Recent developments for MGCAMB

Alex Zucca[1,*]

[1] *Department of Physics, Simon Fraser University,*
8888 University Drive, Burnaby, British Columbia V5A 1S6, Canada

(Dated: October 26, 2018)

In the following notes we describe the new MGCAMB patch.

* azucca@sfu.ca

I. INTRODUCTION

II. THE STRUCTURE OF THE MODIFICATION

III. IMPLEMENTATION OF THE MODIFIED EQUATION

A. Background equations

B. Linear Scalar Perturbations

1. μ - γ parametrization

The conformal Newtonian gauge modified equations are

$$k^2\Psi = -\mu(a,k)4\pi G a^2\left[\rho\Delta + 3(\rho+P)\sigma\right], \tag{1}$$
$$k^2[\Phi - \gamma(a,k)\Psi] = \mu(a,k)12\pi G a^2(\rho+P)\sigma, \tag{2}$$

where the Newtonian gauge potentials Ψ and Φ are related to the synchronous gauge potentials η and h through

$$\Psi = \dot{\alpha} + \mathcal{H}\alpha, \tag{3}$$
$$\Phi = \eta - \mathcal{H}\alpha. \tag{4}$$

Here, $\alpha = (\dot{h} + 6\dot{\eta})/2k^2$, $\mathcal{H} = \dot{a}/a$ and overdots represent derivatives w.r.t. the conformal time τ. CAMB works in synchronous gauge so we have to convert the equations (1) and (2) in the synchronous gauge. To do so we use the equations (3) and (4) in the modified Einstein equations to obtain the variables $\mathcal{Z} \equiv \dot{h}/2k$ and $\sigma^* \equiv k\alpha$. We start by computing α,

$$\alpha = \left\{\eta + \frac{\mu a^2}{2k^2}\left[\gamma\rho\Delta + 3(\gamma-1)(\rho+P)\sigma\right]\right\}\frac{1}{\mathcal{H}}, \tag{5}$$

where we set $8\pi G \equiv 1$ and where the gauge invariant quantity $\rho\Delta$ is defined as

$$\rho\Delta = \rho\delta + \frac{3\mathcal{H}}{k}\rho(1+w)\theta. \tag{6}$$

With the quantity α at hand we can easily compute the quantity σ^* according to

$$\sigma^* \equiv k\alpha. \tag{7}$$

The next step is to obtain $\dot{h}$ by subtracting $\dot{\eta}$ from α. We first rewrite equation (5) as

$$\begin{aligned}\eta &= \mathcal{H}\alpha - \frac{\mu a^2}{2k^2}\left\{\gamma\rho\Delta + 3(\gamma-1)\rho(1+w)\sigma\right\}\\ &= \mathcal{H}\alpha - \frac{\mu a^2}{2k^2}\Gamma,\end{aligned} \tag{8}$$

where we defined

$$\Gamma = \gamma\rho\Delta + 3(\gamma-1)\rho(1+w)\sigma. \tag{9}$$

Taking the derivative w.r.t τ of the equation for η we obtain

$$\dot{\eta} = \dot{\mathcal{H}}\alpha + \mathcal{H}\dot{\alpha} - \frac{\dot{\mu}}{2k^2}\Gamma + \frac{\mu}{2k^2}\dot{\Gamma}. \tag{10}$$

We now need to compute the quantity $\dot{\Gamma}$. We can clearly see that we will need to evaluate the quantity $(\rho\Delta)^{\cdot}$. To do so we use the energy-momentum tensor conservation equations (in synchronous gauge),

$$\dot{\delta} = -(1+w)\left(\theta + \frac{\dot{h}}{2}\right) - 3\mathcal{H}\left(\frac{\delta P}{\delta\rho} - w\right)\delta, \tag{11}$$

$$\dot{\theta} = -\mathcal{H}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{\delta P/\delta\rho}{1+w}k^2\delta - k^2\sigma, \tag{12}$$

and combine them to obtain

$$(\rho\Delta)^{\cdot} = -3\mathcal{H}\rho\Delta - (1+w)\rho\theta\left[1+\frac{3}{k^2}(\mathcal{H}^2-\dot{\mathcal{H}})\right] - 3\mathcal{H}\rho(1+w)\sigma - (1+w)\rho k\mathcal{Z}. \tag{13}$$

