- algorithm to compute Lie bracket with variable order
- algorithm to determine the involtivity of a distribution

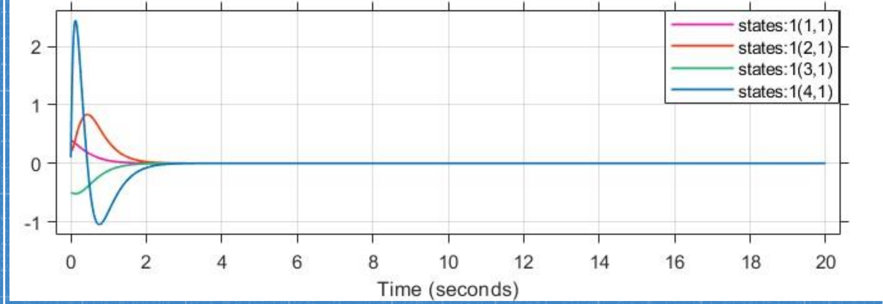
Method description

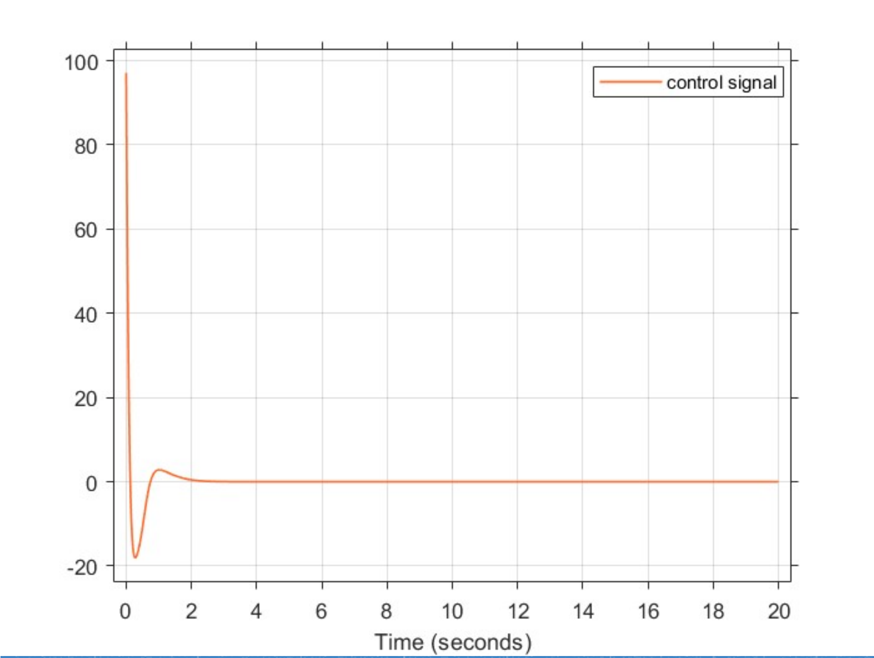
- consider the system $\dot{x} = f(x) + G(x)u$
- bring the system to controller canonical form

