Adaptive Whitening Filters for Small Target Detection*

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ABSTRACT

This paper studies the performance of the two dimensional least mean square adaptive filter as a prewhitening filter for detection systems. In two dimensional infrared sensor data, the clutter is correlated and much wider in spatial extent than the signal of interest. The two dimensional adaptive filter can be trained to adapt and predict the clutter, thereby enabling the error channel output to contain the signal of interest in white noise. Performance of the adaptive prewhitener, in terms of local signal to clutter ratio's(LSCR) and the gain obtained is described. The gain in LSCR due to this augmenting filter, is shown to depend on the statistics of the background clutter, in particular on the local mean. It is shown that, as the amount of color in the background clutter increases, the performance of the conventional matched filter performance degrades much more than the performance of a detector based on the augmenting prewhitener.

1. INTRODUCTION

In this paper we describe the performance of adaptive whitening filters when used for small target detection in two dimensional image data. The problem of small object detection from cluttered image scenes is of interest in a number of applications ranging from that of infrared target detection to medical imaging. In these applications the object of interest is of a very small spatial extent and is masked by clutter of a much larger spatial extent. In this paper we look at some adaptive techniques used to cancel such correlated clutter from two dimensional infrared image data.

Most detection systems for such small targets model the underlying clutter. A number of image representation models ranging from the white noise driven representations to the minimum variance prediction representations have been proposed.[1,2] Prediction based methods which use general image models and estimate the linear prediction coefficients at each pixel have been applied to object detection.[3] For infrared clutter, other models and detection schemes have been developed. Takken et. al.[4–6] developed a spatial filter based on least mean square optimization. The filter was designed to maximize the signal

*This work was supported by the NSF I/UC Research Center on Ultra High Speed Integrated Circuits and Systems (ICAS) at the University of California, San Diego, The Office of Naval Research and the National Science Foundation under grant #ECD89-16669 and the U.S. Naval Command, Control and Surveillance Center Independent Research Program.

to clutter ratio for a given fixed clutter environment. Chen et. al.[7] modeled the underlying clutter and noise from infrared sensors as a whitened Gaussian random process and developed a constant false alarm rate detector using the generalized maximum likelihood ratio.

Among the adaptive approaches, Wang[8] and Adridges et. al.[9] use an adaptive technique to handle any deviation of the input noise statistics. The adaptation is done by estimating the covariance matrix of the noise from the input data and then minimizing a cost function. The calculation of the noise covariance matrix parameters at each update makes this approach computationally expensive. Other infrared systems[10,11] apply multi-dimensional matched filtering techniques to multispectral data and need to either estimate the noise statistics or use more than one band of data for improvement in signal to clutter ratio.

However almost all these approaches are based on an underlying model and the performance of detection systems degrades when the input data does not conform to the model assumed. One dimensional adaptive filters have been applied to narrow band prediction and detection problems with success.[12,13] For broad band signals embedded in narrow band interference, the residuals of such adaptive filters have been used for detection in non-stationary noise without directly using a model of the underlying noise process.[12,15] These one dimensional adaptive filtering algorithms have recently been extended to two dimensions.[16] A detection system based on these two dimensional adaptive filters for target detection has been proposed.[17] This system used the two dimensional least mean square(TDLMS) algorithm to predict and cancel the correlated clutter in image data.

In this paper we describe the performance of such a whitening filter based detection system. We show that in highly structured clutter the filter is able to predict and cancel the clutter and improve the performance of the detection system. It is shown that the gain is dependent on the local statistics of the data. We also describe a recursive adaptive estimation scheme that merges the whitening and detection stages. An exponential windowing of data used for the estimation of the noise autocorrelation allows the algorithm to deal with non-stationary input noise. A relation between the windowing parameter and the detection probability under constant false alarm rate conditions is found for Gaussian data.

