Linjär Algebra
Skalärprodukt

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Problem 1: Let $\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Calculate the scalar product $\mathbf{v} \cdot \mathbf{w}$.

Problem 2: For $\mathbf{a} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, find $\mathbf{a} \cdot \mathbf{b}$.

Problem 3: Given $\mathbf{x} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, determine $\mathbf{x} \cdot \mathbf{y}$.

Problem 4: Find the scalar product of $\mathbf{p} = \begin{bmatrix} 0 \\ -5 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

Problem 5: Calculate $\mathbf{m} \cdot \mathbf{n}$ where $\mathbf{m} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{n} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

Problem 6: For $\mathbf{c} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ and $\mathbf{d} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$, compute $\mathbf{c} \cdot \mathbf{d}$.

Problem 7: Given $\mathbf{s} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $\mathbf{t} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$, find $\mathbf{s} \cdot \mathbf{t}$.

Problem 8: Calculate the scalar product of $\mathbf{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$.

Problem 9: Let $\mathbf{e} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$ and $\mathbf{f} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Find $\mathbf{e} \cdot \mathbf{f}$.

Problem 10: For $\mathbf{g} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\mathbf{h} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, compute $\mathbf{g} \cdot \mathbf{h}$.

Problem 11:

Let $\mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Now, also consider the vectors

$$\mathbf{B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$\mathbf{F} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \mathbf{G} = \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}, \mathbf{H} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}.$$

a) Begin by calculating the length of all vectors A-I.

b) Now graph all vectors A-I in an xy-coordinate system.

tip: sketch your coordinate system so that each vector is relatively large, so that you can clearly distinguish each vector from oneother.

c) What do you think the angle is between the vector A and each of the vectors B-I, individually?

d) Now calculate the scalar product of A with each of the vectors B-I, individually. Use the *component wise* formula of the scalar product for this. Does your answer make sense? Remember that the scalar product is a measure of 'nearness'.

e) Now calculate, again, the scalar product of A with each of the vectors B-I. This time, instead us the *geometric* formula of the scalar product. Use the angles you've guessed in c). You answers here should be the same as in d).

f) Now to verify that the angles you guessed in c., confirm them all using the formula

$$\alpha = \arccos\left(\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right)$$

Remember that you can look up arccos values easily using wolframalpha.com

g)** Repeat the procedure of subproblem a)-d) but now, instead of vector B-I, compare A with the vectors

$$\mathbf{K} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \\ \frac{1}{2} \end{pmatrix}, \mathbf{L} = \begin{pmatrix} \frac{1}{2} \\ \\ \frac{\sqrt{3}}{2} \end{pmatrix}, \mathbf{M} = \begin{pmatrix} \frac{-1}{2} \\ \\ \frac{\sqrt{3}}{2} \end{pmatrix} \mathbf{N} = \begin{pmatrix} \frac{-\sqrt{3}}{2} \\ \\ \frac{1}{2} \end{pmatrix},$$

$$\mathbf{O} = \begin{pmatrix} \frac{-\sqrt{3}}{2} \\ \\ \frac{-1}{2} \end{pmatrix}, \mathbf{P} = \begin{pmatrix} \frac{-1}{2} \\ \\ \frac{\sqrt{-3}}{2} \end{pmatrix} \mathbf{Q} = \begin{pmatrix} \frac{1}{2} \\ \\ \frac{-\sqrt{3}}{2} \end{pmatrix} \mathbf{R} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \\ \frac{-1}{2} \end{pmatrix},$$