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NONLINEAR INDIRECT ADAPTIVE CONTROL OF A QUARTER CAR ACTIVE SUSPENSION

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Abstract

In this paper, an indirect adaptive controller is applied to the vehicle active suspension with a hydraulic actuator. Inner loop controller, which is a nonlinear adaptive controller, is designed to control the force generated by the nonlinear hydraulic actuator. The spool valve dynamics of a hydraulic actuator is reduced using a singular perturbation technique. The parameter estimation error used in an indirect adaptation is decreased asymptotically. The absolute velocity of a sprung mass is damped down by its negatively proportional term(sky-hook damper) adopted as an outer loop controller.

1. Introduction

A suspension control has been studied actively in last decades. Many approaches[1~9] have been developed to control the vertical motion of a car. In [3,4], an optimal control approaches are developed for suspension control. An adaptive control strategies are also proposed in [2,5,8,9]. In [1,6~9], the importance of controlling the hydraulic actuator is emphasized for fully active suspension control. In [1], a hydraulic active suspension model was developed for analysis, design, control law optimization, and diagnostic strategies development. In [9], a nonlinear adaptive controller was proposed to control the hydraulic actuator of the active suspension system. The adaptive approaches in [2,5,8,9] are based on the direct adaptation methods. In a direct adaptive system driven by the output tracking error, it is difficult to choose the controller gain and the adaptation gain. Because the small transient state error caused by the large controller gain slow down the adaptation speed, and thus the convergence speed of the steady state error is worse. On the other hand, an oscillation of the parameter estimates due to the large adaptation gain influences the transient response. Therefore a suitable selection of those gains is important to improve the performance of an active suspension system.

To ease this problem, indirect adaptive controller is investigated to control the nonlinear hydraulic actuator. The spool valve dynamics will be canceled by using a singular perturbation technique, and then nonlinear adaptive controller is designed to control the hydraulic actuator. It is shown that the performance of the parameter adaptation is nearly independent of that of the output tracking. The simulation results are presented to show the effectiveness and validity of the proposed approach.

2. System Model and Problem Formulation

The quarter car body is represented by the sprung mass m_2 and tire and wheel by the unsprung mass m_1 as shown in Fig. 1. To investigate the effects of the active suspension control, consider only the spring and the hydraulic actuator mounted between the sprung and unsprung masses. The spring stiffness is k_2. The tire is modeled as a spring with stiffness k_1 and its damping is assumed to be negligible[9]. The hydraulic force F_a is generated by the hydraulic actuator acting as an active force generating system. F_f is the frictional force due to the interaction between the walls of actuator chambers and the actuator seal.

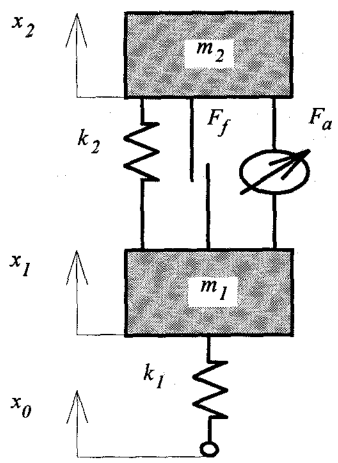

Fig. 1. Quarter car suspension model.

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The dynamic differential equations of the quarter car suspension system shown in Fig.1 can be expressed as follows[8,9]:

$$m_1\ddot{x}_1 = k_1(x_0 - x_1) - k_2(x_1 - x_2) - F_a + F_f \tag{1}$$

$$m_2\ddot{x}_2 = k_2(x_1 - x_2) + F_a - F_f \tag{2}$$

$$\dot{F}_a = -a_1F_a - a_2A_P(\dot{x}_2 - \dot{x}_1) + a_3\sqrt{P_S - \mathrm{sgn}(X_V)F_a / A_P} \cdot X_V \tag{3}$$

$$\dot{X}_V = \frac{1}{\varepsilon}(-X_V + u) \tag{4}$$

where $a_1 = a_2A_PC_{tm}$, $a_2 = 4\beta_eA_P / V_t$, $a_3 = a_2C_dwA_P\sqrt{1/\rho}$ in (3) and ε is a time constant of the spool valve dynamics in (4). u is a control input to the servo valve. In the above suspension model, there is no passive damper (shock absorber). Therefore the desired hydraulic force of the actuator consists of a passive damping term and a term negatively proportional to the vertical sprung mass velocity as follows [9]:

$$F_d = -C_{sh}\dot{x}_2 + B_s(\dot{x}_2 - \dot{x}_1) \tag{5}$$

where C_{sh} is a sky-hook damping coefficient and B_s is a passive damping coefficient.