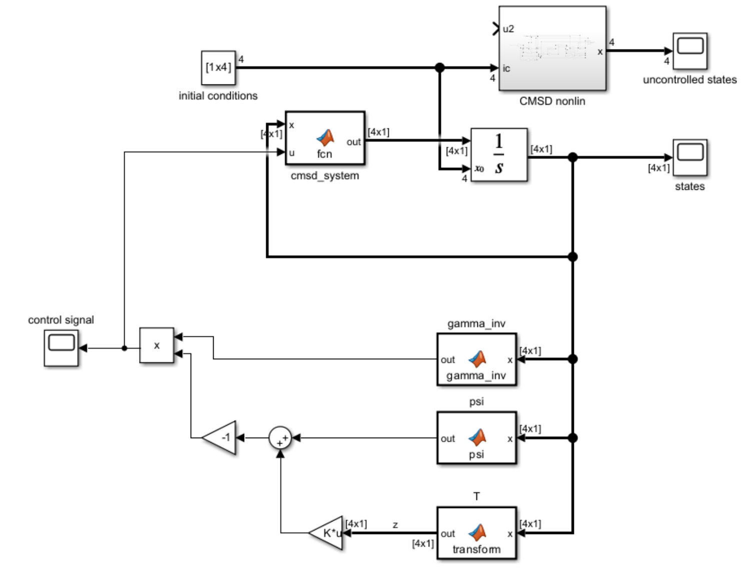
  $\dot{z} = Az + B[\psi(x) + \gamma(x)u]$, where $z = T(x)$ is a system transform
- control law $u = \gamma^{-1}(x)(-\psi(x) + v)$ transforms the initial systems into the linear system $\dot{z} = Az + Bv$
- control the linear system using state feedback linear control

Results


uncontrolled states:1(1)
uncontrolled states:1(2)
uncontrolled states:1(3)
uncontrolled states:1(4)
Time (seconds)


states:1(1,1)
states:1(2,1)
states:1(3,1)
states:1(4,1)
Time (seconds)


control signal
Time (seconds)

