LECTURE 2
Fourier Analysis of video signals, Spatio - Temporal Sampling

- Fourier transform over multidimensional space
- Frequency domain characterization of video signals
- Frequency response of the HVS
- Spatio-Temporal Sampling
- Reconstruction of Continuous Video from Samples

Continuous space signals

$$f(\mathbf{x}) \quad \mathbf{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_k \end{bmatrix}$$

Cont.
Space
signal

$$F(\mathbf{w}) \quad = \int f(\mathbf{x}) \quad \exp(-j\mathbf{w}^T\mathbf{x})\, d\mathbf{x}$$

Cont.
Space

FT

Discrete space signals

$$f(\mathbf{n}) \quad \mathbf{x} = [n_1 \quad n_2 \quad \ldots \quad n_k] \in \quad Z^k$$ Disc. Space signal

$$F_d(\mathbf{w}) \quad = \sum f(\mathbf{n}) \quad \exp(-j\mathbf{w}^T\mathbf{n})$$ Disc. Space

FT

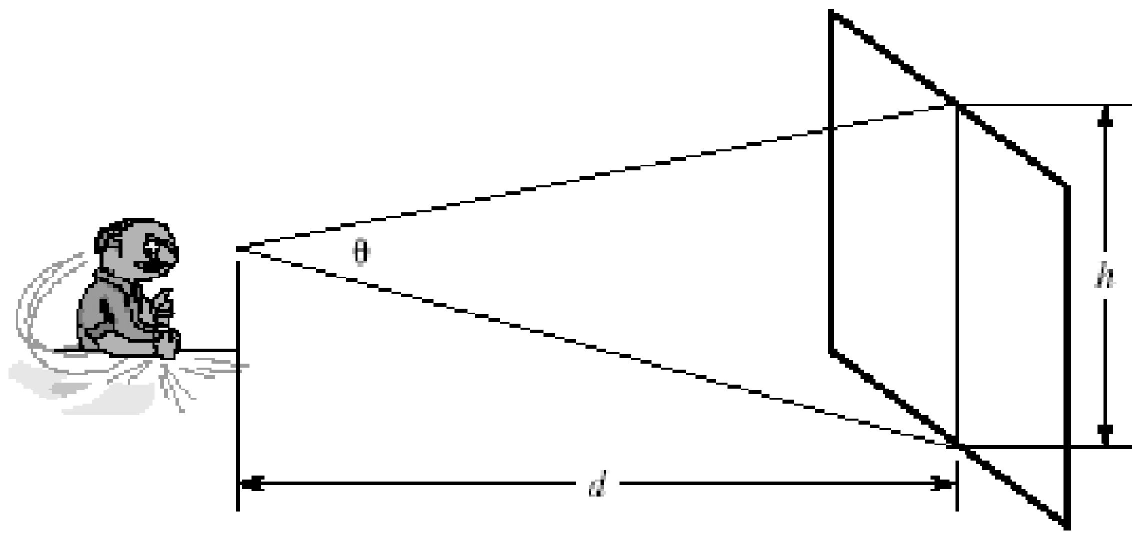
Frequency domain characterization of video signals-1

- Spatial frequency: measures how fast the image intensity changes in the image plane. Can be completely characterized by the variation freq. in two orthogonal directions (f_x: cycles/horizontal unit distance, f_y : cycles/vertical unit distance). It can also be specified by magnitude and angle of change

Problems: Percieved speed of change depends on the viewing distance.

Frequency domain characterization of video signals-2

- Angular frequency



$$\theta = 2\arctan(h/2d)(\text{radian}) \approx \text{h/2d}(\text{radian}) = \frac{180}{\pi}\frac{h}{d}(\text{degree})$$

$$\mathrm{f}_\theta = \frac{\mathrm{f}_s}{\theta} = \frac{\pi}{180}\frac{d}{h}\mathrm{f}_s(\text{cycle/degree})$$

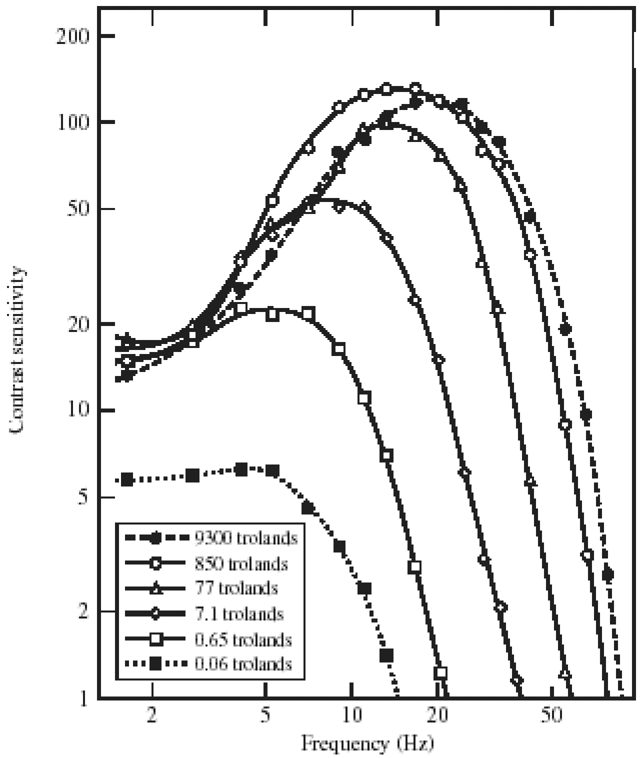
Frequency domain characterization of video signals-3

