### VIDEO-COMPUTER EVALUATION OF HOLOGRAPHIC INTERFEROGRAMS

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ABSTRACT: A simple introduction into optical holography and holographic interferometry is presented. The evaluation of the displacement vector of an arbitrary point of any opaque object from the base point was studied in a case of a simply cantilever. The use of video-computer evaluation of the greatness of displacement enables very easy the determination of the value of deformation with very good precision.

#### 1 Introduction

In the process of lectures of non contact measurements in the course of applied physics there is solved for the students very interesting problem of non contact measurement by the method of optical holographic interferometry [1], [2]. The students are to obtain the basic informations and practical experimental skills about this measuring method during two lectures only. That's why we give to our students the basic theoretical knowledge and we realize the simple record of a hologram on a silver-halide medium and reconstruction of the holographic image after photographic development of the hologram.

For recording and development of the hologram we use a holographic film Agfa-Gevaert 10E75. As the source of coherent light we have He-Ne laser LA 1001 of the output energy 80 mW. For simply measurement of displacement by holographic interferometry the object in the form of a desk was used. If we apply a two exposure technique of holographic interferometry, we record at first the object (a desk) before deformation and than the record after the deformation, on the some hologram. After development and by illumination of the hologram by laser beam we can see through the hologram the reconstructed holographic image of the object by the eye or by video camera. We could see that the reconstructed image is covered by the interference fringes that indicate the

greatness and the direction of deformations of the object. And if we observe a holographic image (interferogram) through a hologram by CCD camera which is connected with a computer we can in simple cases very easy evaluate the deformations (displacement vector) in the single points of the object.

## 2 Holography

Holography is a technique for recording and reconstructing light waves. The wave which is to be recorded is called the object wave. In order to reconstruct, that is, produce a facsimile of the object wave, it is sufficient to reproduce its complex amplitude  $a_1$ , at one plane in space [1], [2]. We can write

$$a_1 = A_1 \exp\left(i\varphi_1\right) \tag{1}$$

where  $A_1$  is real amplitude of the light wave which is connected with the square rood of the intensity  $I_1$  of the wave and  $\varphi_1$  is the phase of this wave and describes the position of the object wave in the space. Once this has been reproduced, the light propagating away from this plane will be identical of the original object wave. The distributions of both real amplitude and phase in the plane must be recorded; however, photographic film or any other detector responds only to irradiance. The object wave irradiance is  $I_1 = a_1 a_1^*$ , which is a real quantity, so film exposed to  $a_1$  can record the distribution of real amplitude  $A_1$ , but the distribution of phase will be lost [1], [2].

It is clear from the discussion that interferometry can be used to convert a phase distribution into an irradiance pattern, which can be recorded on photographic film. This is the basis of Gabor's invention of holography, which he described in detail in 1949 [3], [4]. He proposed to form an interference pat-tern by adding a coherent reference wave to the object wave. This interference pattern can be recorded on film. When the film is developed and illuminated appropriately, it diffracts light in a manner such that the complex amplitude  $a_1$  is reproduced at the plane of the film. All this can be accomplished using the simple system shown in Fig. 1a. A plane wave of monochromatic light passes through a photographic transparency, which is the object. Part of the light will be diffracted by whatever image is recorded on the transparency, and part of the light will pass through the transparency without being scattered. The light which is diffracted is the object wave  $a_1$  and the undiffracted portion of the light serves as the reference wave  $a_0$ . We can again write

$$a_0 = A_0 \exp\left(i\varphi_0\right) \tag{2}$$

where  $A_0$  is the real amplitude and  $\varphi_0$  is the phase of the reference wave.