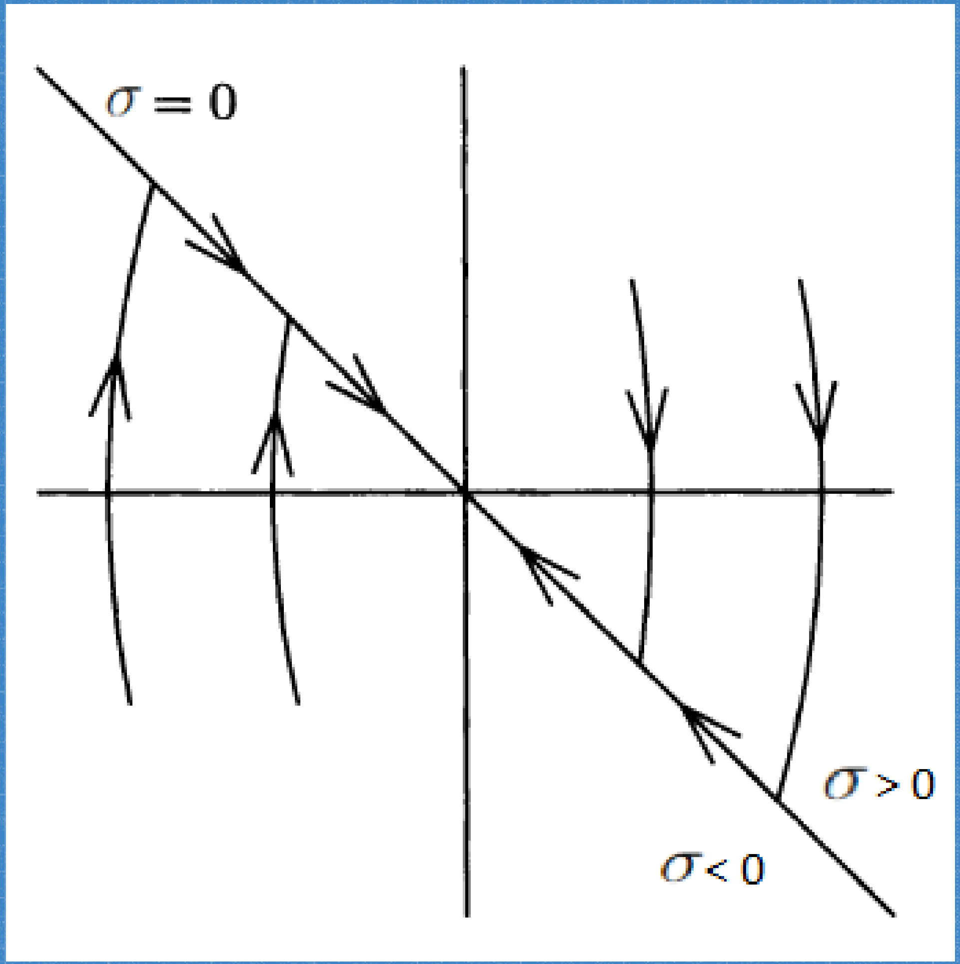

[1x4]
initial conditions
CMSD nonlin
uncontrolled states
fcn
cmsd_system
states
control signal
gamma_inv
psi
T
transform
K*u
-1

Sliding mode control

Method description

- consider the system $\dot{x} = f(x,t) + g(x,t)u$
- construct an n-dimensional sliding surface $\sigma(x)$

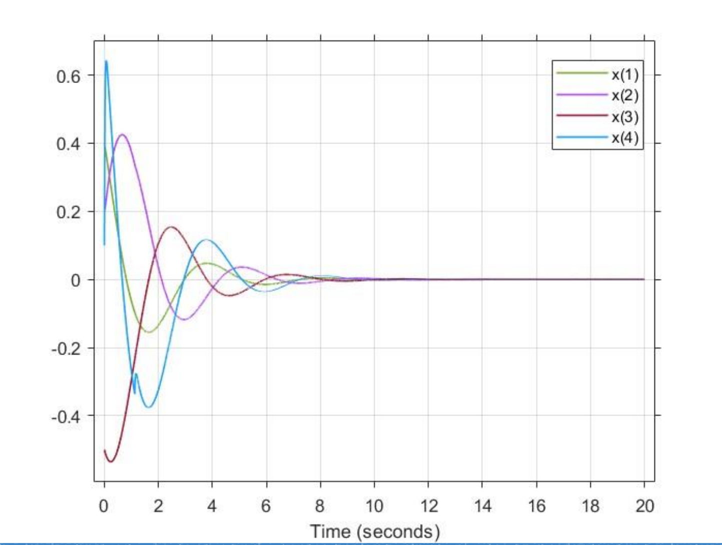
- compute the 2 control signals
- $u_{eq} = -\gamma^{-1}(x)\beta(x)$
- $u_{switch} = -\gamma^{-1}(x)\rho\ \mathrm{sgn}(\sigma)$