Notice that the equations above hold for uncoupled fluids or for the overall fluid in the Universe. In our case we have uncoupled CDM, massless neutrinos and massive neutrinos, while baryons and photons are interacting. However the photon-baryon fluid is uncoupled and hence satisfies the equations above. Two more comments on the equation (13). First we can rewrite the term $\mathcal{H}^2 - \dot{\mathcal{H}}$, using the Friedmann equations,

$$\mathcal{H}^2 = \frac{\rho_{\rm tot}a^2}{3}, \tag{14}$$

$$\dot{\mathcal{H}} = -\frac{1}{6}\rho_{\rm tot}a^2(1+3w_{\rm tot}), \tag{15}$$

as

$$\mathcal{H}^2 - \dot{\mathcal{H}} = \frac{\rho_{\rm tot}a^2}{2}(1+w_{\rm tot}). \tag{16}$$

Note the distinction between w and $w_{\rm tot}$: the first comes from the contribution of the perturbed quantities only, while the second one takes into account the contributions of Dark Energy as well. Second, the quantity $\mathcal{Z}$, that we are trying to evaluate from $\dot{\eta}$, can be eliminated using $k\mathcal{Z} = k^2\alpha - 3\dot{\eta}$.

Another thing to notice is that $\dot{\alpha}$ can be eliminated from equation (10) using the Poisson equation

$$\dot{\alpha} = -\mathcal{H}\alpha - \frac{\mu a^2}{2k^2}\left[\rho\Delta + 3\rho(1+w)\sigma\right], \tag{17}$$

Using the information provided above one can obtain the expression for $\dot{\eta}$,

$$\begin{aligned}\dot{\eta} = \frac{1}{2}\frac{a^2}{\frac{3}{2}\rho a^2\mu\gamma(1+w)+k^2}\Bigg\{&\rho(1+w)\mu\gamma\theta\left[1+\frac{3}{2}\frac{\rho_{\rm tot}a^2}{k^2}(1+w_{\rm tot})\right] + \rho\Delta\left[\mathcal{H}\mu(\gamma-1) - \dot{\mu}\gamma - \mu\dot{\gamma}\right]\\ &+ 3\mu(1-\gamma)\rho(1+w)\dot{\sigma} + k^2\alpha\left[\rho\mu\gamma(1+w) - 2\left(\frac{\mathcal{H}^2-\dot{\mathcal{H}}}{a^2}\right)\right]\\ &+ 3(1+w)\rho\sigma\left[\mathcal{H}(\gamma-1)(3w+2)\mu - \dot{\mu}(\gamma-1) - \dot{\gamma}\mu + \mu(1-\gamma)\frac{\dot{w}}{1+w}\right]\Bigg\}.\end{aligned} \tag{18}$$

CAMB uses different variables, namely the fluxes q and the anisotropic stress defined as

$$(1+w)\theta = kq, \quad \frac{3}{2}(1+w)\sigma = \Pi. \tag{19}$$

The equation for $\dot{\eta}$ can be written in terms of the variables q and Π,

$$\begin{aligned}\dot{\eta} = \frac{1}{2}\frac{a^2}{\frac{3}{2}\rho a^2\mu\gamma(1+w)+k^2}\Bigg\{&\rho\mu\gamma kq\left[1+\frac{3}{2}\frac{\rho_{\rm tot}a^2}{k^2}(1+w_{\rm tot})\right] + \rho\Delta\left[\mathcal{H}\mu(\gamma-1) - \dot{\mu}\gamma - \mu\dot{\gamma}\right]\\ &+ 2\mu(1-\gamma)\rho\dot{\Pi} + k^2\alpha\left[\rho\mu\gamma(1+w) - 2\left(\frac{\mathcal{H}^2-\dot{\mathcal{H}}}{a^2}\right)\right] + 2\rho\Pi\left[\mathcal{H}(\gamma-1)(3w+2)\mu - \dot{\mu}(\gamma-1) - \dot{\gamma}\mu\right]\Bigg\}.\end{aligned} \tag{20}$$