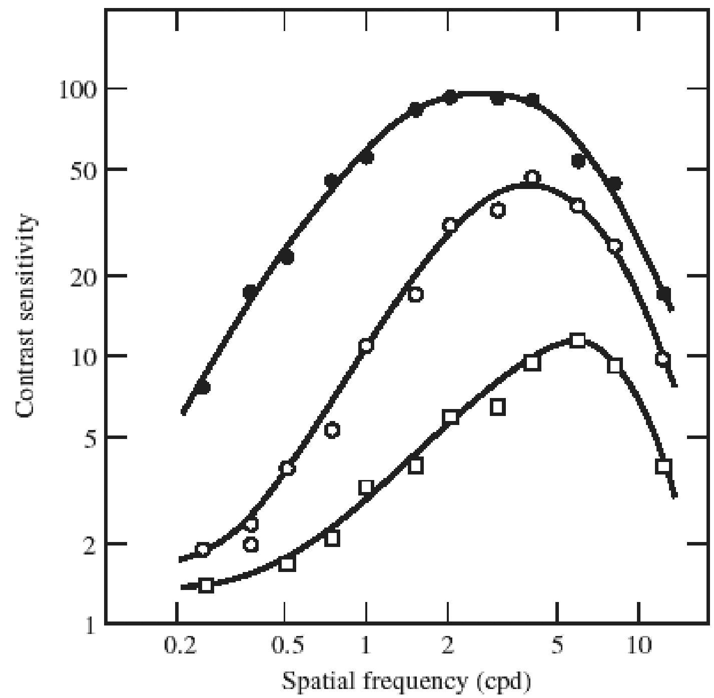
- Temporal frequency: measures temporal variation (cycles/s). In a video, the temporal frequency is spatial position dependent.
- Temporal frequency caused by linear motion:

  object moving with (vx,vy)

$$f_t = -(v_x f_x + v_y f_y)$$

Frequency Response of HVS-1





Temporal response

Spatial response

Frequency Response of HVS-2

- Responses obtained by experiments
- Critical flicker frequency: The lowest frame rate at which the eye does not perceive flicker. Provides guideline for determining the frame rate when designing a video system.

2D Fourier Transform

Continuous Signals

$$S_c(F_1, F_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s_c(x_1, x_2) e^{-j(F_1 x_1 + F_2 x_2)} dx_1 dx_2$$

$$s_c(x_1, x_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_c(F_1, F_2) e^{j(F_1 x_1 + F_2 x_2)} dF_1 dF_2$$

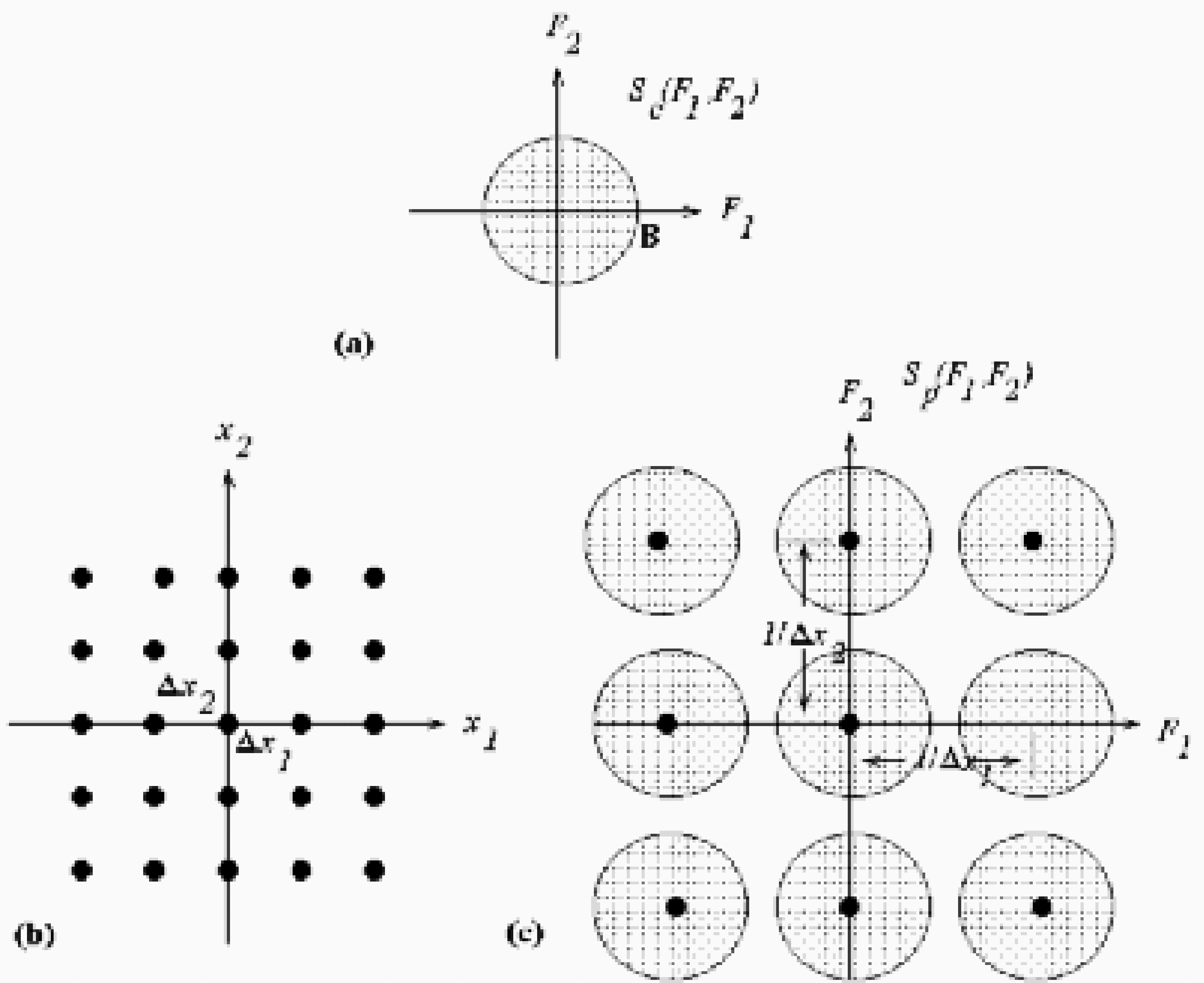
Discrete Signals

$$S(f_1, f_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} s(n_1, n_2) e^{-j(f_1 n_1 + f_2 n_2)}$$

$$s(n_1, n_2) = \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} S(f_1, f_2) e^{j(f_1 n_1 + f_2 n_2)} df_1 df_2$$

