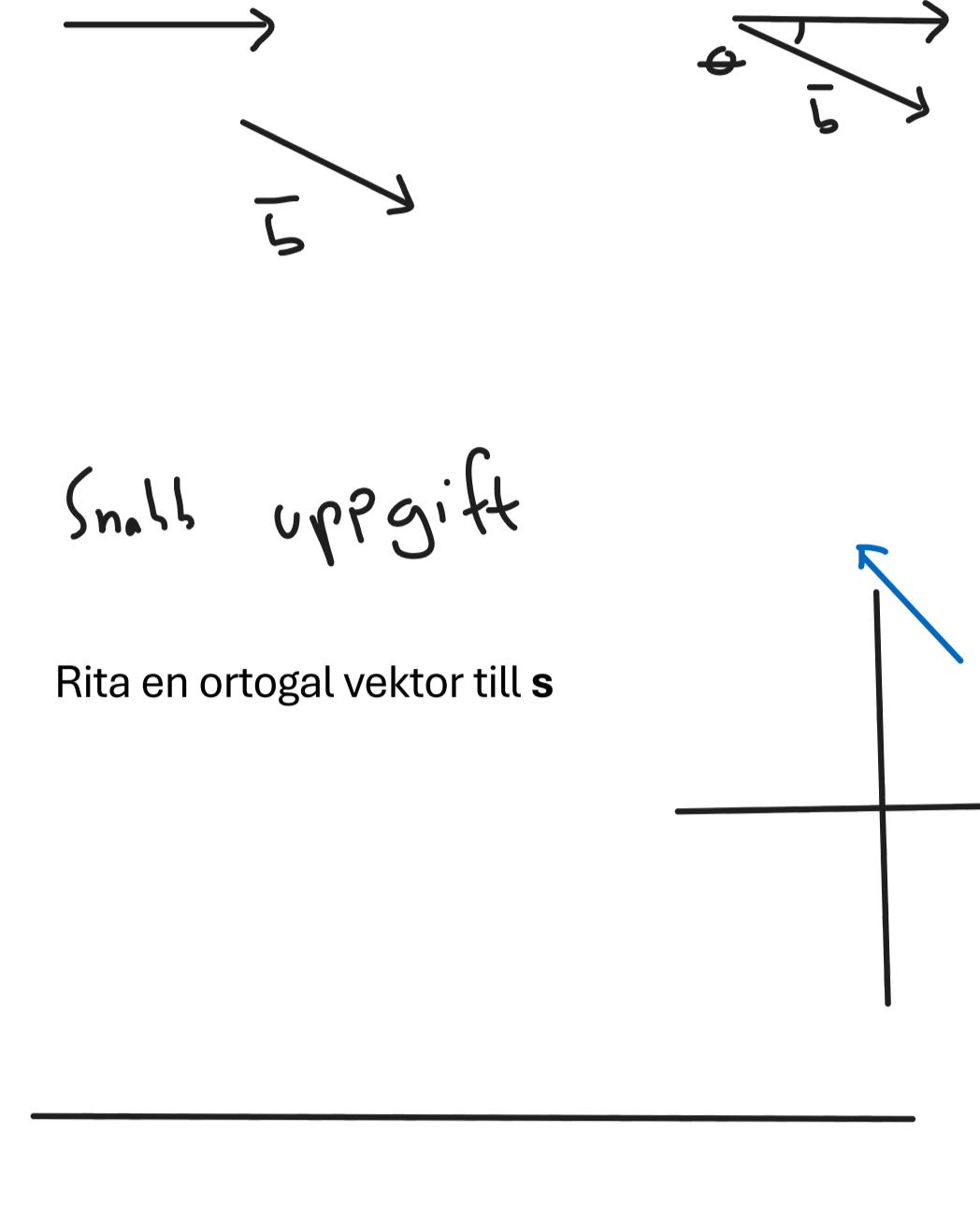


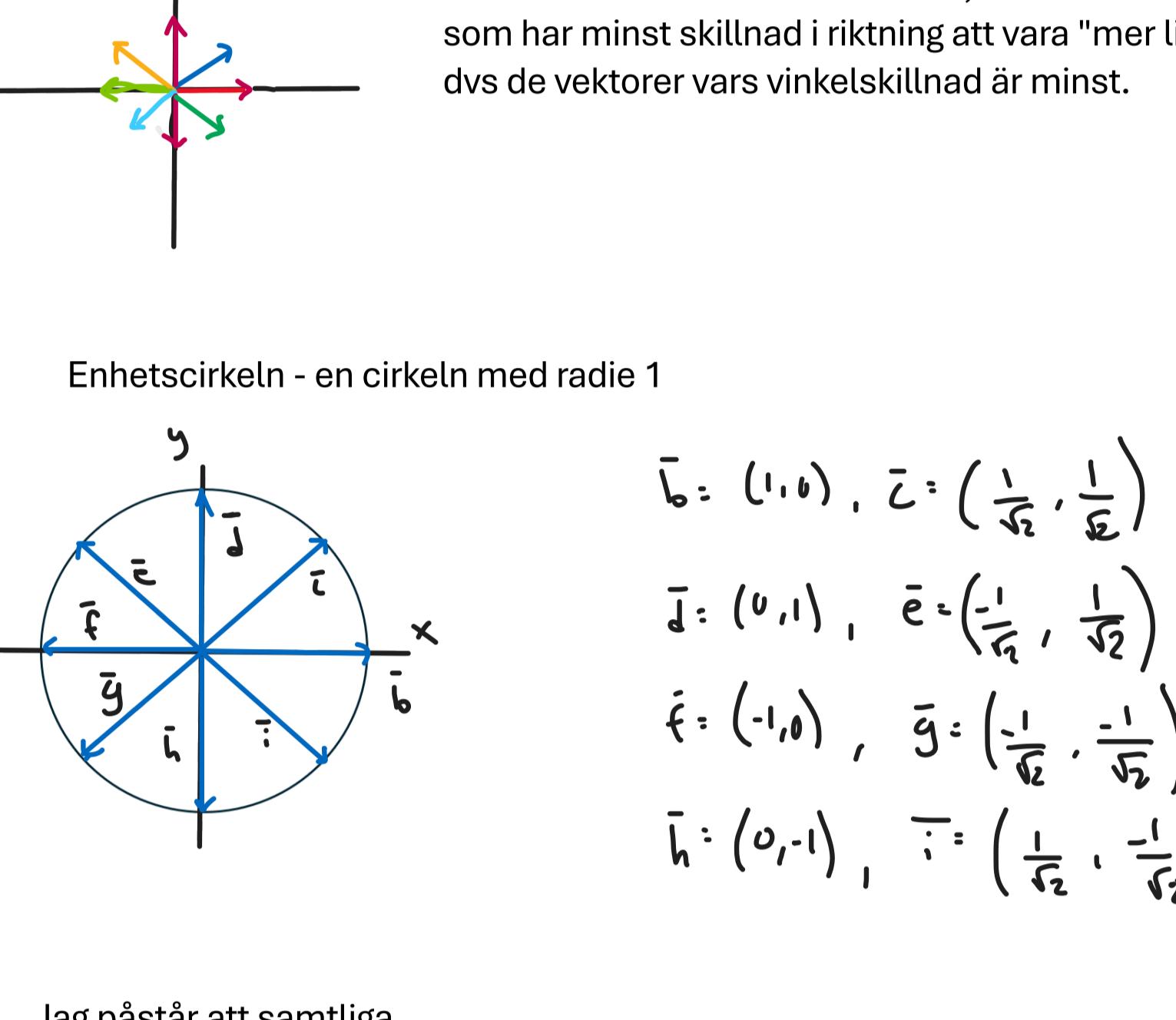
Parallella vektorer

Alla vektorer, oavsett riktning, som pekar i gemensam riktning kallas för **parallella**.