One more clarification about the comment below the equation (16). The equation for $\dot{\eta}$ can be written by explicitly including the contribution of Dark Energy,

$$\begin{aligned}\dot{\eta} = \frac{1}{2}\frac{a^2}{\frac{3}{2}\mu\gamma\sum_i\rho_i a^2(1+w_i)+k^2}\Bigg\{&\mu\gamma\sum_i\rho_i(1+w_i)\theta_i\left[1+\frac{3}{2}\frac{\sum_i\rho_i(1+w_i)a^2 + \rho_{\rm DE}(1+w_{\rm DE})a^2}{k^2}\right]\\ &+ \sum_i\rho_i\Delta_i\left[\mathcal{H}\mu(\gamma-1) - \dot{\mu}\gamma - \mu\dot{\gamma}\right] + 3\mu(1-\gamma)\sum_i\rho_i(1+w_i)\dot{\sigma}_i\\ &+ k^2\alpha\left[(\mu\gamma-1)\sum_i\rho_i(1+w_i) - \rho_{\rm DE}(1+w_{\rm DE})\right] - 3\left(\dot{\mu}(\gamma-1)+\dot{\gamma}\mu\right)\sum_i\rho_i(1+w_i)\sigma_i\\ &+ 3\mu(\gamma-1)\sum_i\rho_i(1+w_i)\sigma_i\left[\mathcal{H}(3w_i+2) - \frac{\dot{w}_i}{1+w_i}\right]\Bigg\},\end{aligned} \tag{21}$$

where $i = \mathrm{CDM}, \nu, r, (\gamma - b)$. We notice a correction term in the coefficients for θ_i and α given by $\rho_{\mathrm{DE}}(1 + w_{\mathrm{DE}})$. If Dark Energy is the cosmological constant, i.e. $w_{\mathrm{DE}} \equiv -1$, then this correction term vanishes. However, if we want to explore different background scenarios where $w_{\mathrm{DE}} \neq 1$, then the correction term is necessary. For the sake of completeness the above equation above, using CAMB variables, becomes

$$\begin{aligned}
\dot{\eta} = \frac{1}{2}\frac{a^2}{\frac{3}{2}\mu\gamma\sum_i \rho_i a^2(1+w_i) + k^2} \Bigg\{ &\mu\gamma \sum_i \rho_i q_i \left[1 + \frac{3}{2}\frac{\sum_i \rho_j(1+w_j)a^2 + \rho_{\mathrm{DE}}(1+w_{\mathrm{DE}})a^2}{k^2}\right] \\
&+ \sum_i \rho_i \Delta_i \left[\mathcal{H}\mu(\gamma - 1) - \dot{\mu}\gamma - \mu\dot{\gamma}\right] + 2\mu(1-\gamma)\sum_i \rho_i \dot{\Pi}_i \\
&+ k^2\alpha \left[(\mu\gamma - 1)\sum_i \rho_i(1+w_i) - \rho_{\mathrm{DE}}(1+w_{\mathrm{DE}})\right] - 2\left(\dot{\mu}(\gamma-1) + \dot{\gamma}\mu\right)\sum_i \rho_i \Pi_i \\
&+ 2\mu(\gamma-1)\mathcal{H}\sum_i \rho_i \Pi_i (3w_i + 2)\Bigg\},
\end{aligned} \tag{22}$$

In the implementation of the code we give the following name to the variables:

$$\mathrm{dgpi} = \sum_i \rho_i a^2 \Pi_i, \tag{23}$$

$$\mathrm{dgrho} = \sum_i \rho_i a^2 \delta_i, \tag{24}$$

$$\mathrm{dgq} = \sum_i \rho_i a^2 q_i, \tag{25}$$

$$\mathrm{MG\_rho\_delta} = \sum_i \rho_i a^2 \Delta_i, \tag{26}$$

$$\mathrm{dgpi\_w\_sum} = \sum_i \rho_i a^2 \Pi_i (3w_i + 2), \tag{27}$$

$$\mathrm{pidot\_sum} = \sum_i \rho_i a^2 \dot{\Pi}_i \tag{28}$$

2. μ - Σ parametrization

Another commonly used parametrization used to investigate phenomenological modifications of gravity is given by the μ-Σ parametrization. The modified equations (in Newtonian gauge) are