The control objective is, for a given desired force F_d, to generate a control input u for the system (1)-(4) so that the actual force generated in actuator F_a tracks F_d as close as possible. In order to satisfy this objective and simplify the design of a nonlinear controller, singular perturbation technique will be introduced to reduce the hydraulic actuator dynamics in (3)-(4)

3. Nonlinear Indirect Adaptive Controller

In this section, nonlinear indirect adaptive controller is designed to control the hydraulic actuator force introduced in (1)-(4). In order to simplify the controller design, the spool valve dynamics in (4) is canceled by using a singular perturbation technique[11]. And thus the control input u can be equal to the spool valve position X_V. The spool valve dynamic equation in (4) can be rewritten as follows:

$$\varepsilon\dot{X}_V + X_V = u. \tag{6}$$

The control input is designed as follows:

$$u = (1 + K_{pf})u_s - K_{pf}X_V \tag{7}$$

where $K_{pf}(\geq 0)$ is a design parameter, u_s is slow control with respect to the time. Substituting the above into (6)

$$\varepsilon\dot{X}_V + (1 + K_{pf})X_V = (1 + K_{pf})u_s \tag{8}$$

is obtained. Let us define $\varepsilon_o \equiv \varepsilon / (1 + K_{pf})$ as the perturbation constant. The equations (3) and (4) can now be written with respect to ε_0 as follows:

$$\dot{F}_a = -a_1F_a - a_2A_P(\dot{x}_2 - \dot{x}_1) + a_3\sqrt{P_S - \mathrm{sgn}(X_V)F_a / A_P} \cdot X_V, \tag{9}$$

$$\varepsilon_o\dot{X}_V + X_V = u_s. \tag{10}$$

In the above equation, if $\varepsilon_o = 0$, then the following relationship is satisfied in (9)

$$\bar{X}_V = \bar{u}_s \tag{11}$$

where $\bar{X}_V$, $\bar{u}_s$ are the variables defined at $\varepsilon_o = 0$. Substituting (11) into (9), we obtain

$$\dot{\bar{F}}_a = -a_1\bar{F}_a - a_2A_P(\dot{\bar{x}}_2 - \dot{\bar{x}}_1) + a_3\sqrt{P_S - \mathrm{sgn}(\bar{X}_V)\bar{F}_a / A_P} \cdot \bar{u}_s. \tag{12}$$

The above equation is called a quasi-steady state system. Using a fast time scale $\tau = t / \varepsilon_o$ and Tichonov's theorem[11], we obtain the following relationships

$$X_V = \bar{X}_V + \eta + O(\varepsilon_0), \qquad \frac{d\eta}{d\tau} + \eta = 0 \tag{13}$$

where $\eta(\tau)$ is a boundary layer correction which satisfies the boundary layer equation (13). It can be easily shown that $\eta(\tau)$ decays exponentially in fast time scale. In the actual system, the spool valve constant ε is designed to satisfy $0 < \varepsilon << 1$. Therefore $\varepsilon_o(= \varepsilon / 1 + K_{pf})$ the perturbation constant can be chosen as small as possible, and $\eta + O(\varepsilon_0)$ may be negligibly small. Thus the equation (12) can be written as follows:

$$\dot{F}_a = -a_1F_a - a_2A_P(\dot{x}_2 - \dot{x}_1) + a_3\sqrt{P_S - \mathrm{sgn}(X_V)F_a / A_P} \cdot u_s. \tag{14}$$