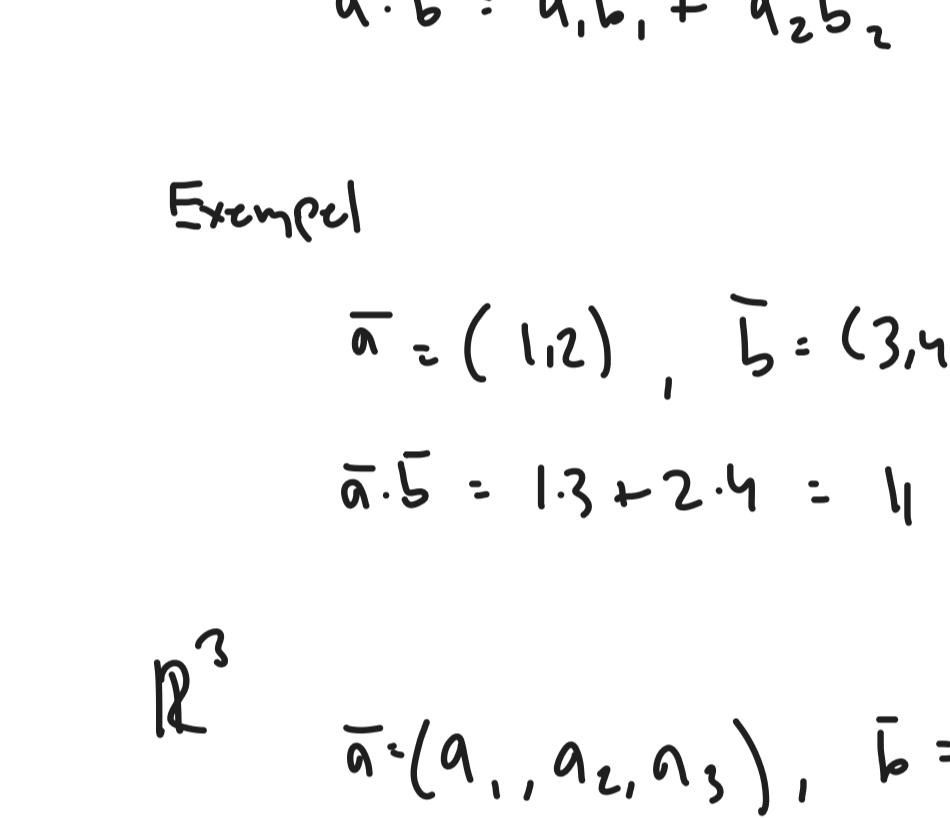
Antiparallella vektorer

Vektorer som pekar i direkt motsatt riktning anses vara **antiparallella**. Magnituderna spelar återigen ingen roll.

