Marginal Entrants and Trade-Liberalization Effects Across Models of Imperfect Competition

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Abstract

When should we expect a trade shock to create pro-competitive effects? In this paper, we investigate this in setups with firm heterogeneity and a linear demand with horizontal product differentiation. Our main finding is that the characterization of marginal entrants completely determines whether pro-competitive effects arise across standard settings of monopolistic competition (i.e., à la Krugman, Melitz, and Chaney/short-run Melitz) and Cournot (with free and restricted entry). This result holds independently of the assumptions on the rest of the firms, and is particularly stark in Cournot, where marginal entrants comprise merely one firm (the last entrant). We also provide conditions on marginal entrants across market structures that lead to pro-competitive, anti-competitive, or null effects following a unilateral trade liberalization.

JEL codes: F10, F12, D43, L13.

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1 Introduction

Trade shocks do not always lead to pro-competitive effects in canonical trade models. Moreover, it is difficult to identify the conditions leading to this, since these models differ across multiple dimensions (e.g., ex-ante vs. given heterogeneity, free vs. restricted entry, negligible vs. non-negligible firms).

In this paper, we put forth the role of marginal entrants in the identification of trade-liberalization effects across models of imperfect competition. Our focus is on pro-competitive effects, defined as reductions of prices and markups by domestic firms, and increases in the domestic survival productivity cutoff. In contrast, if a trade shock does not entail pro-competitive effects, it is understood that it only reduces the mass of domestic firms, leaving the domestic competitive conditions unaltered and, hence, not affecting the behavior of active domestic firms.

By applying the analysis to standard settings of monopolistic competition (à la Krugman, Melitz, and Chaney/short-run Melitz) and oligopoly (Cournot with free and restricted entry), our main findings are twofold. First, marginal entrants' features completely determine whether pro-competitive effects are created following a trade shock, independently of the assumptions on the rest of the firms. Second, once marginal entrants are characterized equivalently across models, they generate the same qualitative outcomes after a trade shock.

Throughout this paper, the analysis is based on a linear demand. This demand system is appropriate since it is ubiquitous under both oligopolistic and monopolistic competition. Moreover, for both market structures, studies utilizing it have led to different outcomes across settings.

The first market structure we consider is monopolistic competition, which is formalized in Section 2. Our framework is a generalization of Melitz and Ottaviano (2008) that allows for a broader set of distributional assumptions. Thus, as in that study, there is free entry and firms do not know their productivity before entering the industry. On the contrary, unlike that study, we suppose that each firm gets a draw from some *firm-specific* productivity distribution after paying an entry cost.

Specifying the distribution at this level serves two purposes. First, particular distributional assumptions make it possible to encompass canonical trade models as special cases. Specifically, a setting as in Melitz arises where all firms obtain a productivity draw from the same distribution, with Krugman's constituting a limiting case with negligible heterogeneity. Moreover, once that firm-specific productivity distributions are allowed for, a setting à la Chaney/short-run Melitz can also be encompassed: this arises in our framework when firms obtain productivity draws from degenerate distributions. Thus, rather than conceiving Chaney/short-run Melitz as a setting with a pool of heterogeneous incumbents knowing their productivity, in our setup this corresponds to a scenario where, by paying an entry cost, each firm obtains a specific productivity draw with probability one. While this might seem like an unusual way to interpret this setting, it translates ex-post concepts (i.e., those holding in the market) into equivalent ex-ante ones. This plays a key role in undertaking a unified analysis, by making it possible to establish mappings between conditions and outcomes that are directly comparable across setups and models.

Second, firm-specific productivity distributions enables us to partition firms and evaluate the role of each group in producing specific outcomes, where groups are defined by the range of productivities that firms potentially can get. By this, it is determined that a specific set of firms, which we denominate **marginal industry entrants (MIEs)**, play a crucial role. They correspond to the set of firms through which extensive-margin adjustments at the *industry* take place. Formally, it is the set of the firms with the least expected profits among those that pay the entry cost to have a variety and draw of productivity assigned.

Why do MIEs play such an essential role regarding market outcomes? Their profits characterize the zero-expected-profits condition. Depending on the MIEs' features, they might completely pin down the domestic choke price, which acts as a single sufficient statistic for the domestic firms' decisions. When this is the case, if any trade shock under consideration does not affect the expected profits of MIEs, then domestic prices, quantities, markups, and the survival productivity cutoff at home are not affected. Thus, the model generates no pro-competitive effects and, instead, adjusts exclusively through variations in the mass of domestic firms that pay the entry cost.

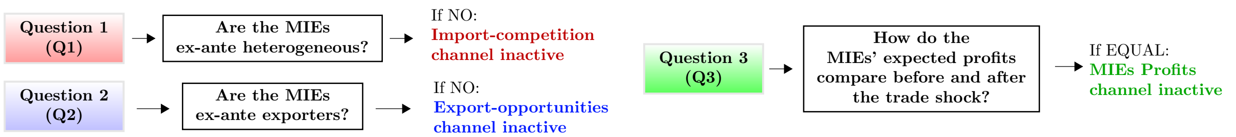
In Section 3, we identify channels that create pro-competitive effects in monopolistic competition, along with the conditions for their activation. With this purpose, we define an *import-competition channel* and an *export-opportunities channel*. Intuitively, the import-competition channel acts through the exposure of domestic firms to lower prices and entry by foreign firms. On the other hand, the export-opportunities channel works through increases in expected profits due to better business opportunities, which induces the creation of firms that serve their domestic market, thereby generating greater competition domestically.

To isolate these channels, we consider a small economy in the sense of Demidova and Rodríguez-Clare (2009; 2013), with wages exogenously given as in Melitz (2018). This establishes that any shock in the small country is negligible for the domestic conditions of foreign countries and, in particular, their choke prices. By keeping trading partners' choke prices fixed, a trade shock in the small country does not simultaneously affect its export conditions. Thus, we are able to directly identify the import-competition and export-opportunities channels through reductions in inward and outward trade barriers, respectively.

Our analysis shows that MIEs' features determine which of these channels are active. In particular, two of the MIEs' characteristics are crucial. First, whether MIEs are ex-ante exporters (i.e., if MIEs expect to become exporters for some productivity draws that occur with positive probability). Second, whether MIEs are ex-ante homogeneous (i.e., if they obtain productivity draws from the same distribution).

The activation of channels according to the MIEs' features is summarized by questions Q1 and Q2 in Figure 1. The third condition, Q3, is always satisfied in monopolistic competition, since smoothness assumptions ensure that the MIEs' expected profits are the same in each equilibrium (i.e., zero). Q1 indicates that *it is necessary that domestic MIEs are ex-ante heterogeneous for the import-competition channel to be active.* On the contrary, if MIEs are ex-ante homogeneous, there is only one choke price consistent with zero expected profits, and this is independent of import trade costs. Additionally, Q2 states that *the export-opportunities channel is activated when domestic MIEs are ex-ante exporters*, so that, before paying the entry cost, they expect to serve both the domestic and the foreign market for a range of productivity draws. Intuitively, this follows because, otherwise, the MIEs' expected profits would not be affected by the trade costs for exporting.

Figure 1. *Conditions for Activation of the Channels*



Note: MIEs are the set of domestic firms with lowest expected profits among those that pay the entry cost. In the case of Cournot, they comprise only one firm (the last entrant) and ex-ante and ex-post features coincide.

In Section 4, we present two applications of the results obtained under monopolistic competition. First, we apply them to analyze the channels operating in setups à la Krugman, Melitz, and Chaney/short-run Melitz. In the Melitz and Krugman settings, all firms (and, hence, the MIEs) are ex-ante alike, implying that the import-competition channel is shut. Instead, since for some productivity draws firms would eventually export, all firms are ex-ante exporters and the export-opportunities channel is active. As for the Chaney/short-run Melitz setting, given that firms obtain productivity draws from a degenerate distribution, ex-ante and ex-post concepts coincide. Moreover, since in this framework the productivity distribution is atomless ex-post (i.e., the measure of firms at the market with the same productivity is zero), all firms, including the MIEs, are heterogeneous and, so, pro-competitive effects are created through the import-competition channel. However, under the standard assumption of selection into exporting, the MIEs exclusively serve the domestic market and, so, the export-opportunities channel is inactive.

The second application we consider refers to unilateral trade liberalizations between two large countries, i.e., a reduction in outward trade barriers in some country H. Unlike the case of a small country, feedback effects are created since changes in the domestic conditions of H have an impact on the choke price of the foreign country. Thus, when there is a decrease in the outward trade costs of H, each country is affected by the channel operating in a small economy *and* changes in export conditions. Based on the conditions Q1 and Q2 in Figure 1, different combinations of MIEs' features activate different channels and we establish the conditions that a unilateral liberalization generates pro-competitive, anti-competitive, and null effects in H. The results we find rationalize that, for instance, the Melitz and Krugman settings create anti-competitive effects while the Chaney/short-run Melitz generate pro-competitive effects, as shown by Melitz and Ottaviano (2008) for the Pareto case.

In Section 5, we analyze the Cournot model. We consider a setup with free entry and where firms know the productivity they would get assigned by entering the industry according to a productivity order. Drawing a parallel with monopolistic competition, this setup can be conceived as firms obtaining some specific productivity draw with probability one after paying the entry cost. Thus, ex-ante and ex-post features of firms coincide.

We show that, just as with monopolistic competition, the features of the MIEs identify which channels are operative. In particular, the MIEs collapse to the least-productive firm that is active in the domestic market, which we refer to as the last entrant. Thus, *through the characterization of merely one firm it is possible to anticipate whether pro-competitive effects are created after a trade shock.*

Under this setup, the last entrant can potentially earn positive profits. Therefore, the impact of trade shocks on the domestic market also requires a comparison of the profits garnered by the last entrant of each equilibrium. We refer to the effects caused by this as the MIEs profits channel and show that, as long as the profits of the last entrants before and after the trade shock are equal, this channel is inactive. This is indicated by Q3 in Figure 1.

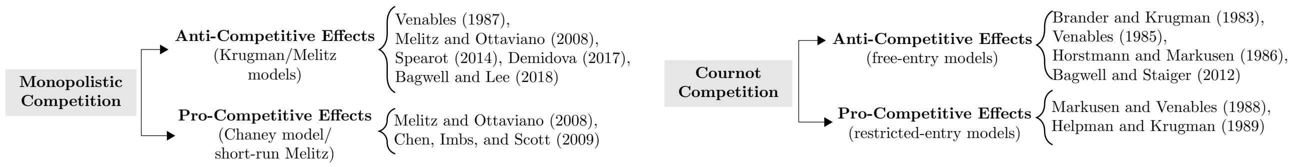
In Section 7, we apply the results of the Cournot model. First, we investigate the pervasive case in which the number of firms is approximated by a real number to make zero profits hold. The main conclusion we derive is that *when the integer number of firms is assumed away the effects of trade shocks are identified exactly as in monopolistic competition.* Specifically, Q3 is always satisfied and, so, their impact is determined through Q1 and Q2 in Figure 1, where firms' features coincide both ex-ante and ex-post. As a corollary, the conditions on MIEs for pro-competitive, anti-competitive, and null effects in a unilateral liberalization between two large countries are the same as in monopolistic competition.

Second, we obtain conclusions for a setup with restricted entry. This case arises in a free-entry setting when a trade shock does not induce extensive-margin adjustments (e.g., when the productivity distribution is dispersed enough). Our main result refers to a reduction in import trade costs, where we show that the MIEs profits channel is the only one operating and always leads to pro-competitive effects.

Our paper makes a methodological contribution to the identification of trade-liberalization effects across models of imperfect competition. Establishing a relationship between model assumptions and specific outcomes across these models is not immediately obvious. It depends on a modeler's choices such as ex-ante vs. given heterogeneity, free vs. restricted entry, negligible vs. non-negligible firms, and it is not always clear what the implications of these modeling choices are. In some cases counterintuitive outcomes arise, such as the Metzler paradox (i.e., the existence of anti-competitive effects in the liberalizing country after a unilateral liberalization).

In particular, our paper is relevant for studies utilizing models with a linear demand, such as those included in Figure 2. The figure also illustrates how a unilateral liberalization between two large countries leads to different outcomes in the liberalizing country depending on the setting considered. Our contribution in this respect is highlighting the role of marginal entrants in the identification of pro-competitive effects. In this sense, we think it constitutes a step towards finding common grounds across models, which has revealed itself as a difficult enterprise (Markusen and Venables 1988; Helpman and Krugman 1989).

Figure 2. *Effects of Unilateral Liberalizations in the Liberalizing Country: Examples of Studies with a Linear Demand*



2 Monopolistic-Competition Model

We conceive a world economy with a set $\mathcal{C}$ of potentially asymmetric countries. Throughout the text we employ the convention that, for any variable, a subscript ij refers to i as the origin country and j as the destination country. Also, all the proofs and derivations of this paper are presented in Appendix A.

Each country has a unitary measure of identical agents supplying one unit of labor inelastically. This is the only factor of production, and firms can hire workers within the country at wage w_i for $i \in \mathcal{C}$.

Moreover, there are two sectors. One consists of a homogeneous good supplied under perfect competition, with a possibly country-specific technology that displays constant returns to scale. The price of this good is taken as a numéraire and is freely traded and produced in each country in equilibrium, implying that wages are pinned down by the competitive sector. The other sector consists of a horizontally differentiated good with a continuum of varieties and it is the focus of our analysis.

2.1 Demand Side

For the description of the setup, we consider countries i and j such that $i, j \in \mathcal{C}$. Let $\overline{\Omega}$ be the set of all the potentially conceivable varieties that might be produced in the industry. A representative consumer from country i has the utility function,

$$U_i := q_0 + \alpha_i \int_{\omega \in \overline{\Omega}} q(\omega) \, \mathrm{d}\omega - \frac{\gamma_i}{2} \int_{\omega \in \overline{\Omega}} [q(\omega)]^2 \, \mathrm{d}\omega - \frac{\eta_i}{2} \left[\int_{\omega \in \overline{\Omega}} q(\omega) \, \mathrm{d}\omega \right]^2,$$

where $\alpha_i, \gamma_i, \eta_i > 0$, and q_0 and $q(\omega)$ denote the consumption of the homogeneous good and variety ω, respectively. We assume that income is high enough such that there is consumption of both goods in equilibrium.

Denote by $\Omega_{ij} := [0, M_{ij}]$ the set of varieties produced in i and consumed in j, and $\Omega_j := [0, M_j]$ the set of total varieties consumed in j, where $M_j := \sum_{i \in \mathcal{C}} M_{ij}$. Usual optimization procedures determine that the demand in country j for a variety $\omega \in \Omega_{ij}$ is given by

$$q_{ij}(\omega) := \frac{\alpha_j}{\gamma_j + \eta_j M_j} - \frac{1}{\gamma_j} p_{ij}(\omega) - \frac{\eta_j}{\gamma_j} \frac{\mathbb{P}_j}{\gamma_j + \eta_j M_j},$$

where $\mathbb{P}_j := \sum_{i \in \mathcal{C}} \int_{\omega \in \Omega_{ij}} p_{ij}(\omega) \, \mathrm{d}\omega$.

The choke price of a variety ω in j, defined as the infimum price that makes demand zero, is denoted $p_j^{\max}$ and given by

$$p_j^{\max}(\mathbb{P}_j, M_j) := \frac{\alpha_j \beta_j + \mathbb{P}_j}{\beta_j + M_j}, \tag{1}$$

where $\beta_j := \gamma_j / \eta_j$. Equation (1) establishes that the choke price is the same for all varieties, irrespective of their country of origin. We can use it to express demand as

$$q_{ij}(\omega) = \frac{p_j^{\max} - p_{ij}(\omega)}{\gamma_j}.$$

This expression shows that the choke price can be interpreted as a measure of toughness of the competitive environment in j: increases in the mass of firms serving j and decreases in the price of its active firms decrease the choke price which, in turn, lowers the demand of variety ω.

The price elasticity of the demand in j is given by $\varepsilon_{ij}(\omega) = \frac{p_{ij}(\omega)}{p_j^{\max} - p_{ij}(\omega)}$ and satisfies that $\frac{\partial \varepsilon_{ij}(\omega)}{\partial p_{ij}(\omega)} = \frac{p_j^{\max}}{\left[p_j^{\max} - p_{ij}(\omega)\right]^2} > 0$. For future reference, we define linear and relative markups by $\mu := p - c$ and $m := \frac{p}{c}$, respectively.

2.2 Supply Side

Consider countries i and j such that $i, j \in \mathcal{C}$. In each i, there is a set $\overline{\Omega}_i$ of potential single-product firms that are of zero measure. Each firm has the possibility of entering the industry by paying a sunk fixed entry cost $F_i^E > 0$. When a firm pays the entry cost, it gets assigned a unique variety ω and a draw of productivity φ from some firm-specific cdf D_i^{ω}.

Among the firms from i that pay F_i^E, each has to decide whether to serve country j. If a firm does so, it has to incur a country-specific fixed cost $f_{ij} \geq 0$. Moreover, given a productivity draw φ, the marginal cost of production is $c_i(\varphi) := \frac{w_i}{\varphi}$ and the firm incurs in trade costs.

The usual way to incorporate trade costs in monopolistic competition is through iceberg trade costs. On the other hand, oligopoly models under a linear demand usually do it through additive trade costs. Due to this, it is necessary to make a choice and we opt to take the additive form as a baseline case. The case of iceberg trade costs is relegated to Appendix D, where we show that all the propositions we state regarding the conditions for activation and deactivation of channels follow verbatim. Thus, the functional form chosen for trade costs does not affect the results.

Incorporating this, the costs in i to have one unit arrive at destination j are $c_{ij}^{\tau}(\varphi) := c_i(\varphi) + \tau_{ij}$, where τ_{ij} are trade costs such that $\tau_{ii} := 0$. To facilitate proofs, and to highlight that the results do not depend on the form of trade costs, we state results by using c_{ij}^{τ} whenever possible. Moreover, exploiting that there is a one-to-one relation between $c_i(\varphi)$ and φ, we characterize the model in terms of marginal costs rather than productivity.

At market j, firms make a decision on quantities $q_{ij}(\omega) \in [0, \overline{q}_j]$. Given a choice for the quantities, prices are $p_{ij}(\omega) \in [0, \overline{p}_j]$, with $\overline{p}_j \in \mathbb{R}_{++} \cup \{\infty\}$ greater than or equal to the demand's choke price of country j. We suppose that markets are segmented, such that firms can sell at a different price in each country. Also, we assume that the home country constitutes the most profitable market of each potential firm. This ensures that, as is standard in the literature, any firm that is active in at least one country necessarily serves its domestic market.

2.3 Distributional Assumptions and Definitions

Next, we define a baseline framework for monopolistic competition. This is used to derive the main propositions and, primarily, consists of the incorporation of a partition of firms and specific characterizations of D_i^{ω}. We only dispense with it for a specific result regarding the import-competition channel and for the numerical exercises we present to illustrate results.

Throughout the paper, we compare the equilibrium outcomes in two different scenarios, defined by the trade costs $\boldsymbol{\tau}^* := (\tau_{ij}^*)_{i,j \in \mathcal{C}}$ and $\boldsymbol{\tau}^{**} := (\tau_{ij}^{**})_{i,j \in \mathcal{C}}$. A particular subset of firms, which we have referred to as MIEs, plays a critical role for the results we obtain in this paper.

We define them formally as follows.

Definition 2.1: ***Marginal industry entrants*** *(MIEs) are the set of domestic firms with the lowest expected profits among those that pay the entry cost in a given equilibrium.*

In monopolistic competition, MIEs are simply the group of firms that have zero expected profits. The definition is stated in more general terms to apply to the oligopoly case, where this is not necessarily the case.

Furthermore, we suppose that firms can be partitioned. This assumption has the goal of ensuring that MIEs always belong to one group, so that we can directly obtain results by characterizing their specific group. This is in contrast to, rather, first determining the range of productivity distribution that defines MIEs in each equilibrium and then deriving the results.

Specifically, for each country $i \in \mathcal{C}$, we partition $\overline{\Omega}_i$ into groups $\mathcal{I}$, $\mathcal{E}$, and $\mathcal{N}$, where each comprises what we denominate insiders, entrants, and non-active firms, respectively. We suppose that each group θ has a total mass of firms $\overline{M}_i^{\theta}$. Moreover, M_i^{θ} is the mass of incumbents from i and group θ that pay the entry cost, and M_{ij}^{θ} corresponds to the set of firms from i that belong to the group θ and serve j.

The labels insiders, entrants, and non-active firms reflect the role these groups play during a trade liberalization. The set $\mathcal{I}$ includes those firms which pay the entry cost under both $\boldsymbol{\tau}^{*}$ and $\boldsymbol{\tau}^{**}$ and have a productivity distribution that ensures they are always active in the domestic market. At the other extreme, the group $\mathcal{N}$ consists of those firms that are inactive in the industry under both vectors of trade costs. As for $\mathcal{E}$, it constitutes the group of firms at which extensive-margin adjustments at the industry take place.

In addition, we define two variants of the setup that allow us to encompass standard versions of monopolistic competition used in the literature. They are based on alternative assumptions regarding D_i^{ω} for firms belonging to $\mathcal{E}$, which recall is the productivity distribution from which a firm ω belonging to $\mathcal{E}$ obtains a productivity draw after paying an entry cost. First, we define a *non-degenerate variant*, where D_i^{ω} corresponds to an atomless distribution that is identical for each firm ω in $\mathcal{E}$. This characterization arises in a setting à la Melitz, where all firms are ex-ante alike and there is heterogeneity only ex-post. Second, we define a *degenerate variant*, where D_i^{ω} for each firm ω in $\mathcal{E}$ is firm-specific and degenerate. This entails that each firm obtains a productivity draw from a distribution that concentrates all its probability mass at one point, and makes it possible to encompass a setting à la Chaney/short-run Melitz. Since this variant differs from how we usually model this setting, next we proceed to explain some of its implications.

In the degenerate variant, firms do not face any intrinsic uncertainty before paying the entry cost. This is because, since D_i^{ω} has all its probability mass at one productivity value, firms are able to anticipate the productivity draw that they would obtain. Thus, in this variant, ex-ante and ex-post characterizations of firms coincide. While this might seem like an unusual way to interpret a setup with firms having some given features, expressing all these models in ex-ante terms is what allows us to generalize results and unify the analysis across them.

Also, in the degenerate variant, the introduction of a firm-specific cdf D_i^{ω} determines that we need to distinguish between the productivity distribution that each firm considers ex-ante and the productivity distribution generated ex-post. This is in contrast with the non-degenerate variant, where D_i^{ω} is the same for each firm and, so, it generates the exact same productivity distribution ex-post.

To distinguish between these distributions, we exploit the fact that, in both setup variants, it is possible to determine the productivity distribution of all potential firms in each country and group. Moreover, we specify them in terms of marginal costs. Specifically, let G_i be the cdf that describes the marginal-cost distribution of the mass of all potential firms, $\overline{M}_i$, with density g_i and support $[\underline{c}_i, \overline{c}_i]$ where $\underline{c}_i \in \mathbb{R}_+$ and $\overline{c}_i \in \mathbb{R}_+ \cup \{\infty\}$. Likewise, we denote by G_i^{θ} be the marginal-cost cdf of all potential firms belonging to θ, $\overline{M}_i^{\theta}$, with density g_i^{θ} and support $\left[\underline{c}_i^{\theta}, \overline{c}_i^{\theta}\right]$. Consistent with the role of each group, we suppose that the subsets $\left[\underline{c}_i^{\theta}, \overline{c}_i^{\theta}\right]$ for $\theta \in \{\mathcal{I}, \mathcal{E}, \mathcal{N}\}$ determine a partition of $[\underline{c}_i, \overline{c}_i]$ and, as a result, the subsets of marginal costs do not overlap. In this way, each group can be ordered according to their expected profits.

Before delving into some properties of these cdfs, we formalize the two variants of the setup that we consider.

Definition 2.2: *The following are two setup variants for monopolistic competition, according to the characterization of firms belonging to $\mathcal{E}$ for country $i \in \mathcal{C}$.*

- ***Non-degenerate setup****: D_i^{ω} is the same for each firm ω in $\mathcal{E}$ and, so, the distribution of marginal costs that it entails coincides with the productivity distribution ex post $G_i^{\mathcal{E}}$. Moreover, we suppose that this distribution is atomless.*
- ***Degenerate setup****: D_i^{ω} for each firm ω belonging to $\mathcal{E}$ is degenerate and firm-specific. Moreover, it is either different for almost all firms, so that each obtains a different productivity draw, or the same for almost all, in which case almost all get an identical productivity draw.*

The limiting case of both variants is the same and we explicitly incorporate it in the degenerate setup to avoid a taxonomy of cases.

In the degenerate variant, assumptions on D_i^{ω} determine specific properties for the marginal-costs distribution of firms in $\mathcal{E}$ ex-post, $G_i^{\mathcal{E}}$. First, if almost all firms have a different D_i^{ω}, then $G_i^{\mathcal{E}}$ is necessarily atomless: since the set of firms obtaining same productivity draws is of measure zero, almost all firms have different marginal costs ex post. On the other hand, if almost all firms belonging to $\mathcal{E}$ have the same D_i^{ω}, they obtain an identical productivity draw and, so, $G_i^{\mathcal{E}}$ is degenerate, thus accumulating all its probability mass at one specific marginal cost.

Next, we define two features that applied to $\mathcal{E}$ allow us to characterize MIEs. They are crucial for the activation and deactivation of the channels. The scenarios in which they hold under each setup variant are indicated later, when we analyze their role in the different results we obtain.

Definition 2.3: *Consider firms from $i \in \mathcal{C}$ that belong to some group θ. Then:*

- ***Firms in θ are ex-ante exporters*** *when there exists a subset of marginal costs in $\left[\underline{c}_i^{\theta}, \overline{c}_i^{\theta}\right]$ such that a non-zero measure of firms in θ would eventually serve some country $k \in \mathcal{C} \setminus \{i\}$.*
- ***Firms in θ are ex-ante heterogeneous*** *when any subset of firms in θ that obtain productivity draws from the same D_i^{ω} has measure zero.*

2.4 Equilibrium

Consider country $i \in \mathcal{C}$. To derive the equilibrium, we suppose that $\overline{M}_i^{\mathcal{E}}$ is large enough such that, in equilibrium, not all the firms in $\mathcal{E}$ pay the entry cost. Moreover, except for the limiting case where all the probability mass is concentrated at one point, we assume that for some strict subset of highest marginal costs in $\left[\underline{c}_i^{\mathcal{E}}, \overline{c}_i^{\mathcal{E}}\right]$ firms do not find it profitable to serve the domestic market ex post, and for some other they do. Joint with the assumption that the home country is the most profitable market, this implies that only a strict subset of firms in $\mathcal{E}$ serve at least a market.

We start by describing the optimal decisions for active firms. Since firms from a specific country with the same marginal cost solve the same optimization problem, we index the solutions by this variable. Optimal prices and quantities in $j \in \mathcal{C}$ of an active firm from $i \in \mathcal{C}$ with marginal costs c are given by:

$$p_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} + c_{ij}^{\tau}}{2}, \tag{2}$$

$$q_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} - c_{ij}^{\tau}}{2\gamma_j}. \tag{3}$$

As for the firms that do not pay either the entry cost or the fixed cost to serve j, they set quantities equal to zero through a price greater than or equal to the choke price of that market. In turn, the linear and relative markups set in j are given by $\mu_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} - c_{ij}^{\tau}}{2}$ and $m_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} + c_{ij}^{\tau}}{2c_{ij}^{\tau}}$, respectively.