Consequently, the above equation (14) can be regarded as the reduced order model of the equations (3) and (4), and can be represented as the following general nonlinear form

$$\dot{x} = f(x) + g(x)u_s \tag{15}$$

where $f(x) = -a_1 F_a - a_2 A_P(\dot{x}_2 - \dot{x}_1)$, $g(x) = a_3\sqrt{P_S - \mathrm{sgn}(X_V)F_a / A_P}$ and $x = F_a$. The above (15) can be parameterized as

$$\dot{x} = \theta^T Y_r \tag{16}$$

where $Y_r^T = \left[F_a \quad A_P(\dot{x}_2 - \dot{x}_1) \quad \sqrt{P_S - \mathrm{sgn}(X_V)F_a / A_P} \cdot u_s\right]$ and $\theta^T = [-a_1 \quad -a_2 \quad a_3]$ are regressor and real parameter vector, respectively. The slow control u_s has a general feedback linearizing structure as follows:

$$u_s = \frac{1}{\hat{g}(x)}\left[\ -\hat{f}(x) + \nu\ \right] \tag{17}$$

where $\hat{f}(x)$ and $\hat{g}(x)$ are constructed with the estimates of the $f(x)$ and $g(x)$, respectively. For simplicity, it is assumed that $\hat{g}(x)$ is lower bounded. In the above, the outer loop controller ν is designed as follows:

$$\nu = \ \dot{x}_d - k_d x_e, \quad x_d = -C_{sh}\dot{x}_2 \tag{18}$$

where $x_e = x - x_d$, and x_d is a reference force of the hydraulic actuator. Substituting (17) and (18) into (15), we obtain the following closed-loop error equation

$$\begin{aligned} \dot{x}_e + k_d x_e &= [f(x) - \hat{f}(x)] + [g(x) - \hat{g}(x)]u_s \\ &= \tilde{f}(x) + \tilde{g}(x)u_s \quad = \tilde{\theta}^T Y_r \end{aligned} \tag{19}$$

where $\tilde{\theta}(= \theta - \hat{\theta})$ is a parameter estimates error vector. In case of the ideally controlled hydraulic actuator, the actual force x exerted by the actuator may be equal to the desired force x_d. In the above equation (19), the output force error x_e converges to zero exponentially fast under the assumption of precisely known parameters. However, it is very difficult to know the parameters exactly. Therefore, in this paper, the parameter adaptation law is also introduced to overcome this difficulty. For estimation error $p_e = \hat{x} - x$, choosing the parameter estimation states $\hat{x}$ as

$$\dot{\hat{x}} = \hat{f}(x) + \hat{g}(x)u_s, \tag{20}$$

the estimation error equation is obtained as follows:

$$\dot{p}_e = \tilde{f}(x) + \tilde{g}(x)u_s = \tilde{\theta}^T Y_r. \tag{21}$$

However, the Eq. (21) as well as (20) is difficult to satisfy the stability of an adaptive system. To solve this problem, the estimation state equation is chosen as follows:

$$\dot{\hat{x}} = -\alpha_1(\hat{x} - x) + \hat{f}(x) + \hat{g}(x)u_s \tag{22}$$

where $\alpha_1(>0)$ is a design parameter. Using a parameterization technique, the above equation can be rewritten as follows:

$$\dot{\hat{x}} = -\alpha_1(\hat{x} - x) + \hat{\theta}^T Y_r. \tag{23}$$

Using the equations (16) and (23), the estimation error equation can be satisfied as the following

$$\dot{p}_e + \alpha_1 p_e = -\tilde{\theta}^T Y_r. \tag{24}$$

It is easy to obtain the estimation error $p_e(= \hat{x} - x)$ used in the parameter adaptation law. It is shown that the estimation error converges to zero and the estimates error belong to a bounded ball. The positive definite function is chosen as follows:

$$V = \frac{1}{2}\left\{\ p_e^2 \ + \ \tilde{\theta}^T \Gamma_2^{-1} \tilde{\theta}\ \right\} \tag{25}$$

where $\Gamma_2^T = \Gamma_2 > 0$. And its time derivative is as follows:

$$\dot{V} = -\alpha_1 p_e^2 + \tilde{\theta}^T\left[-Y_r p_e + \Gamma_2^{-1}\dot{\tilde{\theta}}\right]. \tag{26}$$

In the above, the second term of the right hand side will be canceled by the following

$$\dot{\tilde{\theta}} = \Gamma_2 \cdot Y_r \cdot p_e. \tag{27}$$

Substituting (27) into (26), the following inequality is satisfied.