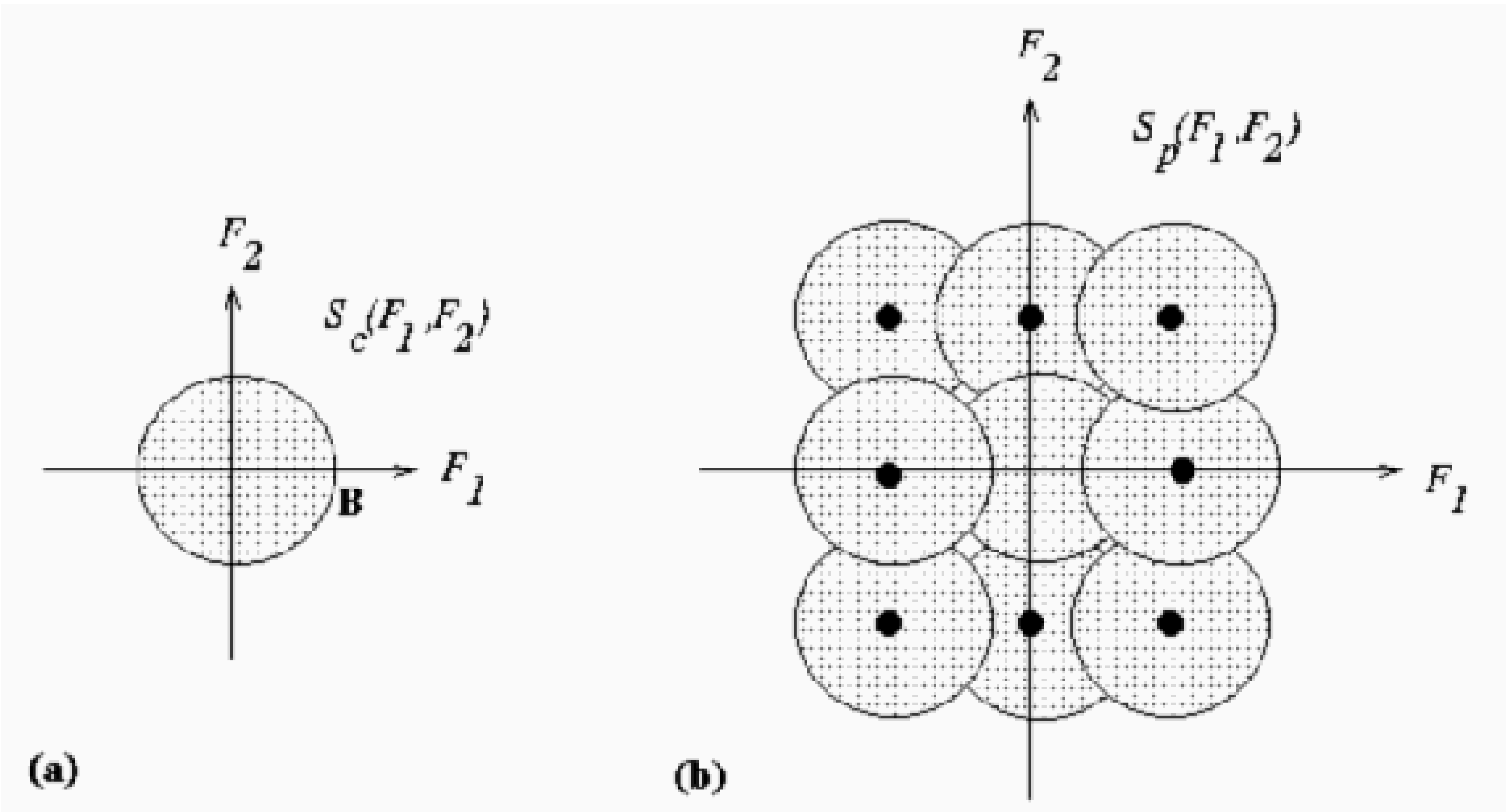
2-D sampling on a rectangular grid

$$S(F_1\Delta x_1, F_2\Delta x_2) = \frac{1}{\Delta x_1 \Delta x_2}\sum_{k_1}\sum_{k_2} S_c(F_1 - \frac{k_1}{\Delta x_1}, F_2 - \frac{k_1}{\Delta x_2})$$



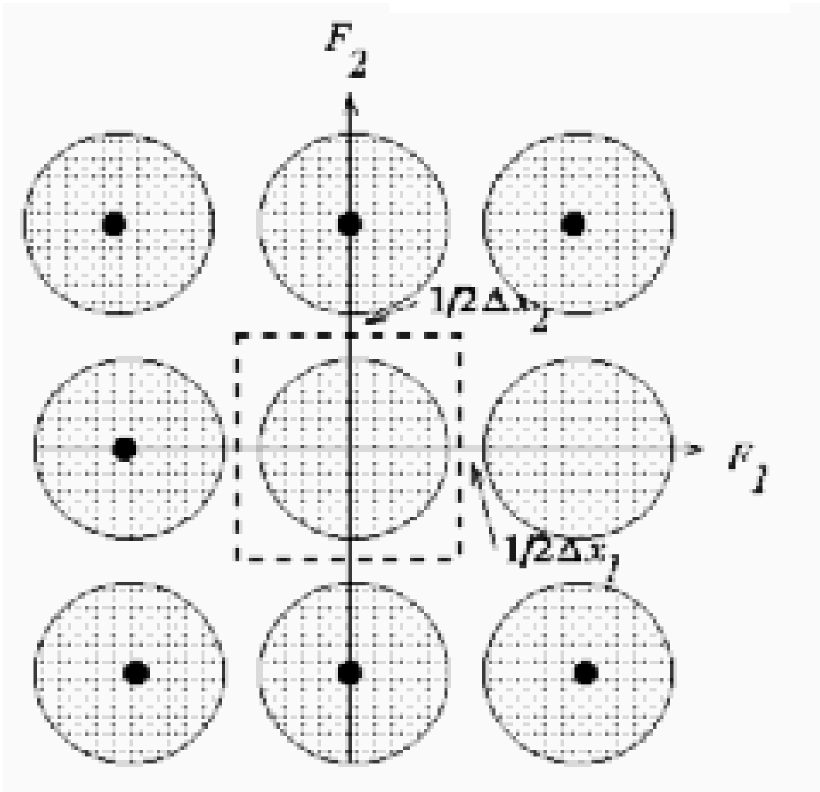
F_2
$S_c(F_1,F_2)$
F_1
B
(a)
x_2
Δx_2
x_1
Δx_1
(b)
F_2
$S_p(F_1,F_2)$
$1/\Delta x_2$
F_1
$1/\Delta x_1$
(c)