Regarding optimal profits of a firm with c_{ij}^{τ} that is active in j, they are

$$\pi_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{\left(p_j^{\max} - c_{ij}^{\tau}\right)^2}{4\gamma_j} - f_{ij}. \tag{4}$$

For trade costs $\boldsymbol{\tau}^*$ or $\boldsymbol{\tau}^{**}$, we denote the equilibrium values of any variable by a superscript $*$ and $**$, respectively. Moreover, we denote the marginal-cost cutoff to serve $j \in \mathcal{C}$ in each of these scenarios by c_{ij}^* and c_{ij}^{**}.

Up to this point, the equilibrium characterization is the same irrespective of specific assumptions regarding D_i^{ω}. However, the rest of equilibrium conditions differ for the non-degenerate and degenerate variants. This affects the description of the marginal-cost cutoffs, and the market-clearing and zero-expected-profits conditions. Due to this, next we outline only the main conditions that are necessary for explaining subsequent results. Specifically, we focus on the conditions implied by free entry, since those constitute the key determinants for the activation and deactivation of channels. Instead, we relegate the characterization of any other condition to Appendix A.1.

For the case of a non-degenerate setup, free entry determines the following. Denote $\widetilde{\pi}_{ji}^{\theta}$ the optimal expected profits in i of a firm from j belonging to θ. It can be shown that, under this variant, the marginal-cost cutoff of a firm from j to serve i can be expressed as a function of $\left(p_i^{\max *}; \tau_{ji}^*\right)$. Moreover, since we consider that MIEs belong to $\mathcal{E}$ under $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$, any firm belonging to $\mathcal{I}$ satisfies

$$\sum_{i \in \mathcal{C}} \widetilde{\pi}_{ji}^{\mathcal{I}}\left(p_i^{\max *}; \tau_{ji}^*\right) > F_j^E \text{ and } \sum_{i \in \mathcal{C}} \widetilde{\pi}_{ji}^{\mathcal{I}}\left(p_i^{\max **}; \tau_{ji}^{**}\right) > F_j^E,$$

while, for firms belonging to $\mathcal{E}$,

$$\sum_{i \in \mathcal{C}} \widetilde{\pi}_{ji}^{\mathcal{E}}\left(p_i^{\max *}; \tau_{ji}^*\right) = \sum_{i \in \mathcal{C}} \widetilde{\pi}_{ji}^{\mathcal{E}}\left(p_i^{\max **}; \tau_{ji}^{**}\right) = F_j^E. \tag{FE-ND}$$

To understand why these equations hold and which group of firms are the MIEs in each scenario, we make use of the entry-order mechanism according to profitability that holds as an equilibrium property in monopolistic competition. First, after all firms in $\mathcal{I}$ enter the industry and before any firm of $\mathcal{E}$ becomes active, firms in $\mathcal{E}$ have positive expected profits. Moreover, in the non-degenerate variant, all firms in $\mathcal{E}$ are ex-ante homogeneous, thus obtaining productivity draws from the same cdf D_i^{ω} and sharing the same expected profits. Thus, since we consider equilibria where MIEs belong to $\mathcal{E}$, it is determined that, *in a non-degenerate variant, all firms belonging to $\mathcal{E}$ that pay the entry cost are MIEs.* Consequently, (FE-ND) holds.

Now, consider the degenerate setup. Under that variant, the description of firms paying the entry cost is analogous. However, since each firm obtains a specific productivity draw with probability one, expected profits become ipso fact deterministic and, thus, firms make entry decisions based on (4). Due to this, any firm with marginal costs c that belongs to $\theta \in \{\mathcal{I}, \mathcal{E}\}$ and is not a MIE has profits that satisfy

$$\sum_{i\in\mathcal{C}} \mathbb{1}_{\left(c\leq c^{*}_{ji}\right)}\pi_{ji}\left(p^{\max *}_{i}, c; \tau^{*}_{ji}\right) > F^{E}_{j} \text{ and } \sum_{i\in\mathcal{C}} \mathbb{1}_{\left(c\leq c^{**}_{ji}\right)}\pi_{ji}\left(p^{\max **}_{i}, c; \tau^{**}_{ji}\right) > F^{E}_{j}.$$

Regarding the MIEs, we have assumed that home constitutes the most profitable market of each potential firm. In addition, since each firm obtains a specific productivity draw with probability one, a necessary condition for a MIE to pay the entry cost is anticipating that it is active in, at least, the domestic market (and, potentially, other countries). Thus, the marginal costs of MIEs for each trade costs coincide with the marginal-cost cutoffs at home (i.e., c^{*}_{jj} and c^{**}_{jj} for country j), establishing that *MIEs in the degenerate variant correspond to the least-productive firms that are active in the domestic market.* Formally, given non-negative profits in a set of countries $\mathcal{F}^{*}$ and $\mathcal{F}^{**}$, the MIEs for each set of trade costs have profits that satisfy

$$\sum_{i\in\mathcal{F}^{*}} \pi_{ji}\left(p^{\max *}_{i}, c^{*}_{jj}; \tau^{*}_{ji}\right) = \sum_{i\in\mathcal{F}^{**}} \pi_{ji}\left(p^{\max **}_{i}, c^{**}_{jj}; \tau^{**}_{ji}\right) = F^{E}_{j}. \tag{FE-D}$$

3 Channels in Monopolistic Competition

In this section, we inquire upon the conditions activating and deactivating channels that create pro-competitive effects. This is done by only modifying assumptions relating to the set $\mathcal{E}$ (and, hence, the MIEs) of our country of interest, H. Instead, for rest of the countries, the different propositions we state are independent of the specific characterization of foreign firms. Thus, we leave unspecified whether foreign countries' firms are characterized by a degenerate or a non-degenerate setup.[1]

For the following description, we refer to some generic equilibrium by using a superscript $*$. Regarding notation, for a firm with marginal cost c, we denote by $p^{*}_{ij}(c)$ and $q^{*}_{ij}(c)$ the solutions (2) and (3), and by $\mu^{*}_{ij}(c)$ and $m^{*}_{ij}(c)$ the equilibrium markups.

In the different propositions, our focus is on the creation of pro-competitive effects. They refer to the impact of a trade shock on each active domestic firm with marginal costs c regarding $p^{*}_{HH}(c)$, markups $\mu^{*}_{HH}(c)$ and $m^{*}_{HH}(c)$, and the marginal-cost cutoff c^{*}_{HH}. Since the choke price acts as a sufficient statistic for the determination of these variables, this entails that we concentrate on how the domestic choke price is impacted. We also provide results regarding $M^{\mathcal{E}*}_{H}$ and each $q^{*}_{HH}(c)$ to highlight the adjustment process of the model.

We say that a channel generates pro-competitive effects when the domestic choke price

[1] In Appendix A.1, we show that a reduction in the inward trade barriers of H always triggers a more aggressive pricing behavior and/or entry of foreign firms. This is all that we need to analyze how trade shocks affect outcomes regarding domestic firms, which is the focus in the propositions we state.

decreases, which implies that domestic prices, markups, and marginal-cost cutoff become lower. On the contrary, when we say that a channel is inactive, these variables and domestic quantities do not change and, instead, only $M_H^{\mathcal{E}*}$ is affected.

We define two channels. The first one is the *import-competition channel*, which acts through the exposure of domestic firms to a reduction of prices and entry by foreign firms. The second one is the *export-opportunities channel* and it works through increases in expected profits due to better export access, which induces the creation of firms that serve their domestic market and, hence, triggers greater competition domestically.

In order to isolate the import and export channel, we suppose that H is a small economy in the sense of Demidova and Rodríguez-Clare (2009; 2013). This establishes that any shock affecting H does not affect the domestic conditions of any foreign country. Consequently, they do not impact $\left(p_j^{\max *}\right)_{j\in\mathcal{C}\setminus\{H\}}$ or the mass of foreign firms that pay the entry cost.[2] Notice that, nonetheless, the model still allows for extensive-margin adjustments of foreign firms through variations in their marginal-cost cutoffs, $\left(c_{jH}^*\right)_{j\in\mathcal{C}\setminus\{H\}}$.

By considering reductions in inward and outward trade barriers in a small economy, it is possible to directly isolate each channel. Specifically, these trade shocks identify, respectively, the impact on the domestic market from tougher import competition and a better export access exclusively. Otherwise, in the case of large countries, shocks that impact H would change the trading partners' competitive conditions, $\left(p_j^{\max *}\right)_{j\in\mathcal{C}\setminus\{H\}}$, thereby affecting H's export conditions simultaneously. Importantly, as demonstrated in the applications of this paper, once we identify the conditions for activating each channel, results for large countries can also be obtained. This requires us to add the effects from changes in the export conditions as an additional channel, which operates in the same fashion as the export-opportunities channel.

3.1 Deactivating the Import-Competition Channel: Ex-Ante Homogeneity of MIEs

Next, we show that, when MIEs are ex-ante homogeneous, the import-competition channel is shut and only entails variations in the mass of domestic firms.

Proposition 3.1

Consider a world economy with an arbitrary number of countries, where H is a small economy. Let trade costs $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ be such that $\tau_{jH}^* \geq \tau_{jH}^{**}$ for each $j \neq H$ with strict inequality for at

[2] This assumption arises in equilibrium in a framework where foreign countries have a continuum of trading partners and H is part of it. Thus, any change in the domestic conditions of H has a negligible impact on the expected profits of foreign firms. Consequently, $\left(p_j^{\max *}\right)_{j\in\mathcal{C}\setminus\{H\}}$ and the mass of foreign firms that pay the entry cost are not affected by shocks affecting H and can be treated as parameters for the analysis. See Alfaro (2019).

least one country. Suppose a monopolistic competition market structure, where MIEs under $\boldsymbol{\tau}^*$ belong to $\mathcal{E}$ and there exists some boundary condition such that MIEs also belong to $\mathcal{E}$ under $\boldsymbol{\tau}^{**}$. Then, if the firms from H belonging to $\mathcal{E}$ are ex-ante homogeneous:

- $p_H^{\max *} = p_H^{\max **}$ *and* $c_{HH}^* = c_{HH}^{**}$,
- *for firms that are active in both equilibria,* $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, *and* $\mu_{HH}^{**}(c)$ *are the same as in the equilibrium with* $\boldsymbol{\tau}^*$, *and*
- $M_H^{\mathcal{E}**} < M_H^{\mathcal{E}*}$.

Notice that the proposition does not contain any assumption regarding insider firms, determining that any characterization of them is compatible with the result. In other words, it is only the ex-ante homogeneity of MIEs which determines that the import-competition channel is inactive. To explain why this is so, next we investigate when ex-ante homogeneity of MIEs holds in each setup variant.

In the non-degenerate scenario, recall that MIEs correspond to the whole subset of firms in $\mathcal{E}$ that pay the entry cost. In that case, ex-ante homogeneity of firms always holds since each firm in $\mathcal{E}$ obtains a productivity draw from the same cdf D_i^{ω}. Thus, ex-ante homogeneity of MIEs shuts the import-competition channel because there is only one choke price consistent with the zero-expected-profits condition, (FE-ND), and this equation is independent of import trade costs.

As for the degenerate variant, MIEs comprise the subset of least-productive firms that serve its domestic market (and, potentially, other countries). Moreover, given that firms obtain productivity draws from a degenerate distribution, checking whether ex-ante homogeneity holds is equivalent to analyzing if there is ex-post homogeneity. In particular, this feature arises when all MIEs share the same productivity, which is rationalized by each MIE obtaining a productivity draw from the same degenerate D_i^{ω}. Mathematically, the fact that the import-competition channel is inactive follows the same logic as the non-degenerate case: since MIEs are homogeneous, the marginal-cost cutoff is the same before and after the trade shock. Thus, incorporating this fact, (FE-D) identifies the choke price, and this equation is independent of import trade costs.

The mechanism in both variants can be rationalized intuitively as follows. When import trade costs become lower, there is an increase in quantities supplied by foreign firms. This makes competition tougher, which reduces the expected profits of domestic firms and, thereby, induces the exit of the firms with the least expected profits (i.e., the MIEs). Thus, the model adjusts by dropping as many of these ex-ante homogeneous MIEs as necessary until zero expected profits hold again. Overall, this determines that the effect of tougher import competition is completely offset by the exit of MIEs, without inducing any variation in the choke price.

3.2 Activating the Import-Competition Channel: Ex-Ante Heterogeneity of MIEs

The following proposition establishes that, when MIEs are ex-ante heterogeneous, the import-competition channel is active and generates pro-competitive effects.

Proposition 3.2

Consider a world economy with an arbitrary number of countries, where H is a small economy. Let trade costs $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ be such that $\tau^*_{jH} \geq \tau^{**}_{jH}$ for each $j \neq H$ with strict inequality for at least one country. Suppose a monopolistic competition market structure, where MIEs under $\boldsymbol{\tau}^*$ belong to $\mathcal{E}$, and there is some boundary condition such that MIEs also belong to $\mathcal{E}$ under $\boldsymbol{\tau}^{**}$. Then, if the firms from H belonging to $\mathcal{E}$ are ex-ante heterogeneous:

- $p_H^{\max **} < p_H^{\max *}$ *and* $c^{**}_{HH} < c^{*}_{HH}$,
- *for firms that are active in both equilibria,* $p^{**}_{HH}(c)$, $q^{**}_{HH}(c)$, $m^{**}_{HH}(c)$, *and* $\mu^{**}_{HH}(c)$ *decrease relative to the equilibrium with* $\boldsymbol{\tau}^*$, *and*
- $M_H^{\mathcal{E}**} < M_H^{\mathcal{E}*}$.

Notice that this result cannot arise in a non-degenerate variant since, in that scenario, firms belonging to $\mathcal{E}$ (and, hence, the MIEs) are always ex-ante homogeneous. Instead, in the degenerate variant, ex-ante heterogeneity occurs when almost all firms have a different degenerate D_i^ω, so that each obtains a different productivity draw. This corresponds to a scenario with an atomless ex-post distribution of marginal costs among firms from i (i.e., an atomless $G_i^{\mathcal{E}}$), since the absence of atoms can only arise if the set of firms obtaining a productivity draw from the same distribution is of measure zero.

Intuitively, when MIEs are ex-ante heterogeneous, the adjustment to redress excess supply is different from the case of ex-ante homogeneity. The reason is that the critical choke price which induces exit now varies across MIEs and, so, the exit of domestic firms cannot be accomplished without it varying. In addition, since the choke price changes, this establishes that any excess of supply is eliminated through both variations in the quantities produced by all active firms and the exit of MIEs.

3.3 Magnitude of the Import-Competition Channel

Proposition 3.1 states that the import-competition channel is inactive when MIEs are ex-ante homogeneous, while Proposition 3.2 indicates that it is active when MIEs are ex-ante heterogeneous. Nonetheless, these results are silent about the magnitude of the import-competition channel according to the degree of MIEs' heterogeneity. Next, we investigate this.

The main result we prove is that, even though the whole distribution of productivity matters when the import-competition channel is active, the degree of heterogeneity of MIEs plays a distinctive role: unlike what happens with the rest of active firms, *if the ex-ante heterogeneity of MIEs is negligible, the pro-competitive effects due to the import-competition channel are negligible too.*

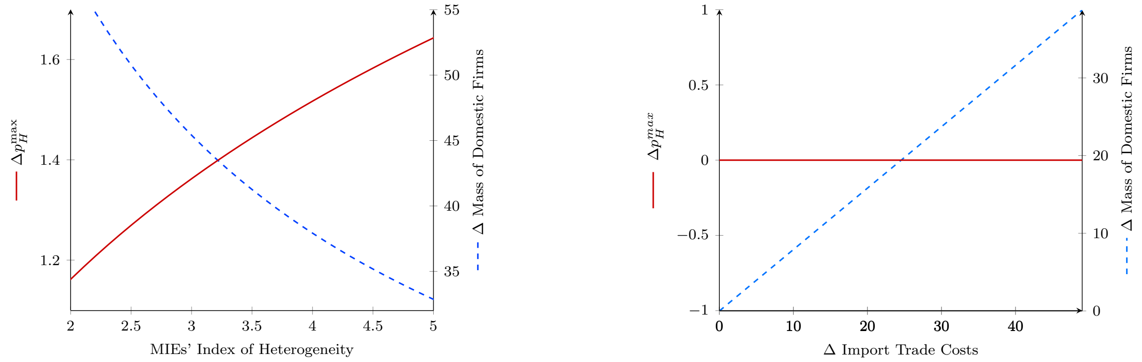
We relegate a formal proof of this to Appendix B since it requires us to dispense with the partition of firms. This is necessary to prove the result because, given that the whole distribution affects the magnitude of pro-competitive effects, the part of the distribution corresponding to the MIEs has to be isolated.

Instead, we resort to some graphical illustrations stemming from numerical exercises presented in Appendix C.1. In these graphs we use the degenerate setup variant, which determines that the MIEs correspond to the set of least-productive firms that are active in the domestic market.

In Figure 3a, we depict the type of adjustment for ex-ante heterogeneous firms, which entails that the import-competition channel is active. To clearly demonstrate the impact of the MIEs' degree of heterogeneity, we consider an increase (rather than a decrease) in inward trade costs. This enables us to isolate the role of MIEs by comparing domestic economies that are identical before the trade shock but differ in terms of the pool of most-productive inactive firms. The graph indicates that, consistent with Proposition 3.2, for a given increase in inward trade costs, part of the adjustment takes place through the choke price and leads to pro-competitive effects. The figure also reveals that, nevertheless, as MIEs become less heterogeneous (i.e., productivity draws get less dispersed), more of the adjustment takes place through the mass of MIEs. In fact, by decreasing the degree of heterogeneity, the model converges smoothly to a limit with an adjustment as in Figure 3b. This figure depicts a scenario with ex-ante homogeneity which, consistent with Proposition 3.1, determines that the choke price is not affected by reductions in inward trade barriers. Instead all of the adjustment is through the mass of MIEs.

Figure 3. *Variations in Inward Trade Costs in a Small Economy - Monopolistic Competition*

(a) Ex-Ante Heterogeneous MIEs **(b)** Ex-Ante Homogeneous MIEs



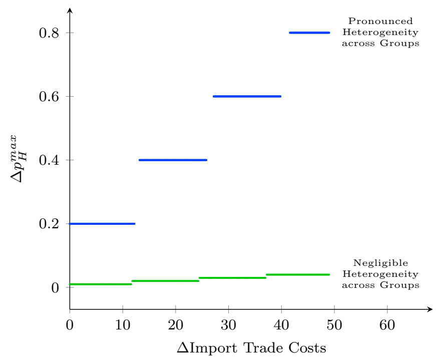
Note: In Figure 3a, a given increase in import trade costs is considered.

Given issues related to numerical convergence, it is not possible to depict within the same graph how the model converges smoothly to a limit that takes place exclusively through extensive-margin adjustments. However, it is possible to do so based on productivity distributions that exhibit mass points. This also makes it possible to include both ex-ante homogeneity and ex-ante heterogeneity of MIEs within the same framework.

Specifically, suppose that each firm obtains a productivity draw with probability one, implying that ex-ante and ex-post qualifications coincide. Moreover, assume that there are several groups where, within each of them, firms share the same productivity and, so, they are homogeneous. Nonetheless, among groups, firms have different levels of productivity, determining that they are heterogeneous across groups.

In Figure 4, we depict the adjustment that takes place when there is an increase in import trade costs. We compare two distributions in H with an identical characterization of active firms in the initial situation, but where the set of inactive firms in each of them differs according to the degree of heterogeneity between groups. The figure contains two cases distinguished by whether heterogeneity between the groups of inactive firms is large (blue lines) or small (green lines).

Figure 4. *The MIEs' Degree of Heterogeneity and the Choke Price*



The first conclusion we can obtain is in relation to the case of homogeneous MIEs. In the graph, there is an adjustment through homogeneous MIEs when the variation in import trade costs is such that MIEs belong to the same group before and after the trade shock. Graphically, by focusing on either the green or blue lines, the effect on the choke price is demonstrated by variations along any of the different horizontal line segments. Thus, for those variations of trade costs, the choke price does not vary and the model is adjusting through variations in the mass of MIEs.

The second conclusion is regarding the case of heterogeneous MIEs. An adjustment with heterogeneous MIEs occurs when the increase in import trade costs is such that MIEs belong to different groups. This case is graphically depicted by a discontinuous increase in the choke price at either the green or blue lines. To compare how the degree of heterogeneity affects the variation in the choke price, we exploit that the productivity distributions differ by the pool of inactive firms. In particular, the blue lines correspond to a large heterogeneity between groups, while the green lines assumes this is small.

By comparing these lines, we can see how the magnitude of the discontinuous jump in the choke price depends on the MIEs' degree of heterogeneity. Specifically, if the differences in productivity between the groups is more pronounced (blue-lines case), then greater variations in the choke price are necessary to restore the equilibrium. Otherwise, when differences in marginal costs are negligible (green-lines case), the variation in the choke price is also negligible, thereby implicitly determining that the adjustment takes place almost exclusively through the mass of domestic firms.

3.4 Activating the Export-Opportunities Channel: Ex-Ante Exporting MIEs

In the following part, we inquire upon the conditions to activate the export-opportunities channel. Intuitively, this channel operates creating pro-competitive effects by the positive effect it has on expected profits, which induces the creation of firms that, in particular, serve their domestic market. The next proposition establishes that this occurs when MIEs are ex-ante exporters.

Proposition 3.3

Consider a world economy with an arbitrary number of countries, where H is a small economy. Let trade costs $\boldsymbol{\tau}^{*}$ and $\boldsymbol{\tau}^{**}$ be such that $\tau_{HF}^{**} < \tau_{HF}^{*}$ for some country $F \in \mathcal{C}\backslash\{H\}$. Suppose a monopolistic competition market structure, where MIEs under $\boldsymbol{\tau}^{*}$ belong to $\mathcal{E}$ and there is some boundary condition which ensures that MIEs also belong to $\mathcal{E}$ under $\boldsymbol{\tau}^{**}$. Then:

- *If firms from H belonging to $\mathcal{E}$ are ex-ante exporters in F,*
  - $p_H^{\max *} < p_H^{\max *}$ *and* $c_{HH}^{**} < c_{HH}^{*}$,
  - *for firms that are active in both equilibria, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, and $\mu_{HH}^{**}(c)$ decrease relative to the equilibrium with $\boldsymbol{\tau}^{*}$, and*
  - $M_H^{\mathcal{E}**} > M_H^{\mathcal{E}*}$;
- *If firms from H belonging to $\mathcal{E}$ are not ex-ante exporters in F, $p_H^{\max *}$ does not vary and neither the prices, quantities, marginal-cost cutoff, and masses of firms serving H do so.*

Taking into account how the channel intuitively operates, the fact that MIEs necessarily have to be ex-ante exporters for pro-competitive effects to emerge is straightforward. To see this, notice that, since markets are segmented, better export access does not modify the decisions on domestic prices and quantities of firms that are active before and after the trade shock. Instead, this shock affects domestic conditions if it produces entry of firms and some of them serve the domestic market. Nonetheless, for this to occur, the MIEs, which are the firms through which extensive-margin adjustments take place, necessarily have to be impacted by the shock, and this is equivalent to assuming that they are ex-ante exporters.

Given this, next, we investigate when MIEs are ex-ante exporters in each setup variant. In the non-degenerate variant, this takes place when firms in $\mathcal{E}$ make entry decisions anticipating that, for some subset of marginal costs, they would eventually export. Thus, new export opportunities increase a firm's expected profits, thereby inducing entry to the industry. Eventually, while not all of the firms survive, some of them become active in the domestic market, making conditions at home tougher. Mathematically, this can appreciated through (FE-ND) where, given an increase in profitability due to better export access and the fact that trading partners' chokes prices are not affected because of the small-economy assumption, zero expected profits can only be restored if the domestic choke price varies. Otherwise, when firms in $\mathcal{E}$ serve exclusively the domestic market, the same condition reveals that the choke price is identified with independence of export trade costs.

Concerning the degenerate variant, recall that ex-ante and ex-post concepts coincide, and that MIEs are the subset of least-productive firms that serve the domestic market. Taking into account the latter, if MIEs are exporters then any other domestic firm would be exporting too since, by definition of what the MIEs are, the rest of the firms are more productive. In other words, a scenario with MIEs that are ex-ante exporters implies there is no selection into exporting, which is at odds with the vast empirical evidence documenting that only a subset of firms export. On the contrary, if we want to reflect selection into exporting, MIEs must not be ex-ante exporters, which determines that profits of MIEs identify the choke price without being affected by new export opportunities and, so, the export-opportunities channel is inactive.

4 Applications to Monopolistic Competition

In this section, we apply the results obtained for monopolistic competition. First, in Section 4.1, we establish assumptions to generate settings à la Krugman, Melitz, and Chaney/short-run Melitz, and determine which channels are active in each of them. Our main conclusion is that the Melitz and Krugman settings operate exclusively through the export-opportunities channel, while the Chaney/short-run Melitz framework does it through the import-competition channel.

Then, in Section 4.2, we apply the results to unilateral liberalizations between two large economies. In particular, we establish conditions that generate specific competitive effects in the liberalizing country.

4.1 Krugman, Melitz, and Chaney/Short-Run Melitz Settings

To obtain standard settings of monopolistic competition, it is necessary that $\mathcal{E}$ constitutes the only group in the economy, since there is no partition of firms in any of them. This is accomplished by setting $\underline{c}_i = \underline{c}_i^{\mathcal{E}}$ and $\overline{c}_i = \overline{c}_i^{\mathcal{E}}$, so that $\mathcal{I}$ and $\mathcal{N}$ are empty sets.

The following are the critical assumptions that are necessary to generate each case.

- **Melitz and Krugman**. The Melitz setting corresponds to a non-degenerate variant where $\mathcal{E}$ constitutes the only group and firms are ex-ante exporters. The Krugman model is the limiting case where each firm gets a productivity draw from a distribution exhibiting negligible heterogeneity.
- **Chaney/short-run Melitz**. It corresponds to a degenerate setup where $\mathcal{E}$ constitutes the only group, there is selection into exporting, and $G_i^{\mathcal{E}}$ is atomless (i.e., D_i^{ω} is different for almost all firms).

By identifying the features of MIEs that these assumptions imply, we can apply the propositions obtained for monopolistic competition and establish which channels are operating in each setting.

First, consider the Melitz setting. This is a particular case of the non-degenerate setup, determining that all firms which pay the entry cost constitute the group of MIEs and they are ex-ante alike. Thus, the MIEs are ex-ante homogeneous, which implies that the import-competition channel is always inactive and, so, Proposition 3.1 applies. Moreover, this proposition also holds in Krugman, since it is the limiting case of Melitz with negligible heterogeneity.

In addition, in Melitz it is assumed that for some productivity draws firms would be eventually exporters, thereby generating selection into exporting ex post. In that model, this can only be generated if each firm considers that it exports with a positive probability, determining that MIEs are ex-ante exporters. Therefore, Proposition 3.3 applies. In the case of the Krugman model, the proposition also holds since, to have two-way trade, it is necessary that in each country at least one firm exports and, due to firm symmetry, all active firms would export too.

As for the Chaney/short-run Melitz setting, since it constitutes a special case of the degenerate variant, MIEs correspond to the subset of least-productive firms that serve the domestic market. Moreover, the marginal-costs distribution of firms ex-post is atomless, which is only possible if the set of firms with identical D_i^{ω} has measure zero. Thus, all firms, and in particular the MIEs, are ex-ante heterogeneous, which implies that Proposition 3.2 holds and the import-competition channel is always active.

Furthermore, to generate selection into exporting in the Chaney/short-run Melitz setting, it is necessary to assume that the least-productive firm serves exclusively its domestic market. Thus, unlike what occurs in the Melitz setting, this property implies that MIEs are not ex-ante exporters. Therefore, by applying Proposition 3.3, it is determined that the export-opportunities channel is inactive.