$$\dot{V} = -\alpha_1 p_e^2 \ \le \ 0. \tag{28}$$

Therefore it can be easily shown that V, $\dot{V}$ are uniformly bounded, and thus p_e, $\dot{p}_e$, $\tilde{\theta}$, and $\dot{\tilde{\theta}}$ are also bounded. Since p_e and $\dot{V}$ are uniformly continuous, $\dot{V}$, $p_e \to 0$ are satisfied by Barbalat's lemma [10, pp.210]. The time derivative of (28) is as follows:

$$\ddot{V} = -2\alpha_1 p_e \dot{p}_e \ \le \ 0. \tag{29}$$

And thus $\ddot{V}$ is bounded, and $\dot{p}_e$ is uniformly continuous, and $p_e \to 0$ is satisfied. Therefore

$$\lim_{t\to\infty}\int_0^t \dot{p}_e(T)dT = \lim_{t\to\infty} p_e(t) - p_e(0) = -p_e(0) < \infty \tag{30}$$

is also satisfied. Introducing Barbalat's lemma[10] into (29) and (30), it can be shown that $\dot{p}_e$ converges to zero. Since p_e, $\dot{p}_e \to 0$, the estimation error system (24) can be written as follows:

$$\dot{p}_e + \alpha_1 p_e = -\tilde{\theta}^T Y_r \to 0 . \tag{31}$$

And thus, the closed-loop error equation (19) can also be written as follows:

$$\dot{x}_e + k_d x_e = \tilde{\theta}^T Y_r \to 0 . \tag{32}$$

As the results, $x_e, \dot{x}_e \to 0$ is satisfied in the above sense. It is shown from the results in (31) and (32) that the value of the controller gain k_d does not affect the estimation error system.

In case of not using $\dot{F}_d (= \dot{x}_d)$, the outer loop controller (18) is reconstructed as follows:

$$v = -k_d x_e \tag{33}$$

Then the closed-loop error equation (19) can now be rewritten as follows:

$$\dot{x} + k_d x = k_d x_d + \tilde{\theta}^T Y_r . \tag{34}$$

Choosing $\tilde{\theta}^T Y_r = 0$, the above (34) is rewritten as follows:

$$\mu \cdot \dot{x} + x = x_d \tag{35}$$

where $\mu = 1/k_d$. When the value of k_d is large enough to satisfy $\mu \cdot \dot{x} = 0$, the force tracking error x_e is negligibly small. Therefore it is important to choose the controller gain of k_d large enough within the range in which the data overflow does not arise. It was shown that the selection of the controller gain k_d is simple compared with the direct adaptive system driven by the output tracking error.

4. Simulation Results

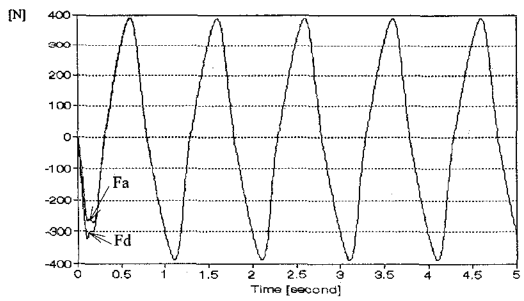
In this section, the proposed nonlinear adaptive controller is tested for 1/4 active suspension to show its effectiveness and validity. The parameter values used in the simulation are the same as the experimental values in [8,9]. It is shown that these values are presented in Table 1. The spool valve constant ε equals 0.003[9]. The road profile adopted in the simulation is chosen as a sinusoidal with a frequency of 1*Hz* and magnitude of 2.54*cm*. The desired force of the actuator is the same as that presented in [9]. The sky-hook damping coefficient C_{sh} is 3000*N/m/sec*, and the passive damping coefficient B_s is 1000 *N/m/sec*.

Table. 1 Real parameter values

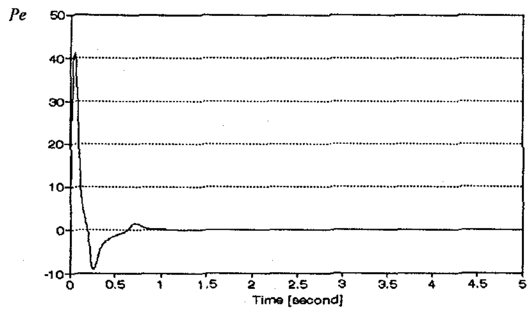
m_2	290 kg	$\lvert F_{fmax} \rvert$	250 N
m_1	59 kg	a_1	3.35e-4 m^2
k_2	16812 N/m	a_2	15.125e9 N/m^3
k_1	190000 N/m	a_3	5.176e5 $N/\left(m^{1/2} kg^{1/2}\right)$
P_S	10342500 Pa	A_P	3.35e-4 m^2