Aliasing



Reconstruction from samples on a rectangular grid

$$S_r(F_1,F_2)=\begin{cases}\Delta x_1\Delta x_2 S(F_1\Delta x_1,F_2\Delta x_2) & \text{for } |F_1|<\dfrac{1}{2\Delta x_1} \text{ and } |F_2|<\dfrac{1}{2\Delta x_2}\\ 0 & \text{otherwise}\end{cases}$$

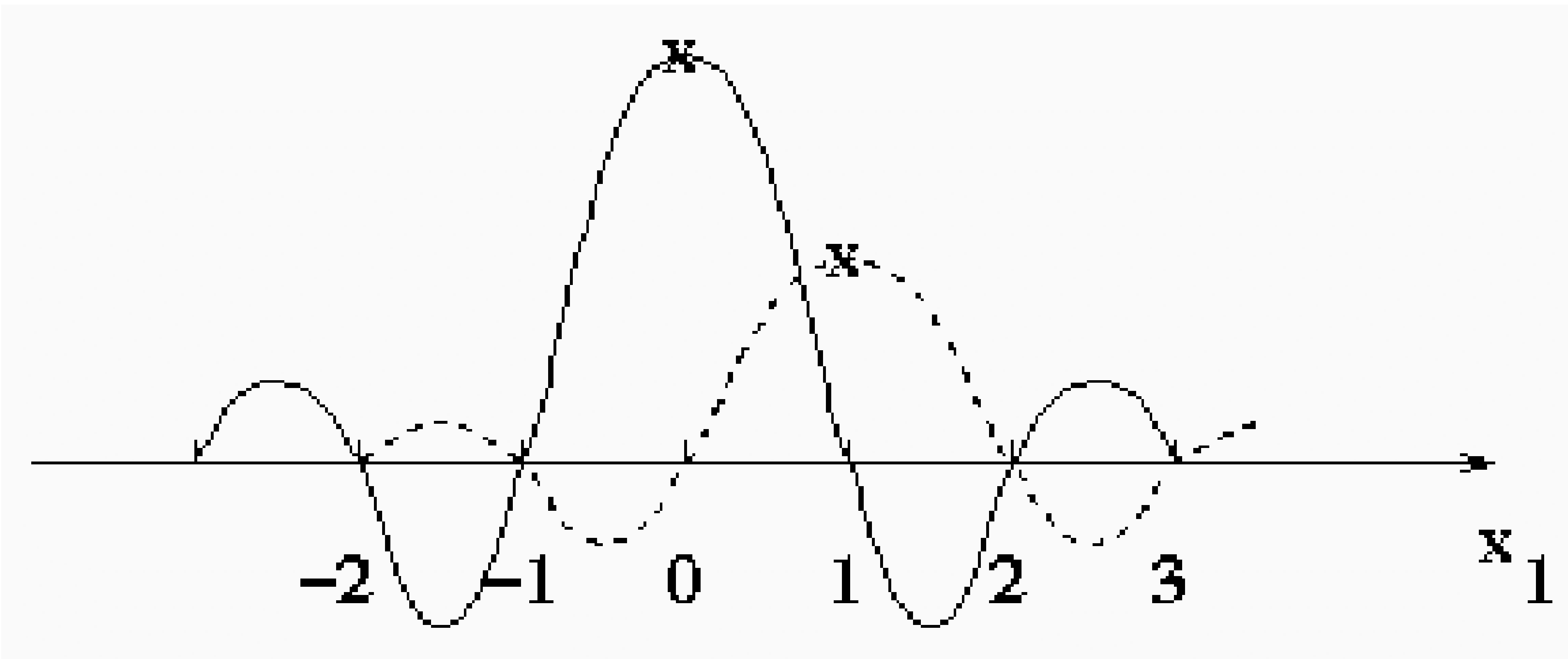


Space-domain filtering

$$s_r(x_1, x_2) = \Delta x_1 \Delta x_2 \sum_{n1} \sum_{n2} s(n_1, n_2) h(x_1 - n_1 \Delta x_1, x_2 - n_2 \Delta x_2)$$

$$h(x_1, x_2) = \frac{\sin(\frac{\Pi}{\Delta x_1} x_1) \sin(\frac{\Pi}{\Delta x_2} x_2)}{\frac{\Pi}{\Delta x_2} x_1 \frac{\Pi}{\Delta x_2} x_2}$$

Bandlimited interpolation



Extension to multidimensions

- Basics of lattice theory
- Sampling over lattices
- Sampling of video signals
- Sampling structure conversion

Basics of lattice theory-1

- A lattice, is the set of all possible vectors that can be represented as integer-weighted combinations of a set of K linearly independent basis vectors $\mathbf{v}_k \in \Re^K, k \in \{1,....K\}$

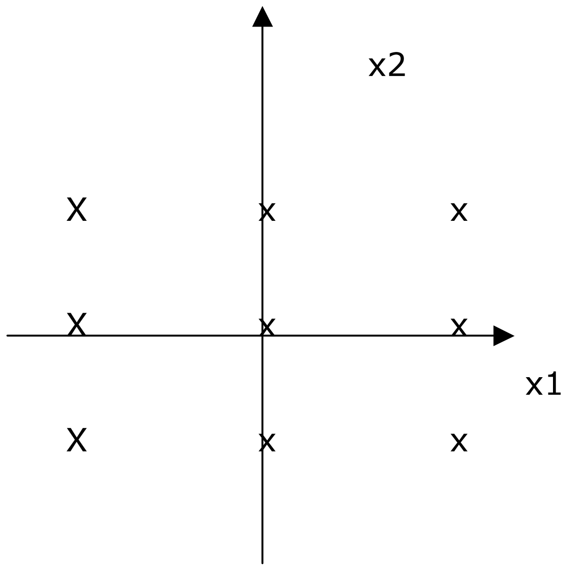
$$\Lambda = \left\{ \mathbf{x} \in \Re^K \middle| \mathbf{x} = \sum_{k=1}^{K} n_k \mathbf{v}_k, \forall n_k \in Z \right\}$$