4.2 Unilateral Liberalizations between Two Large Countries

In this part, we consider a world consisting of two large countries, denoted by H and F, and a reduction in inward trade barriers in H as a trade shock. Our focus is on the impact of this trade shock on the competitive conditions of H and F, along with the behavior of their domestic firms. The experiment allows us to simultaneously investigate the impact due to better export access (effects in F) and tougher import competition (effects in H) when there are two large countries.

Unlike the case where H is a small country, this experiment creates feedback effects that need to be taken into account. Specifically, dispensing with the small-country assumption entails that the competitive conditions in F (i.e., $p_F^{\max}$) are affected by trade shocks in H which, in turn, creates indirect effects. Thus, H and F are affected by the channel operating in a small economy *and* changes in their export conditions.

To incorporate this, we define the *export-conditions channel*, which captures the direct and indirect effects on the country's domestic conditions caused by changes in its trading partner's choke price. Remarkably, *the export-conditions channel is activated and deactivated by the same conditions as those for the export-opportunities channel.* Therefore, we can apply Proposition 3.3 to analyze whether pro-competitive effects are created by the export-conditions channel. Intuitively, this follows because, even though each channel is triggered by a different variable, both operate through variations in expected export profits. Therefore, the fact that the export-opportunities channel acts through a reduction in outward trade costs, whereas the export-conditions channel does it through changes in the foreign choke price, is inconsequential.

To formalize the results, we consider an infinitesimal variation in inward trade barriers in H. Our focus is on the behavior of domestic firms and all the results follow by determining how $p_H^{\max}$ and $p_F^{\max}$ are impacted, since the choke price is a sufficient statistic for domestic prices, markups, and marginal-cost cutoff.

We relegate a formal derivation of the results to Appendix A.2. Instead, next, we show how by using the propositions for a small country we can assess whether pro-competitive, anti-competitive, or null effects are created in each country. To accomplish this, we proceed in two steps: first, we decompose the total effect on channels and, then, apply the conditions on MIEs for the activation and deactivation of them (i.e., Propositions 3.1, 3.2, and 3.3).

Regarding the decomposition of channels, irrespective of the setup variant that is considered, it is possible to show that the equilibrium conditions in each of them determine reduced-form equations $p_H^{\max}\left(p_F^{\max};\tau_{FH}\right)$ and $p_F^{\max}\left(p_H^{\max};\tau_{FH}\right)$. Thus, the equilibrium is given by a pair

$(p_H^{\max *}, p_F^{\max *})$ such that,

$$p_H^{\max *} = p_H^{\max}\left(p_F^{\max *}; \tau_{FH}\right), \tag{5a}$$

$$p_F^{\max *} = p_F^{\max}\left(p_H^{\max *}; \tau_{FH}\right). \tag{5b}$$

This implies that the system (5) is independent of any other endogenous variable such as the mass of incumbents. By keeping some variables constant, these functions make it possible to split the total effect on each choke price into different channels. Specifically, consider country F, which is the country that faces new export opportunities. Then, the total effect on its choke price is given by

$$\frac{\mathrm{d}p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\mathrm{d}\tau_{FH}} = \underbrace{\lambda \frac{\partial p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\partial \tau_{FH}}}_{\text{export-opportunities channel}} + \underbrace{\lambda \frac{\partial p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\partial p_H^{\max}} \frac{\partial p_H^{\max *}\left(p_F^{\max *}\right)}{\partial \tau_{FH}}}_{\text{export-conditions channel}}, \tag{6}$$

where λ is a multiplier of effects which captures that, given feedback effects, a trade shock creates indirect effects through the impact on each trading partner's choke price. Under stability conditions of the system (5), it can be shown that $\lambda > 0$.

Equation (6) indicates that the total impact on $p_F^{\max *}$ is determined by the total effects due to the direct impact of τ_{FH} (the export-opportunities channel, as in the small-country case) and the total effect by the direct impact of τ_{FH} on $p_H^{\max *}$ (the export-conditions channel, arising due to the large-countries assumption).

Regarding country H, which is the one that faces tougher import competition, the total effect can be split in the following way:

$$\frac{\mathrm{d}p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\mathrm{d}\tau_{FH}} = \underbrace{\lambda \frac{\partial p_H^{\max *}\left(p_F^{\max *}\right)}{\partial \tau_{FH}}}_{\text{import-competition channel}} + \underbrace{\lambda \frac{\partial p_H^{\max *}\left(p_F^{\max *}\right)}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\partial \tau_{FH}}}_{\text{export-conditions channel}}. \tag{7}$$

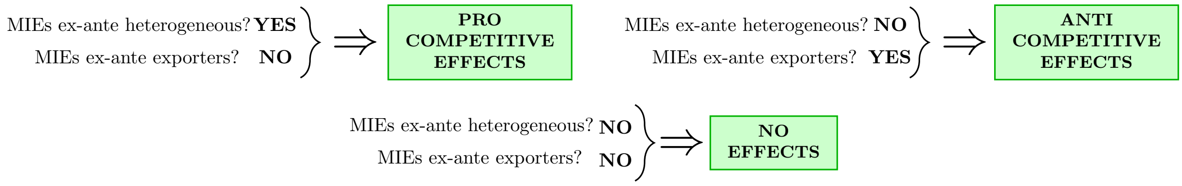
Equation (7) establishes that the total impact on $p_H^{\max *}$ is composed of the total effects due to the direct impact of τ_{FH} (the import-competition channel, as in the small-country case) and the total effects caused by the direct impact of τ_{FH} on $p_F^{\max *}$ (the export-conditions channel, arising due to the large-countries assumption).

For each term of equations (6) and (7), the assumptions on MIEs for the deactivation and activation of each channel establish their signs which, in turn, lead to specific outcomes.

Next, we illustrate its use by focusing on the effects in H, which enables us to determine a set of sufficient conditions on MIEs that generate the specific outcomes in the studies stated in the introduction (i.e., Figure 2). In particular, they identify a set of conditions that leads to anti-competitive effects, which in the literature has been known as the Metzler paradox.

The conditions are presented in Figure 5 and establish a definite sign for (7). For instance, they rationalize the results in Melitz and Ottaviano (2008), where anti-competitive effects arise in a Melitz setting, but pro-competitive effects take place in the Chaney/short-run Melitz framework.

Figure 5. *Unilateral Liberalizations with Two Large Countries in Monopolistic Competition: Conditions for Effects in H*



Next, we explain in terms of channels how these assumptions on MIEs generate the outcomes.

- **Anti-Competitive Effects**: this arises if MIEs in both countries are ex-ante homogeneous (i.e., the import-competition channel is inactive) and ex-ante exporters (i.e., the export channels are active). These assumptions hold, for instance, in the Melitz and Krugman settings. Thus, the existence of anti-competitive effects in H follows because better export opportunities for F create tougher competitive conditions in F which, in turn, entails worse export conditions for H. Given that the import-competition channel is shut, this is the only effect that operates in equilibrium in H.
- **Pro-Competitive Effects**: this holds if MIEs in H are ex-ante heterogeneous (i.e., the import-competition channel is active) and are not ex-ante exporters (i.e., the export channels are inactive). Under these assumptions, pro-competitive effects are generated because H is only affected by an exposure to tougher import competition. As an example, these conditions hold in the Chaney/short-run Melitz setting.
- **Null Competitive Effects**: this holds if MIEs in H are ex-ante homogeneous (i.e., the import-competition channel is inactive) and not ex-ante exporters (i.e., the export channels are inactive). Two examples where these assumptions hold are the following. First, a non-degenerate setup where the set of MIEs only serve the domestic market. Second, a degenerate setup where MIEs share the same productivity and there is selection into exporting, so that MIEs do not export.

5 The Cournot Model

In this section, we outline the setup of the Cournot model and derive its equilibrium. We consider a setup where each firm knows its productivity, making the model akin to the degenerate variant of monopolistic competition with respect to heterogeneity. Therefore, the model can be reinterpreted as one where each firm pays an entry cost to obtain a specific productivity draw with probability one. Due to this, ex-ante features of MIEs coincide with their actual characteristics and, except when we establish a direct connection with monopolistic competition, we omit any ex-ante or ex-post qualification.

The incorporation of an integer number of firms modifies the analysis relative to monopolistic competition in three respects. First, since firms can influence market conditions, they take strategic considerations into account when making decisions. Second, extensive-margin adjustments at the market have discontinuous effects and it is not necessarily the case that the least-productive active firm earns exactly zero profits. Finally, the MIEs comprise only one firm and we refer to it as the *last entrant*, which corresponds to the least-profitable firm that is active in the domestic industry. Similar to the remark above, we only refer to last entrants as MIEs when we make a link with monopolistic competition.

The conclusions that emerge from the propositions we state below are: (i) the fact that the last entrant does not necessarily break even introduces a new channel to the analysis, and (ii) the inclusion of strategic considerations per se does not affect the conclusions regarding competitive effects of monopolistic competition.

The new channel introduced in (i) is what we denominate *the MIEs profits channel*. We show that this channel is inactive as long as the profits of the last entrant of each equilibrium are the same, independently of whether these profits are zero. In addition, conclusion (ii) is a consequence that, when MIEs are characterized equivalently across models, the same conditions prevail for the activation and deactivation of channels as in monopolistic competition. In particular, when the integer number of firms is assumed away, the same characterization of results as in monopolistic competition is obtained.

5.1 Setup and Optimal Variables

The framework is similar to monopolistic competition in some respects. In particular, incorporating that there is a discrete number of varieties and firms, it has a demand and supply side as in Section 2.1 and Section 2.2. On the contrary, given that MIEs collapse to the last entrant, we need not establish any partition of firms to identify them and, hence, obtain results.

We suppose that each firm ω gets a productivity draw from a degenerate D_i^{ω}, which determines a marginal cost c_{ω}. Thus, by allowing for D_i^{ω} of the last entrant in each equilibria being equal or different, we are able to encompass the cases of homogeneous and heterogeneous MIEs.

The inverse demand for a variety ω produced in i and sold in j is $p_{ij}(\omega) = \alpha_j - \gamma_j q_{ij}(\omega) - \eta_j \mathbb{Q}_j$ with $\mathbb{Q}_j := \sum_{i \in \mathcal{C}} \mathbb{Q}_{ij}$ and $\mathbb{Q}_{ij} := \sum_{\omega \in \Omega_{ij}} q_{ij}(\omega)$. We denote the marginal cost of a firm ω inclusive of trade costs by $c_{\omega}^{\tau_{ij}} := c_{\omega} + \tau_{ij}$. In this framework each firm is able to influence $p_j^{\max}(\mathbb{Q}_j)$ through its choice of quantities. Thus, the best-response quantities in j of an active firm from i with marginal costs c_ω are

$$q_{ij}^{BR}\left(\mathbb{Q}_j^{-\omega};c_\omega^{\tau_{ij}}\right):=\frac{\alpha_j-\eta_j\mathbb{Q}_j^{-\omega}-c_\omega^{\tau_{ij}}}{2\left(\gamma_j+\eta_j\right)}, \tag{8}$$

where $\mathbb{Q}_j^{-\omega}$ is the sum of quantities supplied in j by all firms except ω.

To establish a direct link with monopolistic competition, we reexpress the optimal variables in terms of the choke price. To do this, we exploit that there is a one-to-one relation between $p_j^{\max}$ and $\mathbb{Q}_j$, given by $p_j^{\max}\left(\mathbb{Q}_j\right)=\alpha_j-\eta_j\mathbb{Q}_j$. Thus, the inverse demand is $p_{ij}\left(\omega\right)=p_j^{\max}\left(\mathbb{Q}_j\right)-\gamma_j q_{ij}\left(\omega\right)$, which determines that the optimal quantities and prices as functions of the choke price are:

$$\begin{aligned} q_{ij}\left[p_j^{\max}\left(\mathbb{Q}_j\right);c_\omega^{\tau_{ij}}\right] &:= \frac{p_j^{\max}\left(\mathbb{Q}_j\right)-c_\omega^{\tau_{ij}}}{2\gamma_j+\eta_j}, \\ p_{ij}\left[p_j^{\max}\left(\mathbb{Q}_j\right);c_\omega^{\tau_{ij}}\right] &:= \frac{p_j^{\max}\left(\mathbb{Q}_j\right)\left(\gamma_j+\eta_j\right)+\gamma_j c_\omega^{\tau_{ij}}}{2\gamma_j+\eta_j}. \end{aligned} \tag{9}$$

Moreover, optimal linear markups are $\mu\left[p_j^{\max}\left(\mathbb{Q}_j\right);c_\omega^{\tau_{ij}}\right]:=\frac{\gamma_j+\eta_j}{2\gamma_j+\eta_j}\left[p_j^{\max}\left(\mathbb{Q}_j\right)-c_\omega^{\tau_{ij}}\right]$, while relative markups are $m\left[p_j^{\max}\left(\mathbb{Q}_j\right);c_\omega^{\tau_{ij}}\right]:=\frac{\gamma_j}{2\gamma_j+\eta_j}+\frac{\gamma_j+\eta_j}{2\gamma_j+\eta_j}\frac{p_j^{\max}\left(\mathbb{Q}_j\right)}{c_\omega^{\tau_{ij}}}$. In turn, optimal profits in j of an active firm ω from i are

$$\pi_{ij}\left[p_j^{\max}\left(\mathbb{Q}_j\right);c_\omega^{\tau_{ij}}\right]:=\frac{\gamma_j+\eta_j}{\left(2\gamma_j+\eta_j\right)^2}\left[p_j^{\max}\left(\mathbb{Q}_j\right)-c_\omega^{\tau_{ij}}\right]^2-f_{ij}. \tag{10}$$

5.2 Entry Process and Reindex of Variables

Given that multiple equilibria can arise in oligopoly models, additional structure is required for its characterization. To keep the model in line with monopolistic competition and only allow for differences in MIEs, we suppose that firms enter following a productivity order.[3] This assumption holds as an equilibrium property in monopolistic competition and is usually incorporated in oligopoly models to ignore equilibria where less-productive firms crowd out more productive ones. In the international-trade literature, it is a maintained assumption in, for instance, Feenstra and Ma (2007), Atkeson and Burstein (2008), Eaton et al. (2012), Edmond et al. (2015), and Gaubert and Itskhoki (2018).

To have a well-defined order, we follow a similar approach to the one used in the literature of oligopolies with firm heterogeneity (e.g., Eaton et al. 2012 and Gaubert and Itskhoki 2018). This determines that we can define a cost index which establishes a strict order for all potentially conceivable firms in each market. To accomplish this, we suppose that country-specific fixed costs are strictly positive and do not depend on the country of origin. The former rules out that a set of firms earning zero profits provide zero quantities which, otherwise, would make

[3]More generally, a profitability ranking could be defined as the criteria to order firms. Since, in our case, firm heterogeneity is exclusively due to efficiency, productivity and profitability rankings are equivalent.

the equilibrium indeterminate.

Also, for the study of the import-competition channel, we assume that entry costs are the same in each country. This has the goal of avoiding a taxonomy of cases for the proofs regarding the origin country of the least-productive firm in each market. In fact, given the assumption on market fixed costs and that all active firms serve at least their domestic market, we directly suppose they are zero. We restore the assumption $F_i^E > 0$ when we study the export-opportunities channel where, to assess the role of better export access, last entrants have to necessarily be domestic and, so, entry costs do not affect the order of entry. For the applications, where we suppose that last entrants are domestic to have a definition of MIEs in line with monopolistic competition, the assumption is inconsequential.

All this determines that firms differ only by the marginal cost of delivering to a market. Moreover, incorporating that each variety in $\overline{\Omega}$ corresponds to a specific potential firm, so that $\overline{\Omega} = \cup_{k \in \mathcal{C}} \overline{\Omega}_i$, we can establish an order which applies to each firm that could potentially be active in the industry.

We formalize the entry order through an order relation. For each country $i \in \mathcal{C}$, we use $c_\omega^{\tau_{ji}}$ as the cost index of a firm ω from $j \in \mathcal{C}$. Given the existence of trade costs, the cost incurred by a firm supplying one unit depends on the market being served. Thus, there is a different order relation for each country. Formally, we define $\succsim_i$ on $\overline{\Omega}$ such that $\omega'' \succsim_i \omega'$ iff $c_{\omega''}^{\tau_{ji}} \geq c_{\omega'}^{\tau_{ki}}$, where $\omega'' \in \overline{\Omega}_j$, $\omega' \in \overline{\Omega}_k$, and $j, k \in \mathcal{C}$. Notice that, since $\succsim_i$ is defined on $\overline{\Omega}$, it orders all the conceivable firms in the world. While this order relation establishes a strict order for firms with different cost indexes, it does not do so when they are equal. For the purposes of our paper, any order among equivalent firms provides same results. Thus, from now on, we suppose there is some arbitrary order among the firms having the same cost index. Also, we suppose that, if after a variation in the trade costs some firms end up having the same cost index, the order of the status quo is preserved. These assumptions define $\succsim_i$ as a strict order among firms.[4]

Given the definition of $\succsim_i$, we are able to construct an order-preserving bijective function that allows us to reindex all the variables in a one-to-one fashion. Formally, for each country i, there exists a mapping $\omega \mapsto r_i(\omega)$ that orders the elements of $\overline{\Omega}$ according to $\succsim_i$: given $\omega'', \omega' \in \overline{\Omega}$, the mapping is such that $r_i(\omega'') \geq r_i(\omega')$ iff $\omega'' \succsim_i \omega'$. In words, this means that firms with greater index r_i have greater unit costs to serve i. We denote by N_i the total number

[4]Formally, establishing an entry process with a strict order means that we need to endow $(\overline{\Omega}, \succsim_i)$ with a linear order. This is what allows us to obtain an order-preserving bijective function that reindexes all the variables in a one-to-one fashion, i.e., that the function is order isomorphic (see, for instance, Ok (2007), Section B.2). Without assuming an arbitrary order between firms with a same cost index, $(\overline{\Omega}, \succsim_i)$ is only a complete preordered set (i.e., complete and transitive), implying that the equivalence classes defined by $\succsim_i$ are not necessarily singletons. Once that we assume an arbitrary order among the firms belonging to a same equivalence class, we are allowed to extend the complete preorder to a linear order. This is because it satisfies the additional property of $\succsim_i$ being antisymmetric, so that the equivalence classes are singletons.

of active firms in i, and $\Omega_i := \left\{\omega \in \overline{\Omega} : r_i(\omega) \leq N_i\right\}$ the set of active firms in i. Likewise, we denote N_{ji} the number of firms from j which are active in i, and $\Omega_{ji} := \left\{\omega \in \overline{\Omega}_j : r_i(\omega) \leq N_i\right\}$ the set of active firms in i from j. Notice that both N_i and N_{ji} play the dual role of total number of firms and index of the last entrant.

To keep notation simple, when we use r_i as an index, we implicitly assume that it is relative to the set $\overline{\Omega}$. Likewise, the index r_{ji} is relative to the set $\overline{\Omega}_j$. In addition, we occasionally omit country subscripts for trade costs when only the order of the firm is relevant. Thus, for instance, we denote the cost index of the last entrant in i by $c^{\tau}_{N_i}$.

5.3 Equilibrium

In each equilibrium, we suppose that there is at least one domestic and one foreign firm active. Consider countries $i, j \in \mathcal{C}$. Given optimal quantities as in (9), the total quantity supplied by firms from j to i is given by $\mathbb{Q}_{ji}(\mathbb{Q}_i; \tau_{ji}) := \sum_{\omega \in \Omega_{ji}} q_{ji}\left[p^{\max}_i(\mathbb{Q}_i), c^{\tau_{ji}}_{\omega}\right]$. Using this, the equilibrium at the market stage for a given Ω_i requires that

$$\sum_{j \in \mathcal{C}} \mathbb{Q}_{ji}(\mathbb{Q}_i; \tau_{ji}) = \mathbb{Q}_i, \tag{NE}$$

so that the optimal quantities chosen by each firm are consistent with the aggregate quantities.

Firms serve each country as long as they anticipate positive profits. This implies that, for country i, the following inequalities have to hold:

$$\pi_{ji}\left(p^{\max}_i(\mathbb{Q}_i); c^{\tau_{ji}}_{\omega}\right) \geq 0 \text{ for all } \omega \in \Omega_{ji} \text{ and } j \in \mathcal{C},$$
$$\text{for any } q_\omega > 0,\ \pi_{ji}\left[p^{\max}_i(\mathbb{Q}_i + q_\omega), q_\omega; c^{\tau_{ji}}_{\omega}\right] < 0 \text{ for } \omega \notin \Omega_{ji} \text{ and } j \in \mathcal{C},$$

where $\pi_{ji}\left[p^{\max}_i(\mathbb{Q}_i + q_\omega), q_\omega; c^{\tau_{ji}}_{\omega}\right] = q_\omega\left(p^{\max}_i(\mathbb{Q}_i + q_\omega) - \gamma_i q_\omega - c^{\tau_{ij}}_{\omega}\right) - f$ is ω's profits for any arbitrary quantity q_ω. Given the entry order, the reindex of variables, and the monotonicity of optimal profits on the index cost, these conditions can be reexpressed as

$$p^{\max}_i(\mathbb{Q}_i) - c^{\tau}_{N_i} \geq \xi_i \tag{FE-C}$$
$$p^{\max}_i\left(\mathbb{Q}_i + q^{N_i+1}\right) - c^{\tau}_{N_i+1} < \xi_i,$$

where $\xi_i := 2\sqrt{\gamma_i f}$ and $q^{N_i+1} := q^{BR}_{\cdot i}\left(\mathbb{Q}_i, c^{\tau}_{N_i+1}\right)$.

6 Channels in the Cournot Model

We proceed to inquire upon the conditions that activate and deactivate the channels by following a similar approach as for monopolistic competition. Specifically, we focus on a country H that we suppose is small, and compare the equilibrium under two vectors of trade costs, $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$.

Also, we continue to denote their equilibrium values with superscripts $*$ and $**$, respectively, and refer to the last entrant of each equilibrium by N_H^* and N_H^{**}, with corresponding profits $\overline{\pi}_H^*$ and $\overline{\pi}_H^{**}$.

Given that firms do not face any inherent uncertainty regarding their productivity, we do not add any ex-ante qualification to describe the features of firms. However, it is worth keeping in mind that thinking of these features as also holding in ex-ante terms is what makes it possible to establish common results across models of imperfect competition.

6.1 The Import-Competition Channel

The next proposition analyzes the effects coming from the import-competition channel when last entrants are homogeneous. Formally, this corresponds to the case where $c_{N_H^*}^{\tau^*} = c_{N_H^{**}}^{\tau^{**}}$, i.e., when the last entrants have the same marginal cost.

Proposition 6.1

Consider a world economy with an arbitrary number of countries, where H is a small economy. Suppose a model à la Cournot and let $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ be such $\tau_{jH}^{**} \leq \tau_{jH}^*$ with strict inequality for at least one country. If the last entrants in H are homogeneous, then:

- *if $\overline{\pi}_H^{**} < \overline{\pi}_H^*$, then*
  - *$p_H^{\max **} < p_H^{\max *}$,*
  - *for firms that are active in both equilibria, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ are lower relative to the equilibrium with $\boldsymbol{\tau}^*$, and*
  - *the set of domestic firms is either the same or some of them exit;*
- *if $\overline{\pi}_H^{**} = \overline{\pi}_H^*$, then*
  - *$p_H^{\max **} = p_H^{\max *}$,*
  - *for domestic firms that are active in both equilibria, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ do not vary relative to the equilibrium with $\boldsymbol{\tau}^*$, and*
  - *some of the domestic firms exit.*

Before interpreting the results, we present the case of heterogeneous last entrants, i.e., when $c_{N_H^*}^{\tau^*} \neq c_{N_H^{**}}^{\tau^{**}}$.

Proposition 6.2

Consider a world economy with an arbitrary number of countries, where H is a small economy. Suppose a model à la Cournot and let $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ be such $\tau_{jH}^{**} \leq \tau_{jH}^*$ with strict inequality for at least one country. If last entrants in H are heterogeneous and $\overline{\pi}_H^{**} \leq \overline{\pi}_H^*$:

- *$p_H^{\max **} < p_H^{\max *}$,*
- *for firms that are active in both equilibria, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ are lower relative to the equilibrium with $\boldsymbol{\tau}^*$, and*
- *the set of domestic firms is either the same or some domestic firms exit.*

Notice that these propositions do not require a characterization of any firm other than the last entrant. This clearly shows the relevance of MIEs for how the model adjusts and, hence, determines outcomes.

The first conclusion we can obtain is that when $\overline{\pi}^{*}_{H} = \overline{\pi}^{**}_{H}$, so that the MIEs profits channel is shut, pro-competitive effects are created if the last entrants are heterogeneous while null competitive effects emerge if they are homogeneous. Put differently, once the MIEs profits channel is shut, the import-competition channel is activated and deactivated in the same fashion as in monopolistic competition. The fact that, for that market structure, no explicit assumption about the MIEs profits channel is stated follows because smoothness assumptions ensure this is always satisfied through the particular case that $\overline{\pi}^{*}_{H} = \overline{\pi}^{**}_{H} = 0$. Thus, this result highlights that, once MIEs are characterized equivalently across models, the Cournot model and monopolistic competition generate the same qualitative outcome regarding the import-competition channel.

While a scenario where $\overline{\pi}^{*}_{H} = \overline{\pi}^{**}_{H}$ could be considered quite a particular case when there is a discrete number of firms, actually, it holds in the pervasive case of the literature where the integer number of firms is assumed away. More generally, $\overline{\pi}^{*}_{H} = \overline{\pi}^{**}_{H}$ is also relevant for scenarios where differences in profits between last entrants are small.

In addition, the proposition establishes that, when $\overline{\pi}^{**}_{H} < \overline{\pi}^{*}_{H}$, the MIEs profits channel reinforces any pro-competitive effect. Nonetheless, it is worth remarking that, if the MIEs profits channel is the only operating channel, the pro-competitive effects created are bounded by the profits of the last entrant: in any equilibrium, the choke price cannot be lower than the level that makes the last entrant earn exactly zero profits.

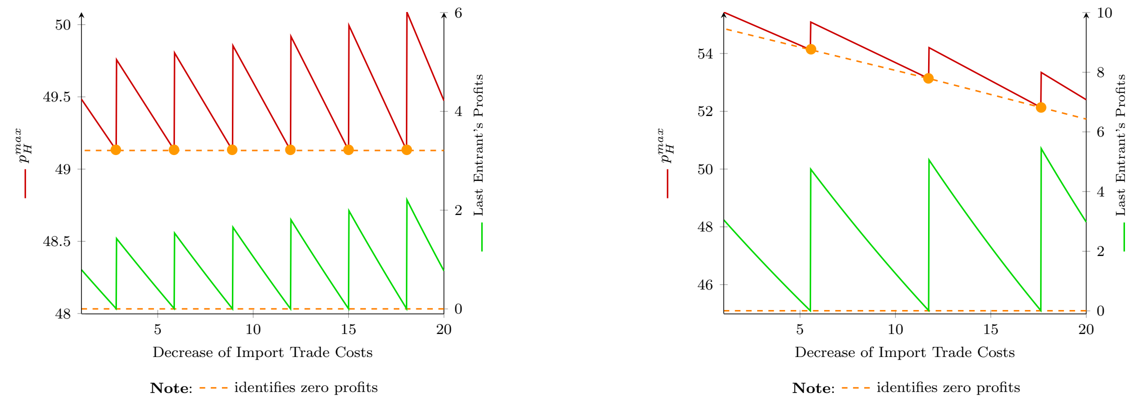
To illustrate the results of the different propositions, in Figure 6 we depict the impact of decreases in inward trade costs, where graphs differ according to whether lasts entrants are homogeneous or heterogeneous. Further details about this numerical example can be found in Appendix C.2.