$$k^2\Psi = -4\pi G a^2 \mu(a,k)\rho\Delta, \tag{29}$$

$$k^2(\Phi + \Psi) = -8\pi G a^2 \Sigma(a,k)\rho\Delta. \tag{30}$$

In the limit of negligible anisotropic stress, $\sigma \equiv 0$, the μ-γ and μ-Σ parametrization are simply related through

$$\Sigma = \frac{\mu(1+\gamma)}{2}. \tag{31}$$

AZ: This parametrization has not been implemented yet in the code

3. Q-R parametrization

Another commonly used parametrization is the Q,R parametrization defined through

$$k^2\Phi = -4\pi G a^2 Q\rho\Delta, \tag{32}$$

$$k^2(\Psi - R\Phi) = -12\pi G a^2 Q(\rho + P)\sigma. \tag{33}$$

In this case we do not use any conversion and rather we compute $\dot{\eta}$ explicitly. As we did with the μ,γ parametrization we convert the equations above to synchronous gauge using the two equations (3) and (4), and obtain an equation for α,

$$\alpha(t) = \left\{ \frac{1}{2k^2} Q a^2 \rho \Delta + \eta \right\} \frac{1}{\mathcal{H}}, \tag{34}$$

where we set $8\pi G \equiv 1$. The equation above allows us to calculate the quantity $\sigma^* \equiv k\alpha$. To obtain an expression for $\dot{\eta}$ we solve the equation above for η and then take a derivative w.r.t τ. Using the equation (13) we obtain

$$\begin{aligned} \dot{\eta} = \frac{1}{2} \frac{a^2}{\frac{3}{2} Q a^2 \sum_i \rho_i (1+w_i) + k^2} \Bigg\{ Q \sum_i k \rho_i q_i \left(1 + \frac{3}{2} \frac{\sum_i \rho_i (1+w_i) a^2 + \rho_{\mathrm{DE}} (1+w_{\mathrm{DE}}) a^2}{k^2} \right) \\ + \sum_i \rho_i \Delta_i \left(\mathcal{H} Q (1-R) - \dot{Q} \right) + k^2 \alpha \left(Q \sum_i \rho_i (1+w_i) - 2 \frac{\mathcal{H}^2 - \dot{H}}{a^2} \right) \Bigg\}. \end{aligned} \tag{35}$$

C. Radiation Streaming Approximation

This has not been developed yet, maybe it is not even possible in the case of modified growth.

IV. TESTS

A. GR limit of the patch

It is important to check that the GR limit of the patch is consistent with the default CAMB. In this section we address this question and we check that the two parametrizations implemented, in the GR limit, are consistent with the default CAMB. The GR limit of the two parametrizations is

$$\mu \equiv 1, \quad \gamma \equiv 1, \tag{36}$$
$$Q \equiv 1, \quad R \equiv 1. \tag{37}$$