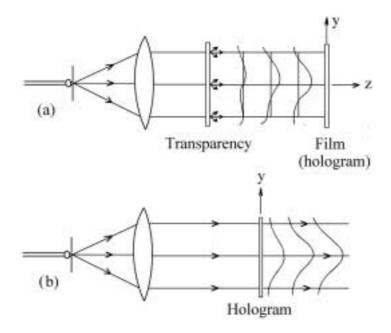


Figure 1: In-line(Gabor)holography. (a) Recording the hologram. (b) Reconstructing the object wave

The developed film is referred to as a Gabor hologram or, alternatively, as an in-line hologram. The intensity distribution  $I_h$  of this hologram can be expressed in the form

$$I_h = (a_1 + a_0)(a_1^* + a_0^*) \tag{3}$$

or better

$$I_h = |a_1|^2 + |a_0|^2 + a_1 a_0^* + a_1^* a_0. (4)$$

The object wave is reconstructed by illuminating the developed hologram with a uniform plane wave of laser light, as shown in Fig. 1b. This is referred to as the reconstruction wave

$$a_2 = A_2 \exp\left(i\varphi_2\right). \tag{5}$$

When the hologram is illuminated by this wave, the complex amplitude will be

$$a = a_2 (a_1 + a_0) (a_1^* + a_0^*) (6)$$

or

$$a = a_2 I_h = a_2 \left( \left| a_1 \right|^2 + \left| a_0 \right|^2 \right) + a_2 a_1 a_0^* + a_2 a_1^* a_0. \tag{7}$$

Considering the reconstruction wave is identical to the reference wave (Fig. 1a, Fig. 1b) that means the relation (2) equals the relation (5), the term  $a_2 (|a_1|^2 + |a_0|^2)$  represents the portion of the reconstruction wave that is just attenuated and transmitted by the hologram, the term  $a_2a_1a_0^*$  is the desired facsimile of  $a_1$ , because the expression of  $a_2a_0^* = A_0^2$  which is the constant value only, that means the goal of recording and

reconstructing the object wave has been attained. The term  $a_2a_1^*a_0$  is a wave that is proportional to the conjugate of the object wave.

We note that in our experiment the hologram is in the phase form, that means the hologram is perfectly transparent "clear" and it has no influence on the intensity of the light during the reconstruction of the object wave.

Generally, off-axis holography, developed by Leith and Upatnieks [5], [6] is used in holographic interferometry. This scheme enables one to spatially separate the three waves produced by diffraction in the reconstruction process. It is based on a common procedure in communication theory, namely, the coding and decoding of signals by modulation of a high-frequency carrier wave. In holography, spatial frequencies are used rather than temporal frequencies, but the concept remains the same [7]. The reference wave used to record an off-axis hologram propagates the different angular direction from the object wave. Fig. 2a illustrates the formation of an off-axis hologram using a plane reference wave.

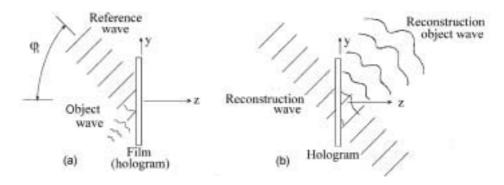


Figure 2: Off-axis (Lenith-Upatnieks) holography. (a) Recording the hologram, (b) Reconstructing the object wave

To reconstruct the object wave, the hologram is illuminated by a plane wave travelling in the same direction as the original reference wave (Fig. 2b).

A typical optical system for recording off-axis holograms is shown in Fig. 3a. The system is quite simple.

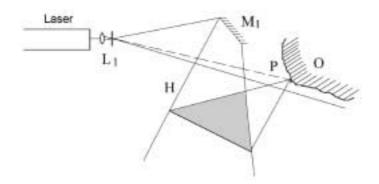


Figure 3a: Recording the hologram

A parallel laser beam is transformed by the lens  $L_1$  on a divergent beam illuminating an opaque object O and a mirror  $M_1$ . A diffusely reflected light from the surface of the object O interferes with the reflected light from the mirror  $M_1$  in some part of the space and in this interference part of space we put a plane hologram. Mostly the hologram is in the form of photographic desk or film provided with the emulsion of high resolution power. The lens  $L_1$  is usually a microscope objective and the expanded beam is filtered by a pinhole spatial filter. By recording the hologram the mirror  $M_1$  is usually in the form of an uncoated glass so that only part of the incident light is reflected. In this case the energy passing on the hologram from the mirror is practically the some like the energy from the opaque object O.