The first aspect that can be appreciated by presenting the last entrant's profits and the choke price within the same figure is that the choke price follows the pattern of the last entrant's profits. This illustrates graphically that, by characterizing a small subset of firms (or, in the Cournot case, one firm), we are able to identify whether pro-competitive outcomes are created.

Regarding competitive effects, unlike monopolistic competition, graphically identifying cases where the effects stem exclusively from the import-competition channel is not immediately visible. It requires us to compare levels of the choke price where variations in trade costs are such that, before and after the trade shock, the last entrant earns the same profits. This ensures that the MIEs profits channel is shut.

Figure 6. *Variations in Inward Trade Costs in a Small Economy - Cournot*

(a) Homogeneous Last Entrants **(b)** Heterogeneous Last Entrants



In each graph of Figure 6, we identify one of these cases through the inclusion of dashed lines that identify when last entrants are having zero profits. Moreover, we utilize orange dots to indicate the equilibrium choke prices corresponding to those situations. In relation to this, it is worth remarking that zero profits constitutes only a particular case where the MIEs profits channel is shut, since all that is needed is that the profits of the last entrant in each equilibrium have the same value. Nonetheless, we have focused on the zero-profits case since it arises under the pervasive case where the integer number of firms is dispensed with.

By comparing the choke prices that keep the last entrant's profits at zero following a trade shock (i.e., the orange dots), in Figure 6a we can observe that the effects in Cournot are the same as in monopolistic competition. Specifically, when last entrants are homogeneous, the import-competition channel is inactive, which means that the choke price does not vary. On the other hand, when last entrants are heterogeneous as in Figure 6b, the comparison of choke prices corresponding to zero profits indicates that the import-competition channel is active and generates pro-competitive effects.

6.2 The Export-Opportunities Channel

Next, we study the conditions under which the export-opportunities channel is active. Since this case generates effects through entry of firms due to a better access to foreign markets, the proposition needs to be established by supposing that the least-profitable firms in each equilibrium are domestic.

Proposition 6.3

Consider a world economy with an arbitrary number of countries where H is a small economy. Suppose a model à la Cournot and let $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ be such that $\tau_{HF}^{**} < \tau_{HF}^{*}$ for some country $F \in \mathcal{C} \setminus \{H\}$. If in H the least profitable firms are domestic and the last entrant serves both H and F, then:

- *if there are extensive-margin adjustments of domestic firms and* $\overline{\pi}_H^{**} \leq \overline{\pi}_H^{*}$*, then*
  - $p_H^{\max **} < p_H^{\max *}$,
  - *for firms that are active in both equilibria,* $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, *and* $\mu_{HH}^{**}(c)$ *are lower relative to the equilibrium with* $\boldsymbol{\tau}^{*}$*, and*
  - *some inactive firms from* H *become active;*
- *if there are no extensive-margin adjustments of domestic firms, then there are no changes in* H.

From the proposition we can conclude that, in order for better export access to generate pro-competitive effects, two conditions are necessary: the last entrant has to be an exporter and at least one firm has to enter following the trade shock. As in monopolistic competition, this reflects how the mechanism intuitively operates: tougher domestic competitive conditions are created only if better export access induces some firms to enter and they serve the home market.

Notice that, in monopolistic competition, the conditions for activating the export-opportunities channel do not include that there has to be entry of a firm. This is because the assumption is always satisfied in monopolistic competition: smoothness assumptions determine that there are always extensive-margin changes after a trade shock. Consequently, the proposition establishes that, once we make assumptions in line with monopolistic competition, the Cournot model behaves similarly regarding the activation and deactivation of the export-opportunities channel. Thus, it only requires to determine whether last entrants are exporters.

In relation to this, given that the Cournot model resembles the Chaney/short-run Melitz setting, it also implies that the same remark as in that variant holds: export opportunities generate pro-competitive effects only if there is no selection into exporting, which conflicts with the extensive empirical evidence documenting that only a subset of firms export.

7 Applications of the Cournot Model

Utilizing the results from the Cournot model, next we consider several applications. The first one explores the consequences of assuming away the integer number of firms. The main conclusion we derive is that, under this assumption, the model behaves exactly as in monopolistic competition regarding the activation and deactivation of channels. This is a consequence that, under this assumption, the two additional conditions that always hold in monopolistic competition also prevail in Cournot: the MIEs profits channel is inactive and there are always extensive-margin adjustments.

After this, we study the case of restricted entry. This corresponds to a scenario where, following a trade shock, there are no extensive-margin adjustments. For this case, we conclude that the import-competition channel and export-opportunities channels are always inactive, while the MIEs profits channel is always active and generates pro-competitive effects.

Finally, we consider the case of unilateral liberalizations between two large countries. In particular, regarding our applications, the results indicate that, when the integer constraint is assumed away, the same mapping as in monopolistic competition between the MIEs' features and outcomes for a unilateral liberalization between two large countries can be established. In addition, under restricted entry, there are always pro-competitive effects.

In all the applications, we assume that the set of least-productive firms are domestic. This ensures the model is consistent with monopolistic competition, which enables us to make a comparison with it.

7.1 Assuming Away the Integer Constraint

Formally, assuming away the integer constraint means that the measure of the last entrant is a real number, $N_H \in \mathbb{R}_{++}$. This is usually incorporated into oligopoly models with the goal of making zero profits always hold, which facilitates the determination of results.[5]

Introducing this into the Cournot model has two implications. First, it ensures that $\overline{\pi}_H^{**} = \overline{\pi}_H^{*}$ and therefore, by construction, the MIEs profits channel is always inactive. Notice that, even though profits happen to be zero, what matters for this conclusion is simply that the last entrant in each equilibrium garners the same profits. On the other hand, the specific value which establishes the equality is inconsequential.

Second, ignoring the integer constraint entails that there are always changes at the extensive margin. This occurs because, after any trade shock, the real part of N_H always adjusts in order to ensure that $\overline{\pi}_H^{**} = \overline{\pi}_H^{*}$. This affects Proposition 6.3, by implying that it is only necessary to determine whether last entrants are exporters to know if the export-opportunities channel is active.

Both facts allows us to conclude that, *when the integer constraint is assumed away, the activation and deactivation of the import-competition channel and export-opportunities channel are determined by exactly the same conditions as in monopolistic competition.* Specifically, taking into account that the MIEs collapse to the last entrant and that ex-ante and ex-post features coincide, homogeneity of last entrants shuts the import-competition channel while their heterogeneity reactivates it; furthermore, the export-opportunities channel is active when last entrants are exporters and is shut if they are not.

[5] N_H can be defined formally as $\widetilde{N}_H + \delta$, where $\widetilde{N}_H \in \mathbb{N}$ is the integer part of N_H and $\delta \in \mathbb{R}_{++}$. The term $1 + \delta$ can be interpreted as the measure of the last entrant. After any trade shock and given the value $\widetilde{N}_H$ of the new equilibrium, assuming the integer number of firms away means that δ always adjusts to ensure that the mass of the last entrant is consistent with zero profits.

7.2 Restricted Entry

The case of restricted entry can be analyzed as a special case of our free-entry framework. This arises when, following a trade shock, there are no extensive-margin adjustments. This could be ensured by, for instance, supposing large differences between active and inactive firms regarding productivity, such that no firm enters or exits after a trade shock.

Regarding the conditions for the activation and deactivation of channels, the following can be established. First, given that there are no extensive-margin adjustments, *the export-opportunities channel is always inactive*. This follows by applying Proposition 6.3.

Second, by definition of a restricted-entry scenario, the last entrant is the same before and after any trade shock. Thus, trivially, last entrants are homogeneous across equilibria. Moreover, in Appendix A.4 we prove that, when there are no extensive-margin adjustments of domestic firms and the last entrant under $\boldsymbol{\tau}^*$ is domestic, $\overline{\pi}_H^{**} < \overline{\pi}_H^{*}$ is always satisfied after a decrease in inward trade barriers in H. Both facts determine that *the import-competition channel is inactive while the MIEs profits channel is active*. Therefore, by applying Proposition 6.1, reductions in inward trade barriers always lead to pro-competitive effects.[6]

To illustrate the implications of the last fact, we can make use of any of the graphs in Figure 6, where a reduction in import trade costs is considered. Given some trade costs at the initial equilibrium, the application of the results requires that we bound the variation in trade costs such that there are no changes at the extensive margin.[7] Graphically, regarding the effect on the choke price, this corresponds to any continuous segment between two consecutive orange dots. Along that range of variations in trade costs, the number of active firms does not vary, which explains why there are no discontinuous jumps. Moreover, the negative slope along that segment indicates that the trade shock reduces the last entrant's profits, thus demonstrating that the MIEs profits channel is active and creates pro-competitive effects.

7.3 Unilateral Liberalizations between Two Large Countries

Next, as in Section 4.2 for monopolistic competition, we consider a scenario with two large countries, H and F, and study the effects on each of them following a reduction in inward trade barriers in H. Recall that, when there is a trade shock between two large countries, each country is affected simultaneously by the channel operating in a small economy *and* changes in its export conditions. This is a consequence that changes in the domestic conditions of H

[6] Additionally, in Appendix A.4, we prove a more general result which indicates that, under restricted entry, $p_H^{\max **} < p_H^{\max *}$ even if the last entrant under $\boldsymbol{\tau}^*$ is not domestic.

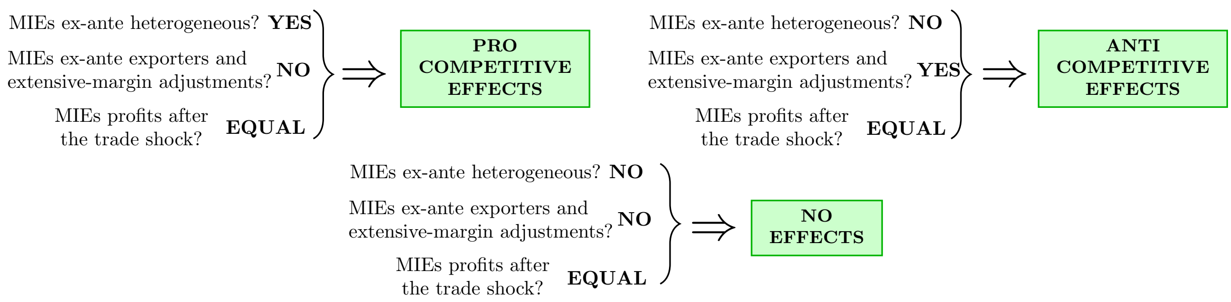
[7] Notice that, either Figure 6a or Figure 6b can be used, even though they refer to cases where last entrants are labeled either homogeneous or heterogeneous. This is because, once that the variation in trade costs is bounded, we always move along a segment where the last entrant is the same before and after the trade shock.

have an impact on the competitive conditions of F (i.e., $p_F^{\max *}$). Our goal is to compare results relative to the monopolistic-competition model and, also, rationalize outcomes obtained in the literature.

We begin by considering a scenario where the integer number of firms is dispensed with. Under this assumption, we have showed that exactly the same conditions on MIEs (i.e., the last entrant) as in monopolistic competition activate and deactivate each channel. This follows because MIEs always break even under both market structures and, so, the MIEs profits channel is inactive. Consequently, *when the integer constraint is ignored, the analysis of a unilateral liberalization between two large countries is exactly the same as in monopolistic competition.* Thus, the impact on $p_F^{\max *}$ and $p_H^{\max *}$ can be decomposed as in (6) and (7), respectively. In addition, the same conditions on MIEs determine the sign of each term, which makes it possible to determine how domestic decisions are impacted.

Focusing on the effects on H, as we did for monopolistic competition, in Figure 7 we establish conditions on the last entrant that ensure specific outcomes in H. This allows us to rationalize the outcomes in the studies references in the introduction (see Figure 2). The conditions are stated for the general case (i.e., irrespective of whether the integer constraint is assumed away) and, with the goal of establishing a direct link with monopolistic competition, expressed in terms of MIEs and ex-ante features.

Figure 7. *Unilateral Liberalizations with Two Large Countries in Cournot: Conditions for Effects in H*



Note: In the case of Cournot, MIEs comprise only one firm (the last entrant) and ex-ante and ex-post features coincide.

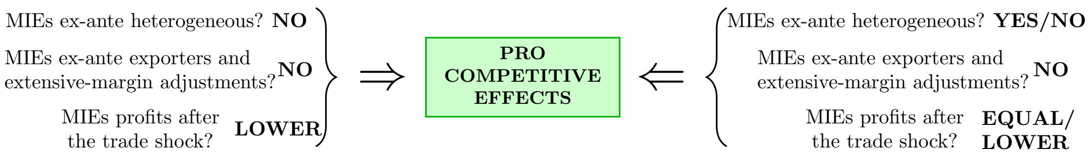
The conclusions from Figure 7 are threefold. First, when the integer constraint is assumed away, we already established that there are always extensive-margin adjustments and that the last entrant in each equilibrium earns the same profits. Incorporating this into the analysis of a unilateral liberalization between two large countries, we are able to conclude that *the mapping between features of the MIEs and outcomes in Figure 7 is identical to that in Figure 5 for monopolistic competition.*

Second, some of the features of MIEs that always hold in monopolistic competition do not necessarily prevail in the Cournot model. Specifically, in the Cournot model, it is not necessarily the case that there are changes at the extensive margin and that MIEs have the same profits before and after a trade shock. The assumptions on last entrants in Figure 7 are generalized to incorporate this feature. In this way, for instance, we can conclude that the same outcomes emerge when last entrants are having same profits, irrespective of whether they satisfy zero profits.

The third conclusion is related to the existence of anti-competitive effects. This has been frequently obtained in the literature, as the studies references in the introduction reveal. In particular, a common set of assumptions in the oligopoly literature of the 1980s consisted of homogeneity of firms and assuming away the integer constraint. Figure 7 indicates that these assumptions lead to anti-competitive effects. In terms of channels, it establishes that the same mechanisms as in the Krugman and Melitz settings operate to determine the outcome: the import-competition and MIEs profits channels are inactive, while export-related channels are active. Therefore, the result is explained by the worse export conditions in H which, in turn, are created by the tougher domestic conditions in F due to the better export opportunities there.

Depending on the circumstances, we might have reasons to believe that an anti-competitive outcome does not capture the situation under analysis. In fact, in the literature, the existence of anti-competitive effects after a unilateral liberalization has been referred to as the Metzler paradox. Due to this, in Figure 8, we indicate a set of sufficient conditions on last entrants that ensure the existence of pro-competitive effects. They are based on the deactivation of export-related channels, which determines that there are no feedback effects between countries. We proceed to explain their implications.

Figure 8. *Unilateral Liberalizations with Two Large Countries: Conditions for Pro-Competitive Effects in H*



Note: In the case of Cournot, MIEs comprise only one firm (the last entrant) and ex-ante and ex-post features coincide. The conditions on the right-hand side allow for any combination of answers, except for simultaneously "No" for ex-ante homogeneity and "Equal" for profits, in which case there are null competitive effects.

On the one hand, the set of the conditions given on the left of the graph has restricted entry as a special case. Consequently, in line with the studies in the introduction, this establishes that *restricted entry guarantees the existence of pro-competitive effects.*

On the other hand, the conditions stated on the right of the graph provide more general results under the same principle: once that export-related channels are deactivated, there are no feedback effects and the impact in H is the same as the one arising when H is a small country. Thus, Propositions 6.1 and 6.2 apply, which determines that anti-competitive effects can be ruled out and, except for the case where both last entrants are homogeneous and have the same profits after the unilateral liberalization, pro-competitive effects emerge.

8 Conclusion

This paper highlights the role of marginal entrants in identifying trade-liberalization outcomes in models of imperfect competition. We showed that the characterization of marginal entrants determines whether a trade shock leads to pro-competitive effects in standard versions of monopolistic competition (i.e., Krugman, Melitz, and Chaney) and oligopoly (Cournot under free and restricted entry). In addition, once marginal entrants are characterized equivalently across models, they generate the same qualitative outcomes after a trade shock.

We think our findings can be useful for informing model choice, since they make it possible to anticipate particular outcomes generated by each model. Additionally, our decomposition of results into channels may be relevant for researchers that analyze the impact from promoting import competition or better export conditions in isolation, rather than a trade liberalization. In particular, our conclusions indicate which settings are more suitable for each of these policies if the goal is to analyze pro-competitive effects. Specifically, a setting à la Melitz is more appropriate for capturing mechanisms operating through export-related channels, while Chaney/short-run Melitz is better suited to reflect the effects of tougher import competition.

Finally, for future work, the role of marginal entrants may be worth investigating in other classes of models. And, while we have focused on pro-competitive effects, it might be that they play a role in the identification of other outcomes across imperfect-competition models.

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Appendices—For Online Publication

The structure of the appendices is as follows. In Appendix A we include derivations for some of the expressions in the main part of the paper and the proofs of the propositions. The remaining appendices include additional results. In Appendix B, we study the magnitude of the effects coming through the import-competition channel when it is active. Appendix C shows some numerical exercises that illustrated different results in the main part of the paper. In Appendix D, we show that the propositions for monopolistic competition hold under the assumption of iceberg trade costs. Finally, in Appendix E, we outline some conditions for existence and uniqueness of the equilibrium.

A Derivations and Proofs

A.1 Monopolistic Competition

The framework for the model under monopolistic competition is that of Section 2.3. For some of the proofs, it is necessary to distinguish between the degenerate and non-degenerate variants as defined in that section. This is because the description of the density of active firms with a specific marginal cost is different in each case. Also, in some proofs, we exploit that the limiting case of the non-degenerate setup coincides with the limiting case of the degenerate distribution.

Regarding notation, we keep indicating the equilibrium values of any variable under $\boldsymbol{\tau}^*$ or $\boldsymbol{\tau}^{**}$ by a superscript $*$ and $**$, respectively, and refer to some generic equilibrium by using $*$ as superscript. Furthermore, regarding trade costs, we define the import trade costs of H by $\boldsymbol{\tau}_{\cdot H} := (\tau_{jH})_{j\in\mathcal{C}\backslash\{H\}}$ and the export trade costs of H by $\boldsymbol{\tau}_{H\cdot} := (\tau_{Hj})_{j\in\mathcal{C}\backslash\{H\}}$.

For each firm from country i belonging to group θ, we define its minimum marginal cost to serve j by $c_{ij}^{\theta*} := \min\left\{c_{ij}^*, \overline{c}_i^\theta\right\}$. This variable captures that $c_{ij}^{\theta*} = \overline{c}_i^\theta$ when all the firms belonging to θ are active in j. Also, for group θ and equilibrium $*$ we define $G_{ij}^{\theta*} := G_i^\theta\left(c_{ij}^{\theta*}\right)$ and $g_{ij}^{\theta*} := g_i^\theta\left(c_{ij}^{\theta*}\right)$ where, to incorporate the limiting cases where the distribution concentrates all its probability mass at one point, we think of the density function as given by the Dirac delta function.

Moreover, $\mathbb{C}_{ij}^{\tau,\theta*} := \int_{\underline{c}_i^\theta}^{c_{ij}^{\theta*}} c_{ij}^\tau \, \mathrm{d}G_i^\theta(c)$ and, when we refer to a domestic firm from i, we omit the superscript τ and refer to it as $\mathbb{C}_{ii}^{\theta*}$. Finally, if all firms from i belonging to a group θ are active in j, we simply use the notation $\mathbb{C}_{ij}^\theta$ to emphasize that $c_{ij}^{\theta*} = \overline{c}_i^\theta$.

Lemma A.1. *Given a mass of incumbents M_i^θ and $i \in \mathcal{C}$, the density of active firms belonging to θ with marginal costs c is $M_i^\theta g_i^\theta(c)$ in the non-degenerate variant, and $\overline{M}_i^\theta g_i^\theta(c)$ in the degenerate variant of monopolistic competition.*

Proof of Lemma A.1. For both variants, the density of firms from i belonging to θ that are active in j and have marginal costs c is $M_{ij}^\theta \frac{g_i^\theta(c)}{G_{ij}^{\theta*}}$. In the case of the non-degenerate variant, we know that $M_{ij}^\theta = M_i^\theta G_{ij}^{\theta*}$ and, so, the result follows. Regarding the degenerate variant, G_{ij}^θ describes the distribution of the mass $\overline{M}_i^\theta$ of firms and, out of this, only a mass $M_{ij}^\theta = \overline{M}_i^\theta G_{ij}^{\theta*}$ is active in j. Therefore, the result follows. ■

Next, we characterize the marginal-cost cutoff for serving each market. We do it by distinguishing between the set of countries that are served by the least-productive firms that are active in the domestic market.

Lemma A.2. *In either the non-degenerate or degenerate variant of monopolistic competition, suppose that the least-productive firms from $i \in \mathcal{C}$ that are active in at least one country only serve their home market. Then, the cutoff to serve $j \in \mathcal{C}\backslash\{i\}$ is given by*

$$c_{ij}^*\left(p_j^{\max}; \tau_{ij}\right) := p_j^{\max} - \tau_{ij} - \xi_{ij}, \tag{ZCP}$$

where $\xi_{ij} := 2\sqrt{\gamma_j f_{ij}}$. For the non-degenerate variant, the same condition applies for $j = i$. For the degenerate variant, the condition for $j = i$ is also (ZCP) but with $\xi_{ii} := 2\sqrt{\gamma_i\left(f_{ii} + F_i^E\right)}$. Instead, if for the degenerate *variant the least-productive firms from i have non-negative optimal profits in its domestic country and a set $\mathcal{F}$ of foreign countries, then c^*_{ij} for $j \in \{i\} \cup \mathcal{F}$ is given by some c^*_{ii} that satisfies*

$$\frac{\left(p_i^{\max *} - c^*_{ii}\right)^2}{4\gamma_i} + \sum_{k\in\mathcal{F}} \frac{\left(p_k^{\max *} - c^*_{ii} - \tau_{ik}\right)^2}{4\gamma_k} = F_i^E + f_{ii} + \sum_{k\in\mathcal{F}} f_{ik}. \tag{ZCP2}$$

Proof of Lemma A.2. Given optimal profits (4), the value c^*_{ij} which is consistent with zero profits satisfies $\frac{\left[p_j^{\max} - \left(c^*_{ij} + \tau_{ij}\right)\right]^2}{4\gamma_j} = f_{ij}$. Working out the expression, we obtain (ZCP). For the case of the non-degenerate variant, only a strict subset of firms that pay the entry cost become active in at least one market and they exclusively serve their domestic market. Hence, (ZCP) also applies to $i = j$. In the degenerate variant, we know that firms become active in the industry as long as they have nonnegative expected profits. Thus, regarding the case where the least-productive firms only serve their home market, since D_i^ω is degenerate, c^*_{ii} is the value that satisfies

$$\frac{\left(p_i^{\max} - c^*_{ii}\right)^2}{4\gamma_i} - f_{ii} = F_i^E,$$

which determines a function as in (ZCP) but with $\xi_{ii} := 2\sqrt{\gamma_i\left(f_{ii} + F_i^E\right)}$. Regarding the degenerate variant when the least-productive firms from i are serving i and a set of foreign countries $\mathcal{F}$, (ZCP2) is obtained by applying the same logic but taking into account that now the firm serves all countries $j \in \{i\} \cup \mathcal{F}$. ■

Next, we characterize the equilibrium condition at the market stage, i.e., for a given set of firms that paid the entry cost.

Lemma A.3. *In either the non-degenerate or degenerate variant of monopolistic competition, the equilibrium at the market stage in $j \in \mathcal{C}$ is given by a $p_j^{\max *}$ which satisfies*

$$\sum_{i\in\mathcal{C}} \Phi_{ij}\left(p_j^{\max *};\tau_{ij}\right) + 2\beta_j p_j^{\max *} = 2\beta_j\alpha_j. \tag{MS}$$

Moreover, if in country i the least-productive firms serving j belong to $\mathcal{E}$, then $\Phi_{ij}\left(p_j^{\max *};\tau_{ij}\right) := \Phi_{ij}^{\mathcal{I}}\left(p_j^{\max *};\tau_{ij}\right) + \Phi_{ij}^{\mathcal{E}}\left(p_j^{\max *};\tau_{ij}\right)$ with $\Phi_{ij}^{\theta} := M_i^{\theta}\left(G_{ij}^{\theta *} p_j^{\max *} - \mathbb{C}_{ij}^{\tau,\theta *}\right)$ for the non-degenerate variant and the same expression for the degenerate variant but substituting M_i^θ by $\overline{M}_i^\theta$.

Proof of Lemma A.3. In any of the variants, the equilibrium at the market stage in j requires that $p_j^{\max *}$ is a fixed point of (1). Define $\mathbb{P}_{ij} := \int_{\omega\in\Omega_{ij}} p_{ij}(\omega)\,\mathrm{d}\omega$. As we show below, by evaluating the expressions in equilibrium in any of these models, we can obtain functions $\mathbb{P}_j\left(p_j^{\max};\boldsymbol{\tau}_{\cdot j}\right) := \sum_{i\in\mathcal{C}} \mathbb{P}_{ij}\left(p_j^{\max};\tau_{ij}\right)$ and $M_j\left(p_j^{\max};\boldsymbol{\tau}_{\cdot j}\right) := \sum_{i\in\mathcal{C}} M_{ij}\left(p_j^{\max};\tau_{ij}\right)$. Thus, the equilibrium condition in j is given by a value $p_j^{\max *}$ such that

$$p_j^{\max *} = \frac{\alpha_j\beta_j + \mathbb{P}_j\left(p_j^{\max *};\boldsymbol{\tau}_{\cdot j}\right)}{\beta_j + M_j\left(p_j^{\max *};\boldsymbol{\tau}_{\cdot j}\right)}.$$

Working out the expression, this becomes

$$p_j^{\max *}\sum_{i\in\mathcal{C}} M_{ij}\left(p_j^{\max *};\tau_{ij}\right) - \sum_{i\in\mathcal{C}} \mathbb{P}_{ij}\left(p_j^{\max *};\tau_{ij}\right) + p_j^{\max *}\beta_j = \alpha_j\beta_j,$$

and, so, by defining

$$\frac{\Phi_{ij}\left(p_j^{\max *};\tau_{ij}\right)}{2} := p_j^{\max *} M_{ij}\left(p_j^{\max *};\tau_{ij}\right) - \mathbb{P}_{ij}\left(p_j^{\max *};\tau_{ij}\right), \tag{11}$$

(MS) is obtained.

Now, consider the non-degenerate variant. Suppose that in i the least-productive firms that serve j belong to $\mathcal{E}$. For given $M_i^{\mathcal{I}}$ and $M_i^{\mathcal{E}}$, then,

$$\begin{aligned}\mathbb{P}_{ij}\left(p_j^{\max};\tau_{ij}\right) &:= M_i^{\mathcal{I}}\int_{\underline{c}_i^{\mathcal{I}}}^{\overline{c}_i^{\mathcal{I}}} p_{ij}\left(p_j^{\max};c,\tau_{ij}\right)\mathrm{d}G_i^{\mathcal{I}}(c) + M_i^{\mathcal{E}}\int_{\underline{c}_i^{\mathcal{E}}}^{c_{ij}^*} p_{ij}\left(p_j^{\max};c,\tau_{ij}\right)\mathrm{d}G_i^{\mathcal{E}}(c),\\ M_{ij}\left(p_j^{\max};\tau_{ij}\right) &:= M_i^{\mathcal{I}} + M_i^{\mathcal{E}}G_{ij}^{\mathcal{E}*}.\end{aligned}$$

Using these definitions, optimal prices (2), and the characterization of active firms, we can express $\Phi_{ij}=\Phi_{ij}^{\mathcal{I}}+\Phi_{ij}^{\mathcal{E}}$ as

$$\Phi_{ij}^{\theta}\left(p_j^{\max *};\tau_{ij}\right) := M_i^{\theta}\left(G_{ij}^{\theta *}p_j^{\max *} - \mathbb{C}_{ij}^{\tau,\theta *}\right). \tag{12}$$

The same derivation applies to the limiting case where the heterogeneity in $\mathcal{E}$ is negligible.

