Kapitel 2

Mängdlära & Sannolikhet

Mängdlära

Introduction to Sets

Definition:

A set is a well-defined collection of distinct objects or elements. Sets are fundamental in mathematics and statistics, providing a framework for organizing and analyzing data.

In a set, each element is unique, meaning it appears only once. Duplicate elements are not allowed within a set.

Notation:

We represent sets using curly braces "$\{\}$", and list their elements separated by commas. For

Examples:

Set A: $\{1, 2, 3, 4, 5\}$

Set B: $\{$apple, orange, banana$\}$

Null Set (Empty Set):

The null set, denoted by $\emptyset$, is a set with no elements. It plays a crucial role in set theory.

$\emptyset : \{\}$

Examples:

Let C represent the set of all months with 31 days.

$$C = \{\text{January, March, May, July, August, October, December}\}$$

Let D be the set of colors Amir likes.

$$D = \{\text{red, orange, yellow, green, blue, indigo, violet}\}$$

Let E be the set of the three largest cities in Sweden.

$$E = \{\text{Stockholm, Malmö, Göteborg}\}$$

Cardinality of Sets

Definition:

The cardinality of a set is the count of its elements. We denote the cardinality of set A as $|A|$.

Examples:

Let $G = \{2, 4, 6, 8, 10\} \rightarrow |G| = 5$.

Let $H = \{$apple, orange, banana, peach$\} \rightarrow |H| = 4$.

Let $I = \{$red, green, blue$\} \rightarrow |I| = 3$.

Let $J = \{\} \rightarrow |J| = 0$.

Intersections of Sets

Definition:

The intersection of two sets, A and B, denoted by $A \cap B$, is the set of all elements that are both in A and in B.

Examples:

Let $S = \{2, 4, 6, 8\}$ and $T = \{3, 6, 9\}$. The intersection $S \cap T = \{6\}$.

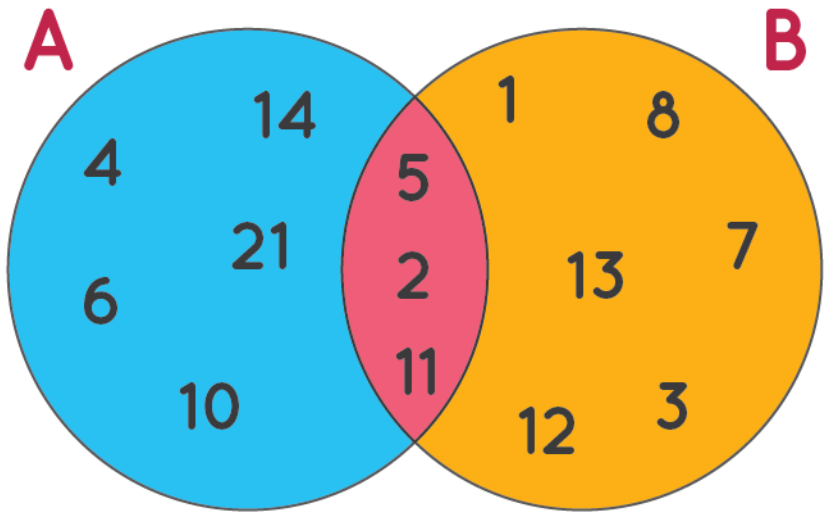
Let $U = \{a, b, c, d\}$ and $V = \{c, d, e\}$. The intersection $U \cap V = \{c, d\}$.

Let $W = \{$dog, cat, rabbit$\}$ and $X = \{$monkey$\}$. The intersection $W \cap X = \{\}$.

When the intersection of two sets is the null set, we call those sets ***disjoint****.*

A = {2, 4, 5, 6, 10, 11, 14, 21}

B = {1, 2, 3, 5, 7, 8, 11, 12, 13}



$$A \cap B = \{5,2,1\}$$

$$|A \cap B| = 3$$

Unions of Sets

Definition:

The union of two sets, A and B, denoted by $A \cup B$, is the set of all elements that are in A, or in B, or in both.

Examples:

Let $K = \{1, 2, 3\}$ and $L = \{3, 4, 5\}$. The union of sets $K \cup L = \{1, 2, 3, 4, 5\}$.