After recording and developing the hologram we put the hologram in its original place and instead the original mirror  $M_1$  we give in the some place the wholly reflected mirror  $M_2$ . Then if we get out the object O we can see trough the hologram the virtual image of the object in the original place as by the record of the hologram (Fig. 3b).

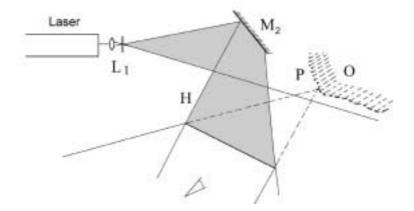


Figure 3b: Reconstructing the object wave

As it was described earlier the hologram is formed by exposing and developing the holographic plate or film and the reconstruction wave is usually identical to the original reference wave. Then three-dimensional virtual image of the object can be seen by an observer in the some position as the original object. The process of recording and reconstruction in off-axis holography can be mathematically described similarly as in the case of in-line holography.

Holograms can be recorded on any photographic emulsion whose high resolution and transmittance versus exposure characteristics are appropriate. Commercial holographic emulsions are available on acetate film or on glass plates. The glass plates are usually preferred in holographic interferometry because of their dimensional stability.

## 3 Holographic interferometry

Trough the use of off-axis holography, one can produce three-dimensional images of diffusely reflecting objects which appear to be overlaid by interference fringes that are indicative of deformation, displacement, or rotation of the object. Similarly, in the case of transparent objects, fringe patterns can be formed which are indicative of changes in refractive index or object thickness. This type of interferometry is possible because a light wave scattered by an object can be holographically recorded and reconstructed with such precision that it can be compared iterferometrically with light scattered by the same object at another time.

Alternatively, it can be compared interferometrically with a second holographic reconstruction or light scattered by the object. Accordingly, we define holographic interferometry as the interferometric comparison of two or more waves, at least one of which is holographically reconstructed. The composite of these two or more waves will be referred to as a holographic interferogram. The term interferogram with no modifying adjective will denote a pattern of interference fringes recorded on photographic film or formed on a two-dimensional viewing screen, on the retina of the eye or on the chip of CCD camera.

The production of a holographic interferogram is simple in concept and in practice. As an illustration, suppose that we wish to determine the response of some mechanical component to the application of a force. The component is mounted in a loading fixture and placed in the object position of an off axis holographic system such as that shown in Fig. 3a. With the component in an original, unstressed condition a holographic exposure of the film plate is made. The desired force is then applied, and a second holographic exposure of the same film plate is made. When such a doubly exposed hologram is illuminated with a duplicate of the reference wave, the holographic interferogram can be viewed. The result is a fascinating and appealing display. An observer looking through the hologram sees the three-dimensional image of the component overlaid with a pattern of interference fringes. The fringes appear to be localized in space-sometimes on the object, sometimes in front of it or behind it. Futhermore, the fringes have a dynamic character in that they appear to change position and be altered in shape as the observer moves his or her head about to vary the viewing direction.

Holographic interferometry has found many applications which require only qualitative interpretation of fringe patterns, particularly in the field of nondestructive testing. Applications to metrology and stress analysis require quantitative evaluation of interferograms. In it procedures for determining the vector displacements of points on the surface of an object will be developed.