As for the degenerate variant, the same expression (11) holds, and (12) is satisfied by substituting M_i^{θ} by $\overline{M}_i^{\theta}$. ■

Lemma A.4. *$p_{ij}\left(p_j^{\max};c_{ij}^{\tau}\right)$, $q_{ij}\left(p_j^{\max};c_{ij}^{\tau}\right)$, c_{ij}^* given by either (ZCP) or (ZCP2), $m_{ij}\left(p_j^{\max};c_{ij}^{\tau}\right)$, and $\mu_{ij}\left(p_j^{\max};c_{ij}^{\tau}\right)$ are increasing in $p_j^{\max}$.*

Proof of Lemma A.4. Taking derivatives of each function: $\frac{\partial p_{ij}(\cdot)}{\partial p_j^{\max}}=\frac{1}{2}$, $\frac{\partial q_{ij}(\cdot)}{\partial p_j^{\max}}=\frac{1}{2\gamma_j}$, $\frac{\partial m_{ij}(\cdot)}{\partial p_j^{\max}}=\frac{1}{2c_{ij}^{\tau}}$, and $\frac{\partial \mu_{ij}(\cdot)}{\partial p_j^{\max}}=\frac{1}{2}$. If c_{ij}^* is given by (ZCP) then $\frac{\partial c_{ij}^*(\cdot)}{\partial p_j^{\max}}=1$. In case c_{ij}^* is given by (ZCP2), then $\frac{\partial c_{ij}^*(\cdot)}{\partial p_j^{\max *}}=\left(\frac{p_j^{\max *}-c_{ii}^*-\tau_{ij}}{2\gamma_j}\right)\left(\frac{p_i^{\max *}-c_{ii}^*}{\gamma_i}+\sum_{k\in\mathcal{F}}\frac{p_k^{\max *}-c_{ii}^*-\tau_{ik}}{2\gamma_k}\right)^{-1}>0$. ■

Lemma A.5. *c_{ij}^* given by either (ZCP) or (ZCP2) is decreasing in τ_{ij}.*

Proof of Lemma A.5. If c_{ij}^* is given by (ZCP) then $\frac{\partial c_{ij}^*(\cdot)}{\partial \tau_{ij}}=-1$. If c_{ij}^* is given by (ZCP2), then $\frac{\partial c_{ij}^*(\cdot)}{\partial \tau_{ij}}=-\left(\frac{p_j^{\max *}-c_{ii}^*-\tau_{ij}}{2\gamma_j}\right)\left(\frac{p_i^{\max *}-c_{ii}^*}{\gamma_i}+\sum_{k\in\mathcal{F}}\frac{p_k^{\max *}-c_{ii}^*-\tau_{ik}}{2\gamma_k}\right)^{-1}<0$. ■

Lemma A.6. *Suppose that the least-productive firms from i that are active in j belong to $\mathcal{E}$. Then, at the market stage of either the non-degenerate or degenerate variant of monopolistic competition, $\frac{\partial\Phi_{ij}^{\theta *}}{\partial p_j^{\max *}}>0$ and $\frac{\partial\Phi_{ij}^{\theta *}}{\partial \tau_{ij}}<0$ for $\theta\in\{\mathcal{E},\mathcal{I}\}$. In addition, $\frac{\partial\Phi_{ij}^{\theta *}}{\partial p_j^{\max *}}=-\frac{\partial\Phi_{ij}^{\theta *}}{\partial \tau_{ij}}$.*

Proof of Lemma A.6. At the market stage, $M_i^{\mathcal{I}}$ and $M_i^{\mathcal{E}}$ are given. We begin by establishing some additional calculations regarding $\mathbb{C}_{ij}^{\tau,\theta *}:=\int_{\underline{c}_i^{\theta}}^{c_{ij}^{\theta *}} c_{ij}^{\tau}\,\mathrm{d}G_i^{\theta}(c)$. If $c_{ij}^{\theta *}=\overline{c}_i^{\theta}$, then all firms in θ are active and, so, $\frac{\partial\mathbb{C}_{ij}^{\tau,\theta *}}{\partial p_j^{\max *}}=0$ and $\frac{\partial\mathbb{C}_{ij}^{\tau,\theta *}}{\partial \tau_{ij}}=\int_{\underline{c}_i^{\theta}}^{\overline{c}_i^{\theta}}\frac{\partial c_{ij}^{\tau}}{\partial\tau_{ij}}\mathrm{d}G_i^{\theta}(c)=1>0$. If $c_{ij}^{\theta *}=c_{ij}^*$, then $\frac{\partial\mathbb{C}_{ij}^{\tau,\theta *}}{\partial p_j^{\max *}}=\left(c_{ij}^*+\tau_{ij}\right)g_{ij}^{\theta *}\frac{\partial c_{ij}^*}{\partial p_j^{\max *}}$ and $\frac{\partial\mathbb{C}_{ij}^{\tau,\theta *}}{\partial \tau_{ij}}=\left(c_{ij}^*+\tau_{ij}\right)g_{ij}^{\theta *}\frac{\partial c_{ij}^*}{\partial \tau_{ij}}+G_{ij}^{\theta *}$.

As for $\Phi_{ij}^{\theta *}$, in the non-degenerate variant, if $c_{ij}^{\theta *}=\overline{c}_i^{\theta}$ then $\Phi_{ij}^{\theta *}:=M_i^{\theta}\left(p_j^{\max *}-\mathbb{C}_{ij}^{\tau,\theta *}\right)$ and, so, $\frac{\partial\Phi_{ij}^{\theta *}}{\partial p_j^{\max *}}=-\frac{\partial\Phi_{ij}^{\theta *}}{\partial \tau_{ij}}=M_i^{\theta}$. The same result is obtained for the degenerate variant by substituting M_i^{θ} with $\overline{M}_i^{\theta}$. Consider now $c_{ij}^{\theta *}=c_{ij}^*$. For the non-degenerate variant, $\Phi_{ij}^{\theta *}:=M_i^{\theta}\left(G_{ij}^{\theta *}p_j^{\max *}-\mathbb{C}_{ij}^{\tau,\theta *}\right)$. Performing the calculations, $\frac{\partial\Phi_{ij}^{\theta *}}{\partial p_j^{\max *}}=-\frac{\partial\Phi_{ij}^{\theta *}}{\partial \tau_{ij}}=M_i^{\theta}\left(G_{ij}^{\theta *}+g_{ij}^{\theta *}\xi_{ij}\right)$. Regarding the degenerate variant, the same result holds when (ZCP) is satisfied if M_i^{θ} is substituted by $\overline{M}_i^{\theta}$. Moreover, when (ZCP2) holds, $\frac{\partial\Phi_{ij}^{\theta *}}{\partial p_j^{\max *}}=\overline{M}_i^{\theta}\left(G_{ij}^{\theta *}+g_{ij}^{\theta *}\frac{\partial c_{ii}^*}{\partial p_j^{\max *}}\left(p_j^{\max *}-c_{ii}^*-\tau_{ij}\right)\right)>0$ which uses that $\frac{\partial c_{ii}^*}{\partial p_j^{\max *}}>0$ by Lemma A.4. Also, $\frac{\partial\Phi_{ij}^{\theta *}}{\partial \tau_{ij}}=\overline{M}_i^{\theta}\left(-G_{ij}^{\theta *}+g_{ij}^{\theta *}\frac{\partial c_{ii}^*}{\partial \tau_{ij}}\left(p_j^{\max *}-c_{ii}^*-\tau_{ij}\right)\right)<0$ since $\frac{\partial c_{ii}^*}{\partial \tau_{ij}}<0$ by Lemma A.5. By the same lemmas, it also follows that $\frac{\partial c_{ii}^*}{\partial p_j^{\max *}}=-\frac{\partial c_{ii}^*}{\partial \tau_{ij}}$, which implies that $\frac{\partial\Phi_{ij}^{\theta *}}{\partial p_j^{\max *}}=-\frac{\partial\Phi_{ij}^{\theta *}}{\partial \tau_{ij}}$. ■

Lemma A.6 implies that the same signs for the effects of $p_j^{\max *}$ and τ_{ij} on $\Phi_{ij}^{\theta *}$ hold irrespective of the variant of the model considered. This explains why, in the main part of the paper, we focused on country H without describing the market structure for the rest of the countries. Specifically, by defining, $\Phi_{-H}\left(p_H^{\max};\boldsymbol{\tau}_{\cdot H}\right):=\sum_{j\in\mathcal{C}\setminus\{H\}}\Phi_{jH}\left(p_H^{\max};\tau_{jH}\right)$, next we prove all the propositions by using that $\frac{\partial\Phi_{-H}^*}{\partial p_H^{\max *}}>0$ and $\frac{\partial\Phi_{-H}^*}{\partial \tau_{jH}}<0$ for any $j\in\mathcal{C}\setminus\{H\}$.

Before proving the propositions concerning the import-competition channel, we provide some bounds for $\boldsymbol{\tau}_{\cdot H}^{**}$ such that, when the MIEs belong to $\mathcal{E}$ under $\boldsymbol{\tau}_{\cdot H}^{*}$, they also belong to $\mathcal{E}$ under $\boldsymbol{\tau}_{\cdot H}^{**}$. They are based on the cases and assumptions where the main propositions apply.

Lemma A.7. *Consider the scenarios of Propositions 3.1 and 3.2. If, in the non-degenerate variant of monopolistic competition or the limiting case,*

$$\overline{M}_H^{\mathcal{I}}\left(p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+\Phi_{-H}\left(p_H^{\max *};\boldsymbol{\tau}_{\cdot H}^{**}\right)+2\beta_H p_H^{\max *}<2\beta_H\alpha_H \tag{13}$$

holds and, in the degenerate variant,

$$\overline{M}_H^{\mathcal{I}}\left(\underline{c}_H^{\mathcal{E}}+\xi_{HH}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+\Phi_{-H}\left(\underline{c}_H^{\mathcal{E}}+\xi_{HH};\boldsymbol{\tau}_{\cdot H}^{**}\right)+2\beta_H\left(\underline{c}_H^{\mathcal{E}}+\xi_{HH}\right)<2\beta_H\alpha_H \tag{14}$$

holds, then the MIEs in the equilibrium with $\boldsymbol{\tau}_{\cdot H}^{**}$ belong to $\mathcal{E}$.

Proof of Lemma A.7. Consider the non-degenerate variant. Condition (MS) in H for trade costs $\boldsymbol{\tau}_{\cdot H}^{*}$ can be expressed as:

$$\overline{M}_H^{\mathcal{I}}\left(p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+M_H^{\mathcal{E}*}\left(G_{HH}^{\mathcal{E}*}p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{E}*}\right)+\Phi_{-H}\left(p_H^{\max *};\boldsymbol{\tau}_{\cdot H}^{*}\right)+2\beta_H p_H^{\max *}=2\beta_H\alpha_H. \tag{15}$$

We also know that $\frac{\partial\Phi_{-H}^{*}}{\partial\tau_{iH}}<0$ for $i\in\mathcal{C}\backslash\{H\}$ by Lemma A.6 and, so,

$$\overline{M}_H^{\mathcal{I}}\left(p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+M_H^{\mathcal{E}*}\left(G_{HH}^{\mathcal{E}*}p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{E}*}\right)+\Phi_{-H}\left(p_H^{\max *};\boldsymbol{\tau}_{\cdot H}^{**}\right)+2\beta_H p_H^{\max *}>2\beta_H\alpha_H. \tag{16}$$

Next, we prove that $c_{HH}^{**}\in\left[\underline{c}_H^{\mathcal{E}},\overline{c}_H^{\mathcal{E}}\right]$. We do this by showing that both $c_{HH}^{**}>\overline{c}_H^{\mathcal{E}}$ and $c_{HH}^{**}<\underline{c}_H^{\mathcal{E}}$ lead us to a contradiction. This also proves the result for the limiting case.

Suppose that $c_{HH}^{**}>\overline{c}_H^{\mathcal{E}}$. Condition (MS) in H with trade costs $\boldsymbol{\tau}_{\cdot H}^{**}$ becomes

$$\overline{M}_H^{\mathcal{I}}\left(p_H^{\max **}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+\overline{M}_H^{\mathcal{E}}\left(p_H^{\max **}-\mathbb{C}_{HH}^{\mathcal{E}**}\right)+\Phi_{HH}^{\mathcal{N}}+\Phi_{-H}\left(p_H^{\max **};\boldsymbol{\tau}_{\cdot H}^{**}\right)+2\beta_H p_H^{\max **}=2\beta_H\alpha_H, \tag{17}$$

where $\Phi_{HH}^{\mathcal{N}}>0$ is the additional term of Φ_{HH} corresponding to firms in group $\mathcal{N}$ that become active. Suppose that (ZCP) holds, since otherwise $p_H^{\max **}>p_H^{\max *}$ trivially and the result would follow too. Therefore, $c_{HH}^{*}=p_H^{\max *}-\xi_{HH}$ and $c_{HH}^{**}=p_H^{\max **}-\xi_{HH}$. Since $c_{HH}^{**}>\overline{c}_H^{\mathcal{E}}$ and $c_{HH}^{*}\in\left[\underline{c}_H^{\mathcal{E}},\overline{c}_H^{\mathcal{E}}\right]$, it follows that $p_H^{\max **}>p_H^{\max *}$. In addition, $\overline{M}_H^{\mathcal{E}}\geq M_H^{\mathcal{E}*}$ by definition, and $\frac{\partial\Phi_{-H}^{*}}{\partial p_H^{\max *}}>0$ and $\frac{\partial\Phi_{-H}^{*}}{\partial\tau_{jH}}<0$ for any $j\in\mathcal{C}\backslash\{H\}$ by Lemma A.6. Thus, the left-hand side (LHS) of (17) is greater than the LHS of (15). This implies that (17) cannot hold with an equality, which is a contradiction.

Now, towards a contradiction, suppose that $c_{HH}^{**}<\underline{c}_H^{\mathcal{E}}$. In the equilibrium with trade costs $\boldsymbol{\tau}_{\cdot H}^{**}$, given that $c_{HH}^{**}<\underline{c}_H^{\mathcal{E}}$, (MS) becomes:

$$M_H^{\mathcal{I}**}\left(G_{HH}^{\mathcal{I}**}p_H^{\max **}-\mathbb{C}_{HH}^{\mathcal{I}**}\right)+\Phi_{-H}\left(p_H^{\max **};\boldsymbol{\tau}_{\cdot H}^{**}\right)+2\beta_H p_H^{\max **}=2\beta_H\alpha_H,$$

and, so, combining this expression with (13),

$$\begin{aligned}\overline{M}_H^{\mathcal{I}}\left(p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+\Phi_{-H}\left(p_H^{\max *};\boldsymbol{\tau}_{\cdot H}^{**}\right)+2\beta_H p_H^{\max *}<\\ M_H^{\mathcal{I}**}\left(G_{HH}^{\mathcal{I}**}p_H^{\max **}-\mathbb{C}_{HH}^{\mathcal{I}**}\right)+\Phi_{-H}\left(p_H^{\max **};\boldsymbol{\tau}_{\cdot H}^{**}\right)+2\beta_H p_H^{\max **}.\end{aligned} \tag{18}$$

Given $c_{HH}^{**}<\underline{c}_H^{\mathcal{E}}$ and $c_{HH}^{**}=p_H^{\max **}-\xi_{HH}$, then $p_H^{\max **}<\underline{c}_H^{\mathcal{E}}+\xi_{HH}$. Since MIEs belong to $\mathcal{E}$ when trade costs are $\boldsymbol{\tau}_{\cdot H}^{*}$, then $\underline{c}_H^{\mathcal{E}}+\xi_{HH}<p_H^{\max *}$ which implies that $p_H^{\max **}<p_H^{\max *}$. Also, by Lemma A.6, $\frac{\partial\Phi_{-H}^{*}}{\partial p_H^{\max *}}>0$ and $\left(G_{HH}^{\theta *}p_H^{\max *}-\mathbb{C}_{HH}^{\theta *}\right)$ is increasing in $p_H^{\max *}$. Therefore, the LHS of (18) is greater than its right-hand side (RHS), which is a contradiction.

Consider now the degenerate variant. Condition (MS) with trade costs $\boldsymbol{\tau}_{\cdot H}^{*}$ is

$$\overline{M}_H^{\mathcal{I}}\left(p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+\overline{M}_H^{\mathcal{E}}\left(G_{HH}^{\mathcal{E}*}p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{E}*}\right)+\Phi_{-H}\left(p_H^{\max *};\boldsymbol{\tau}_{\cdot H}^{*}\right)+2\beta_H p_H^{\max *}=2\beta_H\alpha_H,$$

and, since $\frac{\partial \Phi^*_{-H}}{\partial \tau_{jH}} < 0$ for any $j \in \mathcal{C} \backslash \{H\}$ by Lemma A.6, then

$$\overline{M}_H^{\mathcal{I}} \left(p_H^{\max *} - \mathbb{C}_{HH}^{\mathcal{I}}\right) + \overline{M}_H^{\mathcal{E}} \left(G_{HH}^{\mathcal{E}*} p_H^{\max *} - \mathbb{C}_{HH}^{\mathcal{E}*}\right) + \Phi_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^{**}\right) + 2\beta_H p_H^{\max *} > 2\beta_H \alpha_H. \tag{19}$$

The LHS of (19) is continuous and decreasing in $p_H^{\max *}$. Thus, combining (19) and (14), there exists a $p_H^{\max **} \in \left(\underline{c}_H^{\mathcal{E}} + \xi_{HH}, p_H^{\max *}\right)$ such that

$$\overline{M}_H^{\mathcal{I}} \left(p_H^{\max **} - \mathbb{C}_{HH}^{\mathcal{I}}\right) + \overline{M}_H^{\mathcal{E}} \left(G_{HH}^{\mathcal{E}**} p_H^{\max **} - \mathbb{C}_{HH}^{\mathcal{E}**}\right) + \Phi_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right) + 2\beta_H p_H^{\max **} = 2\beta_H \alpha_H,$$

and the result follows. ∎

In all the subsequent proofs, we use the fact that, when H is a small economy, any trade shock in H has a negligible impact on the rest of the world. Thus, $\left(p_j^{\max *}\right)_{j \in \mathcal{C} \backslash \{H\}}$ can be treated as a parameter.

Proof of Proposition 3.1. Consider the non-degenerate setup. Since H is a small economy, $\left(p_j^{\max *}\right)_{j \in \mathcal{C} \backslash \{H\}}$ and, by (FE-ND), then $p_H^{\max *} = p_H^{\max **}$. Since the choke price in H does not vary, then, for firms that are active in both equilibria, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, and $\mu_{HH}^{**}(c)$ have the same value as in the equilibrium with $\boldsymbol{\tau}^*$.

In addition, (MS) in each equilibrium is, respectively,

$$\overline{M}_H^{\mathcal{I}} \left(p_H^{\max *} - \mathbb{C}_{HH}^{\mathcal{I}}\right) + M_H^{\mathcal{E}*} \left(G_{HH}^{\mathcal{E}*} p_H^{\max *} - \mathbb{C}_{HH}^{\mathcal{E}*}\right) + \Phi_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^{*}\right) = 2\beta_H \left(\alpha_H - p_H^{\max *}\right),$$

$$\overline{M}_H^{\mathcal{I}} \left(p_H^{\max **} - \mathbb{C}_{HH}^{\mathcal{I}}\right) + M_H^{\mathcal{E}**} \left(G_{HH}^{\mathcal{E}**} p_H^{\max **} - \mathbb{C}_{HH}^{\mathcal{E}**}\right) + \Phi_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right) = 2\beta_H \left(\alpha_H - p_H^{\max **}\right).$$

By making use of this system of equations, and since the choke price is the same before and after the trade shock:

$$M_H^{\mathcal{E}**} - M_H^{\mathcal{E}*} = \frac{\Phi_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^{*}\right) - \Phi_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right)}{G_{HH}^{\mathcal{E}*} p_H^{\max *} - \mathbb{C}_{HH}^{\mathcal{E}*}},$$

which, by applying Lemma A.6 to the numerator of the RHS, establishes that $M_H^{\mathcal{E}**} < M_H^{\mathcal{E}*}$

As for the degenerate case, ex-ante homogeneity of firms belonging to $\mathcal{E}$ means that they all obtain some productivity draw from the same degenerate distribution. Then, the proof follows verbatim because it is the limiting case of the non-degenerate variant with negligible heterogeneity. ∎

Proof of Proposition 3.2. By assumption, the set $\mathcal{E}$ consists of firms that are ex-ante heterogeneous. This rules out the non-degenerate variant and the limiting case with all its probability mass at one point. Thus, consider the degenerate setup with an atomless distribution, where the assumption holds. We also know that the MIEs belong to $\mathcal{E}$ for trade costs $\boldsymbol{\tau}_{\cdot H}^{*}$ and $\boldsymbol{\tau}_{\cdot H}^{**}$ when (14) holds. Thus, (MS) in H under each vector of trade costs is, respectively,

$$\overline{M}_H^{\mathcal{I}} \left(p_H^{\max *} - \mathbb{C}_{HH}^{\mathcal{I}}\right) + \overline{M}_H^{\mathcal{E}} \left(G_{HH}^{\mathcal{E}*} p_H^{\max *} - \mathbb{C}_{HH}^{\mathcal{E}*}\right) + \Phi_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^{*}\right) = 2\beta_H \left(\alpha_H - p_H^{\max *}\right),$$

$$\overline{M}_H^{\mathcal{I}} \left(p_H^{\max **} - \mathbb{C}_{HH}^{\mathcal{I}}\right) + \overline{M}_H^{\mathcal{E}} \left(G_{HH}^{\mathcal{E}**} p_H^{\max **} - \mathbb{C}_{HH}^{\mathcal{E}**}\right) + \Phi_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right) = 2\beta_H \left(\alpha_H - p_H^{\max **}\right).$$

This implies that

$$\underbrace{\left(M_H^{\mathcal{I}} + 2\beta_H\right)\left(p_H^{\max **} - p_H^{\max *}\right)}_{=:A_1} + \underbrace{\overline{M}_H^{\mathcal{E}} \left[\left(G_{HH}^{\mathcal{E}**} p_H^{\max **} - \mathbb{C}_{HH}^{\mathcal{E}**}\right) - \left(G_{HH}^{\mathcal{E}*} p_H^{\max *} - \mathbb{C}_{HH}^{\mathcal{E}*}\right)\right]}_{=:A_2} = \Phi_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^{*}\right) - \Phi_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right).$$

Suppose that $p_H^{\max **} \geq p_H^{\max *}$. Then, $A_1 \geq 0$ and, by Lemma A.6, $A_2 \geq 0$. Therefore, the LHS is non-negative. Moreover, by Lemma A.6, the RHS is negative, which leads to a contradiction. Hence, $p_H^{\max **} < p_H^{\max *}$.

Since $p_H^{\max **} < p_H^{\max *}$, then, by Lemma A.4 and for firms that are active in both equilibria, c_{HH}^{**}, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ are lower relative to the equilibrium with $\boldsymbol{\tau}_{\cdot H}^{*}$. Moreover, $M_{HH}^{**} < M_{HH}^{*}$ since

$c^{**}_{HH} < c^{*}_{HH}$, which in the degenerate variant determines that $M^{\mathcal{E}**}_{H} < M^{\mathcal{E}*}_{H}$. ■

We begin by establishing some lemmas such that if the MIEs in the equilibrium with τ^{*}_{HF} belong to $\mathcal{E}$, then the MIEs in the equilibrium with τ^{**}_{HF} also belong to $\mathcal{E}$. Basically, the lemmas show that, by choosing values for $\overline{M}^{\mathcal{E}}_{H}$ and $\overline{c}^{\mathcal{E}}_{H}$ that are large enough, this property is ensured.

Lemma A.8. *For the scenarios in Proposition 3.3, consider the non-degenerate variant of monopolistic competition, including the limiting variant with all its probability mass at one point, and let the set of countries be $\mathcal{C}$ with τ^{*}_{HF} and τ^{**}_{HF} such that $\tau^{**}_{HF} < \tau^{*}_{HF}$. Suppose that the MIEs in the equilibrium with τ^{*}_{HF} belong to $\mathcal{E}$ and ex-ante serve H and a set of countries $\mathcal{F}$ that include F. Then, it is always possible to choose a value of $\overline{M}^{\mathcal{E}}_{H}$ large enough such that the MIEs in the equilibrium with τ^{**}_{HF} belong to $\mathcal{E}$.*

Proof of Lemma A.8. Given that MIEs under $\boldsymbol{\tau}^{*}_{\cdot H}$ belong to $\mathcal{E}$, it is satisfied that

$$\int_{\underline{c}^{\mathcal{E}}_{H}}^{p^{\max *}_{H}-\xi_{HH}} \left[\frac{\left(p^{\max *}_{H}-c\right)^{2}}{4\gamma_{H}} - f_{HH}\right] \mathrm{d}G_{H}(c) + \int_{\underline{c}^{\mathcal{E}}_{H}}^{p^{\max *}_{F}-\tau^{*}_{HF}-\xi_{HF}} \left[\frac{\left(p^{\max *}_{F}-c-\tau^{*}_{HF}\right)^{2}}{4\gamma_{F}} - f_{HF}\right] \mathrm{d}G_{H}(c) + \kappa = F^{E}_{H},$$

where $\kappa := \sum_{j\in\mathcal{F}\setminus\{F\}} \widetilde{\pi}^{\mathcal{E}}_{Hj}\left(p^{\max}_{j};\tau_{Hj}\right)$.

Given that expected profits are decreasing in τ_{HF}, then

$$\int_{\underline{c}^{\mathcal{E}}_{H}}^{p^{\max *}_{H}-\xi_{HH}} \left[\frac{\left(p^{\max *}_{H}-c\right)^{2}}{4\gamma_{H}} - f_{HH}\right] \mathrm{d}G_{H}(c) + \int_{\underline{c}^{\mathcal{E}}_{H}}^{p^{\max *}_{F}-\tau^{**}_{HF}-\xi_{HF}} \left[\frac{\left(p^{\max *}_{F}-c-\tau^{**}_{HF}\right)^{2}}{4\gamma_{F}} - f_{HF}\right] \mathrm{d}G_{H}(c) + \kappa > F^{E}_{H}. \quad (20)$$

Moreover, there always exist a $\delta > 0$ such that

$$\int_{\underline{c}^{\mathcal{E}}_{H}}^{\underline{c}^{\mathcal{E}}_{H}+\delta} \left[\frac{\left(\underline{c}^{\mathcal{E}}_{H}+\xi_{HH}-c\right)^{2}}{4\gamma_{H}} - f_{HH}\right] \mathrm{d}G_{H}(c) + \int_{\underline{c}^{\mathcal{E}}_{H}}^{\underline{c}^{\mathcal{E}}_{H}+\delta} \left[\frac{\left(p^{\max *}_{F}-c-\tau^{**}_{HF}\right)^{2}}{4\gamma_{F}} - f_{HF}\right] \mathrm{d}G_{H}(c) + \kappa' < F^{E}_{H}. \quad (21)$$

where $\kappa' := \sum_{j\in\mathcal{F}\setminus\{F\}} \int_{\underline{c}^{\mathcal{E}}_{H}}^{\underline{c}^{\mathcal{E}}_{H}+\delta} \pi_{Hj}\left(p^{\max}_{j},c;\tau_{Hj}\right)\mathrm{d}G_{H}(c)$.