2. LMS ADAPTIVE WHITENING

2.1. Filter Description

The matched filter detection statistic is created as[18]

$$g = \mathbf{s}^T \mathbf{\Phi}^{-1} \mathbf{x} \tag{1}$$

where $\mathbf{s}$ is the known signal template, $\mathbf{\Phi}$ is the autocorrelation matrix of the noise and $\mathbf{x}$ is the input data. This can be split into two stages : a whitening stage and a matching stage.

In the absence of any knowledge of the clutter characteristics and because the clutter may be non-stationary, fixed whitening filters that estimate the autocorrelation by using maximum likelihood methods show a degradation in performance. However since the clutter is known to have a spatially longer autocorrelation length than the signal of interest, a two dimensional adaptive filter can be used as a whitening filter.[17]

The two dimensional filter based on the adaptive least mean square algorithm uses a window of pixels as input to a linear predictor whose weights are updated at every pixel.[16,17] The predicted pixel value at any given iteration is a weighted average of a small window of pixels around it and is given by

$$Y(m,n) = \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} W_j(l,k) X(m-l, n-k) \qquad m,n = 0, ..., M-1 \tag{2}$$

where j is the iteration number. The image is scanned from top to bottom and left to right (lexicographically) and hence j is found as $j = mM + n$. Here X is the input image, Y the predicted image and W_j the weight matrix at the j^{th} iteration. The images are assumed to be square of size $M \times M$ pixels and the weight matrix (hence the window size of the pixels being used to predict) is taken to be square of size $N \times N$. This predicted value is compared with a reference and the residual is the error between the two given by

$$E(m,n) = \epsilon_j = D(m,n) - Y(m,n) \tag{3}$$

where D is the reference image. In the line enhancer configuration this reference is a shifted version of the input image. The weights are updated by using instantaneous estimates of the autocorrelation of the noise and the cross correlation between the reference and the input and then using a steepest descent direction.[16] The update equations then become

$$W_{j+1}(l,k) = W_j + \mu \epsilon_j X(m-l, n-k) \qquad l,k = 0, ..., N-1 \tag{4}$$

where μ is the adaptation parameter. Under stationary conditions, it can be shown that the weights of this filter converge to the solution of the two dimensional Wiener-Hopf equations, provided μ is chosen to be within the bounds required for stability.[19]

Knowing the difference in the correlation lengths of the signal of interest and the clutter in the image it is possible to find bounds on μ such that this TDLMS based adaptive filter predicts the clutter but not the signal of interest.[20] Thus a separation of the signal and the clutter is obtained by the use of such an adaptive filter. The error channel output of this filter (which now ideally contains only the signal of interest and white noise) can be used for a matched filtering operation and thus the detection statistic can be generated.

2.2. Filtering Performance

The performance of such a adaptive clutter whitener(ACW) based detection system was studied. Both the filtering and detection performance for structured background clutter and real infrared data were studied. The filtering performance was characterized by the *local signal to noise ratio(LSCR)* defined in a window of interest. Thus, if the window of interest were a rectangular region defined from (L_x, L_y) to (H_x, H_y), then the LSCR (in dB) is defined as

$$LSCR = 10 \log_{10} \frac{\sum_{i=L_x}^{H_x} \sum_{j=L_y}^{H_y} (s(i,j) - m_w)^2}{\sigma_w^2} \tag{5}$$

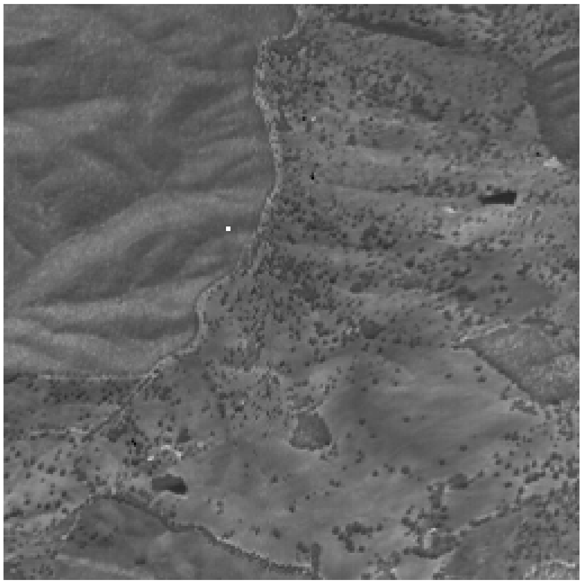
where, m_w is the mean of the noise in the window and σ_w is the variance. And the gain (in dB) provided by such a filter is then given by