## 4 Theory, examination of deformation

For evaluation of holographic interferograms, it becomes important to adopt the concept of the phase, or path change of light waves. If, during our experiments, the

point M of a surface is displaced to the point M', while illuminating the object in the direction of unit vector  $\overrightarrow{n_1}$  and observing it in the direction of unit vector  $\overrightarrow{n_2}$  then the change in the optical path corresponding to the displacement vector  $\Delta \overrightarrow{r}$  may be expressed in the form of a scalar product (Fig. 4)

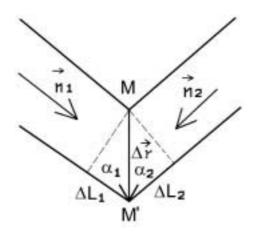


Figure 4: Illustration of the path change in holographic interferometry

$$\Delta L = \Delta L_1 + \Delta L_2 = (\overrightarrow{n}_1 - \overrightarrow{n}_2) \Delta \overrightarrow{r}$$
 (8)

or in simpler expression

$$\Delta L = \Delta r \left(\cos \alpha_1 + \cos \alpha_2\right). \tag{9}$$

The corresponding phase change  $\Delta \varphi$  of light wave reflected is then

$$\Delta \varphi = \frac{2\pi}{\lambda} \Delta L \tag{10}$$

where  $\lambda$  is wavelength of laser used.

In the case of the so-called double-exposure method, when the object is illuminated and two exposures are recorded on the same hologram, i.e. before and after the forced deformation, we may express the complex amplitude of light wave before deformation [1], [2], [8]

$$a_{1,1} = A_1 \exp\left(i\varphi_1\right) \tag{11}$$

and after deformation

$$a_{1,2} = A_1 \exp\left[i\left(\varphi_1 + \Delta\varphi\right)\right] \tag{12}$$

where  $A_1$  is the real amplitude and  $\varphi_1$  the phase of wave corresponding to the point M in space.

In reconstructing the double-exposure hologram both the waves are composed and interfere with one another, i.e. the resulting amplitude is the sum of amplitudes of both reflected waves. Then it holds

$$a = a_{1,1} + a_{1,2}. (13)$$

Since the eye is a quadratic detector which detects intensity, we may write [1], [2], [8]

$$I = a.a^* \tag{14}$$

where  $a^*$  is a complex conjugate value.

Substituting relation (13) in relation (14) we obtain

$$I = (a_{1,1} + a_{1,2}) \left( a_{1,1}^* + a_{1,2}^* \right) \tag{15}$$

or

$$I = |a_{1,1}|^2 + |a_{1,2}|^2 + a_{1,1}a_{1,2}^* + a_{1,1}^*a_{1,2}.$$
(16)

After adjustment we obtain

$$I = 2A_1^2 + A_1^2 \left[ \exp\left(i\Delta\varphi\right) + \exp\left(-i\Delta\varphi\right) \right]$$
 (17)

i.e.

$$I = I_0 \left( 1 + \cos \Delta \varphi \right) \tag{18}$$

where we have put

$$I_0 = 2A_1^2. (19)$$

From relation (18) it becomes evident that intensity I of the image of the reconstructed object is changed periodically in dependence upon the magnitude of deformation  $\Delta \overrightarrow{r}$ , or phase change  $\Delta \varphi$ . For an observer the object seems to be covered with interference fringes pattern, expressing deformations in individual points of the reconstructed image of the object.

As an example let us adduce here the deformation of the pump blade which has been fixed firmly in its lower part and deflected in its right-hand top in the direction perpendicular to the imaginary plane of the blade surface (Fig. 5).

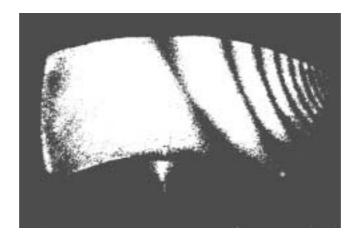


Figure 5: The pump blade loaded in its right-hand top as an example of holographic interferogram

At first, we consider a simple example in which displacement occurs in only one dimension. The object is a cantilever beam with one end fixed to a rigid block. The beam is illuminated with a plane wave traveling in the -z direction, normal to its surface (see Fig. 6a). The light scattered by the cantilever beam is recorded during an initial exposure of an off-axis hologram.