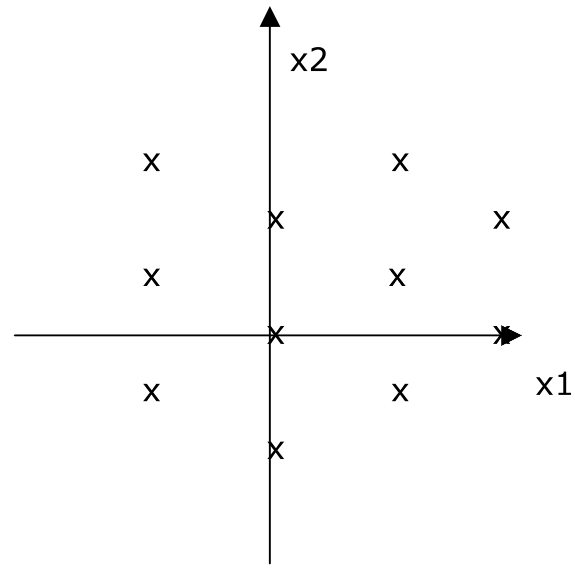
Basics of lattice theory-2

Example:

$$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} \sqrt{3}/2 & 0 \\ 1/2 & 1 \end{bmatrix}$$





Basics of lattice theory-3

- *Unit cell:* Given a lattice, one can find a unit cell such that its translations to all lattic epoints form a partition (nonoverlapping covering) of the entire space
- *Voronoi cell of a lattice:* is the set of all points that are closer to the origin than any other points in the lattice.

Basics of lattice theory-4

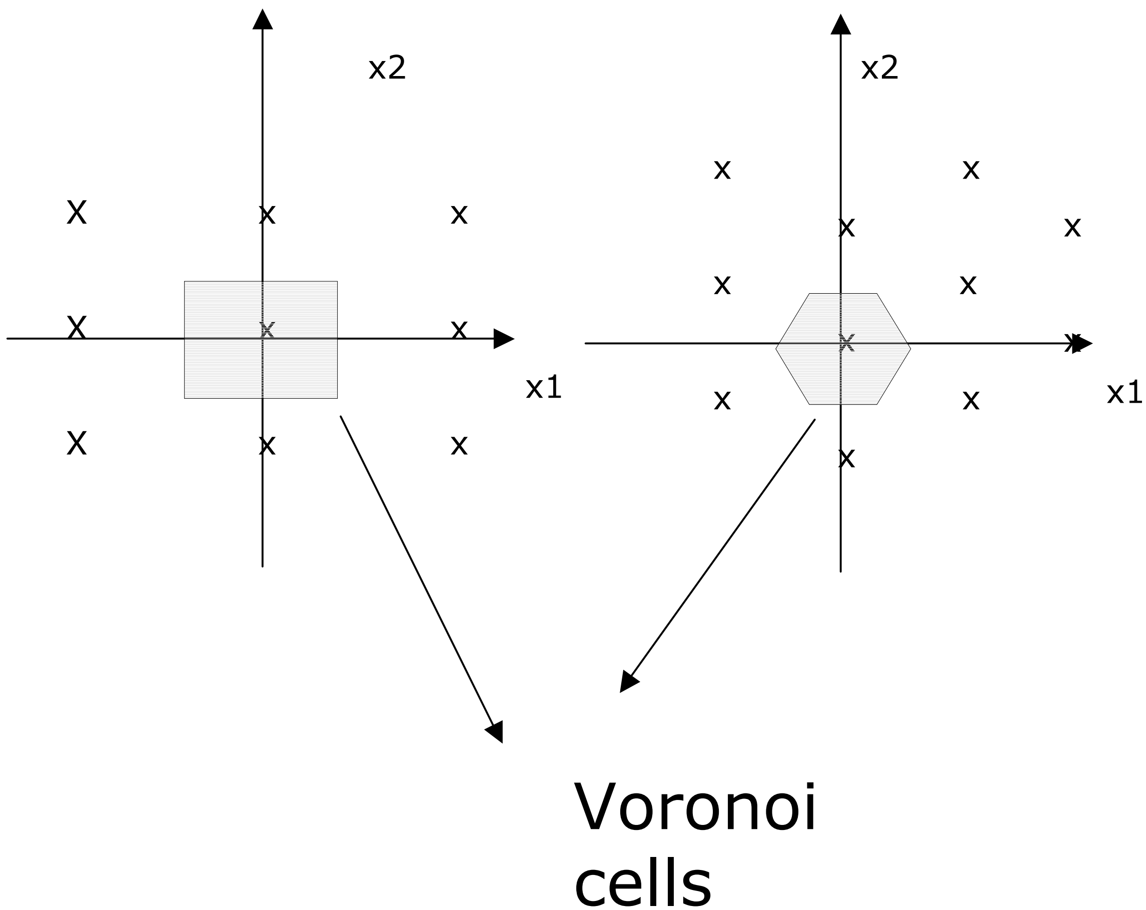
- *Volume of the Unit cell:* $|\det \mathbf{V}|$
- *Sampling density:* $d(\Lambda) = 1/|\det \mathbf{V}|$
- *Reciprocal lattice:* $\mathbf{U} = \mathbf{V}^{T-1}$

Basics of lattice theory-5

Example:

$$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} \sqrt{3}/2 & 0 \\ 1/2 & 1 \end{bmatrix}$$