Since expected profits are continuous and increasing in $p^{\max}_{H}$, by using (20) and (21), we can always find a $p^{\max **}_{H} \in \left(\underline{c}^{\mathcal{E}}_{H}, p^{\max *}_{H}-\xi_{HH}\right)$ such that

$$\int_{\underline{c}^{\mathcal{E}}_{H}}^{p^{\max **}_{H}-\xi_{HH}} \left[\frac{\left(p^{\max **}_{H}-c\right)^{2}}{4\gamma_{H}} - f_{HH}\right] \mathrm{d}G_{H}(c) + \int_{\underline{c}^{\mathcal{E}}_{H}}^{p^{\max *}_{F}-\tau^{**}_{HF}-\xi_{HF}} \left[\frac{\left(p^{\max *}_{F}-c-\tau^{**}_{HF}\right)^{2}}{4\gamma_{F}} - f_{HF}\right] \mathrm{d}G_{H}(c) + \kappa = F^{E}_{H}. \quad (22)$$

Recall that $p^{\max *}_{F} = p^{\max **}_{F}$ due to the fact that H is a small economy. If $p^{\max **}_{H}$ is such that (MS) holds, then the result follows. To show this, given that expected profits are increasing in $p^{\max}_{H}$, (22) determines that $p^{\max **}_{H} < p^{\max *}_{H}$. This establishes that $c^{**}_{HH} < c^{*}_{HH}$. In addition, $\boldsymbol{\tau}^{*}_{\cdot H} = \boldsymbol{\tau}^{**}_{\cdot H}$ and, given trade costs $\boldsymbol{\tau}^{*}_{\cdot H}$, (MS) is

$$\overline{M}^{\mathcal{I}}_{H}\left(p^{\max *}_{H}-\mathbb{C}^{\mathcal{I}}_{HH}\right) + M^{\mathcal{E}*}_{H}\left(G^{\mathcal{E}*}_{HH}p^{\max *}_{H}-\mathbb{C}^{\mathcal{E}*}_{HH}\right) + \Phi_{-H}\left(p^{\max *}_{H};\boldsymbol{\tau}^{*}_{\cdot H}\right) + 2\beta_{H}p^{\max *}_{H} = 2\beta_{H}\alpha_{H},$$

and this implies that

$$\overline{M}^{\mathcal{I}}_{H}\left(p^{\max **}_{H}-\mathbb{C}^{\mathcal{I}}_{HH}\right) + M^{\mathcal{E}*}_{H}\left(G^{\mathcal{E}**}_{HH}p^{\max **}_{H}-\mathbb{C}^{\mathcal{E}**}_{HH}\right) + \Phi_{-H}\left(p^{\max **}_{H};\boldsymbol{\tau}^{**}_{\cdot H}\right) + 2\beta_{H}p^{\max **}_{H} < 2\beta_{H}\alpha_{H}. \quad (23)$$

Therefore, if $\overline{M}^{\mathcal{E}}_{H}$ is large enough, we can always find some $M^{\mathcal{E}**}_{H} \leq \overline{M}^{\mathcal{E}}_{H}$ such that, by substituting $M^{\mathcal{E}*}_{H}$ with $M^{\mathcal{E}**}_{H}$, (23) holds with equality. The proof for the limiting case with all its probability mass at one point follows by noticing that zero profits directly determines $p^{\max **}_{H} < p^{\max *}_{H}$, and then applying the same steps for (23). ■

Lemma A.9. *For the scenarios in Proposition 3.3, consider the degenerate variant of monopolistic competition with atomless distribution $G^{\mathcal{E}}_{H}$, set of countries $\mathcal{C}$ and let τ^{*}_{HF} and τ^{**}_{HF} be such that $\tau^{**}_{HF} < \tau^{*}_{HF}$. Suppose that the MIEs in the equilibrium with τ^{*}_{HF} belong to $\mathcal{E}$ and serve H and a set of countries $\mathcal{F}$ that include F. Then, it is always possible to choose a value of $\overline{c}^{\mathcal{E}}_{H}$ large enough such that the MIEs in the equilibrium with τ^{**}_{HF} belong to $\mathcal{E}$.*

Proof of Lemma A.9. The proof requires us to show that $p^{\max **}_{H} < p^{\max *}_{H}$ and $c^{**}_{HH} > c^{*}_{HH}$. By proving that, then we know that we can always choose a value of $\overline{c}^{\mathcal{E}}_{H}$ large enough such that $\overline{c}^{\mathcal{E}}_{H} > c^{**}_{HH}$ and the result follows.

In equilibrium,

$$\frac{\left(p_H^{\max *}-c_{HH}^*\right)^2}{4\gamma_H}+\sum_{j\in\mathcal{F}}\frac{\left(p_j^{\max *}-c_{HH}^*-\tau_{Hj}^*\right)^2}{4\gamma_j}=F_H^E+\sum_{j\in\mathcal{F}}f_{Hj},$$
$$\frac{\left(p_H^{\max **}-c_{HH}^{**}\right)^2}{4\gamma_H}+\sum_{j\in\mathcal{F}}\frac{\left(p_j^{\max **}-c_{HH}^{**}-\tau_{Hj}^{**}\right)^2}{4\gamma_j}=F_H^E+\sum_{j\in\mathcal{F}}f_{Hj}.$$

This implies that

$$\left[\frac{\left(p_H^{\max *}-c_{HH}^*\right)^2}{4\gamma_H}-\frac{\left(p_H^{\max **}-c_{HH}^{**}\right)^2}{4\gamma_H}\right]+\sum_{j\in\mathcal{F}}\left[\frac{\left(p_j^{\max *}-c_{HH}^*-\tau_{Hj}^*\right)^2}{4\gamma_j}-\frac{\left(p_j^{\max **}-c_{HH}^{**}-\tau_{Hj}^{**}\right)^2}{4\gamma_j}\right]=0. \quad (24)$$

Towards a contradiction, suppose that $p_H^{\max **}\geq p_H^{\max *}$. We know that $\tau_{HF}^{**}<\tau_{HF}^*$ while $\tau_{Hj}^{**}=\tau_{Hj}^*$ and $p_k^{\max *}=p_k^{\max **}$ for $j\in\mathcal{F}\backslash\{F\}$ and $k\in\mathcal{F}$. By Lemmas A.4 and A.5, this determines that $c_{HH}^{**}>c_{HH}^*$ in order for (24) to hold. Regarding (MS),

$$\Phi_{HH}^{\mathcal{I}*}+\Phi_{HH}^{\mathcal{E}*}+\Phi_{-H}^*+2\beta_H p_H^{\max *}=2\beta_H\alpha_H,$$
$$\Phi_{HH}^{\mathcal{I}**}+\Phi_{HH}^{\mathcal{E}**}+\Phi_{HH}^{\mathcal{N}**}+\Phi_{-H}^{**}+2\beta_H p_H^{\max **}=2\beta_H\alpha_H,$$

which determines that

$$\left(\Phi_{HH}^{\mathcal{I}**}-\Phi_{HH}^{\mathcal{I}*}\right)+\left(\Phi_{HH}^{\mathcal{E}**}-\Phi_{HH}^{\mathcal{E}*}\right)+\Phi_{HH}^{\mathcal{N}**}+\left(\Phi_{-H}^{**}-\Phi_{-H}^*\right)+2\beta_H\left(p_H^{\max **}-p_H^{\max *}\right)=0. \quad (25)$$

We know that $\boldsymbol{\tau}_{H\cdot}^{**}$ does not directly affect (MS). Moreover, all the terms in the LHS of (25) are nonnegative. Also, since $p_H^{\max **}\geq p_H^{\max *}$ and $c_{HH}^{**}>c_{HH}^*$, at least one term is positive, determining that the LHS is positive. This contradicts (25) and, therefore, $p_H^{\max **}<p_H^{\max *}$.

Now we want to show that $c_{HH}^{**}>c_{HH}^*$. Suppose not, so that $c_{HH}^{**}\leq c_{HH}^*$. Then, in terms of (MS), this implies that

$$\left(\Phi_{HH}^{\mathcal{I}**}-\Phi_{HH}^{\mathcal{I}*}\right)+\left(\Phi_{HH}^{\mathcal{E}**}-\Phi_{HH}^{\mathcal{E}*}\right)+\left(\Phi_{-H}^{**}-\Phi_{-H}^*\right)+2\beta_H\left(p_H^{\max **}-p_H^{\max *}\right)=0. \quad (26)$$

Given that $p_H^{\max **}<p_H^{\max *}$, then $\Phi_{HH}^{\mathcal{I}**}<\Phi_{HH}^{\mathcal{I}*}$ and $\Phi_{-H}^{**}<\Phi_{-H}^*$ by Lemma A.6. Thus, (26) can only hold if $\Phi_{HH}^{\mathcal{E}**}>\Phi_{HH}^{\mathcal{E}*}$. But, since $p_H^{\max **}<p_H^{\max *}$ and $c_{HH}^{**}\leq c_{HH}^*$, then $\Phi_{HH}^{\mathcal{E}**}<\Phi_{HH}^{\mathcal{E}*}$, which is a contradiction. Therefore, $c_{HH}^{**}>c_{HH}^*$, and the result follows. ■

Proof of Proposition 3.3. We start by considering the non-degenerate variant. Suppose that $\overline{M}_H^{\mathcal{E}}$ is large enough such that, by Lemma A.9, MIEs belong to $\mathcal{E}$ if trade costs are τ_{HF}^* or τ_{HF}^{**}. This determines that (FE-ND) is satisfied and, so,

$$\underbrace{\left[\widetilde{\pi}_{HH}^{\mathcal{E}}\left(p_H^{\max **}\right)-\widetilde{\pi}_{HH}^{\mathcal{E}}\left(p_H^{\max *}\right)\right]}_{=:A_1}+\underbrace{\left[\sum_{j\neq H}\widetilde{\pi}_{Hj}^{\mathcal{E}}\left(p_j^{\max **};\tau_{Hj}^{**}\right)-\sum_{j\neq H}\widetilde{\pi}_{Hj}^{\mathcal{E}}\left(p_j^{\max *};\tau_{Hj}^*\right)\right]}_{=:A_2}=0. \quad (27)$$

By the fact that H is a small economy, $p_j^{\max *}=p_j^{\max **}$ for any $j\in\mathcal{C}\backslash\{H\}$. Moreover, $\widetilde{\pi}_{HF}^{\mathcal{E}}$ is decreasing in τ_{HF}. Thus, since $\tau_{HF}^{**}<\tau_{HF}^*$, then $A_2>0$. Moreover, $\widetilde{\pi}_{HH}^{\mathcal{E}}$ is increasing in $p_H^{\max}$, which means that (27) can only hold if $p_H^{\max **}<p_H^{\max *}$ so that $A_1<0$. Therefore, by Lemma A.4 and for firms that are active in both equilibria, c_{HH}^{**}, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ are lower relative to the equilibrium with τ_{HF}^*. Regarding the mass of incumbents,

$$\overline{M}_H^{\mathcal{I}}\left(p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+M_H^{\mathcal{E}*}\left(G_{HH}^{\mathcal{E}*}p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{E}*}\right)+\Phi_{-H}\left(p_H^{\max *};\boldsymbol{\tau}_{\cdot H}^*\right)=2\beta_H\left(\alpha_H-p_H^{\max *}\right),$$

$$\overline{M}_H^{\mathcal{I}}\left(p_H^{\max **}-\mathbb{C}_{HH}^{\mathcal{I}}\right)+M_H^{\mathcal{E}**}\left(G_{HH}^{\mathcal{E}**}p_H^{\max **}-\mathbb{C}_{HH}^{\mathcal{E}**}\right)+\Phi_{-H}\left(p_H^{\max **};\boldsymbol{\tau}_{\cdot H}^{**}\right)=2\beta_H\left(\alpha_H-p_H^{\max **}\right).$$

which, combining both expressions, becomes

$$\underbrace{\left(M_H^{\mathcal{I}}+2\beta_H\right)\left(p_H^{\max **}-p_H^{\max *}\right)}_{=:B_1}+\underbrace{\Phi_{-H}\left(p_H^{\max **};\tau_{\cdot H}^{**}\right)-\Phi_{-H}\left(p_H^{\max *};\tau_{\cdot H}^{*}\right)}_{=:B_2}=$$
$$M_H^{\mathcal{E}*}\left(G_{HH}^{\mathcal{E}*}p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{E}*}\right)-M_H^{\mathcal{E}**}\left(G_{HH}^{\mathcal{E}**}p_H^{\max **}-\mathbb{C}_{HH}^{\mathcal{E}**}\right).$$

Since $p_H^{\max **}<p_H^{\max *}$, then $B_1<0$. Moreover, by Lemma A.6 and the fact that $\tau_{jH}^{*}=\tau_{jH}^{**}$ for any $j\in\mathcal{C}\backslash\{H\}$, then $B_2<0$. Both facts determine that it is necessary that the RHS is negative. In addition, by Lemma A.6, the term $\left(G_{HH}^{\mathcal{E}*}p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{E}*}\right)$ is increasing in $p_H^{\max *}$. Thus, reexpressing the RHS,

$$\frac{M_H^{\mathcal{E}**}}{M_H^{\mathcal{E}*}}>\frac{G_{HH}^{\mathcal{E}*}p_H^{\max *}-\mathbb{C}_{HH}^{\mathcal{E}*}}{G_{HH}^{\mathcal{E}**}p_H^{\max **}-\mathbb{C}_{HH}^{\mathcal{E}**}}>1,$$

which determines that $M_H^{\mathcal{E}**}>M_H^{\mathcal{E}*}$.

As for the degenerate case where $G_H^{\mathcal{E}}$ has all its probability mass at one point, it holds by following the same steps since it is the limiting case of the degenerate variant. For the degenerate case with atomless $G_H^{\mathcal{E}}$, by Lemma A.9, we know that if $\overline{c}_H^{\mathcal{E}}$ is large enough, MIEs belong to $\mathcal{E}$ under τ_{HF}^{*} and τ_{HF}^{**}. Moreover, in the proof of Lemma A.9 we have already shown that $p_H^{\max *}>p_H^{\max **}$ and $c_{HH}^{*}<c_{HH}^{**}$. Therefore, by Lemma A.4 and for firms that are active in both equilibria, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, and $\mu_{HH}^{**}(c)$ are lower relative to the equilibrium with τ_{HF}^{*}. Finally, since $c_{HH}^{**}>c_{HH}^{*}$, then $M_{HH}^{**}>M_{HH}^{*}$. This implies that $M_H^{\mathcal{E}**}>M_H^{\mathcal{E}*}$ since, in that variant, the set of firms belonging to $\mathcal{E}$ that pay the entry cost coincides with the set of firms that are active in the domestic market.

Finally, notice that if MIEs do not export, then neither (24) or (27) are affected by τ_{HF}. Moreover, (MS) is not directly affected by τ_{HF}. Thus, in both setup variants, $p_H^{\max *}=p_H^{\max **}$ and, so, prices, quantities, markups, the marginal-cost cutoff, and mass of firms serving H do not change either. ■

A.2 Applications of the Monopolistic-Competition Model

In this part, we formalize the results outlined in Section 4.2 for a unilateral liberalization between large countries. We consider a world economy with $\mathcal{C}:=\{H,F\}$, where H and F are large countries. This implies that the market conditions and behavior of firms from one country will have an influence on the other. The experiment consists of an infinitesimal decrease in τ_{FH} when MIEs belong to $\mathcal{E}$.

Irrespective of the specific equilibrium conditions that identify the choke price, we show below that we can always determine reduced-form equations $p_H^{\max}\left(p_F^{\max};\tau_{FH}\right)$ and $p_F^{\max}\left(p_H^{\max};\tau_{FH}\right)$. Thus, the equilibrium is obtained through a pair $\left(p_H^{\max *},p_F^{\max *}\right)$ such that

$$\begin{aligned}p_H^{\max *}&=p_H^{\max}\left(p_F^{\max *};\tau_{FH}\right),\\ p_F^{\max *}&=p_F^{\max}\left(p_H^{\max *};\tau_{FH}\right).\end{aligned}\tag{28}$$

The system (28) can be used to decompose the effects on each choke price in different channels. Specifically, differentiating (28), we obtain

$$\begin{aligned}\frac{\mathrm{d}p_H^{\max *}}{\mathrm{d}\tau_{FH}}&=\frac{\partial p_H^{\max *}\left(p_F^{\max *};\tau_{FH}\right)}{\partial\tau_{FH}}+\frac{\partial p_H^{\max *}\left(p_F^{\max *};\tau_{FH}\right)}{\partial p_F^{\max}}\frac{\mathrm{d}p_F^{\max *}}{\mathrm{d}\tau_{FH}},\\ \frac{\mathrm{d}p_F^{\max *}}{\mathrm{d}\tau_{FH}}&=\frac{\partial p_F^{\max *}\left(p_H^{\max *};\tau_{FH}\right)}{\partial\tau_{FH}}+\frac{\partial p_F^{\max *}\left(p_H^{\max *};\tau_{FH}\right)}{\partial p_H^{\max}}\frac{\mathrm{d}p_H^{\max *}}{\mathrm{d}\tau_{FH}}.\end{aligned}$$

Solving for $\frac{\mathrm{d}p_H^{\max *}}{\mathrm{d}\tau_{FH}}$ and $\frac{\mathrm{d}p_F^{\max *}}{\mathrm{d}\tau_{FH}}$, we obtain expressions (6) and (7), where

$$\lambda := \left(1 - \frac{\partial p_H^{\max *}\left(p_F^{\max *};\tau_{FH}\right)}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}\left(p_H^{\max *},\tau_{FH}\right)}{\partial p_H^{\max}}\right)^{-1}.$$

Next, we determine the signs of each effect for each variant of the monopolistic-competition model. To do this, we characterize the system (28) corresponding to each setup. Then, we establish the sign of the different effects depending on the assumptions on MIEs.

For the non-degenerate variant, condition (FE-ND) for two large countries is:

$$\begin{aligned} \widetilde{\pi}_{HH}^{\mathcal{E}}\left(p_H^{\max *}\right) + \mathbb{1}_{\left(c_{HF}^* \in \left(\underline{c}_H^{\mathcal{E}},\overline{c}_H^{\mathcal{E}}\right)\right)} \widetilde{\pi}_{HF}^{\mathcal{E}}\left(p_F^{\max *}\right) &= F_H^E, \\ \widetilde{\pi}_{FF}^{\mathcal{E}}\left(p_F^{\max *}\right) + \mathbb{1}_{\left(c_{FH}^* \in \left(\underline{c}_F^{\mathcal{E}},\overline{c}_F^{\mathcal{E}}\right)\right)} \widetilde{\pi}_{FH}^{\mathcal{E}}\left(p_H^{\max *};\tau_{FH}\right) &= F_F^E. \end{aligned} \tag{29}$$

The indicator functions in (29) reflect whether the MIEs in each country are ex-ante exporters.

As for the degenerate variant, there are two possible systems of equations that determine (28). First, if choke prices are determined by (FE-D), then:

$$\begin{aligned} \pi_{HH}^{\mathcal{E}}\left(p_H^{\max *},c_{HH}^*\right) + \mathbb{1}_{\left(c_{HH}^*=c_{HF}^*\right)} \pi_{HF}^{\mathcal{E}}\left(p_F^{\max *},c_{FH}^*\right) &= F_H^E, \\ \pi_{FF}^{\mathcal{E}}\left(p_F^{\max *},c_{FF}^*\right) + \mathbb{1}_{\left(c_{FF}^*=c_{FH}^*\right)} \pi_{FH}^{\mathcal{E}}\left(p_H^{\max *},c_{FH}^*;\tau_{FH}\right) &= F_F^E, \end{aligned} \tag{30}$$

where the indicator function reflects whether the MIEs are exporting, in which case (ZCP2) holds. Second, suppose that (28) is determined by (MS). In such a case, the following system of equations holds:

$$\begin{aligned} \Phi_{HH}\left(p_H^{\max *}\right) + \Phi_{FH}\left(p_H^{\max *};\tau_{FH}\right) + 2\beta_H p_H^{\max *} &= 2\beta_H \alpha_H, \\ \Phi_{FF}\left(p_F^{\max *}\right) + \Phi_{HF}\left(p_F^{\max *}\right) + 2\beta_F p_F^{\max *} &= 2\beta_F \alpha_F. \end{aligned} \tag{31}$$

We denote the Jacobian matrix of either (29) or (30) by J^{FE}. Likewise, let the Jacobian matrix of (31) be J^{MS}. Next, we provide a lemma which establishes the sign of each partial effect.

Lemma A.10. *Consider that $\mathrm{d}\tau_{FH} \neq 0$. Suppose that (28) is determined by either (29) or (30). Then, $\frac{\partial p_H^{\max *}(p_F^{\max *})}{\partial \tau_{FH}} = 0$. Moreover, if firms from F belonging to $\mathcal{E}$ are ex-ante exporters, then $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} > 0$ and $\frac{\partial p_F^{\max *}}{\partial p_H^{\max *}} < 0$, and if they are not ex-ante exporters both terms are zero. If firms from H belonging to $\mathcal{E}$ are ex-ante exporters then $\frac{\partial p_H^{\max *}}{\partial p_F^{\max *}} < 0$, and if they are not ex-ante exporters the term is zero. Suppose that (28) is determined by (31), then $\frac{\partial p_H^{\max *}}{\partial \tau_{FH}} > 0$, $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} = 0$, and $\frac{\partial p_H^{\max *}}{\partial p_F^{\max *}} = \frac{\partial p_F^{\max *}}{\partial p_H^{\max *}} = 0$.*

Proof of Lemma A.10. Let $i,j \in \{H,F\}$ with $i \neq j$. For the case of the non-degenerate variant, given (29), $\frac{\partial \widetilde{\pi}_{ji}^{\mathcal{E}}(p_i^{\max *};\tau_{ji})}{\partial p_i^{\max *}} = -\frac{\partial \widetilde{\pi}_{ji}^{\mathcal{E}}(p_i^{\max *};\tau_{ji})}{\partial \tau_{ji}} = \frac{p_i^{\max *} G_{ji}^{\mathcal{E}*} - \mathbb{C}_{ji}^{\tau,\mathcal{E}*}}{2\gamma_i} > 0$, and $\frac{\partial \widetilde{\pi}_{ji}^{\mathcal{E}}(p_i^{\max *};\tau_{ji})}{\partial \tau_{ij}} = 0$. For the degenerate variant, given (30), then $\frac{\partial \pi_{ji}^{\mathcal{E}}\left(p_i^{\max *},c_{ji}^*;\tau_{ji}\right)}{\partial p_i^{\max *}} = -\frac{\partial \pi_{ji}^{\mathcal{E}}\left(p_i^{\max *},c_{ji}^*;\tau_{ji}\right)}{\partial \tau_{ji}} = \frac{p_i^{\max *} - c_{ji}^* - \tau_{ij}}{2\gamma_i} > 0$, and $\frac{\partial \pi_{ji}^{\mathcal{E}}\left(p_i^{\max *},c_{ji}^*;\tau_{ji}\right)}{\partial \tau_{ij}} = 0$. Denote by π_{ij} either the expected profits in (29) or the profits in (30). Then,

$$\frac{\partial p_F^{\max *}\left(p_H^{\max *},\tau_{FH}\right)}{\partial \tau_{FH}} = -\frac{\partial \pi_{FH}}{\partial \tau_{FH}} \left(\frac{\partial \pi_{FF}}{\partial p_F^{\max *}}\right)^{-1} > 0 \text{ and } \frac{\partial p_H^{\max *}\left(p_F^{\max *};\tau_{FH}\right)}{\partial \tau_{FH}} = 0,$$

$$\frac{\partial p_i^{\max *}}{\partial p_j^{\max *}} = \frac{\partial \pi_{ij}}{\partial p_j^{\max *}} \left(\frac{\partial \pi_{ii}}{\partial p_i^{\max *}}\right)^{-1} > 0.$$

Consider now the case where (28) is determined by (31). Then, $\frac{\partial p_H^{\max}}{\partial p_F^{\max}} = \frac{\partial p_F^{\max}}{\partial p_H^{\max}} = 0$ because there is no direct relation between the variables. Moreover, $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} = 0$ because τ_{FH} does not directly affect $p_F^{\max *}$. In addition,

$$\frac{\partial p_H^{\max *}}{\partial \tau_{FH}} = -\frac{\partial \Phi_{FH}^*}{\partial \tau_{FH}} \left(2\beta_H + \frac{\partial \Phi_{HH}^*}{\partial p_H^{\max *}} + \frac{\partial \Phi_{FH}^*}{\partial p_H^{\max *}}\right)^{-1} > 0,$$

where the sign follows by using Lemma A.6. ■

Lemma A.11. *Consider that* $\mathrm{d}\tau_{FH} \neq 0$*. Suppose that* (28) *is determined by either* (29) *or* (30)*. If MIEs in at least one country are not ex-ante exporters, then* $\lambda = 1$*. If MIEs are ex-ante exporters in both countries then* $\lambda > 1$ *iff* $\det J^{FE} > 0$*. If* (28) *is determined by* (31)*, then* $\lambda = 1$*.*

Proof of Lemma A.11. Irrespective of how (28) is determined, we can express

$$\lambda = \left(1 - \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}(p_H^{\max *}, \tau_{FH})}{\partial p_H^{\max}}\right)^{-1}.$$

If MIEs in at least one country are not ex-ante exporters, then either $\frac{\partial p_H^{\max *}(p_F^{\max *};\tau_{FH})}{\partial p_F^{\max}} = 0$ or $\frac{\partial p_F^{\max *}(p_H^{\max *},\tau_{FH})}{\partial p_H^{\max}} = 0$ (or both) and, so, $\lambda = 1$. Denote by π_{ij} either the expected profits in (29) or the profits in (30). In case MIEs are ex-ante exporters in both countries, then

$$\lambda = \frac{\frac{\partial \pi_{HH}}{\partial p_H^{\max}} \frac{\partial \pi_{FF}}{\partial p_F^{\max}}}{\frac{\partial \pi_{HH}}{\partial p_H^{\max}} \frac{\partial \pi_{FF}}{\partial p_F^{\max}} - \frac{\partial \pi_{HF}}{\partial p_F^{\max}} \frac{\partial \pi_{FH}}{\partial p_H^{\max}}},$$

and $\lambda > 1$ iff the denominator is positive. Moreover, J^{FE} is given by

$$J^{FE} := \begin{pmatrix} \frac{\partial \pi_{HH}}{\partial p_H^{\max}} & \frac{\partial \pi_{HF}}{\partial p_F^{\max}} \\ \frac{\partial \pi_{FH}}{\partial p_H^{\max}} & \frac{\partial \pi_{FF}}{\partial p_F^{\max}} \end{pmatrix},$$

and $\det J^{FE} > 0$ iff $\frac{\partial \pi_{HH}}{\partial p_H^{\max}} \frac{\partial \pi_{FF}}{\partial p_F^{\max}} > \frac{\partial \pi_{HF}}{\partial p_F^{\max}} \frac{\partial \pi_{FH}}{\partial p_H^{\max}}$, which holds iff $\lambda = 1$. If (28) is determined by (31), then $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} = 0$ and, so, $\lambda = 1$. ■

With these lemmas, we can establish the effects on choke prices for different assumptions on MIEs. Next, we illustrate this by showing how they are determined for each of the cases stated in Section 4.2.

Anti-Competitive Effects. If MIEs in both countries are ex-ante homogeneous, then (28) is determined by (29) or (30). Moreover, assuming that MIEs are ex-ante exporters in both countries, $\lambda > 1$ iff $\det J^{FE} > 0$ by Lemma A.11. In addition, by using Lemma A.10:

$$\frac{\mathrm{d}p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\mathrm{d}\tau_{FH}} = \underbrace{\lambda \frac{\partial p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}_{\text{export-opportunities channel}>0} + \underbrace{\lambda \frac{\partial p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\partial p_H^{\max}} \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}_{\text{export-conditions channel}=0},$$

$$\frac{\mathrm{d}p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\mathrm{d}\tau_{FH}} = \underbrace{\lambda \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}_{\text{import-competition channel}=0} + \underbrace{\lambda \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial p_F^{* \max}} \frac{\partial p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}_{\text{export-conditions channel}<0}.$$

Notice that, for country F, the export-conditions channel is active but, given that $\frac{\partial p_H^{\max *}(p_F^{\max *})}{\partial \tau_{FH}} = 0$, then the export conditions are not changing. Thus, the effect of this channel is zero.