$$Gain = LSCR_{out} - LSCR_{in}. \tag{6}$$

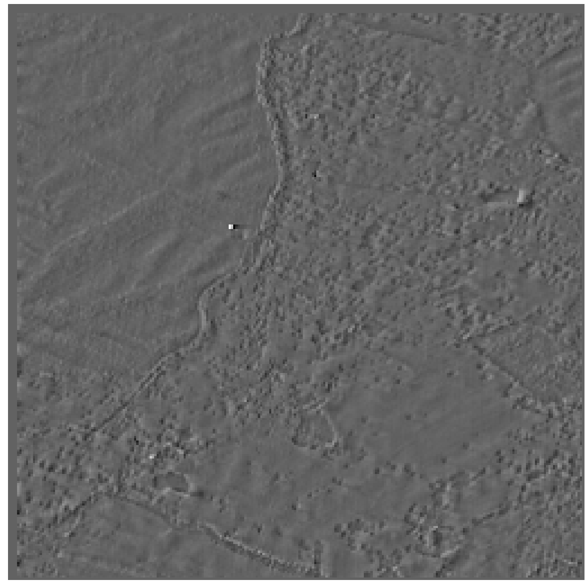
Fig.1a shows the input image used for some of the simulations. This image is part of a 6 channel multispectral data set collected by a NASA Thermal Infrared Multispectral Scanner (TIMS) sensor and is of a rural background over the hills of Adelaide, Australia. An artificial target 2x2 pixels in extent was inserted into this background.

For a very high LSCR the output of the ACW is shown in Fig.1b It is seen that most of the correlated background clutter is predicted and hence the residual contains very little correlated background clutter. The target is not predicted and is present in the error output of the ACW.

The performance of this ACW depends on μ. Fig.2 shows the LSCR gain obtained as a function of the adaptation parameter. It is seen that as μ is increased beyond an optimum and approaches the stability bounds, the performance of the ACW degrades very fast. This is a result of two factors. First, increasing



(a) Input Image



(b) Output from 3x3 TDLMS Filter

Figure 1: **Adelaide Infrared Scene with target inserted at (100,100), Input LSCR = 25.8 dB; Output LSCR = 26.4 dB**