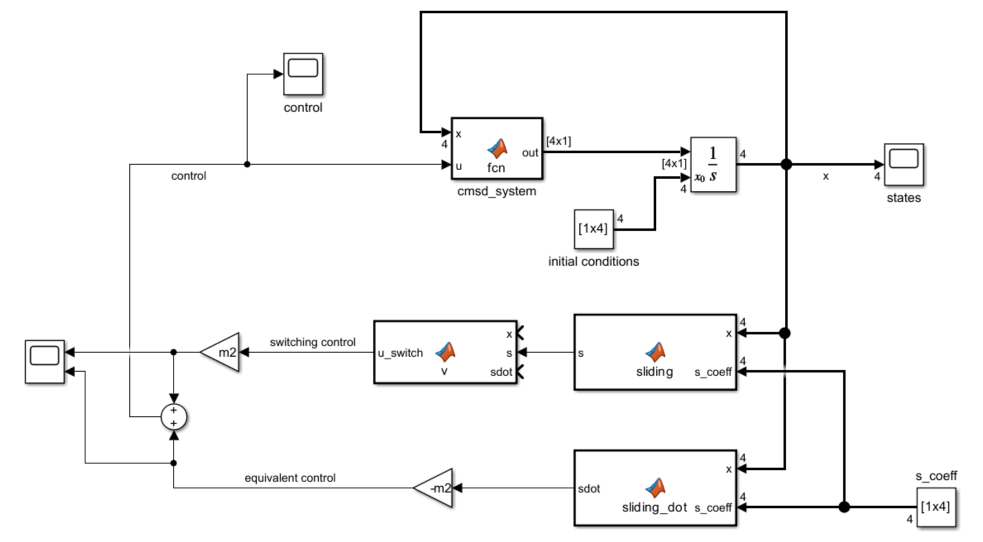


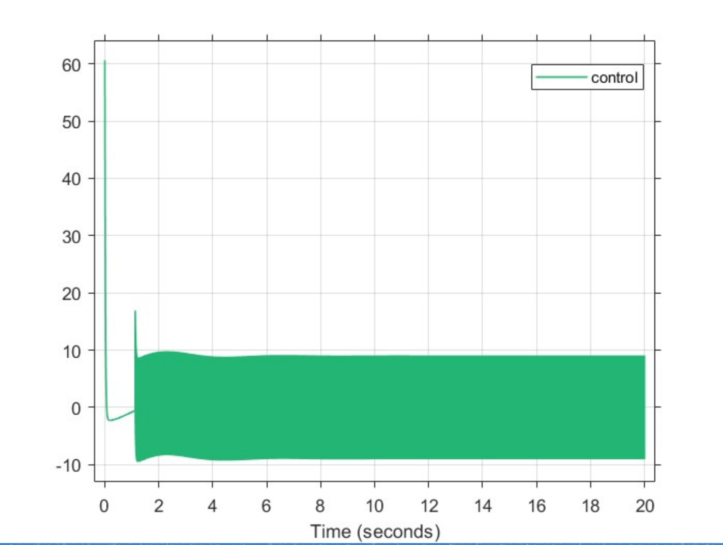
where $\beta(x)\ and\ \gamma(x)$ are uncertainties corresponding to functions $f(x,t)$ and $g(x,t)$ respectively, ρ depends on uncertainty's upper bound