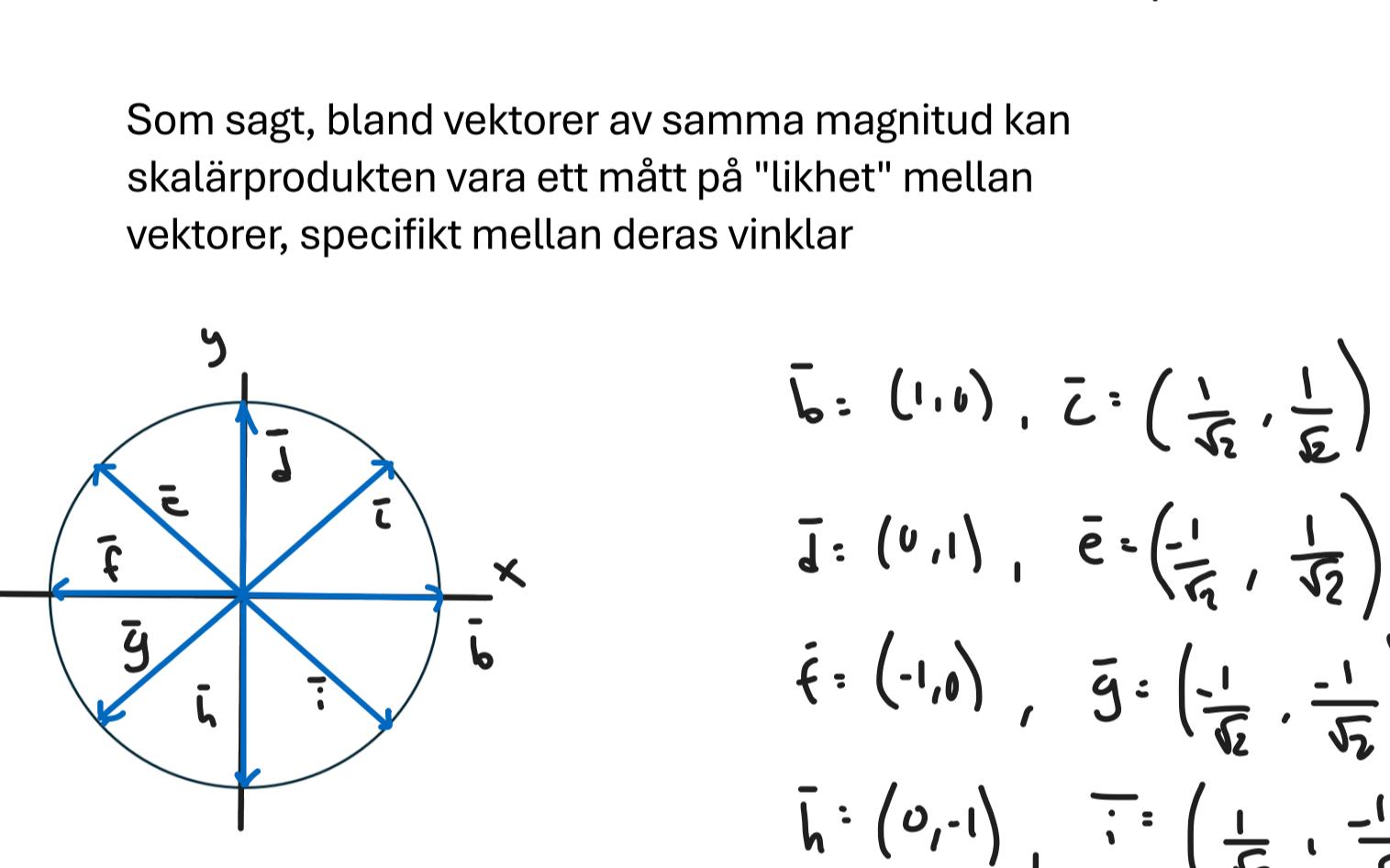


Orthogonala vektorer

Vektorer för vilken den relativata vinkeln är 90 grader, anses vara **orthogonala**.