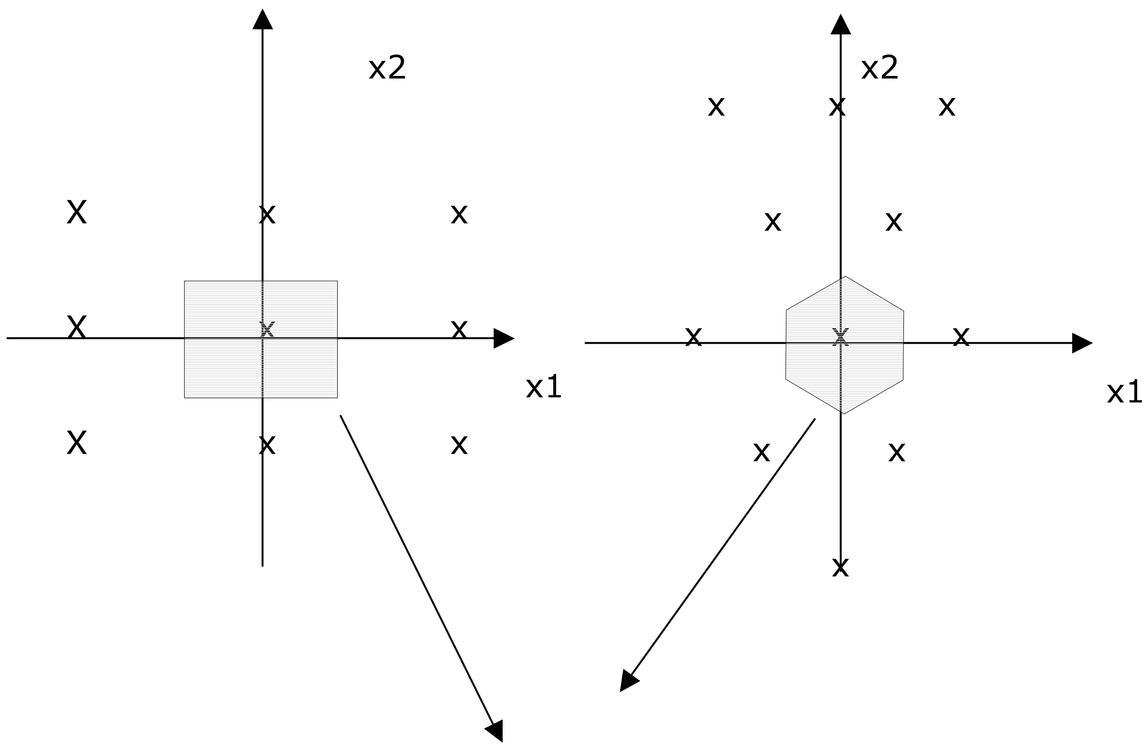


Basics of lattice theory-6

Example:

$$U_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$U_2 = \begin{bmatrix} 2/\sqrt{3} & -1/\sqrt{3} \\ 0 & 1 \end{bmatrix}$$



Voronoi cells of the reciprocal lattice

Sampling over lattices

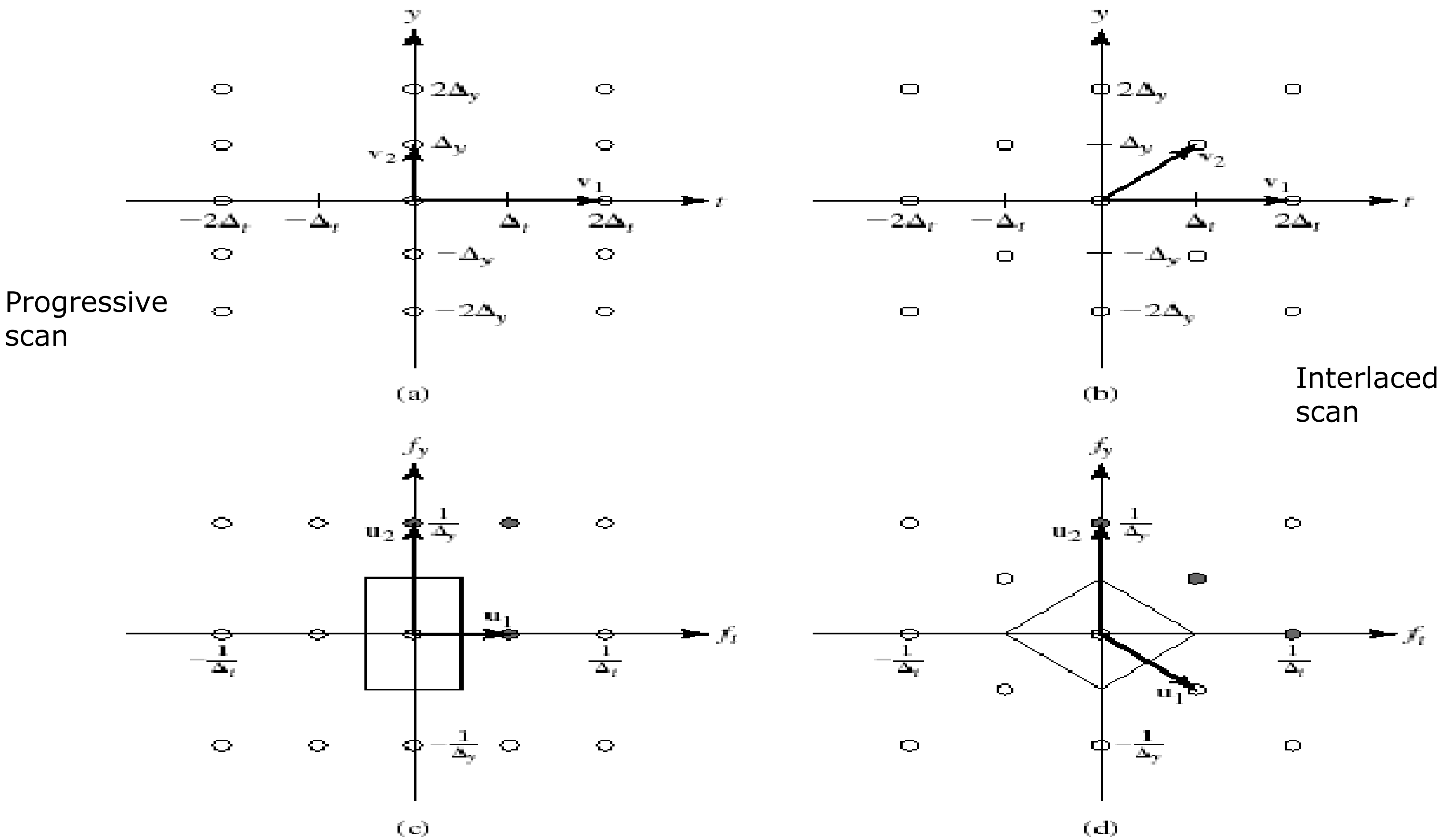
- Given a continous signal $f_c(\mathbf{x}$, a sampled signal over a lattice with a generating matrix $\mathbf{V}$ is