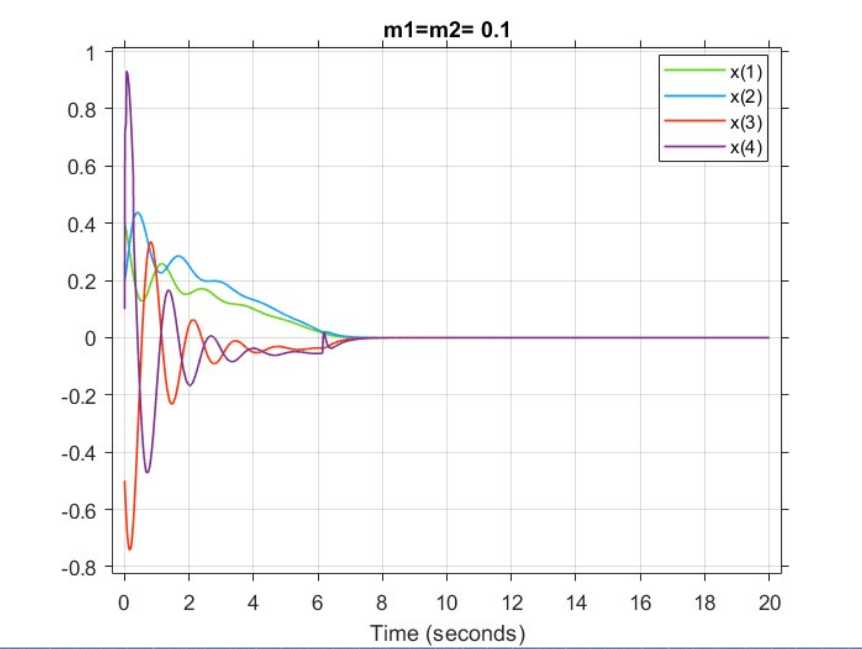
$$u = u_{eq} + u_{switch}$$

Results

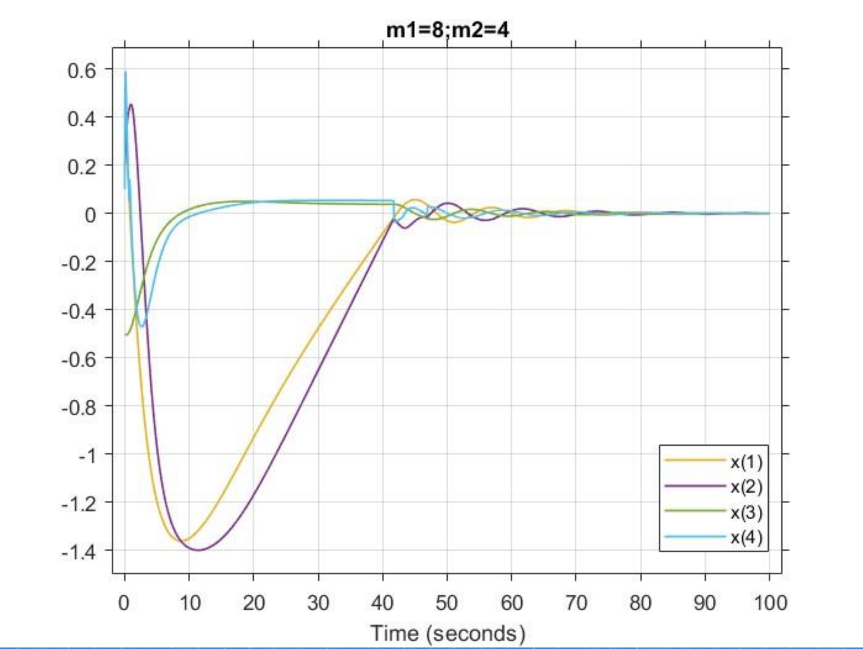




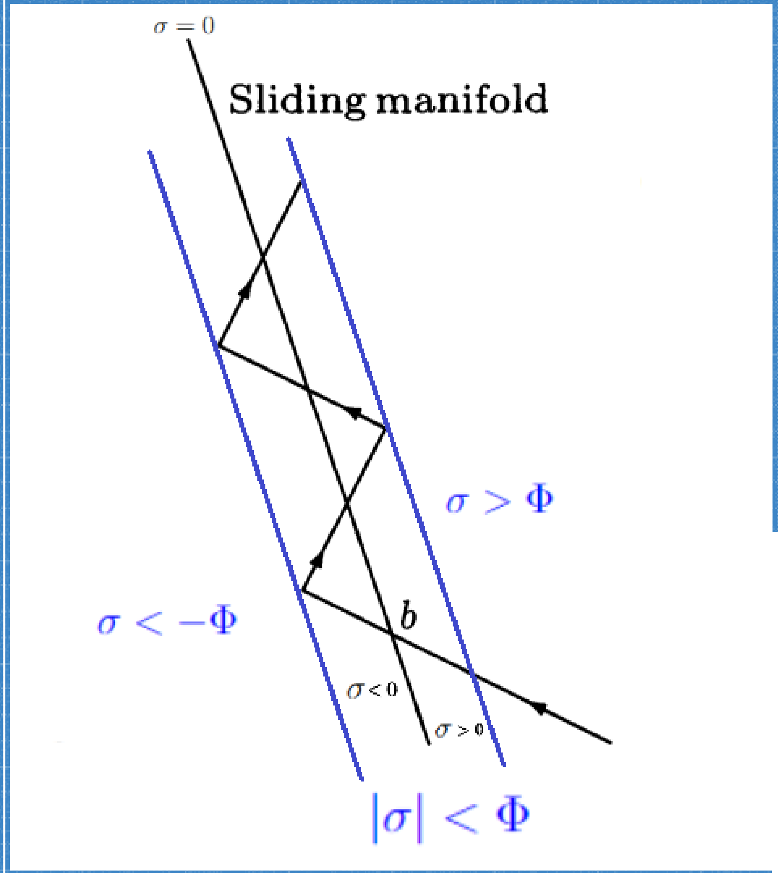


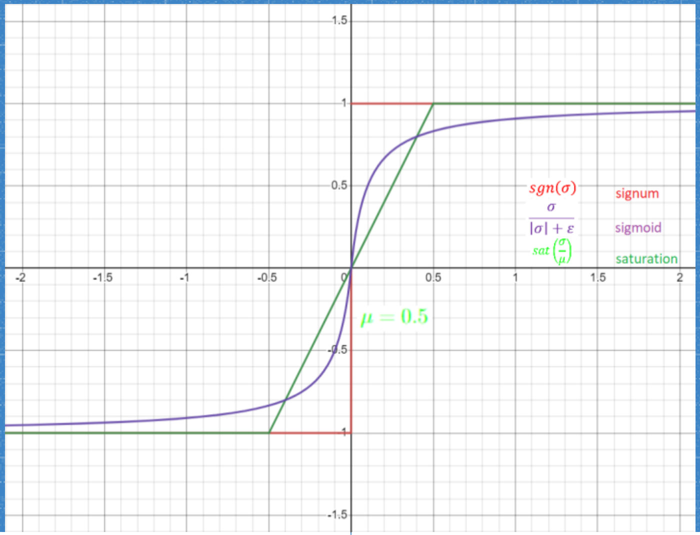