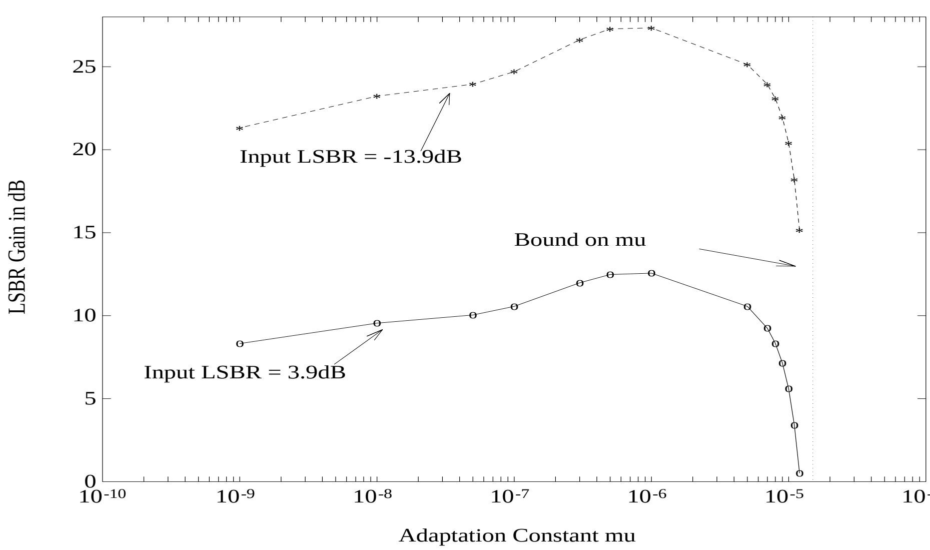


Figure 2: **Gain vs.** μ

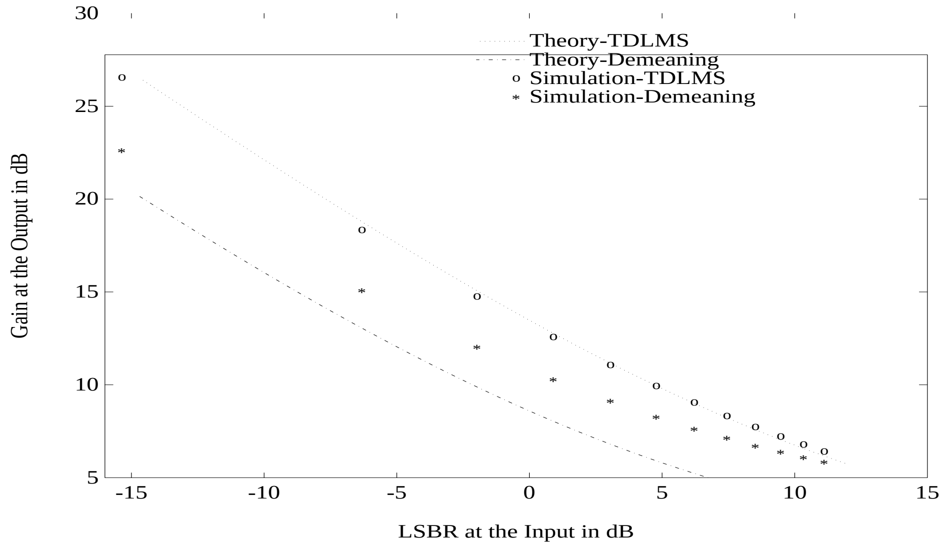


Figure 3: **Gain vs. Input LSCR for target position 100,100**

the μ allows the filter to predict the signal of interest and thus some of the target energy is cancelled in the output. Second, as μ approaches the stability bound of the filter the weight misadjustment noise increases.

It is also seen that as μ is decreased below the optimum, the performance of the ACW degrades, though not as drastically as that in the upper bound case. This degradation is because, with the slower adaptation parameter, the filter is unable to accurately predict the clutter in the environment.

2.3. Comparison with Local Demeaning

It has been shown that the removal of the local mean from an image causes the image statistics to become Gaussian and also removes any spatial correlation, thus "whitening" the image.[7,21] Consequently, the local demeaning filter is an alternative technique for whitening the image. In this section, the comparative performance of the adaptive clutter whitening filter and the local demeaning filter will be obtained.

The amount of LSCR gain obtained by such a system can be calculated theoretically since the target inserted in the image is known completely. If m_l is the mean of the background clutter using a window of the same size as that used for the local demeaning and m_a is the value of background clutter at the pixel of interest. Then the output of the local demeaning filter at that pixel will be $s_n - m_l$, and that of the TDLMS will be $s_n - m_a$, where s_l is the intensity of the pixel with the signal present. The theoretical plots are based on the assumption that the TDLMS has converged fully and is able to predict the background clutter without any leakage of the clutter into the error channel.

Fig.3 shows the gain in the output as a function of the LSCR in the input for a 2×2 target located at (100,100). At this position, $m_l < m_a$. and TDLMS shows more gain than the local demeaning algorithm.