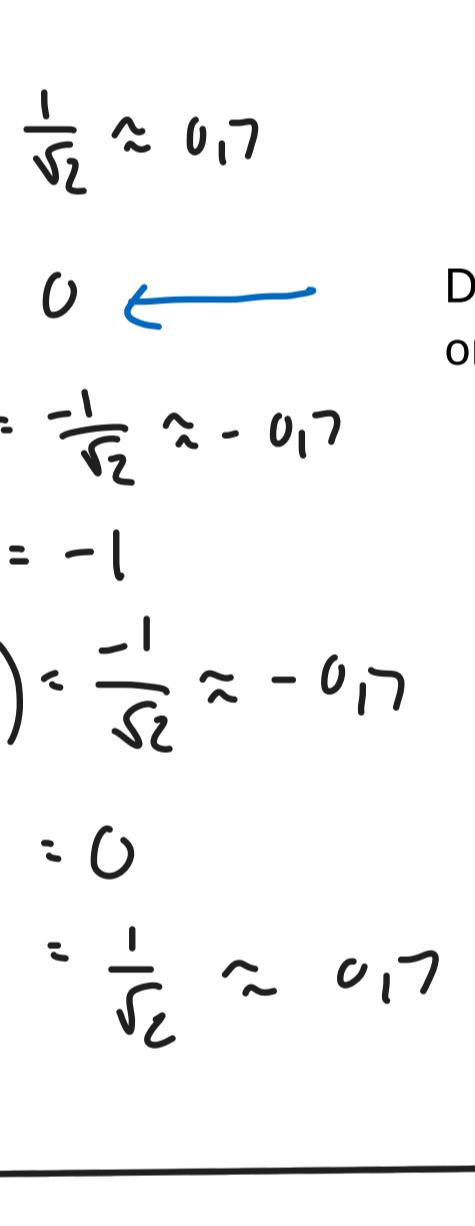


Hur vi definierar vinkelar mellan vektorer



Simpla uppgift

Rita en ortogonal vektor till s

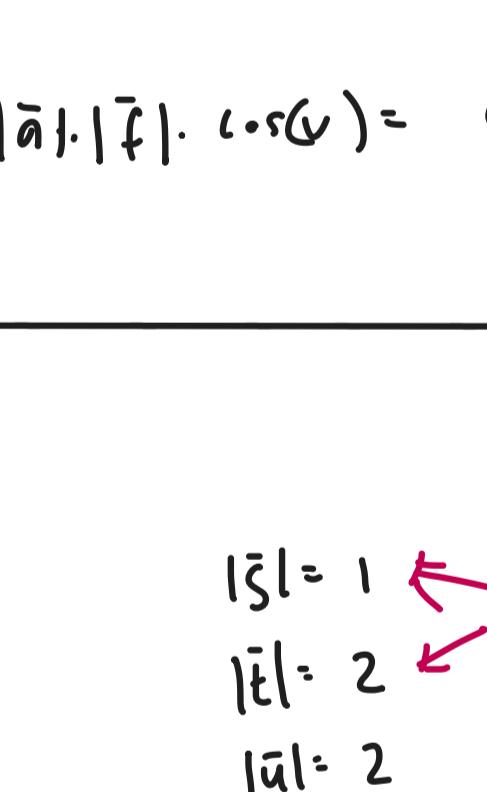


Skalärprodukt

Vi har tidigare lärt oss vektor addition och skalärmultiplikation - nu ska istället titta på skalärprodukt

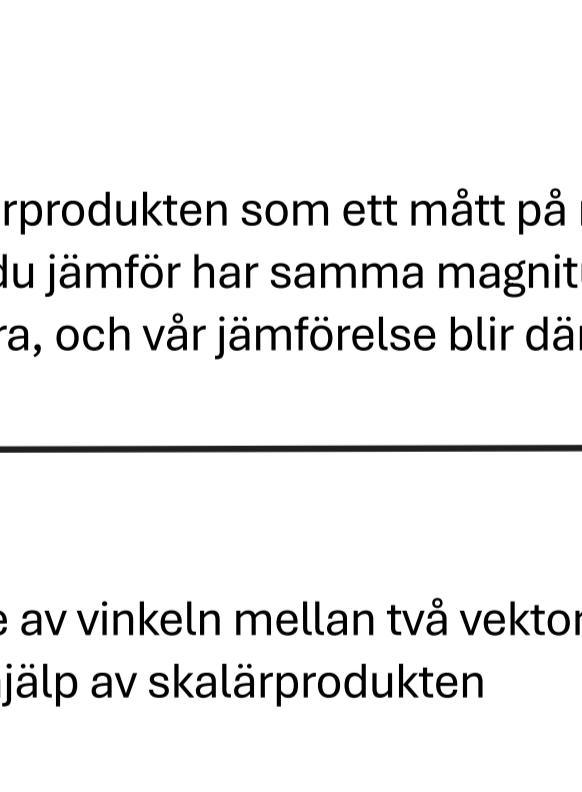
I kort så kommer vi nu lära oss ett sätt att multiplicera vektorer med varandra.

Skalärprodukten kan man använda som ett mått för "likheten" mellan vektorer.



Bland vektorer av samma storlek, anses de vektorer som har minst skillnad i riktning att vara "mer lika" - dvs de vektorer vars vinkelskillnad är minst.

Enhetscirkeln - en cirkel med radie 1



$$\bar{b} = (1, 0), \bar{z} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\bar{d} = (0, 1), \bar{e} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\bar{f} = (-1, 0), \bar{g} = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

$$\bar{h} = (0, -1), \bar{i} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

Jag påstår att samtliga vektorer ovan har längden 1.

$$|\bar{g}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1 \quad \text{VS!}$$