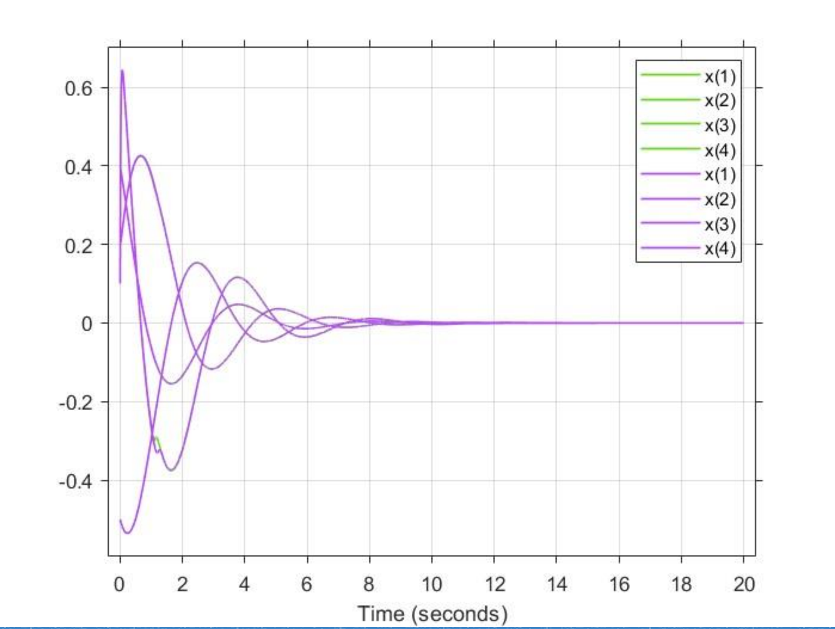
uncertainties

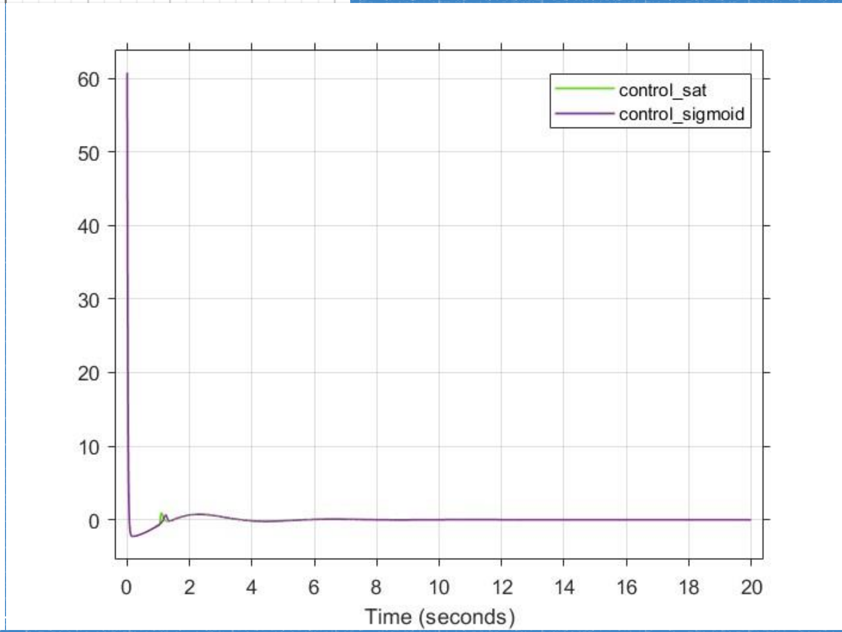


Eliminating *chattering* phenomenon









Backstepping

C O N T R O L

Method description