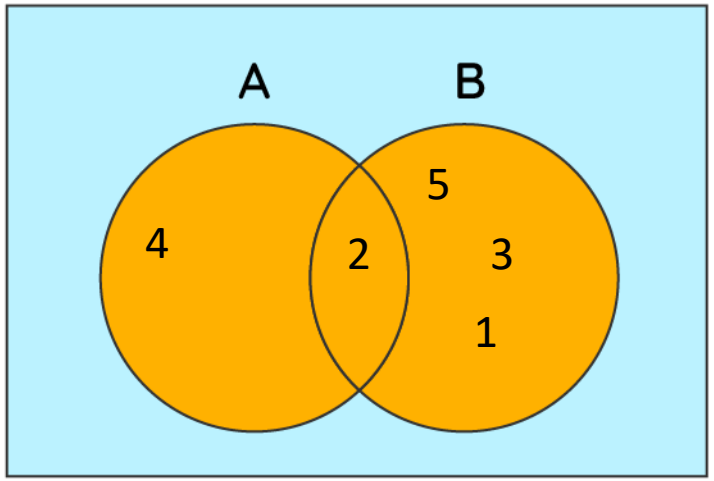
Let $M = \{a, b, c\}$ and $N = \{c, d, e\}$. The union of sets $M \cup N = \{a, b, c, d, e\}$.

Let $O = \{$red, blue, green$\}$ and $P = \{$yellow, green, purple$\}$.
The union of sets $O \cup P$, is $\{$red, blue, green, yellow, purple$\}$.

Let $Q = \{$dog, cat$\}$ and $R = \{\}$. The union of Q and R, $Q \cup R = \{$dog, cat$\}$.

A = {2, 4}

B = {1, 2, 3, 5}



Shaded Portion in Orange is A ∪ B

$$A \cup B = \{1,2,3,4,5\}$$

$$|A \cup B| = 5$$

Complement of Sets

Definition:

The complement of a set A, denoted by $\bar{A}$, represents all elements outside of set A within a universal set, typically denoted as S. It includes all elements not in A.

Example:

Let S denote all possible numbers of an ordinary die. Then $S = \{1, 2, 3, 4, 5, 6\}$.

If we then have $A = \{1, 2\}$ we get that $\bar{A} = \{3, 4, 5, 6\}$

S = {monkey, lion, black widow, cordyceps, tiger, koala, orangutang}

A = {monkey, lion}

$\bar{A}$ = {black widow, cordyceps, koala, orangutang}

Subsets

Definition:

A set A is considered a subset of another set B if every element of A is also an element of B. If all elements of set A are contained within set B, then A is a subset of B, denoted as $A \subseteq B$.

Examples:

Let $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$. Here, A **is** a subset of B, denoted as $A \subseteq B$.

Consider sets $C = \{\text{apple, orange}\}$ and $D = \{\text{apple, orange, banana}\}$. Set C **is** a subset of D, written as $C \subseteq D$.

Let $E = \{\text{red, indigo}\}$ and $F = \{\text{red, blue, green, yellow}\}$. Set E **is not** a subset of F, denoted as $E \subseteq F$.

Consider $G = \{2, 4, 6\}$ and $H = \{1, 2, 3, 4, 5, 6\}$. Here, G **is** a subset of H, thus $G \subseteq H$.

Let $I = \{\text{dog, cat, rabbit}\}$ and $J = \{\text{dog, cat}\}$. Set I **is not** a subset of J, however we do instead have that $J \subseteq I$.

Sannolikhet

Ett experiment är en skeende som leder till ett av flera olika möjliga utfall.

Ett utfall är resultatet av ett experiment.

En sannolikhet är ett mått på hur troligt det är att ett experiment får ett visst utfall.

Ett utfallsrum är de olika möjliga utfall som ett experiment kan leda till.

Exempel 2-1

Kajsa singlar slant med en enkrona. Vad är utfallsrummet och de olika utfallens sannolikheter?

Lösning

Kajsa kan få antingen krona eller klave som resultat av slantsinglingen. Experimentets utfallsrum är alltså {krona, klave}. Eftersom utfallsrummet består av två olika utfall som är exakt lika troliga så är sannolikheten för vart och ett av dem lika med 0,5, eller, om man så vill, 50 %. Formellt uttryckt har vi alltså

$$P(\text{krona}) = 0{,}5$$

och

$$P(\text{klave}) = 0{,}5$$