$$f_s(\mathbf{n}) = f_c(\mathbf{V}\mathbf{n}),\ \mathbf{n} \in Z^K$$

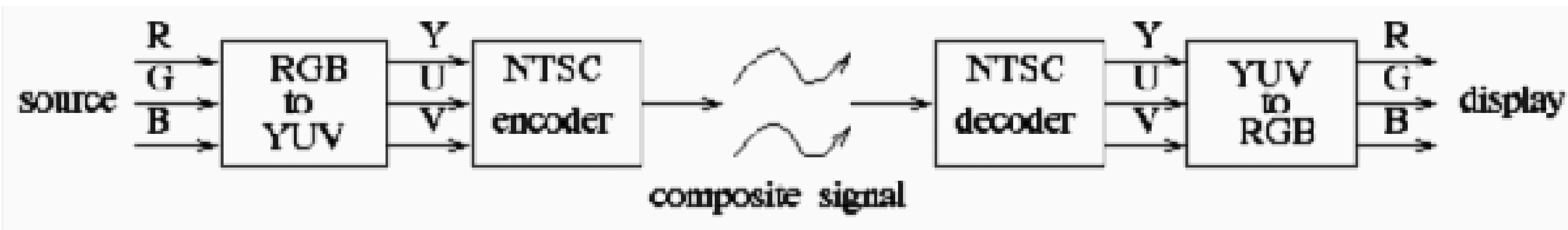
Generalized Nyquist sampling theorem

- It is possible to reconstruct the original continuous signal from the sampled signal perfectly if and only if the nonzero region of the continuous space FT of the original signal is limited within the Voronoi cell of the reciprocal lattice (Fc(w) = 0 for w$\notin \mathcal{V}(\Lambda^*)$)
- Furthermore perfect reconstruction can be achieved by H(w) = 1/d(Λ) for w$\in \mathcal{V}(\Lambda^*)$)

Sampling of video signals



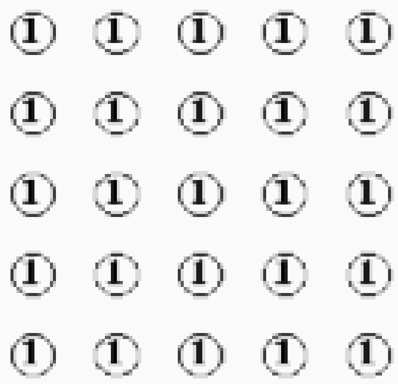
Sampling in analog and digital video



•Analog video is sampled in two dimensions (usually x_2 and t) by means of the scanning process, and

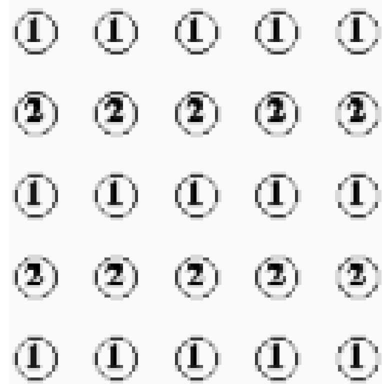
•Digital video is sampled in all three dimensions (x_1,x_2,t)

3-D sampling structures



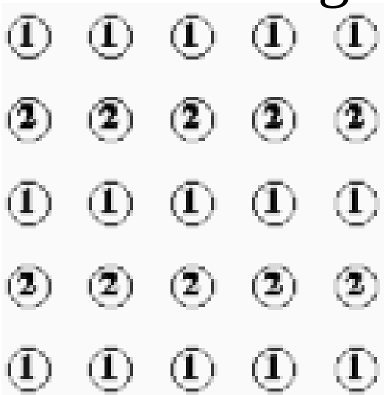
$$\mathbf{V} = \begin{bmatrix} \Delta x_1 & 0 & 0 \\ 0 & \Delta x_2 & 0 \\ 0 & 0 & \Delta t \end{bmatrix}$$

Progressive sampling



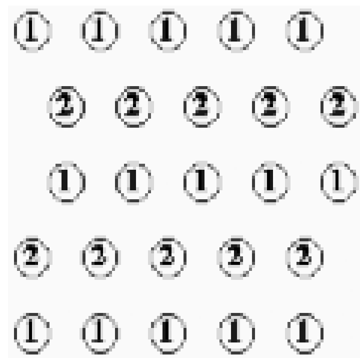
$$\mathbf{V} = \begin{bmatrix} \Delta x_1 & 0 & 0 \\ 0 & 2\Delta x_2 & \Delta x_2 \\ 0 & 0 & \Delta t/2 \end{bmatrix}$$

Interlaced sampling



$$\mathbf{V} = \begin{bmatrix} \Delta x_1 & 0 & 0 \\ 0 & 2\Delta x_2 & \Delta x_2 \\ 0 & 0 & \Delta t/2 \end{bmatrix}$$

Field-Quincunx sampling



$$\mathbf{V} = \begin{bmatrix} \Delta x_1 & \Delta x_1/2 & 0 \\ 0 & 2\Delta x_2 & 0 \\ 0 & 0 & \Delta t \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 0 \\ \Delta x_2 \\ \Delta t/2 \end{bmatrix}$$

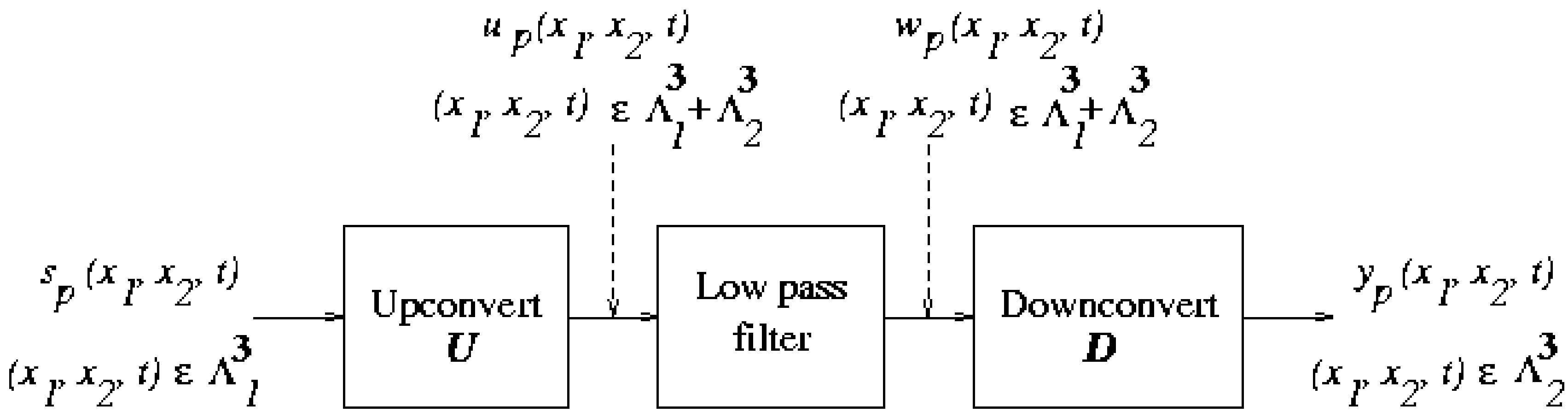
Line-Quincunx sampling

Analog to Digital conversion

- The minimum sampling frequency is 4.2x2
- Sampling frequency should be an integral multiple of line rate
- To sample the composite signal, the sampling frequency should be an integral multiple of the subcarrier frequency. (This simplifies decoding (composite to RGB) of the sampled signal.)
- To sample *component signals*, there should be a single rate for 525/30 and 625/50 systems; i.e., the sampling rate should be an integral multiple of both
- 29.97 x 525 = 15,734 and 25 x 625 =15,625.