- basic method

$$\dot{\eta} = f_a(\eta) + g_a(\eta)\xi$$
$$\dot{\xi} = f_b(\eta, \xi) + g_b(\eta, \xi)u$$

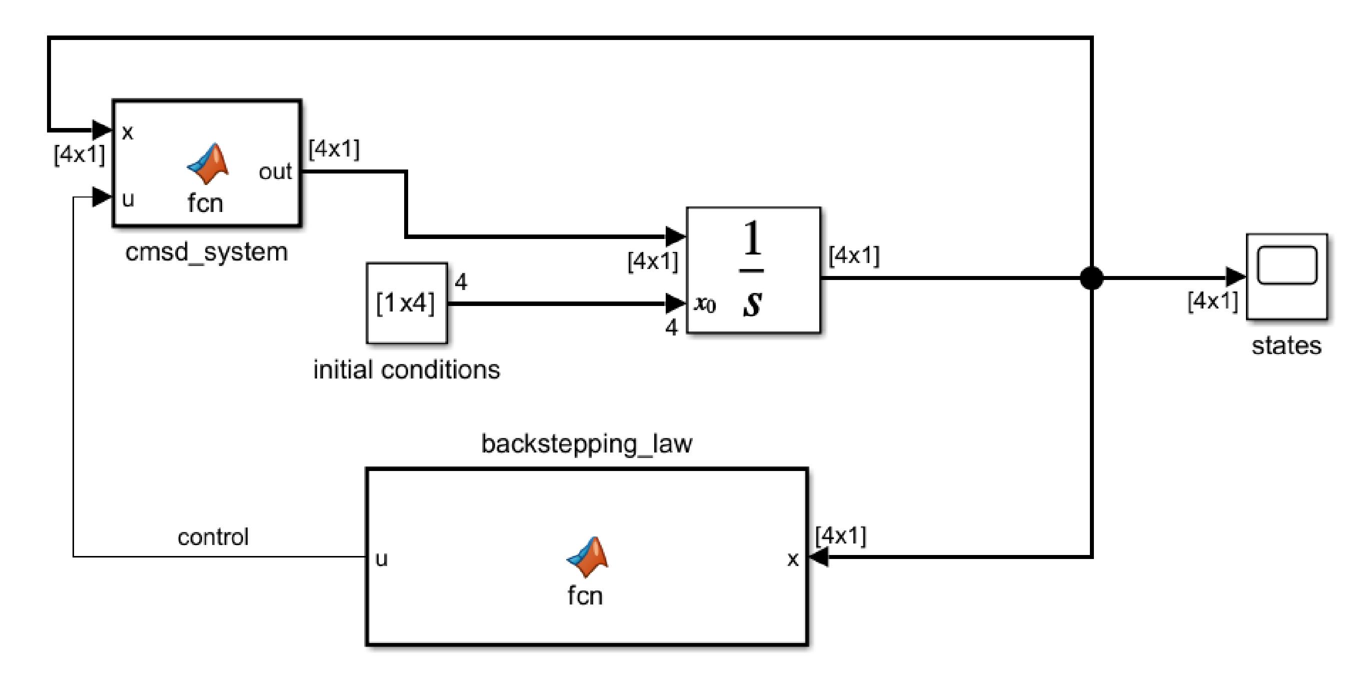
- find $\xi = \phi(\eta)$, that stabilizes the first equation