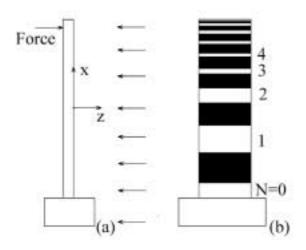


Figure 6: Normal deformation of a cantilever beam: (a) Side view of the beam. Displacement is in the +z direction and illumination in the -z direction. (b) Fringe pattern observed on the front of the beam

The free end of the beam is then displaced slightly in the z direction, and a second exposure is made. Fig. 6b shows what a photograph of the virtual image from this two-exposure hologram would look like. The same pattern could be observed visually. Since displacements are small, each point on the object moves only in the z direction. Initially each point was in the plane z=0. After displacement each point moved to a new position described by z=Z(x). If light travels a distance  $l_0$  from the source to a point on the object and back to the observer (or photograph) before the object is deformed, it will travel a distance  $l_0-2Z(x)$  after the object is deformed. The corresponding optical phase shift is  $\Delta \varphi(x)=(2\pi/\lambda)\left[2Z(x)\right]$ , where  $\lambda$  is the wavelength of the laser light. Note that we have not mentioned the hologram explicitly – its function is simply to record the two wavefronts and reconstruct them simultaneously to form the interferogram shown in Fig. 6b. We next assign fringe order numbers to the bright fringes in this figure. The base has not moved, so the next bright fringes are assigned by the numbers  $N=1,2,3\ldots$  consecutively. The  $N^{th}$  bright fringe corresponds to a phase change  $\Delta \varphi=2\pi N=(2\pi/\lambda)\left[2Z(x)\right]$ ; hence

$$Z\left(x\right) = \frac{N\lambda}{2}.\tag{20}$$

By simply counting the fringes to a given location, the displacement can be calculated using equation (20).

If we use the relation (9) and because of the equation (21), see Fig. 6a

$$\alpha_1 = \alpha_2 = 0 \tag{21}$$

we get

$$\Delta \varphi = 2\pi N = \frac{2\pi}{\lambda} \Delta r.2 \tag{22}$$

that means

$$\Delta r = Z(x) = N\frac{\lambda}{2}.$$
 (23)

In the case that

$$\alpha_1 = \alpha_2 = 60^o \tag{24}$$

then because of the relation

$$\cos 60^o = 0,5 \tag{25}$$

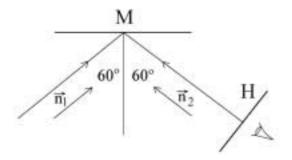


Figure 7: Observation of the illuminated cantilever M through the hologram H

we get

$$\Delta \varphi = 2\pi N = \frac{2\pi}{\lambda} \Delta r \tag{26}$$

and

$$\Delta r = Z\left(x\right) = N\lambda \tag{27}$$

as it is clear from Fig. 7.

# 5 Video-computer evaluation of holographic interferograms

To evaluate the deformations in more details in our last examples, see relations (23) and (27), we can very successfully use the video-computer interferogram processing.

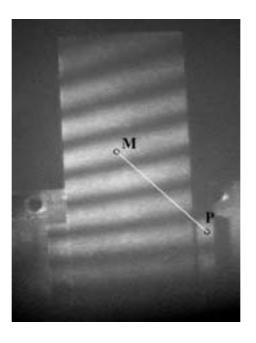


Figure 8: Interferogram of the cantilever mentioned in Fig. 6

In our interferogram of cantilever (Fig. 6) we are to determine the greatness of the displacement of the point M in z direction versus the base point P in the case of normal z direction of illumination and observation, see equation (23). Connecting point M with  $P_1$  (Fig. 8) we get a fundamental axis d of an intensity profile of the interferogram (Fig. 9).