Pro-Competitive Effects. Since MIEs in H are ex-ante heterogeneous and are not ex-ante exporters, then (28) is determined by (31) and, by Lemma A.11, $\lambda = 1$. Moreover, by Lemma A.10,

$$\frac{\mathrm{d}p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\mathrm{d}\tau_{FH}} = \underbrace{\lambda \frac{\partial p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}_{\text{export-opportunities channel}=0} + \underbrace{\lambda \frac{\partial p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\partial p_H^{\max}} \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}_{\text{export-conditions channel}=0},$$

$$\frac{\mathrm{d}p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\mathrm{d}\tau_{FH}} = \underbrace{\lambda \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}_{\text{import-competition channel}>0} + \underbrace{\lambda \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial p_F^{* \max}} \frac{\partial p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}_{\text{export-conditions channel}=0}.$$

Null Competitive Effects. Since MIEs in H are ex-ante homogeneous, then (28) is determined by (29) or (30). Moreover, since MIEs in both countries are not ex-ante exporters, then, by Lemma A.11, $\lambda = 1$. Also, applying Lemma A.10,

$$\frac{\mathrm{d}p_F^{\max *}\left(p_H^{\max *};\tau_{FH}\right)}{\mathrm{d}\tau_{FH}}=\underbrace{\lambda\frac{\partial p_F^{\max *}\left(p_H^{\max *};\tau_{FH}\right)}{\partial\tau_{FH}}}_{\text{export-opportunities channel=0}}+\underbrace{\lambda\frac{\partial p_F^{\max *}\left(p_H^{\max *};\tau_{FH}\right)}{\partial p_H^{\max}}\frac{\partial p_H^{\max *}\left(p_F^{\max *};\tau_{FH}\right)}{\partial\tau_{FH}}}_{\text{export-conditions channel=0}},$$

$$\frac{\mathrm{d}p_H^{\max *}\left(p_F^{\max *};\tau_{FH}\right)}{\mathrm{d}\tau_{FH}}=\underbrace{\lambda\frac{\partial p_H^{\max *}\left(p_F^{\max *};\tau_{FH}\right)}{\partial\tau_{FH}}}_{\text{import-competition channel=0}}+\underbrace{\lambda\frac{\partial p_H^{\max *}\left(p_F^{\max *};\tau_{FH}\right)}{\partial p_F^{*\max}}\frac{\partial p_F^{\max *}\left(p_H^{\max *};\tau_{FH}\right)}{\partial\tau_{FH}}}_{\text{export-conditions channel=0}}.$$

A.3 Cournot Competition

For the following proofs, we use $p_H^{\max}$ as shorthand notation for $p_H^{\max}(\mathbb{Q})$.

Lemma A.12. *$q_{ij}\left(p_j^{\max};c_\omega^\tau\right)$, $p_{ij}\left(p_j^{\max};c_\omega^\tau\right)$, $m_{ij}\left(p_j^{\max};c_\omega^\tau\right)$, and $\mu_{ij}\left(p_j^{\max};c_\omega^\tau\right)$ are increasing in $p_j^{\max}$.*

Proof of Lemma A.12. Taking derivatives of each function: $\frac{\partial q_{ij}(\cdot)}{\partial p_j^{\max}}=\frac{1}{2\gamma_j+\eta_j}>0$, $\frac{\partial p_{ij}(\cdot)}{\partial p_j^{\max}}=\frac{\gamma_j+\eta_j}{2\gamma_j+\eta_j}>0$, $\frac{\partial m_{ij}(\cdot)}{\partial p_j^{\max}}=\frac{\gamma_j+\eta_j}{2\gamma_j+\eta_j}\frac{1}{c_\omega^\tau}>0$ and $\frac{\partial \mu_{ij}(\cdot)}{\partial p_j^{\max}}=\frac{\gamma_j+\eta_j}{2\gamma_j+\eta_j}>0$. ■

Lemma A.13. *$p_{ij}\left(p_j^{\max};c_\omega^\tau\right)$ is increasing in c_ω^τ and $q_{ij}\left(p_j^{\max};c_\omega^\tau\right)$, $m_{ij}\left(p_j^{\max};c_\omega^\tau\right)$, and $\mu_{ij}\left(p_j^{\max};c_\omega^\tau\right)$ are decreasing in c_ω^τ.*

Proof of Lemma A.13. Taking derivatives of each function: $\frac{\partial p_{ij}(\cdot)}{\partial c_\omega^\tau}=\frac{\gamma_j}{2\gamma_j+\eta_j}>0$, $\frac{\partial q_{ij}(\cdot)}{\partial c_\omega^\tau}=-\frac{1}{2\gamma_j+\eta_j}<0$, $\frac{\partial m_{ij}(\cdot)}{\partial c_\omega^\tau}=-\frac{\gamma_j+\eta_j}{2\gamma_j+\eta_j}\frac{p_j^{\max}}{(c_\omega^\tau)^2}<0$, and $\frac{\partial \mu_{ij}(\cdot)}{\partial c_\omega^\tau}=-\frac{2(\gamma_j+\eta_j)}{2\gamma_j+\eta_j}\left(p_j^{\max}-c_\omega^\tau\right)<0$. ■

Lemma A.14. *$\pi_{ij}\left(p_j^{\max},c_\omega^\tau\right)$ is decreasing in $\mathbb{Q}_j^{-\omega}$ and c_ω^τ.*

Proof of Lemma A.14. Profits are given by (10) and, so, $\frac{\partial \pi_{ij}[\cdot]}{\partial \mathbb{Q}_j^{-\omega}}=2\frac{(\gamma_j+\eta_j)}{(2\gamma_j+\eta_j)^2}\left(p_j^{\max}-c_\omega^\tau\right)(-\eta_j)<0$ and $\frac{\partial \pi_{ij}[\cdot]}{\partial c_\omega^\tau}=\frac{\partial \pi_{ij}[\cdot]}{\partial \mathbb{Q}_j^{-\omega}}\frac{1}{\eta_j}<0$. Then, by the Envelope Theorem, optimal profits are decreasing in $\mathbb{Q}_j^{-\omega}$ and c_ω^τ. ■

For the subsequent lemmas, we use the following notation. Given trade costs $\boldsymbol{\tau}_{\cdot H}^{*}$ and $\boldsymbol{\tau}_{\cdot H}^{**}$, we denote the equilibrium values of each variable with superscripts $*$ and $**$, respectively. Also, we use primes as superscripts when we refer to general properties of the equilibrium. We still keep using the index r_i when it is relative to the set $\overline{\Omega}$, and r_{ji} when it is relative to $\overline{\Omega}_j$. Finally, $c_{r_i}^\tau =: c_{r_{ji}}^\tau := c_{r_i}+\tau_{ji}$, depending on the reference set of the index and when it is clear from the context.

Lemma A.15. *Let $\boldsymbol{\tau}'_{\cdot H}$ and $\boldsymbol{\tau}''_{\cdot H}$ be such $\tau''_{jH}\leq\tau'_{jH}$ for each $j\in\mathcal{C}$. If $\left(\mathbb{Q}_j^{-\omega}\right)''>\left(\mathbb{Q}_j^{-\omega}\right)'$ then $\mathbb{Q}''_j>\mathbb{Q}'_j$, where $\mathbb{Q}_j:=\mathbb{Q}_j^{-\omega}+q_{ij}^{BR}\left(\mathbb{Q}_j^{-\omega},c_\omega^\tau\right)$.*

Proof of Lemma A.15. Suppose not, so that $\mathbb{Q}''_j\leq\mathbb{Q}'_j$. Then, by definition, $\mathbb{Q}''_j=\left(\mathbb{Q}_j^{-\omega}\right)''+q_{ij}^{BR}\left[\left(\mathbb{Q}_j^{-\omega}\right)'',c_\omega^{\tau''}\right]$ and $\mathbb{Q}'_j=\left(\mathbb{Q}_j^{-\omega}\right)'+q_{ij}^{BR}\left[\left(\mathbb{Q}_j^{-\omega}\right)',c_\omega^{\tau'}\right]$. Therefore, $\left(\mathbb{Q}_j^{-\omega}\right)''+q_{ij}^{BR}\left[\left(\mathbb{Q}_j^{-\omega}\right)'',c_\omega^{\tau''}\right]\leq\left(\mathbb{Q}_j^{-\omega}\right)'+q_{ij}^{BR}\left[\left(\mathbb{Q}_j^{-\omega}\right)',c_\omega^{\tau'}\right]$. Since $q_{ij}^{BR}\left(\mathbb{Q}_j^{-\omega},c_\omega^\tau\right)$ is given by (8), the inequality holds iff $(2\gamma_j+\eta_j)\left[\left(\mathbb{Q}_j^{-\omega}\right)''-\left(\mathbb{Q}_j^{-\omega}\right)'\right]+\left[c_\omega^{\tau'}-c_\omega^{\tau''}\right]\leq 0$. Since $c_\omega^{\tau''}\leq c_\omega^{\tau'}$ by the assumption on trade costs, this implies that $\left(\mathbb{Q}_j^{-\omega}\right)''\leq\left(\mathbb{Q}_j^{-\omega}\right)'$, which is a contradiction.■

Lemma A.16. *Let $\boldsymbol{\tau}'_{\cdot H}$ and $\boldsymbol{\tau}''_{\cdot H}$ be such $\tau''_{jH}\leq\tau'_{jH}$ for each $j\in\mathcal{C}$. Given N''_H and N'_H such that $N''_H>N'_H$, condition (NE) only holds if $\mathbb{Q}''_H>\mathbb{Q}'_H$.*

Proof of Lemma A.16. By substituting in the optimal quantities given by (9), condition (NE) for trade costs $\boldsymbol{\tau}''_{\cdot H}$ and $\boldsymbol{\tau}''_{\cdot H}$ are given by, respectively, $\sum_{r_H\leq N'_H}\left(\frac{p_H^{\max}(\mathbb{Q}'_H)-c_{r_H}^{\tau'}}{2\gamma_H+\eta_H}\right)=\mathbb{Q}'_H$ and $\sum_{r_H\leq N''_H}\left[\frac{p_H^{\max}(\mathbb{Q}''_H)-c_{r_H}^{\tau''}}{2\gamma_H+\eta_H}\right]=\mathbb{Q}''_H$. Hence,

$$\sum_{r_H\leq N''_H}\left[\frac{p_H^{\max}\left(\mathbb{Q}''_H\right)-c_{r_H}^{\tau''}}{2\gamma_H+\eta_H}\right]-\sum_{r_H\leq N'_H}\left[\frac{p_H^{\max}\left(\mathbb{Q}'_H\right)-c_{r_H}^{\tau'}}{2\gamma_H+\eta_H}\right]=\mathbb{Q}''_H-\mathbb{Q}'{}_H,$$

which, given that $N''_H > N'_H$, can be expressed by

$$\underbrace{\sum_{r_H \leq N'_H} \left[\frac{p_H^{\max}(\mathbb{Q}''_H) - c_{r_H}^{\tau''}}{2\gamma_H + \eta_H} \right] - \sum_{r_H \leq N'_H} \left[\frac{p_H^{\max}(\mathbb{Q}'_H) - c_{r_H}^{\tau'}}{2\gamma_H + \eta_H} \right]}_{=:A_1} + \underbrace{\sum_{r_H > N'_H}^{N''_H} \left[\frac{p_H^{\max}(\mathbb{Q}''_H) - c_{r_H}^{\tau''}}{2\gamma_H + \eta_H} \right]}_{=:A_2} = \mathbb{Q}''_H - \mathbb{Q}'_H.$$

Towards a contradiction, suppose that $\mathbb{Q}''_H \leq \mathbb{Q}'_H$. Then, $p_H^{\max}(\mathbb{Q}''_H) \geq p_H^{\max}(\mathbb{Q}'_H)$ which, given $\tau''_{jH} \leq \tau'_{jH}$ for each $j \in \mathcal{C}$, implies that $A_1 \geq 0$. Moreover, A_2 comprises the optimal quantities of firms that are not active under $\boldsymbol{\tau}'_{\cdot H}$ but are under $\boldsymbol{\tau}''_{\cdot H}$. Thus, since the fixed cost is positive and, hence, the quantities of active firms is strictly positive, then $A_2 > 0$. This determines that the LHS is positive. But, since $\mathbb{Q}''_H \leq \mathbb{Q}'_H$, the RHS is nonpositive, which is a contradiction. ■

Lemma A.17. *For some given trade costs $\boldsymbol{\tau}_{\cdot H}$, suppose two firms r^1 and r^2 with costs indices of serving market H given by $c_{r^1}^{\tau}$ and $c_{r^2}^{\tau}$, respectively, where $c_{r^2}^{\tau} > c_{r^1}^{\tau}$. If the firm with $c_{r^2}^{\tau}$ serves H then the firm with $c_{r^1}^{\tau}$ does so too.*

Proof of Lemma A.17. Let $\mathbb{Q}_H^*$ denote the equilibrium aggregate quantity. Towards a contradiction, suppose that the firm with $c_{r^1}^{\tau}$ does not serve H. Let $\mathbb{Q}_H^1$ and $\mathbb{Q}_H^2$ be the aggregate quantity that the firms r^1 and r^2 face respectively when they have to make an entry decision. By the entry order and that, by Lemma A.16, $\mathbb{Q}$ is increasing in the number of firms, then $\mathbb{Q}_H^1 \leq \mathbb{Q}_H^2$.

We know that $p_H^{\max}\left[\mathbb{Q}_H^2 + q^{BR}\left(\mathbb{Q}_H^2; c_{r^2}^{\tau}\right)\right] - c_{r^2}^{\tau} \geq \xi_H$ but $p_H^{\max}\left[\mathbb{Q}_H^1 + q^{BR}\left(\mathbb{Q}_H^1; c_{r^1}^{\tau}\right)\right] - c_{r^1}^{\tau} < \xi_H$, where q^{BR} is the best response given by equation (8). Given that $\mathbb{Q}_H^1 \leq \mathbb{Q}_H^2$ and $c_{r^2}^{\tau} > c_{r^1}^{\tau}$, and using Lemma A.14, then $p_H^{\max}\left[\mathbb{Q}_H^1 + q\left(\mathbb{Q}_H^1; c_{r^1}^{\tau}\right)\right] - c_{r^1}^{\tau} \geq p_H^{\max}\left[\mathbb{Q}_H^2 + q\left(\mathbb{Q}_H^2; c_{r^2}^{\tau}\right)\right] - c_{r^2}^{\tau} \geq \xi_H$, which implies that $p_H^{\max}\left[\mathbb{Q}_H^1 + q\left(\mathbb{Q}_H^1; c_{r^1}^{\tau}\right)\right] - c_{r^1}^{\tau} \geq \xi_H$, which is a contradiction. ■

Lemma A.18. *Let $\boldsymbol{\tau}_{\cdot H}^*$ and $\boldsymbol{\tau}_{\cdot H}^{**}$ be such $\tau_{jH}^{**} \leq \tau_{jH}^*$ for each $j \in \mathcal{C} \backslash \{H\}$, with strict inequality for at least one country. If $p_H^{\max **} < p_H^{\max *}$, then the number of domestic firms in H with $\boldsymbol{\tau}_{\cdot H}^{**}$ is either lower or the same relative $\boldsymbol{\tau}_{\cdot H}^*$. If $p_H^{\max **} = p_H^{\max *}$, the number is lower.*

Proof of Lemma A.18. The lemma can be proved by showing that there are no new domestic firms serving H when $\mathbb{Q}_H^{**} \geq \mathbb{Q}_H^*$. Formally, this means that $N_{HH}^{**} \leq N_{HH}^*$. Towards a contradiction, suppose that there is entry of some domestic firm h with cost index c_h. Let $\mathbb{Q}_H^h$ be the aggregate quantity that the firm h faces when it makes its entry decision with trade costs $\boldsymbol{\tau}_{\cdot H}^{**}$. First, since, by assumption, h enters when trade costs are $\boldsymbol{\tau}_{\cdot H}^{**}$ but it is not active when they are $\boldsymbol{\tau}_{\cdot H}^*$, applying Lemma A.17 we know that $c_h \geq c_{N_H^*}^{\tau^*}$ and $c_h \leq c_{N_H^{**}}^{\tau^{**}}$. Moreover, given that $\tau_{jH}^{**} \leq \tau_{jH}^*$ for any $j \neq H$, then we know that, for any firm r, $c_r^{\tau^{**}} \leq c_r^{\tau^*}$. Thus, by Lemma A.17, an active firm r^* with trade costs $\boldsymbol{\tau}_{\cdot H}^*$ satisfies that $c_{r^*}^{\tau^*} \leq c_h$ and so $c_{r^*}^{\tau^{**}} \leq c_{N_H^{**}}^{\tau^{**}}$. Therefore $N_H^h \geq N_H^*$, where N_H^h is the number of firms when firm h has to make an entry decision under $\boldsymbol{\tau}_{\cdot H}^{**}$. By Lemma A.16, we know that $\mathbb{Q}_H^h \geq \mathbb{Q}_H^*$, which implies by Lemma A.15 that $\mathbb{Q}_H^h + q^{BR}\left(\mathbb{Q}_H^h, c_h\right) \geq \mathbb{Q}_H^* + q^{BR}\left(\mathbb{Q}_H^*, c_h\right)$. Thus, $p_H^{\max}\left[\mathbb{Q}_H^h + q^{BR}\left(\mathbb{Q}_H^h, c_h\right)\right] - c_h \leq p_H^{\max}\left[\mathbb{Q}_H^* + q^{BR}\left(\mathbb{Q}_H^*, c_h\right)\right] - c_h$ and, since $\mathbb{Q}_H^h + q^{BR}\left(\mathbb{Q}_H^h, c_h\right) \leq \mathbb{Q}_H^{**}$, it implies that firm h would have also been active with trade costs $\boldsymbol{\tau}_{\cdot H}^*$, which is a contradiction.

Now, we show that if, in particular, $\mathbb{Q}_H^{**} = \mathbb{Q}_H^*$ then $N_{HH}^{**} < N_{HH}^*$. Towards a contradiction, suppose not, so that, given that $N_{HH}^{**} \leq N_{HH}^*$, it implies that $N_{HH}^{**} = N_{HH}^*$. Taking the difference between (NE) under each trade cost,

$$\underbrace{\sum_{r_{HH} \leq N_{HH}^{**}} \frac{p_H^{\max}(\mathbb{Q}_H^{**}) - c_{r_{HH}}}{2\gamma_H + \eta_H} - \sum_{r_{HH} \leq N_{HH}^*} \frac{p_H^{\max}(\mathbb{Q}_H^*) - c_{r_{HH}}}{2\gamma_H + \eta_H}}_{=:B_1}$$

$$\underbrace{+\sum_{j \neq H} \left\{ \sum_{r_{jH} \leq N_{jH}^{**}} \left[\frac{p_H^{\max}(\mathbb{Q}_H^{**}) - c_{r_{jH}}^{\tau^{**}}}{2\gamma_H + \eta_H} \right] - \sum_{r_{jH} \leq N_{jH}^*} \left[\frac{p_H^{\max}(\mathbb{Q}_H^*) - c_{r_{jH}}^{\tau^*}}{2\gamma_H + \eta_H} \right] \right\}}_{=:B_2} = \mathbb{Q}_H^{**} - \mathbb{Q}_H^*.$$

Since $\mathbb{Q}_H^{**} = \mathbb{Q}_H^*$ and $N_{HH}^{**} = N_{HH}^*$, then $B_1 = 0$. In addition, by using that $\tau_{jH}^{**} \leq \tau_{jH}^*$ for each $j \in \mathcal{C} \backslash \{H\}$, we can conclude that there is no exit of foreign firms. By assumption, for at least one country k, $\tau_{kH}^{**} < \tau_{kH}^*$.

Therefore, active foreign firms from k are supplying a greater quantity by Lemma A.13. These determine that $B_2 > 0$. But the RHS is zero, which is a contradiction. ■

Lemma A.19. *Let $\boldsymbol{\tau}^*_{\cdot H}$ and $\boldsymbol{\tau}^{**}_{\cdot H}$ be such $\tau^{**}_{jH} \leq \tau^*_{jH}$ for each $j \neq H$, with strict inequality for at least one country. Suppose that the set of MIEs in H are homogeneous. If $\overline{\pi}^{**}_H < \overline{\pi}^*_H$ then $p^{\max **}_H < p^{\max *}_H$, and if $\overline{\pi}^{**}_H = \overline{\pi}^*_H$ then $p^{\max **}_H = p^{\max *}_H$.*

Proof of Lemma A.19. Since MIEs in H are homogeneous, then $c^{\tau^*}_{N^*_H} = c^{\tau^{**}}_{N^{**}_H}$. Hence, if $\overline{\pi}^{**}_H < \overline{\pi}^*_H$, then $p^{\max *}_H - c^{\tau^*}_{N^*_H} < p^{\max **}_H - c^{\tau^{**}}_{N^{**}_H}$ which determines that $p^{\max *}_H < p^{\max **}_H$. By the same token, if $\overline{\pi}^{**}_H = \overline{\pi}^*_H$ then $p^{\max *}_H = p^{\max **}_H$. ■

Lemma A.20. *Let $\boldsymbol{\tau}^*_{\cdot H}$ and $\boldsymbol{\tau}^{**}_{\cdot H}$ be such $\tau^{**}_{jH} \leq \tau^*_{jH}$ for each $j \in \mathcal{C} \backslash \{H\}$, with strict inequality for at least one country. Suppose that the set of MIEs in H are heterogeneous. If $\overline{\pi}^{**}_H \leq \overline{\pi}^*_H$ then $p^{\max **}_H < p^{\max *}_H$.*

Proof of Lemma A.20. Towards a contradiction, suppose not, so that $p^{\max **}_H \geq p^{\max *}_H$ and, hence, $\mathbb{Q}^{**}_H \leq \mathbb{Q}^*_H$. Since $\overline{\pi}^{**}_H \leq \overline{\pi}^*_H$, then we know that $p^{\max **}_H - c^{\tau^{**}}_{N^{**}_H} \leq p^{\max *}_H - c^{\tau^*}_{N^*_H}$ which, given that $p^{\max **}_H \geq p^{\max *}_H$, implies that $c^{\tau^{**}}_{N^{**}_H} \geq c^{\tau^*}_{N^*_H}$. Thus, since $c^{\tau^*}_{N^*_H} \geq c^{\tau^{**}}_{N^*_H}$, then $c^{\tau^{**}}_{N^{**}_H} \geq c^{\tau^{**}}_{N^*_H}$.

Take any active firm when trade costs are $\boldsymbol{\tau}^*_{\cdot H}$. Also, suppose its costs are indexed by $\overline{c}_{\tau^*}$ and $\overline{c}_{\tau^{**}}$ when trade costs are $\boldsymbol{\tau}^*_{\cdot H}$ and $\boldsymbol{\tau}^{**}_{\cdot H}$, respectively. Then, since, by definition, the firm is active when trade costs are $\boldsymbol{\tau}^*_{\cdot H}$, by Lemma A.17 we know that $\overline{c}_{\tau^*} \leq c^{\tau^*}_{N^*_H}$. Moreover, since $\overline{c}_{\tau^{**}} \leq \overline{c}_{\tau^*}$ and $c^{\tau^*}_{N^*_H} \leq c^{\tau^{**}}_{N^{**}_H}$, then $\overline{c}_{\tau^{**}} \leq c^{\tau^{**}}_{N^{**}_H}$. Thus, by applying Lemma A.17, we establish that any active firm in the equilibrium with $\boldsymbol{\tau}^*_{\cdot H}$ is also active when trade costs are $\boldsymbol{\tau}^{**}_{\cdot H}$. Since there is no exit of firms, we know that $N^{**}_H \geq N^*_H$ and condition (NE) is such that

$$\underbrace{\sum_{r_H \leq N^*_H} \left[\frac{p^{\max}_H(\mathbb{Q}^{**}_H) - c^{\tau^{**}}_{r_H}}{2\gamma_H + \eta_H} \right] - \sum_{r_H \leq N^*_H} \left[\frac{p^{\max}_H(\mathbb{Q}^{*}_H) - c^{\tau^{*}}_{r_H}}{2\gamma_H + \eta_H} \right]}_{=:A_1} + \underbrace{\sum_{r_H > N^*_H}^{N^{**}_H} \left[\frac{p^{\max}_H(\mathbb{Q}^{**}_H) - c^{\tau^{**}}_{r_H}}{2\gamma_H + \eta_H} \right]}_{=:A_2} = \mathbb{Q}^{**}_H - \mathbb{Q}^*_H.$$

The term A_1 comprises firms that are active in both equilibria. We know that $c^{\tau^{**}}_{r_H} \leq c^{\tau^*}_{r_H}$ with strict inequality for at least one term, since $\tau^*_{jH} > \tau^{**}_{jH}$ for some $j \in \mathcal{C} \backslash \{H\}$. Moreover, $p^{\max}_H(\mathbb{Q}^{**}_H) \geq p^{\max}_H(\mathbb{Q}^*_H)$. Therefore, $A_1 > 0$. In addition, A_2 comprises firms that enter when trade costs are $\boldsymbol{\tau}^{**}_{\cdot H}$, but are inactive when trade costs are $\boldsymbol{\tau}^*_{\cdot H}$. Thus, since it is possible that $N^{**}_H = N^*_H$, it implies that $A_2 \geq 0$. Both facts determine that the LHS is positive. But, because we were assuming that $\mathbb{Q}^{**}_H \leq \mathbb{Q}^*_H$, the RHS is nonpositive, which is a contradiction. ■

Proof of Proposition 6.1. Consider the case where $\overline{\pi}^{**}_H < \overline{\pi}^*_H$. By Lemma A.19, this implies that $p^{\max **}_H < p^{\max *}_H$. Regarding domestic firms in H that are active in both equilibria, by Lemma A.12 and the fact that $p^{\max **}_H < p^{\max *}_H$, we obtain that $p^{**}_{HH}(c)$, $q^{**}_{HH}(c)$, $m^{**}_{HH}(c)$ and $\mu^{**}_{HH}(c)$ are lower relative to the equilibrium with trade costs $\boldsymbol{\tau}^*_{\cdot H}$. Also, by Lemma A.18, there is no entry of new domestic firms.