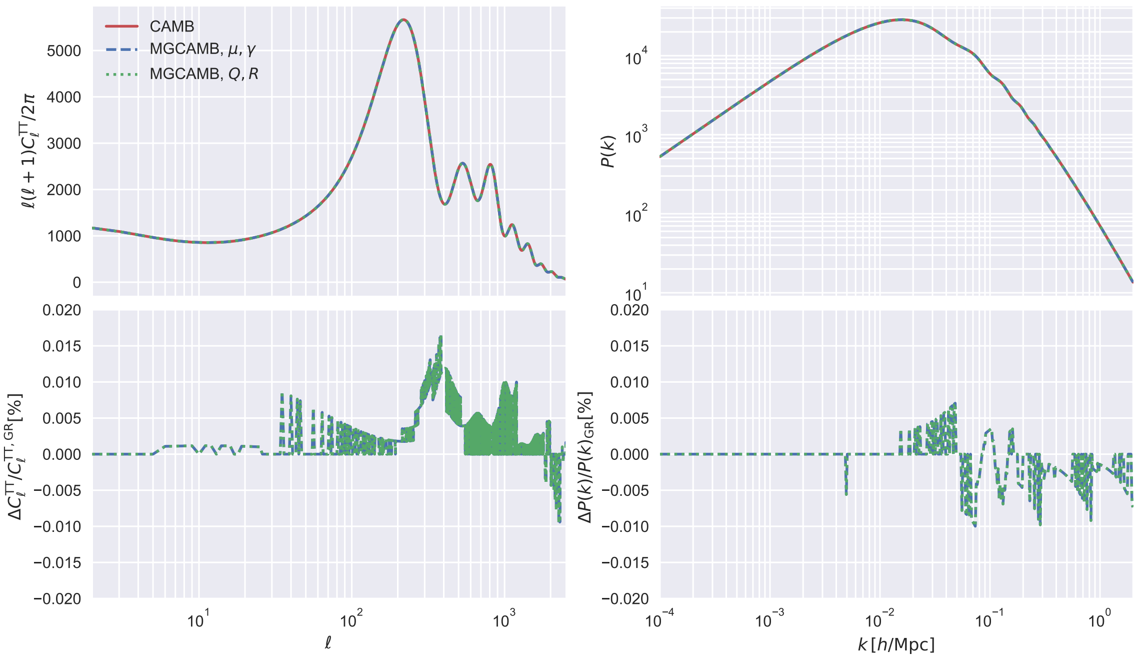

FIG. 1. Comparison of the default CAMB results with the GR limit of the MGCAMB patch with the total mass of neutrinos $m_\nu = 0.060\,\mathrm{eV}$. Upper plots: CMB TT power spectrum and matter power spectrum comparison between default CAMB and the MGCAMB patch in the GR limit. Lower plots: relative difference (in percent) of the CMB TT power spectrum and matter power spectrum between default CAMB and the MGCAMB patch.

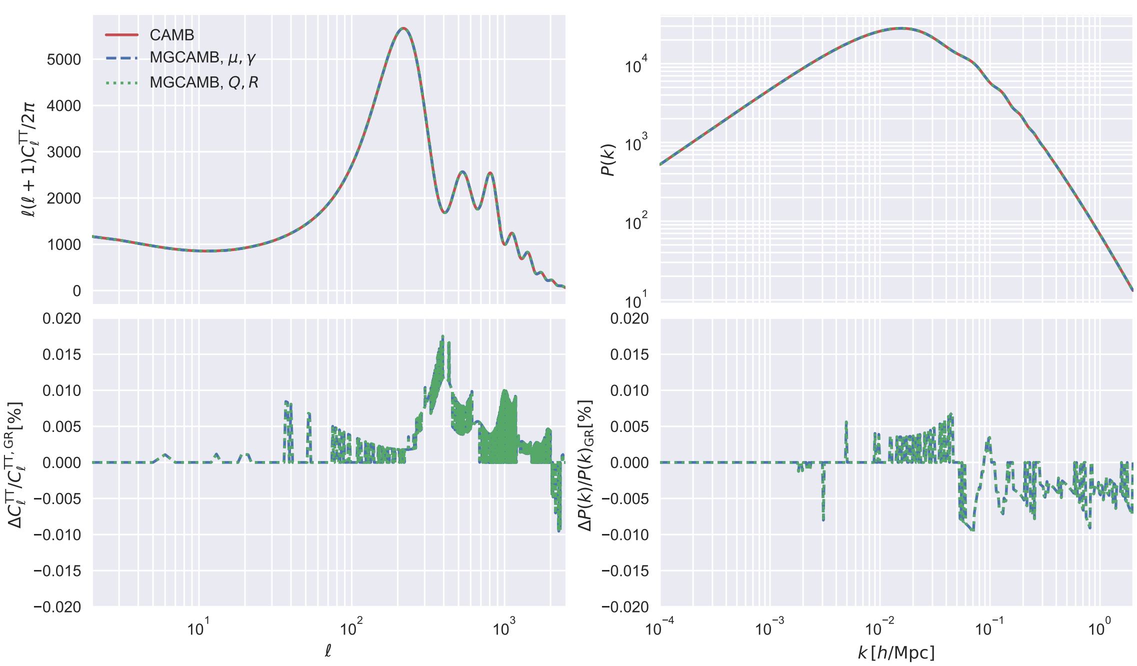

FIG. 2. Comparison of the default CAMB results with the GR limit of the MGCAMB patch with the total mass of neutrinos $m_\nu = 0.120\,\mathrm{eV}$. Upper plots: CMB TT power spectrum and matter power spectrum comparison between default CAMB and the MGCAMB patch in the GR limit. Lower plots: relative difference (in percent) of the CMB TT power spectrum and matter power spectrum between default CAMB and the MGCAMB patch.