However Fig.4 shows the gain in the output as a function of the input LSCR for a target position of (105,105) where $m_l > m_a$, and the local demeaning filter is seen to perform better than the TDLMS. This is primarily because the dominant component of the clutter is a d.c. (in the spatial sense) component which can then be removed by a local demeaning operation across the image.

However for images that do not have a strong d.c. component but have a highly correlated clutter

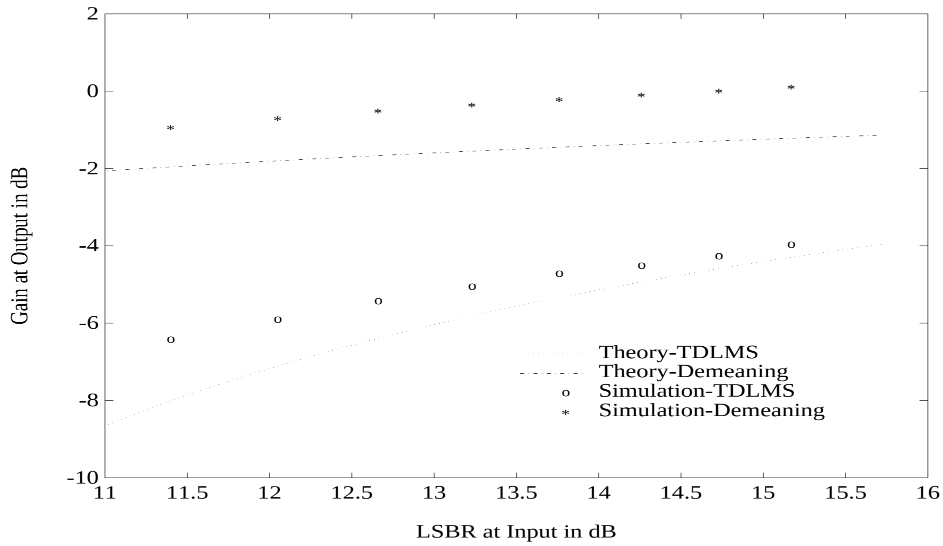
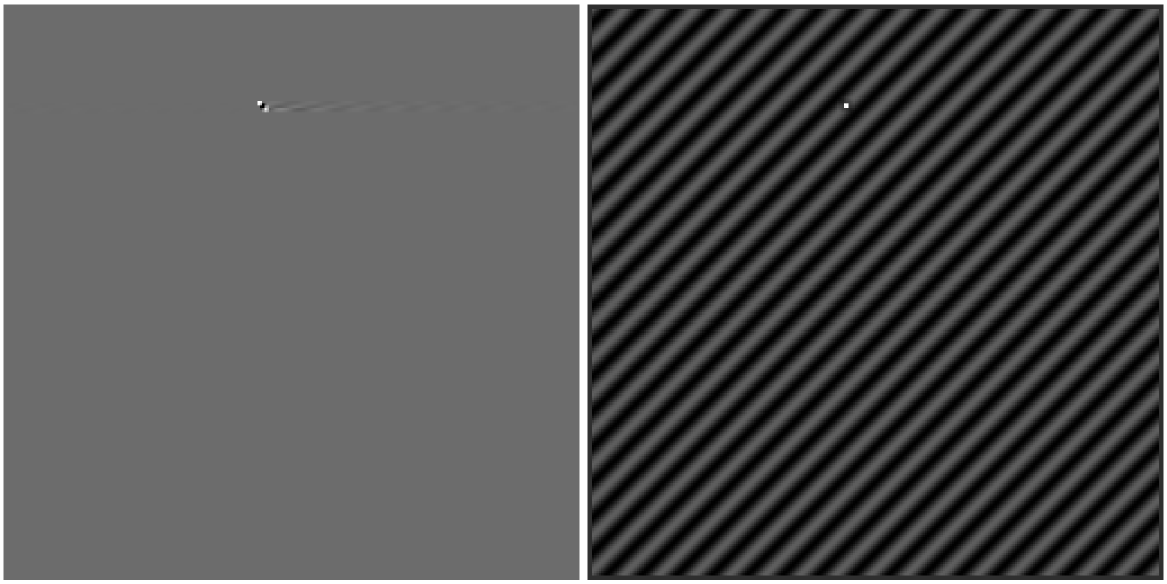