Skalärprodukt - komponentvis

$$\mathbb{R}^2 \quad \bar{a} = (a_1, a_2), \bar{b} = (b_1, b_2)$$

$$\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2$$

Exempel

$$\bar{a} = (1, 2), \bar{b} = (3, 4)$$

$$\bar{a} \cdot \bar{b} = 1 \cdot 3 + 2 \cdot 4 = 11$$

$$\mathbb{R}^3 \quad \bar{a} = (a_1, a_2, a_3), \bar{b} = (b_1, b_2, b_3)$$

$$\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\mathbb{R}^n \quad \bar{a} = (a_1, a_2, \dots, a_n), \bar{b} = (b_1, b_2, \dots, b_n)$$

$$\bar{a} \cdot \bar{b} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

Som sagt, bland vektorer av samma magnitud kan skalärprodukten vara ett mått för "likheten" mellan vektorer, specifikt mellan deras vinklar



$$\bar{b} = (1, 0), \bar{z} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\bar{d} = (0, 1), \bar{e} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\bar{f} = (-1, 0), \bar{g} = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

$$\bar{h} = (0, -1), \bar{i} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

Låt oss nu definiera en vektor $\mathbf{a} = (1, 0)$ och beräkna skalärprodukten mellan den och alla vektorer ovan

$$\bar{a} \cdot \bar{b} = 1 \cdot 1 + 0 \cdot 0 = 1$$

$$\bar{a} \cdot \bar{c} = 1 \cdot \frac{1}{\sqrt{2}} + 0 \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \approx 0,7$$

$$\bar{a} \cdot \bar{d} = 1 \cdot 0 + 0 \cdot 1 = 0$$

$$\bar{a} \cdot \bar{e} = 1 \cdot \left(-\frac{1}{\sqrt{2}}\right) + 0 \cdot \frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \approx -0,7$$

$$\bar{a} \cdot \bar{f} = 1 \cdot \left(-\frac{1}{\sqrt{2}}\right) + 0 \cdot \left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}} \approx -0,7$$

$$\bar{a} \cdot \bar{h} = 1 \cdot 0 + 0 \cdot 0 = 0$$

$$\bar{a} \cdot \bar{i} = 1 \cdot \left(\frac{1}{\sqrt{2}}\right) + 0 \cdot \left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} \approx 0,7$$

Detta resultat är mycket viktigt, alla orthogonala vektorer har skalärprodukten 0

$$\bar{a} \cdot \bar{b} = |\bar{a}| \cdot |\bar{b}| \cdot \cos(0) = 1 \cdot 1 \cdot \cos(0) = 1$$

$$\bar{a} \cdot \bar{c} = |\bar{a}| \cdot |\bar{c}| \cdot \cos(45^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{d} = |\bar{a}| \cdot |\bar{d}| \cdot \cos(90^\circ) = 1 \cdot 0 \cdot \cos(90^\circ) = 0$$

$$\bar{a} \cdot \bar{e} = |\bar{a}| \cdot |\bar{e}| \cdot \cos(135^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(135^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{f} = |\bar{a}| \cdot |\bar{f}| \cdot \cos(180^\circ) = 1 \cdot 1 \cdot \cos(180^\circ) = -1$$

$$\bar{a} \cdot \bar{h} = |\bar{a}| \cdot |\bar{h}| \cdot \cos(225^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(225^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{i} = |\bar{a}| \cdot |\bar{i}| \cdot \cos(270^\circ) = 1 \cdot 0 \cdot \cos(270^\circ) = 0$$

$$\bar{a} \cdot \bar{j} = |\bar{a}| \cdot |\bar{j}| \cdot \cos(315^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(315^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{k} = |\bar{a}| \cdot |\bar{k}| \cdot \cos(360^\circ) = 1 \cdot 1 \cdot \cos(360^\circ) = 1$$

$$\bar{a} \cdot \bar{l} = |\bar{a}| \cdot |\bar{l}| \cdot \cos(45^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{m} = |\bar{a}| \cdot |\bar{m}| \cdot \cos(90^\circ) = 1 \cdot 0 \cdot \cos(90^\circ) = 0$$

$$\bar{a} \cdot \bar{n} = |\bar{a}| \cdot |\bar{n}| \cdot \cos(135^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(135^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{o} = |\bar{a}| \cdot |\bar{o}| \cdot \cos(180^\circ) = 1 \cdot 1 \cdot \cos(180^\circ) = -1$$

