Introduction to Sets

Definition:

A set is a well-defined collection of distinct objects or elements. Sets are fundamental in mathematics and statistics, providing a framework for organizing and analyzing data.

In a set, each element is unique, meaning it appears only once. Duplicate elements are not allowed within a set.

Notation:

We represent sets using curly braces "$\{\}$", and list their elements separated by commas. For

Examples:

Set A: $\{1, 2, 3, 4, 5\}$

Set B: {apple, orange, banana}

Null Set (Empty Set):

The null set, denoted by $\emptyset$, is a set with no elements. It plays a crucial role in set theory.

$\emptyset$: $\{\}$

Examples:

Let C represent the set of all months with 31 days.

$$C = \{\text{January, March, May, July, August, October, December}\}$$

Let D be the set of colors Amir likes.

$$D = \{\text{red, orange, yellow, green, blue, indigo, violet}\}$$

Let E be the set of the three largest cities in Sweden.

$$E = \{\text{Stockholm, Malmö, Göteborg}\}$$

Cardinality of Sets

Definition:

The cardinality of a set is the count of its elements. We denote the cardinality of set A as $|A|$.

Examples:

Let $G = \{2, 4, 6, 8, 10\} \rightarrow |G| = 5$.

Let $H = \{$apple, orange, banana, peach$\} \rightarrow |H| = 4$.

Let $I = \{$red, green, blue$\} \rightarrow |I| = 3$.

Let $J = \{\} \rightarrow |J| = 0$.

Intersections of Sets

Definition:

The intersection of two sets, A and B, denoted by $A \cap B$, is the set of all elements that are both in A and in B.

Examples:

Let $S = \{2, 4, 6, 8\}$ and $T = \{3, 6, 9\}$. The intersection $S \cap T = \{6\}$.

Let $U = \{a, b, c, d\}$ and $V = \{c, d, e\}$. The intersection $U \cap V = \{c, d\}$.

Let $W = \{$dog, cat, rabbit$\}$ and $X = \{$monkey$\}$. The intersection $W \cap X = \{\}$.

*When the intersection of two sets is the null set, we call those sets **disjoint**.*

Unions of Sets

Definition:

The union of two sets, A and B, denoted by $A \cup B$, is the set of all elements that are in A, or in B, or in both.

Examples:

Let $K = \{1, 2, 3\}$ and $L = \{3, 4, 5\}$. The union of sets $K \cup L = \{1, 2, 3, 4, 5\}$.

Let $M = \{a, b, c\}$ and $N = \{c, d, e\}$. The union of sets $M \cup N = \{a, b, c, d, e\}$.

Let $O =$ {red, blue, green} and $P =$ {yellow, green, purple}.
The union of sets $O \cup P$, is {red, blue, green, yellow, purple}.

Let $Q =$ {dog, cat} and $R = \{\}$. The union of Q and R, $Q \cup R =$ {dog, cat}.

Complement of Sets

Definition:

The complement of a set A, denoted by $\bar{A}$, represents all elements outside of set A within a universal set, typically denoted as S. It includes all elements not in A.

Example:

Let S denote all possible numbers of an ordinary die. Then $S = \{1, 2, 3, 4, 5, 6\}$.

If we then have $A = \{1, 2\}$ we get that $\bar{A} = \{3, 4, 5, 6\}$

Subsets

Definition:

A set A is considered a subset of another set B if every element of A is also an element of B. If all elements of set A are contained within set B, then A is a subset of B, denoted as $A \subseteq B$.

Examples:

Let $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$. Here, A **is** a subset of B, denoted as $A \subseteq B$.

Consider sets $C = \{$apple, orange$\}$ and $D = \{$apple, orange, banana$\}$. Set C **is** a subset of D, written as $C \subseteq D$.

Let $E = \{$red, indigo$\}$ and $F = \{$red, blue, green, yellow$\}$. Set E **is not** a subset of F, denoted as $E \subseteq F$.

Consider $G = \{2, 4, 6\}$ and $H = \{1, 2, 3, 4, 5, 6\}$. Here, G **is** a subset of H, thus $G \subseteq H$.

Let $I = \{$dog, cat, rabbit$\}$ and $J = \{$dog, cat$\}$. Set I **is not** a subset of J, however we do instead have that $J \subseteq I$.