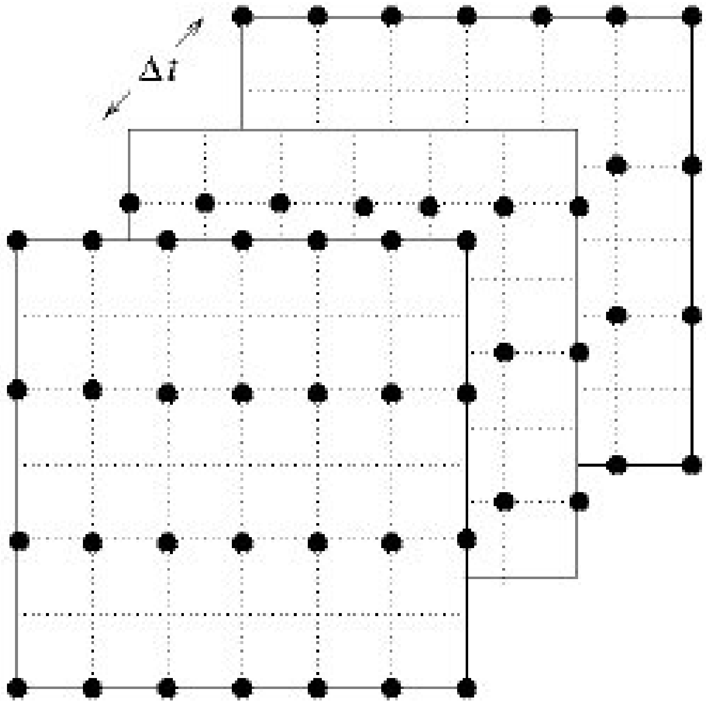
Sampling structure conversion-applications

- Frame rate conversion
- Deinterlacing
- Interlacing
- NTSC-to-PAL conversion
- SIF/CIF conversion
- Data compression



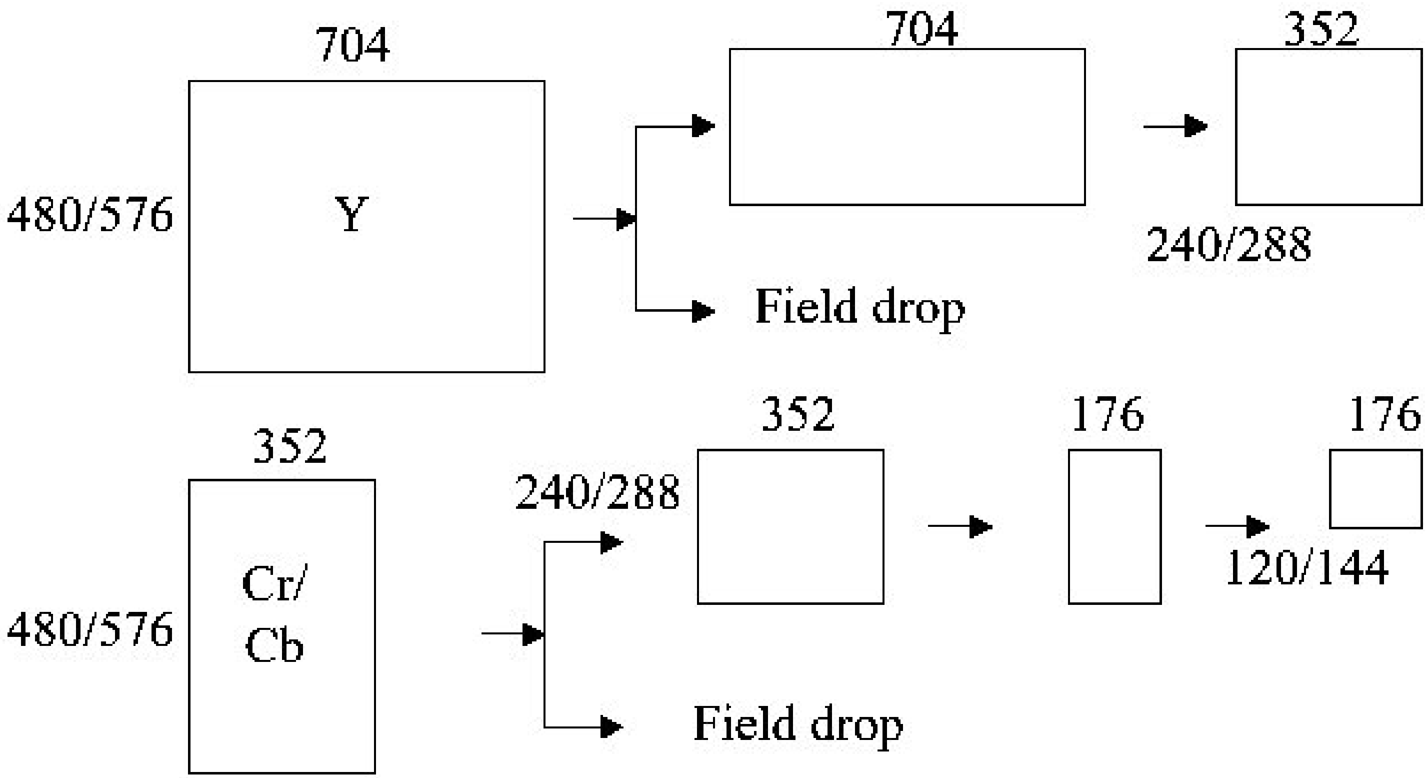
Decomposition of the system for sampling structure conversion.

Deinterlacing



(a) (b)

ITU-R 601 4:2:2 to SIF Conversion



Motion picture -> NTSC