In case of the small controller gain ($k_d = 10$), the simulation results are represented for the useful case of the derivative of the desired force as in Fig.2. Otherwise, the results are shown in Fig.3. In both cases, α_1 in (22) equals 30. The actual force generated by the hydraulic actuator deviates large from the desired force as shown in Fig.3. In case of the large controller gain ($k_d = 1000$), the results are also represented as shown in Fig.4 and Fig.5 for the useful case of $\dot{F}_d (= \dot{x}_d)$ and the otherwise, respectively.

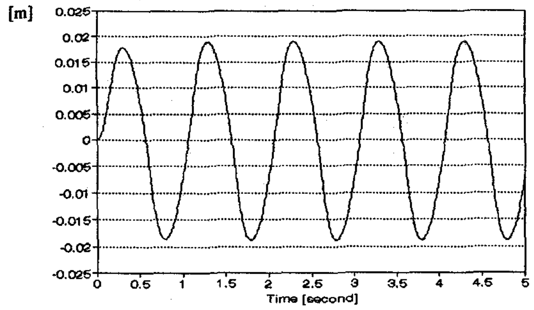
In case of the useful derivative of the desired force shown as in Fig.2 and Fig.4, the tracking performance of the actuator force is not sensitive to the variation of the controller gain. However, In case of not using $\dot{F}_d (= \dot{x}_d)$ as represented in Fig.3 and Fig.5, the force tracking performance is closely dependent on the variation of the controller gain, and the steady state error may be enlarged for the small feedback gain as shown in Fig.3. As a result, this error due to the lack of the anti-vibration force decreases the ride comforts. And thus, considering the inequality $|x - x_d| \le \mu |\dot{x}| = |\dot{x}| / k_d$ from the Eq. (35), the control gain k_d must be chosen as large as possible within the range in which the data overflow does not arise.



(a) The desired force(F_d) and actual force(F_a).

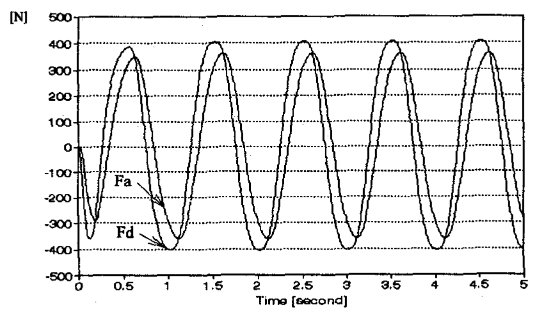


(b) The parameter estimation error.

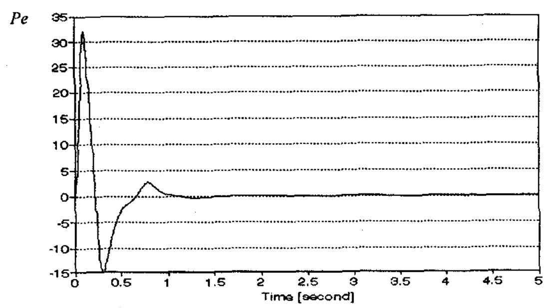


(c) The displacement of active sprung mass: x_2.

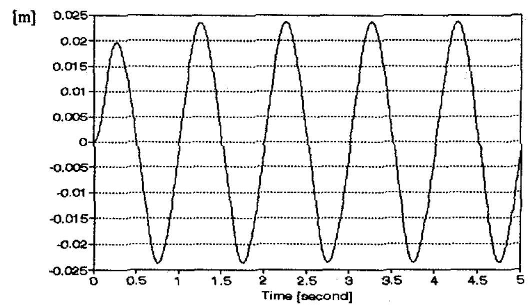
Fig. 2. The desired force derivative used case: $k_d = 10$.



(a) The desired force(F_d) and actual force(F_a).