Figure 4: **Gain vs. Input LSCR for target position 105,105**

Figure 5: **The Input Image with the target at (115,45), Input LSCR=15.3 dB and Color =1**



(a) Output from TDLMS (b) Output from Local Demeaning

Figure 6: **The Performance of the TDLMS and the Local Demeaning filter for structured sinusoidal clutter**

structure, the local demeaning filter fails to remove the background clutter, while the TDLMS is able to predict the correlated clutter. This is seen in Figure 5 which shows the input being a two dimensional sinusoid with a 2×2 target. The TDLMS output shown in Fig.6a is seen to have only the target, while the output of the local demeaning is seen to contain sinusoidal clutter as well(Fig.6b).

Thus we see that the TDLMS based adaptive filter can whiten clutter without also removing the target, when the clutter is structured. However the performance depends on the local statistics of the image. The detection performance of such ACW augmented receivers in the presence of highly structured noise has been characterized and ROC curves plotted using Mont-Carlo methods.[17] It has been shown that as the amount of color in the background clutter increases, the detection performance of the augmented matched filter does not degrade as much as that of the conventional matched filter.

3. ADAPTIVE RECURSIVE DETECTION

3.1. Weighted Estimation

Another approach to the target detection problem consists of estimating the noise autocorrelation matrix and calculating the detection statistic directly from Eq.1. The sampled matrix inversion,[22] the modified sampled matrix inversion[23] and the generalized likelihood ratio[24,25] detectors belong in this class. The estimation is usually done as a maximum likelihood estimation using

$$\mathbf{\Phi} = \sum_{k=0}^{n} \mathbf{x}(k)\mathbf{x}(k)^T. \tag{7}$$

Since the detection is to be done recursively on a stream of data, a detection statistic has to be found at each iteration point, viz.

$$g(n) = \mathbf{s}^T \mathbf{\Phi}^{-1}(n)\mathbf{x}(n) \tag{8}$$

However this estimation procedure assumes the clutter to be stationary and the performance of such a system would degrade if the clutter were non-stationary. In infrared image data, the clutter is known to be non-stationary and a weighted estimation procedure can be used[26] for the autocorrelation as

$$\mathbf{\Phi}(n) = \sum_{k=0}^{n} \lambda^{n-i}\mathbf{x}(k)\mathbf{x}(k)^T \tag{9}$$

where λ is a weighting factor. $\lambda = 0$ corresponds to an instantaneous estimate of the noise and $\lambda = 1$ corresponds to the maximum likelihood estimate for a stationary noise process. The tracking behaviour of such an estimation process for $0 < \lambda < 1$, for one dimensional adaptive prediction has been studied.[27]

With an estimation scheme given by Eq.9, recursive update equations for $g(n)$ and $\mathbf{\Phi}^{-1}(n)$, can be found[26] as

$$\mathbf{\Phi}^{-1}(n) = \lambda^{-1}\mathbf{\Phi}^{-1}(n-1) - \left(\frac{\lambda^{-2}\mathbf{\Phi}^{-1}(n-1)\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{\Phi}^{-1}(n-1)}{1+\lambda^{-1}\mathbf{x}^T(n)\mathbf{\Phi}^{-1}(n-1)\mathbf{x}(n)}\right). \tag{10}$$

and

$$g(n) = \frac{\lambda^{-1}\mathbf{s}^T\mathbf{\Phi}^{-1}(n-1)\mathbf{x}(n)}{1+\lambda^{-1}\mathbf{x}^T(n)\mathbf{\Phi}^{-1}(n-1)\mathbf{x}(n)} \tag{11}$$

This estimation technique windows the data used for the estimation of the noise autocorrelation, thus permitting a matched filter based on such an estimation scheme to change with non-stationary input. However, it is no longer a maximum likelihood estimate and the probability of detection will now change with every iteration.

