Phys 607 Project-3

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I. Theory

The project uses the Markov Chain Monte Carlo methods to estimate two parameters from simulated gravitational wave data with Gaussian noise from binary black hole mergers. Estimated parameters are inclination and coalesence phase. This work made use of two methods of calculations: **(i)** a hand written method which directly employs the use of the Metropolis-Hastings algorithm. The basic idea in this approach is to iteratively propose a new state from the current state and then accepting or rejecting the proposal based on a certain acceptance probability. The acceptance probability depends on the ratio of the posterior probabilities of the current and proposed states. We defined a log likelihood function as well as a prior function which takes input from our simulated data, and then calculate the posterior by adding the prior and likelihood. This method is known as Bayes theorem, and can be represented mathematically below:

$$P(\mathrm{B}|\mathrm{A}) = \text{likelihood}$$
$$P(\mathrm{A}) = \text{Prior}$$
$$\text{Posterior} = P(\mathrm{A}|\mathrm{B}) = P(\mathrm{A}) \times P(\mathrm{B}|\mathrm{A})$$

For our proposal function, we made use of a 2D normal distribution to generate new parameter values. We did this so as to explore the parameter space effectively. In other words, by using a normal distribution, our algorithm explores a range of parameter values centered around the current state, facilitating both local exploration and the potential discovery of distant, high-probability regions. Choosing from a normal distribution for the proposal function allows an appropriate acceptance rate which optimizes exploration efficiency. In addition, we added a Gaussian noise to our data distribution, so it makes sense for us to propose from a normal distribution as well.

(ii) for library implementation we used EnsembleSampler function from emcee library.

We tested both our results by using Gelman Rubin statistics and trace plots.

The formula for the Gelman-Rubin statistic is given by

$$R = \sqrt{\frac{\hat{V}}{W}}$$

Where:

- R is the Gelman-Rubin statistic.
- $\hat{V}$ is the estimated marginal posterior variance.
- W is the average of the within-chain variances.

A value of R close to 1 suggests convergence. We incorporated the Gelman Rubin Statistic for each parameter to access convergence, and an overall R statistic was obtained.

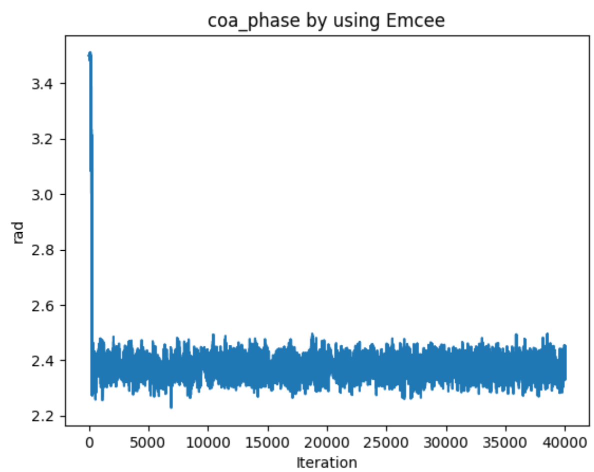

Fig. 1. Chain for coalescence phase with emcee

II. The Data

The data is simulated in frequency series by using the pycbc library. Parameters given to the function is drawn by the population simulation confusion-noise-3g.

We made the data 10^{24} times louder and add Gaussian noise with mean zero and standard deviation 1. Real values for inclination and coalesence phase are 2.7579985692108107 and 2.4092341172487513 respectively.

III. Method

In both methods we used some initial conditions, proposal functions and prior boundaries. Our proposal function is determined by our choice of noise as discussed earlier.

Since the unit of our parameters is radian, prior functions are bounded between zero and 2 pi naturally. However, when we used this boundaries our models did not converge in run time that our computers allowed. Thus, we limit our boundaries to $1.5 - 1.5\pi$ for coalesence phase and $2.0 - 1.5\pi$ for inclination angle.

These choices helped us to keep our runtime as short as 10 minutes.

IV. The Result

R values are calculated for handwritten version in two chains are for the coalescence phase is 0.997 and for the inclination is 0.994. Since these value very close to 1, we can say that our handwritten algorithm converges.

Auto-correlation time in emcee implementation for coalescence phase is 64.34 and for inclination is 66.88.