- calculate u that allows $\xi = \phi(\eta)$

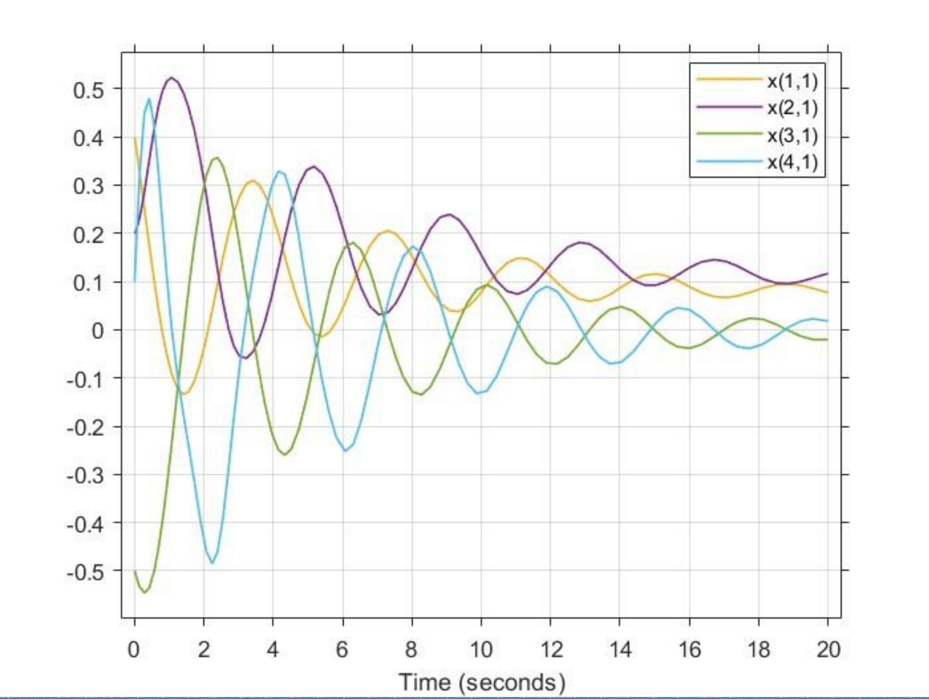
- recursive method

$$\begin{aligned}
\dot{x} &= f_0(x) + g_0(x)z_1 \\
\dot{z}_1 &= f_1(x, z_1) + g_1(x, z_1)z_2 \\
\dot{z}_2 &= f_2(x, z_1, z_2) + g_2(x, z_1, z_2)z_3 \\
&\vdots \\
\dot{z}_{k-1} &= f_{k-1}(x, z_1, \ldots, z_{k-1}) + g_{k-1}(x, z_1, \ldots, z_{k-1})z_k \\
\dot{z}_k &= f_k(x, z_1, \ldots, z_k) + g_k(x, z_1, \ldots, z_k)u
\end{aligned}$$

Results



[4x1]
x
u
fcn
out
[4x1]
cmsd_system
1
s
x_0
[4x1]
[4x1]
[1x4]
4
4
initial conditions
[4x1]
states
backstepping_law
control
u
fcn
x
[4x1]



x(1,1)
x(2,1)
x(3,1)
x(4,1)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0 2 4 6 8 10 12 14 16 18 20
Time (seconds)