3.2. Detection Probabilities

If we assume the input data to be Gaussian under both hypotheses (H_1: signal present and H_0: signal absent), with zero mean noise, the mean under H_1 becomes $m = \mathbf{s}^T\mathbf{\Phi}^{-1}\mathbf{s}$. The probability of detection is then given by[18]

$$P_d = \frac{1}{2} erfc\left[\frac{g_0 - m}{\sigma\sqrt{2}}\right] \tag{12}$$

where, σ is the variance of the process, m is the mean under H_1, g_0 is the threshold used, and $erfc()$ is defined as

$$erfc(x) = \frac{1}{2\pi}\int_x^{\infty} e^{-x^2}dx \tag{13}$$

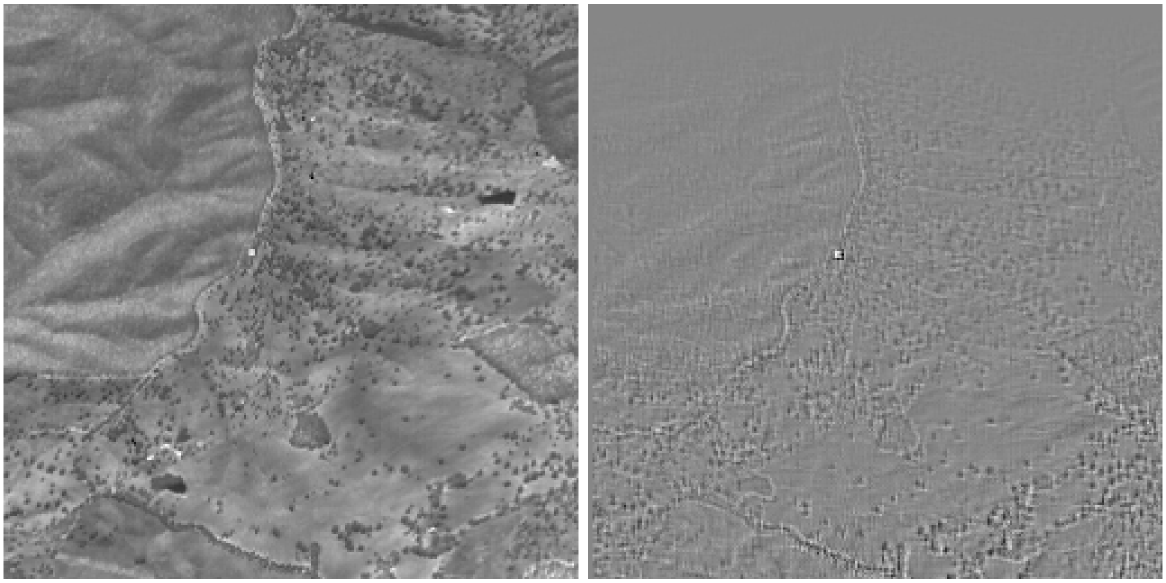
Similarly the probability of false alarm is defined as

$$P_{fa} = \frac{1}{2} erfc\left[\frac{g_0}{\sigma\sqrt{2}}\right] \tag{14}$$

For a non-stationary process, if we assume that at the $(n-1)^{th}$ iteration the autocorrelation was known exactly($\mathbf{\Phi}^*(n-1)$), an optimum value of $P_d(n-1)$ and $P_{fa}(n-1)$ can be found.[26] The mean and the variance for the $(n-1)^{th}$ iteration can then be found as

$$m^*(n-1) = \mathbf{s}^T\mathbf{\Phi}^*(n-1)^{-1}\mathbf{s} \tag{15}$$

$$\sigma^*(n-1) = \left(\mathbf{s}^T\mathbf{\Phi}^*(n-1)^{-1}\mathbf{s}\right)^{\frac{1}{2}} \tag{16}$$



