g)

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$$

$$\text{Det}(A) = 1 \cdot 5 - 2 \cdot 3 = -1 \neq 0 \checkmark$$

$$A^{-1} = \frac{1}{\text{Det}(A)} \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} = -1 \cdot \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$$

$$A A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

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e)

$$A = \begin{pmatrix} 5 & 1 & -2 \\ -2 & 0 & 5 \\ 7 & 2 & 8 \end{pmatrix}$$

$$\text{Det}(A) = 0 + 35 + 8 - 0 - 50 + 16 = 43 - 34 = 9 \neq 0 \checkmark$$

$$\left(\begin{array}{ccc|ccc} 5 & 1 & -2 & 1 & 0 & 0 \\ -2 & 0 & 5 & 0 & 1 & 0 \\ 7 & 2 & 8 & 0 & 0 & 1 \end{array} \right) \underset{2}{\overset{}{\rceil}} \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 8 & 1 & 2 & 0 \\ -2 & 0 & 5 & 0 & 1 & 0 \\ 7 & 2 & 8 & 0 & 0 & 1 \end{array} \right) \begin{array}{c} \rceil 2 \\ \rceil \\ \leftarrow \end{array} -7$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 8 & 1 & 2 & 0 \\ 0 & 2 & 21 & 2 & 5 & 0 \\ 0 & -5 & -48 & -7 & -14 & 1 \end{array} \right) \rceil 3$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 8 & 1 & 2 & 0 \\ 0 & 2 & 21 & 2 & 5 & 0 \\ 0 & 1 & 15 & -1 & 1 & 1 \end{array} \right) \begin{array}{c} \leftarrow \\ \rceil -2 \end{array} -1$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -7 & 2 & 1 & -1 \\ 0 & 0 & -9 & 4 & 3 & -2 \\ 0 & 1 & 15 & -1 & 1 & 1 \end{array} \right) \rceil$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -7 & 2 & 1 & -1 \\ 0 & 1 & 15 & -1 & 1 & 1 \\ 0 & 0 & -9 & 4 & 3 & -2 \end{array} \right) \frac{-1}{9}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 7 & 2 & 1 & -1 \\ 0 & 1 & 15 & -1 & 1 & 1 \\ 0 & 0 & 1 & \frac{-4}{9} & \frac{-1}{3} & \frac{2}{9} \end{array} \right) \begin{array}{c} \leftarrow \\ \rceil -15 \end{array} 7$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \left(2 - \frac{28}{9}\right) & \left(1 - \frac{7}{3}\right) & \left(-1 + \frac{14}{9}\right) \\ 0 & 1 & 0 & (-1 \quad) & (\quad) & (\quad) \\ 0 & 0 & 1 & \frac{-4}{9} & \frac{-1}{3} & \frac{2}{9} \end{array} \right)$$

The rest of the solution is left as
an excercise for the reader

$$\sim \quad \cdots$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{10}{9} & -\frac{12}{9} & \frac{5}{9} \\ 0 & 1 & 0 & \frac{51}{9} & \frac{54}{9} & \frac{-21}{9} \\ 0 & 0 & 1 & -\frac{4}{9} & \frac{-3}{9} & \frac{2}{9} \end{array} \right)$$

$$A^{-1} = \frac{1}{9} \begin{pmatrix} -10 & -12 & 5 \\ 51 & 54 & -21 \\ -4 & -3 & 2 \end{pmatrix}$$