Final Solution

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| 1. (8 scores) Solve the following nonlinear DE. | |
| **Sol. 1.** | Let  The equation becomes  Now |
| 1. (24 scores) Solve the following PDEs.         (just find the general solution)      (just find the general solution) | |
| **Sol. 2.(a)** | Using  and  as a separation constant    We obtain  and  and the boundary condition gives  First,  With  the nontrivial solutions are    Next, we solve  for  and    Now , where  for each  So  Imposing  gives  Hence |
| **Sol. 2.(b)** | Using  and  as a separation constant    The boundary condition gives        So the general solution is |
| **Sol. 2.(c)** | Assume  so that    For (A), using  and  as a separation constant  , and we obtain  With  the nontrivial solutions are      Therefore  where  for each    For (B), we only need to find a particular solution.  Hence |
| 1. (14 scores) Solve the following PDEs. 2. Try to approximate     by *c*0 + *c*1*x* + *c*2*x*2 such that  is minimal.   1. Try to approximate **z** = [3, 1, 3, 5, 3, 5] by *d*0**b0** + *d*1**b1** + *d*2**b2**   where **b0** = [1, 1, 1, 1, 1, 1], **b1** = [1, -1, 1, -1, 1, -1], **b2** = [1, 2, 3, 4, 5, 6]  such that  is minimal. | |
| **Sol. 3.(a)** | Apply Gram-Schmidt process on        Hence |
| **Sol. 3.(b)** | Apply Gram-Schmidt process on        Hence |
| 1. (15 scores) Determine 2. where  means the Fourier transform 3. where \* means the convolution 4. where \* means the convolution | |
| **Sol. 4.(a)** |  |
| **Sol. 4.(b)** |  |
| **Sol. 4.(c)** |  |
| 1. (6 scores) Suppose that *G*(*f*) is the Fourier transform of *g*(*t*) and *G*(*f*) = 0 for |*f*| ≥ 1.   If *g*(−1) = *g*(−1/2) = 1, *g*(0) = 2, *g*(1/2) = *g*(1) = 3, and *g*(*n*/2) = 0 if n is an integer and |*n*| > 2,  try to determine *g*(*t*). | |
| **Sol. 5.** | Apply sampling theorem, since  for , we obtain |
| 1. (15 scores) Suppose that 2. Determine the Jordan-canonical form of **A**. 3. Determine **A**100. 4. Determine  and . | |
| **Sol. 6.(a)** | The characteristic polynomial of A is  The eigenvalues are ,  The eigenvectors  satisfy  The Jordan-canonical form of **A** is |
| **Sol. 6.(b)** |  |
| **Sol. 6.(c)** | , , , |
| 1. (6 scores) Determine the SVD of | |
| **Sol. 7.** | , eigenvalues are  Singular values are  The corresponding normalized eigenvectors are  , eigenvalues are  The corresponding normalized eigenvectors are |
| 1. (12 scores) Suppose that   , ,  otherwise  ,  otherwise   1. Determine the entropy of *X* and the entropy of *Y* (express the solution in terms of ln). 2. Determine the KL divergence from *PY*(*n*) to *PX*(*n*) (express the solution in terms of ln). | |
| **Sol. 8.(a)** | The entropy of  is  The entropy of  is |
| **Sol. 8.(b)** |  |