$$\bar{a} \cdot \bar{p} = |\bar{a}| \cdot |\bar{p}| \cdot \cos(225^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(225^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{q} = |\bar{a}| \cdot |\bar{q}| \cdot \cos(270^\circ) = 1 \cdot 0 \cdot \cos(270^\circ) = 0$$

$$\bar{a} \cdot \bar{r} = |\bar{a}| \cdot |\bar{r}| \cdot \cos(315^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(315^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{s} = |\bar{a}| \cdot |\bar{s}| \cdot \cos(360^\circ) = 1 \cdot 1 \cdot \cos(360^\circ) = 1$$

$$\bar{a} \cdot \bar{t} = |\bar{a}| \cdot |\bar{t}| \cdot \cos(45^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{u} = |\bar{a}| \cdot |\bar{u}| \cdot \cos(90^\circ) = 1 \cdot 0 \cdot \cos(90^\circ) = 0$$

$$\bar{a} \cdot \bar{v} = |\bar{a}| \cdot |\bar{v}| \cdot \cos(135^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(135^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{w} = |\bar{a}| \cdot |\bar{w}| \cdot \cos(180^\circ) = 1 \cdot 1 \cdot \cos(180^\circ) = -1$$

$$\bar{a} \cdot \bar{x} = |\bar{a}| \cdot |\bar{x}| \cdot \cos(225^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(225^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{y} = |\bar{a}| \cdot |\bar{y}| \cdot \cos(270^\circ) = 1 \cdot 0 \cdot \cos(270^\circ) = 0$$

$$\bar{a} \cdot \bar{z} = |\bar{a}| \cdot |\bar{z}| \cdot \cos(315^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(315^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{b} = |\bar{a}| \cdot |\bar{b}| \cdot \cos(360^\circ) = 1 \cdot 1 \cdot \cos(360^\circ) = 1$$

$$\bar{a} \cdot \bar{c} = |\bar{a}| \cdot |\bar{c}| \cdot \cos(45^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{d} = |\bar{a}| \cdot |\bar{d}| \cdot \cos(90^\circ) = 1 \cdot 0 \cdot \cos(90^\circ) = 0$$

$$\bar{a} \cdot \bar{e} = |\bar{a}| \cdot |\bar{e}| \cdot \cos(135^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(135^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{f} = |\bar{a}| \cdot |\bar{f}| \cdot \cos(180^\circ) = 1 \cdot 1 \cdot \cos(180^\circ) = -1$$

$$\bar{a} \cdot \bar{g} = |\bar{a}| \cdot |\bar{g}| \cdot \cos(225^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(225^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{h} = |\bar{a}| \cdot |\bar{h}| \cdot \cos(270^\circ) = 1 \cdot 0 \cdot \cos(270^\circ) = 0$$

$$\bar{a} \cdot \bar{i} = |\bar{a}| \cdot |\bar{i}| \cdot \cos(315^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(315^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{j} = |\bar{a}| \cdot |\bar{j}| \cdot \cos(360^\circ) = 1 \cdot 1 \cdot \cos(360^\circ) = 1$$

$$\bar{a} \cdot \bar{k} = |\bar{a}| \cdot |\bar{k}| \cdot \cos(45^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{l} = |\bar{a}| \cdot |\bar{l}| \cdot \cos(90^\circ) = 1 \cdot 0 \cdot \cos(90^\circ) = 0$$

$$\bar{a} \cdot \bar{m} = |\bar{a}| \cdot |\bar{m}| \cdot \cos(135^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(135^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{n} = |\bar{a}| \cdot |\bar{n}| \cdot \cos(180^\circ) = 1 \cdot 1 \cdot \cos(180^\circ) = -1$$

$$\bar{a} \cdot \bar{o} = |\bar{a}| \cdot |\bar{o}| \cdot \cos(225^\circ) = 1 \cdot \frac{1}{\sqrt{2}} \cdot \cos(225^\circ) = -\frac{1}{\sqrt{2}}$$

$$\bar{a} \cdot \bar{p} = |\bar{a}| \cdot |\bar{p}| \cdot \cos(270^\circ) = 1 \cdot 0 \cdot \cos(270^\circ$$