(a) Target Embedded in Infrared Image Data (b) Detection Statistic

Figure 7: **Typical Input and Output for Recursive Estimation Based Detection with** $\lambda = 0.99$

Now if the detector is designed for a constant false alarm rate operation (CFAR), $P_{fa}(n) = P_{fa}(n-1)$. Further if the rate of non-stationarity is small we can show

$$P_d^*(n) \approx \frac{1}{2} erfc\left[C_d^* + \Delta_d\right] \tag{17}$$

where

$$C_d^* = C_f - \frac{m^*(n-1)}{\sqrt{2}\sigma^*(n-1)} - \frac{\delta\sigma^*(n-1)}{2m^*(n-1)} \tag{18}$$

$$\Delta_d = \frac{1}{\sqrt{2}} \frac{m^*(n-1)}{\sigma^*(n-1)} \left(1 - \frac{1}{\sqrt{\lambda}}\right) + \frac{1}{2} \frac{\sigma^*(n-1)}{m^*(n-1)} (\delta + \Delta) \tag{19}$$

$$\Delta = \frac{\lambda^{-2}\left[\mathbf{s}^T \mathbf{\Phi}^*(n-1)^{-1} \mathbf{\Phi}(n) \mathbf{\Phi}^*(n-1)^{-1} \mathbf{s} - \lambda m^*(n-1)\right]}{1 + \lambda^{-1} \mathbf{x}^T \mathbf{\Phi}^*(n-1)^{-1} \mathbf{x}} \tag{20}$$

and $\delta = \mathbf{s}^T \Delta\mathbf{\Phi}\mathbf{s}$. Here $\Delta\mathbf{\Phi}$ is the change in the *inverse* of the autocorrelation of the non-stationary noise input and is given by

$$\Delta\mathbf{\Phi} = \mathbf{\Phi}^{-1}(n) - \mathbf{\Phi}^{-1}(n-1) \tag{21}$$

Equations 17 to 21 thus define the relationship between the probability of detection (P_d), the rate of non stationarity of the noise ($\Delta\mathbf{\Phi}$) and the weighting factor of the weighted update (λ). A proper choice of λ depends on the autocorrelation matrix of the noise, and can influence the probability of detection.

A two dimensional version of the recursive detection procedure can be formulated by ordering the data matrix into a vector lexicographically. Such a matched filter was used in a noise canceller structure for multi-spectral images. A typical input image (with an artificially inserted target) and the resulting detection statistic are shown in Figures 7a and 7b. It is seen that due to the adaptive estimation of the autocorrelation matrix, the matched filter is able to follow the changes in the statistics of the background clutter.

4. CONCLUSIONS

We have shown that the use of adaptive whitening filters improves the performance of detection systems without direct modeling of the input data characteristics. The performance of the LMS based adaptive filter depends on the local statistics of the image. In cases where the background clutter is non-stationary only because of a changing local mean, these filters may not do as well as the local demeaning algorithm. However in cases where the clutter is highly correlated but does not have a spatial d.c. component, these filters are still able to predict and cancel the clutter which the local demeaning filter is unable to do. Further, in case of detection systems which estimate the noise statistics, the adaptive recursive estimation procedure described in section 3 permits a relationship between the windowing parameter and the probabilities of detection.

The computational complexity of the TDLMS based adaptive filter is $O(w^2)$ where w is the size of the window being used. This makes such systems very attractive for real time detection systems. The recursive estimation scheme described in section 3 needs more computations than the TDLMS filter. With the recursive form being used its computational complexity is on the order of $O(w^4)$ since the size of the autocorrelation matrix itself is $w^2 \times w^2$. However since this technique uses a very structured autocorrelation matrix, QR based algorithms can be applied to speed it up. Further due to the structure in the problem it is possible to formulate a parallel implementation of this algorithm which will lead to very fast throughput.

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