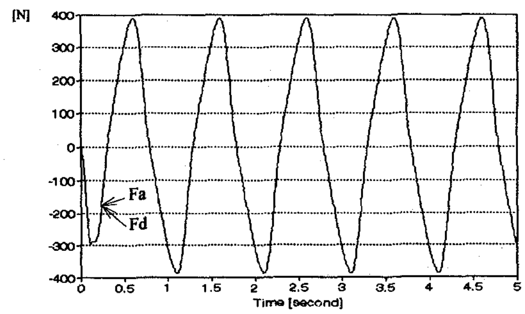


(b) The parameter estimation error.

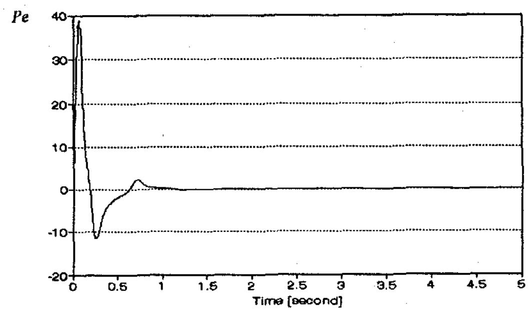


(c) The displacement of active sprung mass: x_2.

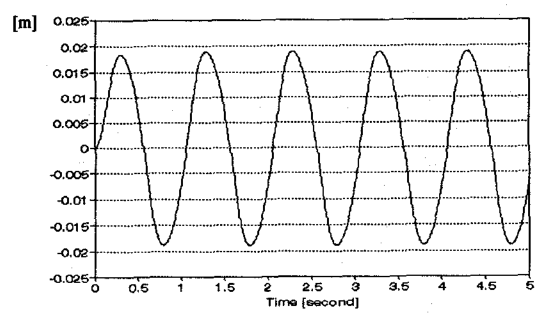
Fig. 3. The desired force derivative no used: $k_d = 10$.



(a) The desired force(F_d) and actual force(F_a).

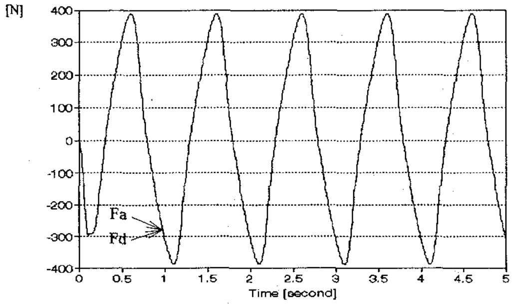


(b) The parameter estimation error.

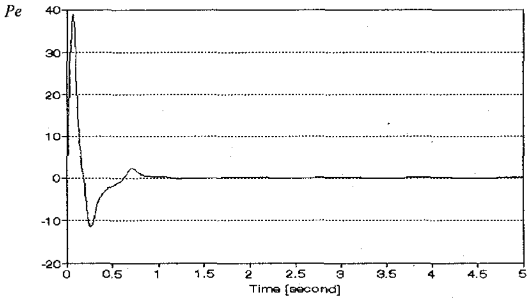


(c) The displacement of active sprung mass: x_2.

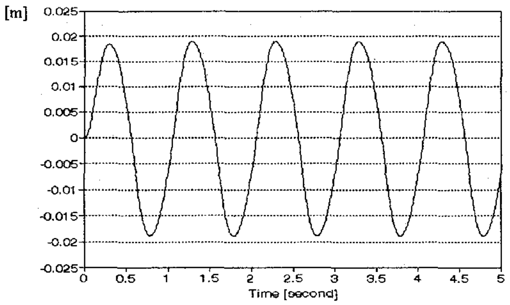
Fig. 4. The desired force derivative used case: $k_d = 1000$.



(a) The desired force(F_d) and actual force(F_a).



(b) The parameter estimation error.



(c) The displacement of active sprung mass: x_2.

Fig. 5. The desired force derivative no used: $k_d = 1000$.

5. Conclusion

In this paper, the nonlinear indirect adaptive controller was proposed to control the hydraulic actuator force of the active suspension system. The spool valve dynamics was reduced by using an singular perturbation strategy. The indirect adaptive law was proposed to estimate the parameter of the reduced hydraulic actuator model. The sky-hook damper was adopted to damp down the vertical oscillation of the car. It was shown that the coupling effect between the adaptation gain and the controller gain is relaxed by the indirect adaptive law introduced in this paper. The reduced order modeling error must be considered to satisfy the global stability of the active suspension, and this will be investigated in the future.

6. References

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