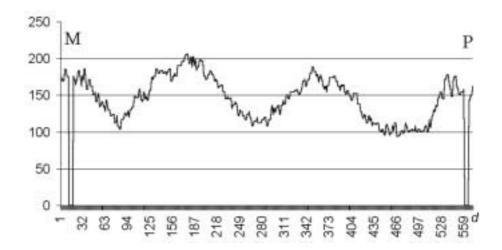


Figure 9: Intensity profile of the interferogram from Fig. 8 between points M and P

Due to the speckle pattern effect we use a method of Fast Fourier transformation and space filtration of high frequencies to improve the cause of function of intensity distribution (Fig. 10).

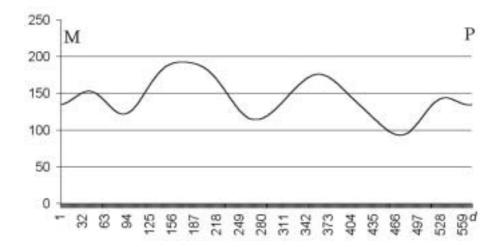


Figure 10: The profile of intensity distribution in the interferogram (Fig. 8) after application of Fast Fourier transformation and space filtration

By derivation of this function (Fig. 11) we get the x position of maximum and minimum of the intensity distribution function in Fig. 10.

Using a simple approximations we can say that the displacement

$$Z(M) \approx 0.7 \frac{\lambda}{4} + 3 \frac{\lambda}{2} + 0.8 \frac{\lambda}{4} = 7.5 \frac{\lambda}{4} = 1185 \text{nm}$$
 (28)

where the first member in equation (28) is connected with the part  $d_1$  in Fig. 11, the second member with the parts  $d_2 + d_3 + d_4$  and the third member with the part  $d_5$ .

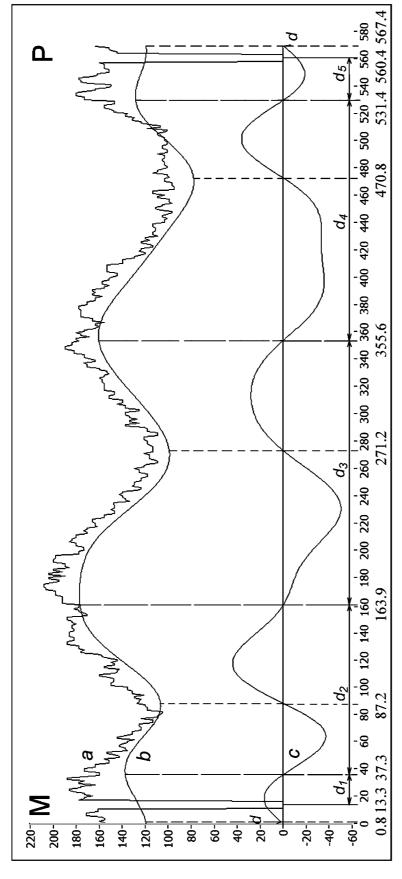


Figure 11: Graphical illustration of the evaluation of displacement of the cantilever in point M from base point P (Fig. 8). By index a is described the intensity distribution in interferogram between points P and M, the index b determines the improve function of intensity distribution from Fig. 10 and by index b is described the derivation of the improve function b

#### 6 Conclusion

It can be say, that using video-computer evaluation of deformations in classical holographic interferometry we can determine very easy the greatness of displacement of an arbitrary point of opaque object from any point of the base with very good precision.

## SOUHRN VIDEO-POČÍTAČOVÉ VYHONOCENÍ HOLOGRAFICKÝCH INTERFEROGRAMŮ

V práci je nastíněn úvod do metody optické holografie a holografické interferometrie. Určení vektoru posuvu libovolného bodu difusně odraženého předmětu od bodu nedeformované části předmětu je provedeno na příkladu jednoduchého nosníku. Pomocí CCD videozáznamu interferogramu zpracovávaného připojeným počítačem je velmi snadno docíleno velmi dobrého, tj. relativně velmi přesného vyhodnocení velikosti deformace.

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