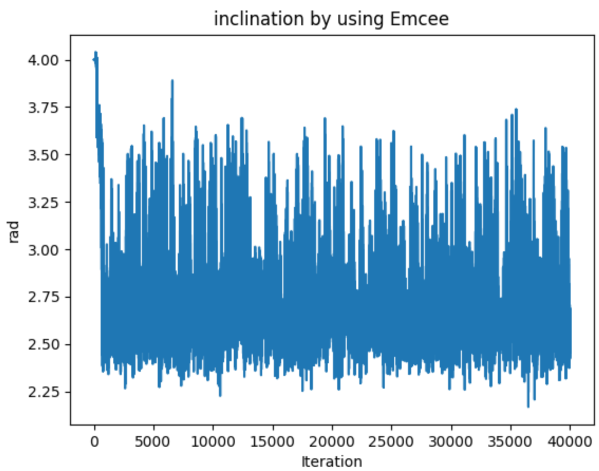

Fig. 2. Chain for inclination with emcee

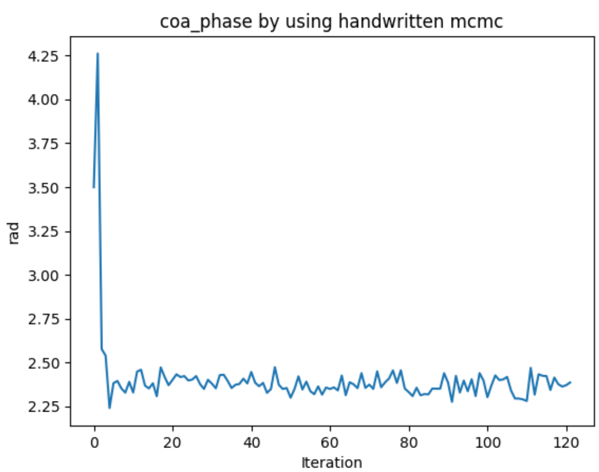

Fig. 3. Chain for coalescence phase with emcee

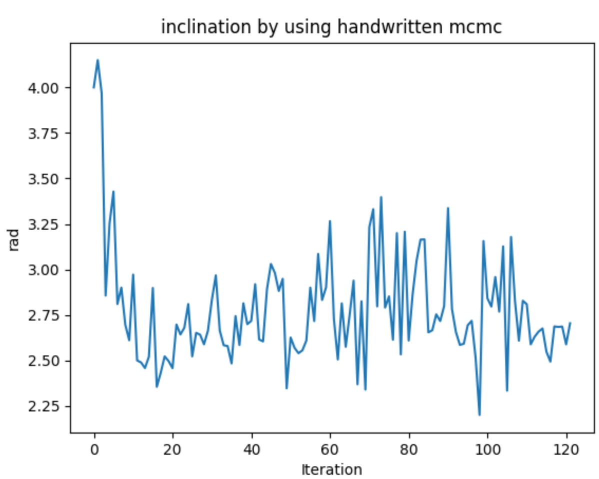

Fig. 4. Chain for coalescence phase with emcee

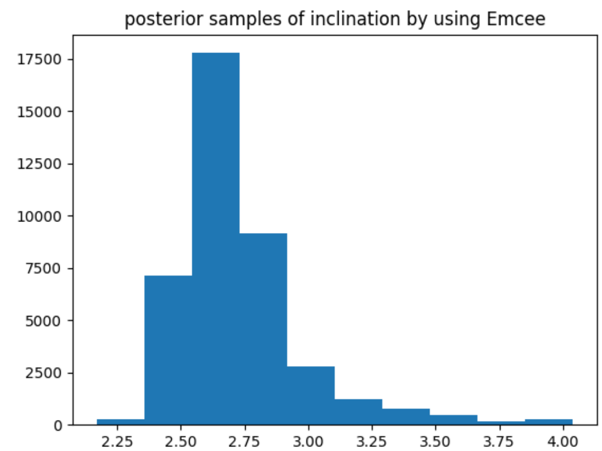

Fig. 5. Posterior samples for inclination with emcee

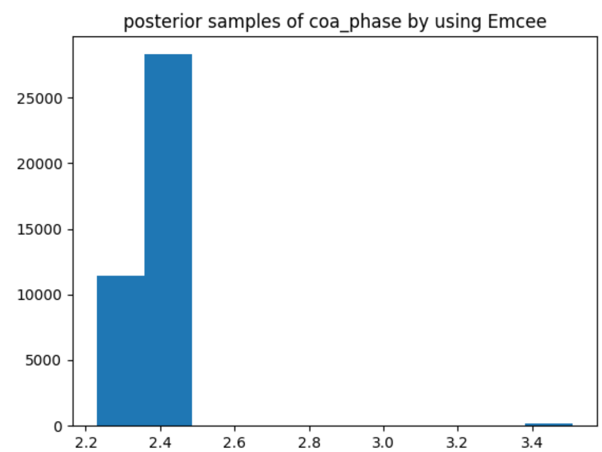

Fig. 6. Posterior samples for coalescence phase with emcee

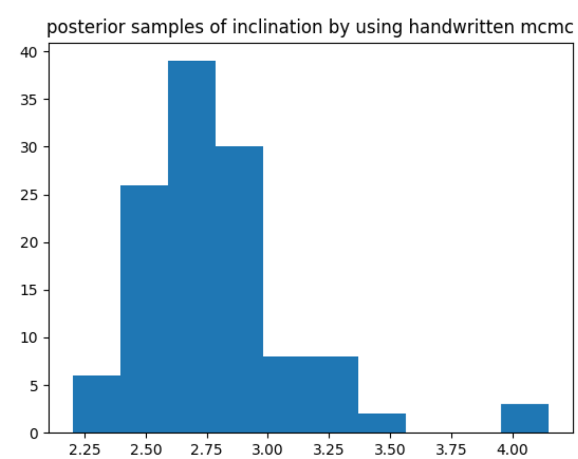

Fig. 7. Posterior samples for inclination with handwritten version

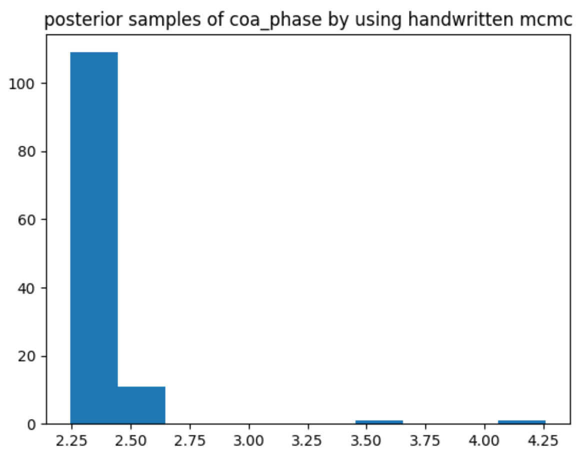

Fig. 8. Chain for coalescence phase with handwritten version