Consider the case with $\overline{\pi}^{**}_H = \overline{\pi}^*_H$. By Lemma A.19, this implies that $p^{\max **}_H = p^{\max *}_H$. Therefore, since $p^{\max **}_H = p^{\max *}_H$, for any domestic firm that is active in H in both equilibria, we get that $p^{**}_{HH}(c)$, $q^{**}_{HH}(c)$, $m^{**}_{HH}(c)$ and $\mu^{**}_{HH}(c)$ have the same value as in the equilibrium with $\boldsymbol{\tau}^*_{\cdot H}$. Also, by Lemma A.18, some of the domestic firms exit. ■

Proof of Proposition 6.2. Since MIEs are heterogeneous and $\overline{\pi}^{**}_H \leq \overline{\pi}^*_H$, applying Lemma A.20 we get that $p^{\max **}_H < p^{\max *}_H$. Then, the proof for the effects on prices, quantities, and markups of active domestic firms follows verbatim the proof of Proposition 6.1. ■

Proof of Proposition 6.3. The profits of the last entrants under $\boldsymbol{\tau}^*_{H\cdot}$ and $\boldsymbol{\tau}^{**}_{H\cdot}$ are, respectively,

$$\frac{\gamma_H + \eta_H}{(2\gamma_H + \eta_H)^2} \left(p^{\max *}_H - c_{N^*_H}\right)^2 + \frac{\gamma_F + \eta_F}{(2\gamma_F + \eta_F)^2} \left(p^{\max *}_F - c^{\tau^*}_{N^*_H}\right)^2 \geq F^E_H + 2f,$$

$$\frac{\gamma_H + \eta_H}{(2\gamma_H + \eta_H)^2} \left(p^{\max **}_H - c_{N^{**}_H}\right)^2 + \frac{\gamma_F + \eta_F}{(2\gamma_F + \eta_F)^2} \left(p^{\max **}_F - c^{\tau^{**}}_{N^{**}_H}\right)^2 \geq F^E_H + 2f.$$

Given that $\overline{\pi}^{*}_{\nu_H} \geq \overline{\pi}^{**}_{\nu_H}$, then

$$\underbrace{\frac{\gamma_H+\eta_H}{(2\gamma_H+\eta_H)^2}\left[\left(p_H^{\max *}-c_{N_H^*}\right)^2-\left(p_H^{\max **}-c_{N_H^{**}}\right)^2\right]}_{=:A_1}+\underbrace{\frac{\gamma_F+\eta_F}{(2\gamma_F+\eta_F)^2}\left\{\left(p_F^{\max *}-c^{\tau^*}_{N_H^*}\right)^2-\left(p_F^{\max **}-c^{\tau^{**}}_{N_H^{**}}\right)^2\right\}}_{=:A_2} \geq 0. \tag{32}$$

Moreover, taking the difference between (NE) for each equilibrium, we obtain

$$\underbrace{\sum_{r_{HH}\leq N^{**}_{HH}} \frac{p_H^{\max}\left(\mathbb{Q}^{**}_H\right)-c_{r_{HH}}}{2\gamma_H+\eta_H} - \sum_{r_{HH}\leq N^{*}_{HH}} \frac{p_H^{\max}\left(\mathbb{Q}^{*}_H\right)-c_{r_{HH}}}{2\gamma_H+\eta_H}}_{=:B_1}$$
$$+\underbrace{\sum_{j\neq H}\left\{\sum_{r_{jH}\leq N^{**}_{jH}}\left[\frac{p_H^{\max}\left(\mathbb{Q}^{**}_H\right)-c^{\tau}_{r_{jH}}}{2\gamma_H+\eta_H}\right]-\sum_{r_{jH}\leq N^{*}_{jH}}\left[\frac{p_H^{\max}\left(\mathbb{Q}^{*}_H\right)-c^{\tau}_{r_{jH}}}{2\gamma_H+\eta_H}\right]\right\}}_{=:B_2} = \mathbb{Q}^{**}_H-\mathbb{Q}^{*}_H. \tag{33}$$

If there are no changes in the extensive-margin adjustments of domestic firms, then the shock does not affect market H and, so, $p_H^{\max *} = p_H^{\max **}$ in order for (33) to hold. Since $\boldsymbol{\tau}^{*}_{H\cdot}$ and $\boldsymbol{\tau}^{**}_{H\cdot}$ can only affect the domestic market through the choke price and H is a small economy, this implies that there are no changes in market H.

Consider now that there are extensive-margin adjustments in the domestic market. First, we rule out that $p_H^{\max *} = p_H^{\max **}$. If that were the case, the RHS would be zero. Moreover, given that the variation in trade costs does not affect the order of firms in H and $p_H^{\max *} = p_H^{\max **}$, then $B_2 = 0$. Thus, it necessarily has to be that $B_1 = 0$. However, since $p_H^{\max *} = p_H^{\max **}$, this can only occur if the number of domestic firms is the same. But then there would no changes in the extensive margin of the domestic firms, which is a contradiction.

Next, we show that $p_H^{\max *} < p_H^{\max **}$ leads to a contradiction. The assumption implies $\mathbb{Q}^{*}_H > \mathbb{Q}^{**}_H$ and, so, the RHS of (33) is negative. Given that $p_H^{\max *} < p_H^{\max **}$ and the order of firms in the domestic market is the same, the same set of foreign firms is active before and after the trade shock. Thus, given that $\mathbb{Q}^{*}_H > \mathbb{Q}^{**}_H$, $B_2 > 0$. This determines that $B_1 < 0$. Also, since $\mathbb{Q}^{*}_H > \mathbb{Q}^{**}_H$, active domestic firms supply more quantities in the equilibrium with $\boldsymbol{\tau}^{**}_{H\cdot}$. Thus, $B_1 > 0$ only if there is exit of at least one domestic firm. This implies that $c_{N_H^*} \geq c_{N_H^{**}}$, with equality if MIEs are homogeneous. Since $p_H^{\max *} < p_H^{\max **}$ and $c_{N_H^*} \geq c_{N_H^{**}}$, then $A_1 < 0$, implying that $A_2 > 0$. But, if that is the case, then $c^{\tau^{**}}_{N_H^{**}} > c^{\tau^*}_{N_H^*}$ which is a contradiction, since $\tau^{**}_{HF} < \tau^{*}_{HF}$.

Given that $p_H^{\max *} > p_H^{\max **}$, the effects on prices, quantities and markups in H follow by Lemma A.12.

Next, we prove that there is no exit of domestic firms which, joint with the assumption that there are extensive-margin adjustments of domestic firms, implies that some inactive firms from H become active. Towards a contradiction, suppose that there is exit of at least one domestic firm. Given that $\mathbb{Q}^{*}_H < \mathbb{Q}^{**}_H$, the RHS of (33) is positive. Moreover, since optimal quantities are given by (9) and $\mathbb{Q}^{*}_H < \mathbb{Q}^{**}_H$, any active domestic firm supplies less quantities in the equilibrium with $\boldsymbol{\tau}^{**}_{H\cdot}$. In addition, we have assumed that there is exit of at least one domestic firm. Hence $B_1 < 0$. This also implies that it necessarily has to be that $B_2 > 0$. But active foreign firms are supplying less quantities in the equilibrium with $\boldsymbol{\tau}^{**}_{H\cdot}$ and the profits of foreign firms are lower in equilibrium. This rules out entry of foreign firms. Thus, $B_2 < 0$, which is a contradiction. ■

A.4 Applications to Cournot Competition

Next, we formalize the case of restricted entry outlined in Section 7. There, we stated that, when there is restricted entry, reductions in inward trade barriers decrease the profits of the last entrant. We proceed to prove this formally.

Proposition A.21

Consider a world economy with an arbitrary number of countries, where H is a small economy. Suppose a model à la Cournot and let $\boldsymbol{\tau}^{*}_{\cdot H}$ and $\boldsymbol{\tau}^{**}_{\cdot H}$ be such $\tau^{**}_{jH} \leq \tau^{*}_{jH}$ with strict inequality for at *least one country. If there are no changes in the set of active firms in H, then $p_H^{\max **} < p_H^{\max *}$. Moreover, if the last entrant under $\boldsymbol{\tau}_{\cdot H}^*$ is domestic, then $\overline{\pi}_H^{**} < \overline{\pi}_H^*$.*

Proof of Proposition A.21. Suppose that there is no change in the set of active domestic firms. Taking the difference between (NE) for each equilibrium yields

$$\underbrace{\sum_{r_{HH} \leq N_{HH}^{**}} \frac{p_H^{\max}\left(\mathbb{Q}_H^{**}\right) - c_{r_{HH}}}{2\gamma_H + \eta_H} - \sum_{r_{HH} \leq N_{HH}^{*}} \frac{p_H^{\max}\left(\mathbb{Q}_H^{*}\right) - c_{r_{HH}}}{2\gamma_H + \eta_H}}_{=:B_1}$$

$$\underbrace{+\sum_{j \neq H} \left\{ \sum_{r_{jH} \leq N_{jH}^{**}} \left[\frac{p_H^{\max}\left(\mathbb{Q}_H^{**}\right) - c_{r_{jH}}^{\tau^{**}}}{2\gamma_H + \eta_H} \right] - \sum_{r_{jH} \leq N_{jH}^{*}} \left[\frac{p_H^{\max}\left(\mathbb{Q}_H^{*}\right) - c_{r_{jH}}^{\tau^{*}}}{2\gamma_H + \eta_H} \right] \right\}}_{=:B_2} = \mathbb{Q}_H^{**} - \mathbb{Q}_H^{*}.$$

Given that $\tau_{jH}^{**} \leq \tau_{jH}^*$ with strict inequality for at least one country, it is easy to see that $\mathbb{Q}_H^{**} = \mathbb{Q}_H^*$ cannot be part of an equilibrium. Thus, towards a contradiction, suppose that $\mathbb{Q}_H^{**} < \mathbb{Q}_H^*$, so that $p_H^{\max **} > p_H^{\max *}$. When this is the case, $B_1 > 0$ and $B_2 > 0$, but the RHS is negative, which is a contradiction. Thus, $p_H^{\max **} < p_H^{\max *}$.

Given that the last entrant is domestic when trade costs are $\boldsymbol{\tau}_{\cdot H}^*$, we have two possibilities regarding the last entrant under $\boldsymbol{\tau}_{\cdot H}^{**}$: it could either be domestic or foreign. If it is domestic, then the fact that $p_H^{\max **} > p_H^{\max *}$ implies that $\overline{\pi}_H^{**} < \overline{\pi}_H^*$. Suppose that the last entrant is foreign. Then, by using that $p_H^{\max **} > p_H^{\max *}$, this implies that $\overline{\pi}_H^* = \overline{\pi}_{HH}^* > \overline{\pi}_{HH}^{**} \geq \overline{\pi}_{FH}^{**}$, and the result follows. ■

B Magnitude of the Import-Competition Channel

In Proposition 3.2, we have established that the import-competition channel is activated when MIEs are ex-ante heterogeneous. In addition, in Section 3.3, we sketched out some conclusions regarding the magnitude of the import-competition channel when this is active. In particular, we claimed that, even though the whole distribution of productivity matters when the import-competition channel is active, the degree of heterogeneity of MIEs plays a distinctive role: when their heterogeneity is negligible, the effect on the choke price is also negligible.

Next, we prove this claim. The proof requires to dispense with the partition of firms considered in the main part of the paper. By doing so, we are able to exactly isolate the subset of firms that comprise the MIEs for each vector of trade costs.

Consider a monopolistic competition model with a small economy H, where D_H^ω is degenerate and determines an atomless productivity distribution at the market. In other terms, the set of firms with the same D_H^ω is of measure zero. Also, given that D_H^ω is degenerate, ex-ante and ex-post features of firms coincide.

We suppose that in H there are two possible distributions of marginal costs at the country level, whose cdfs are $\underline{\underline{G}}_H$ or $\overline{\overline{G}}_H$ and both have support $[\underline{c}_H, \overline{c}_H]$. In terms of notation, for any variable $\cdot$ we indicate its corresponding equilibrium value under each distribution by $\underline{\underline{\cdot}}$ and $\overline{\overline{\cdot}}$, respectively.

Moreover, we compare the equilibrium under two vectors of trade costs, $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$, where $\tau_{jH}^* > \tau_{jH}^{**}$ for each $j \neq H$. To isolate the role of MIEs, we proceed as it follows. First, we suppose that H behaves identically under $\boldsymbol{\tau}^{**}$, irrespective of whether the cdf is $\underline{\underline{G}}_H$ or $\overline{\overline{G}}_H$. This is accomplished by supposing that the marginal-cost cutoff of domestic firms is given by c_{HH}^{**} under both $\underline{\underline{G}}_H$ and $\overline{\overline{G}}_H$ when trade costs are $\boldsymbol{\tau}^{**}$, and that these distributions are identical for $c \leq c_{HH}^{**}$ with cdf $\underline{\underline{G}}_H = \overline{\overline{G}}_H =: G_H$ and density g_H. Second, for $c \in [c_{HH}^{**}, \kappa]$ where $\kappa < \overline{c}_H$ and $\kappa > \max\left\{\underline{\underline{c}}_{HH}^*, \overline{\overline{c}}_{HH}^*\right\}$, we suppose that $\overline{\overline{g}}_H(c) > \underline{\underline{g}}_H(c)$.[8] Intuitively, the fact that $\overline{\overline{g}}_H(c) > \underline{\underline{g}}_H(c)$ for $c \in [c_{HH}^{**}, \kappa]$ means that MIEs are more concentrated when the cdf is $\overline{\overline{G}}_H$. Therefore,

[8]Implicitly, this is valid by supposing that there is a set of inactive firms under both equilibria such that we can shift the distribution, without affecting the distribution of firms that are active in each equilibrium.

under the distribution $\overline{\overline{G}}_H$, the firms are less heterogeneous. In the limit, when $\overline{\overline{g}}_H$ is the Dirac Delta function, the distribution would be degenerate and MIEs homogeneous.

In words, these assumptions ensure that H is identical under both distributions of marginal costs when trade costs are $\boldsymbol{\tau}^{**}$, but they differ exclusively by the dispersion of marginal costs of inactive firms.

Proposition B.1

Consider a world economy with an arbitrary number of countries, where H is a small economy. Suppose a monopolistic-competition model with two possible cdfs of marginal costs at the country level given by $\underline{\underline{G}}_H$ and $\overline{\overline{G}}_H$ defined as above. Consider trade costs $\boldsymbol{\tau}^{*}$ and $\boldsymbol{\tau}^{**}$, where $\tau^{*}_{jH} > \tau^{**}_{jH}$ for each $j \neq H$, such that the marginal-cost cutoff of domestic firms under $\boldsymbol{\tau}^{**}$ is c^{**}_{HH}. Then, $\left|\Delta \overline{\overline{p}}^{\max}_H\right| < \left|\Delta \underline{\underline{p}}^{\max}_H\right|$ where $\Delta p^{\max}_H := p^{\max **}_H - p^{\max *}_H$.

Proof of Proposition B.1. We denote the equilibrium values for each vector of trade costs by superscripts $*$ and $**$ respectively. By assumption, when import trade costs are $\boldsymbol{\tau}^{**}_{\cdot H}$, the distributions of marginal costs for active firms are the same under $\underline{\underline{G}}_H$ and $\overline{\overline{G}}_H$. Therefore, the equilibrium condition (MS) is

$$\overline{M}_H \int_{\underline{c}_H}^{c^{**}_{HH}} \left(p^{\max **}_H - c\right) g_H(c)\,\mathrm{d}c + \Phi_{-H}\left(p^{\max **}_H; \boldsymbol{\tau}^{**}_{\cdot H}\right) = 2\left(\alpha_H - p^{\max **}_H\right)\beta_H, \tag{34}$$

which determines that $p^{\max **}_H$ and c^{**}_{HH} are the same under $\underline{\underline{G}}_H$ and $\overline{\overline{G}}_H$.

Suppose now that import trade costs are $\boldsymbol{\tau}^{*}_{\cdot H}$. Condition (MS) for each distribution is, respectively,

$$\overline{M}_H \left[\int_{\underline{c}_H}^{c^{**}_{HH}} \left(\overline{\overline{p}}^{\max *}_H - c\right) g_H(c)\,\mathrm{d}c + \int_{c^{**}_{HH}}^{\overline{\overline{c}}^{*}_H} \left(\overline{\overline{p}}^{\max *}_H - c\right) \overline{\overline{g}}_H(c)\,\mathrm{d}c\right] + \Phi_{-H}\left(\overline{\overline{p}}^{\max *}_H; \boldsymbol{\tau}^{*}_{\cdot H}\right) = 2\left(\alpha_H - \overline{\overline{p}}^{\max *}_H\right)\beta_H,$$

$$\overline{M}_H \left[\int_{\underline{c}_H}^{c^{**}_{HH}} \left(\underline{\underline{p}}^{\max *}_H - c\right) g_H(c)\,\mathrm{d}c + \int_{c^{**}_{HH}}^{\underline{\underline{c}}^{*}_H} \left(\underline{\underline{p}}^{\max *}_H - c\right) \underline{\underline{g}}_H(c)\,\mathrm{d}c\right] + \Phi_{-H}\left(\underline{\underline{p}}^{\max *}_H; \boldsymbol{\tau}^{*}_{\cdot H}\right) = 2\left(\alpha_H - \underline{\underline{p}}^{\max *}_H\right)\beta_H,$$

and they imply that

$$\begin{aligned}
&\overline{M}_H \left[\int_{\underline{c}_H}^{c^{**}_{HH}} \left(\overline{\overline{p}}^{\max *}_H - c\right) g_H(c)\,\mathrm{d}c + \int_{c^{**}_{HH}}^{\overline{\overline{c}}^{*}_H} \left(\overline{\overline{p}}^{\max *}_H - c\right) \overline{\overline{g}}_H(c)\,\mathrm{d}c\right] + \Phi_{-H}\left(\overline{\overline{p}}^{\max *}_H; \boldsymbol{\tau}^{*}_{\cdot H}\right) + 2\beta_H \overline{\overline{p}}^{\max *}_H = \\
&\overline{M}_H \left[\int_{\underline{c}_H}^{c^{**}_{HH}} \left(\underline{\underline{p}}^{\max *}_H - c\right) g_H(c)\,\mathrm{d}c + \int_{c^{**}_{HH}}^{\underline{\underline{c}}^{*}_H} \left(\underline{\underline{p}}^{\max *}_H - c\right) \underline{\underline{g}}_H(c)\,\mathrm{d}c\right] + \Phi_{-H}\left(\underline{\underline{p}}^{\max *}_H; \boldsymbol{\tau}^{*}_{\cdot H}\right) + 2\beta_H \underline{\underline{p}}^{\max *}_H.
\end{aligned} \tag{35}$$

Next, we show that $\overline{\overline{p}}^{\max *}_H > \underline{\underline{p}}^{\max *}_H$ and $\overline{\overline{p}}^{\max *}_H = \underline{\underline{p}}^{\max *}_H$ lead us to a contradiction. Suppose that $\overline{\overline{p}}^{\max *}_H > \underline{\underline{p}}^{\max *}_H$. By condition (ZCP), $\underline{\underline{c}}^{*}_{HH} = \underline{\underline{p}}^{\max *}_H - \xi_{HH}$ and $\overline{\overline{c}}^{*}_{HH} = \overline{\overline{p}}^{\max *}_H - \xi_{HH}$, which implies that $\overline{\overline{c}}^{*}_{HH} > \underline{\underline{c}}^{*}_{HH}$. Since $\frac{\partial \Phi^{*}_{jH}}{\partial p^{\max}_H} > 0$ for each $j \neq H$, then $\Phi_{-H}\left(\overline{\overline{p}}^{\max *}_H; \boldsymbol{\tau}^{*}_{\cdot H}\right) > \Phi_{-H}\left(\underline{\underline{p}}^{\max *}_H; \boldsymbol{\tau}^{*}_{\cdot H}\right)$. Finally, by assumption, $\overline{\overline{g}}_H(c) > \underline{\underline{g}}_H(c)$ for $c \in [c^{**}_{HH}, \kappa]$. But, then, the LHS of (35) is always greater than the RHS and the equality cannot hold.

Now consider that $\overline{\overline{p}}^{\max *}_H = \underline{\underline{p}}^{\max *}_H =: p^{\max *}_H$. This implies that $\overline{\overline{c}}^{*}_{HH} = \underline{\underline{c}}^{*}_{HH} =: c^{*}_{HH}$ and the equality in (35) can only hold if $c^{**}_{HH} = c^{*}_{HH}$ since $\overline{\overline{g}}_H > \underline{\underline{g}}_H$. This determines that $p^{\max **}_H = p^{\max *}_H$. But, if that is the case, then

$$\overline{M}_H \left[\int_{\underline{c}_H}^{c^{**}_{HH}} \left(p^{\max *}_H - c\right) g_H(c)\,\mathrm{d}c\right] + \Phi_{-H}\left(p^{\max *}_H; \boldsymbol{\tau}^{*}_{\cdot H}\right) = \overline{M}_H \int_{\underline{c}_H}^{c^{**}_{HH}} \left(p^{\max **}_H - c\right) g_H(c)\,\mathrm{d}c + \Phi_{-H}\left(p^{\max **}_H; \boldsymbol{\tau}^{**}_{\cdot H}\right). \tag{36}$$

The first terms on each side are equal since $p^{\max **}_H = p^{\max *}_H$. But then, since $\frac{\partial \Phi^{*}_{jH}}{\partial \tau_{jH}} < 0$ for each $j \neq H$, we have that $\Phi_{-H}\left(p^{\max **}_H; \boldsymbol{\tau}^{**}_{\cdot H}\right) > \Phi_{-H}\left(p^{\max *}_H; \boldsymbol{\tau}^{*}_{\cdot H}\right)$. Therefore, (36) cannot hold with equality, which is a contradiction. Thus, $\overline{\overline{p}}^{\max *}_H < \underline{\underline{p}}^{\max *}_H$. ■

C Numerical Illustrations

In this appendix, we expand upon the numerical illustrations presented in the main part of the paper. They all refer to the experiment of a variation in import trade costs in a small economy, and were introduced with the goal comparing the mechanism of adjustment under different assumptions regarding the MIEs.

C.1 Monopolistic Competition

Next, we provide further details of the two exercises used in Section 3.3. These two numerical exercises have some features in common and are based on the approach in Appendix B. This entails the following.

First, we do not assume that there is a partition of firms in terms of insiders, entrants, and non-active firms. Specifically, we suppose that $\mathcal{I}$ and $\mathcal{N}$ are empty sets, so that $\mathcal{E}$ is the only group in the economy. Moreover, as in that appendix, we establish a productivity distribution which distinguishes between always-active firms and the rest. This serves the purpose of isolating assumptions that only affect the MIEs' productivity distribution, rather than a whole group. Finally, we consider that each firm obtains a productivity draw from a degenerate distribution, so that we do not need to make any ex-ante qualification when referring to the MIEs' features. Moreover, this distribution is not necessarily different for each firm, allowing us to consider the possibility that MIEs are either ex-ante homogeneous or ex-ante heterogeneous.

We present two types of results, distinguished by the nature of the marginal-cost distribution of all potential firms in the country. In Appendix C.1.1, we consider an atomless distribution, while in Appendix C.1.2 the distribution has atoms.

C.1.1 Continuous Productivity Distributions

We compare the outcomes for several domestic productivity distributions. They have the property of coinciding for the set of active firms but differing for those which are inactive. Specifically, we consider a Pareto distribution for active firms and a uniform distribution for the rest.

To implement this, it is necessary to ensure that, before the trade shock, the set of active firms for the different distributions we compare is the same and coincides with the portion that has a Pareto distribution. We achieve this by calibrating the trade costs of the foreign firms such that, initially, the domestic distributions for active firms are identical. Then, we consider increases in trade costs so that the behavior of the domestic economy in each case differs exclusively by the productivity distribution of the MIEs.

Also, the use of a uniform distribution allows us to measure the degree of heterogeneity of MIEs through the length of the support. Thus, we refer to it as the index of heterogeneity and denote it by l_{me}. For a given measure of firms, a greater length determines that the levels of productivity are more dispersed and, hence, MIEs more heterogeneous.

Algorithm Description.

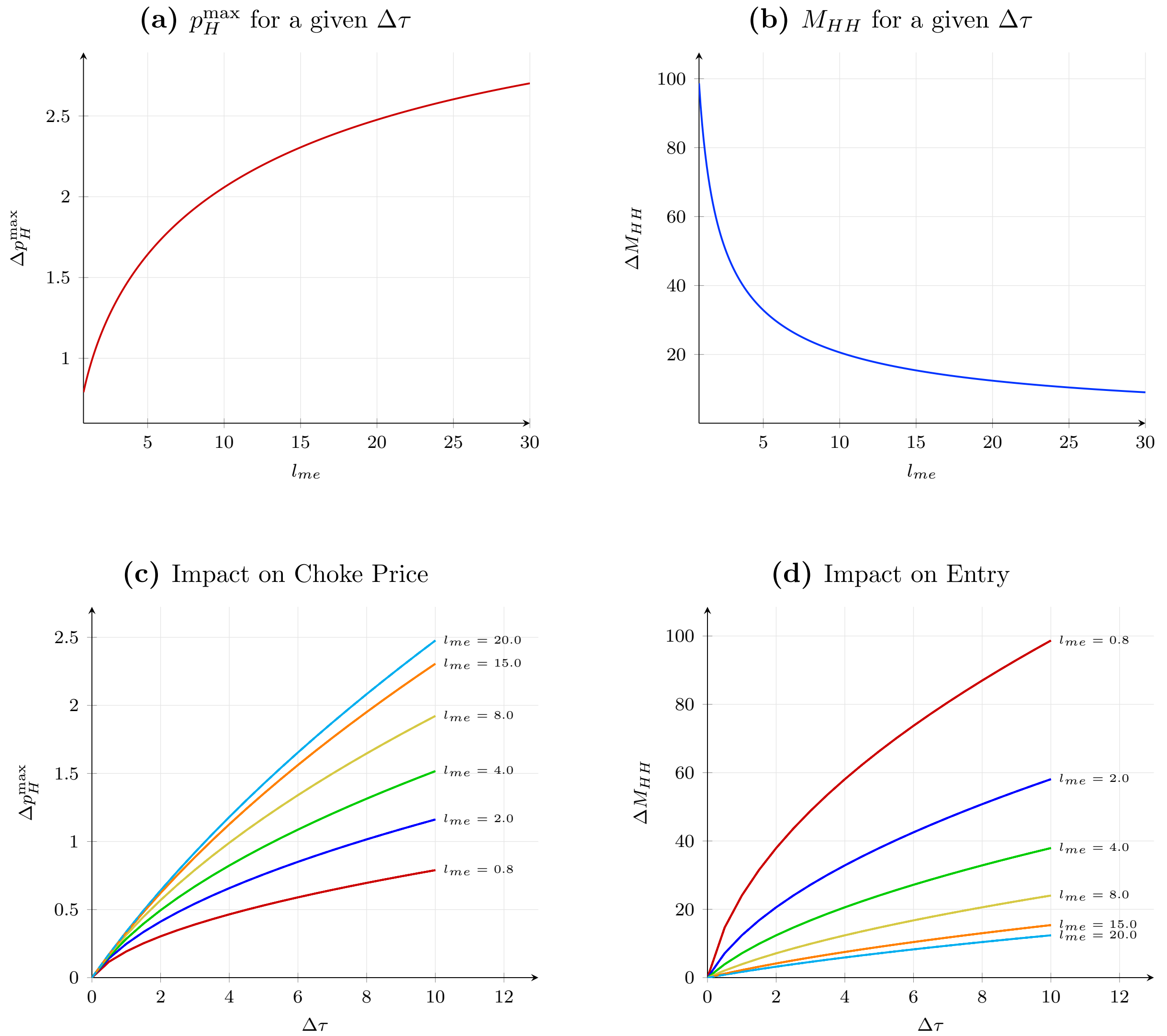
[1] Break down the distribution of domestic firms into always-active firms and the rest of the firms. Set a function that returns the trade costs that make all the firms in the group with a Pareto distribution serve the market, while ensuring the rest of the firms are inactive.

[2] Set a vector of trade costs greater than the value obtained in the previous step. Create a function that, for some l_{me}, calculates the equilibrium at the market stage for all the values of trade costs.

[3] Set a vector of values for l_{me} and create a function that returns the equilibrium for this vector.

We present the results of the simulations and then proceed to its analysis.

Figure 9. *Unilateral Liberalization in a Small Economy: Monopolistic Competition with a Continuous Distribution*



Note: l_{me} refers to the length of the uniform distribution. In Figures 9c and 9d, the choke price and measure of domestic MIEs are normalized by expressing them as a difference relative to its initial value.

The figures allow us to illustrate several results pertaining to the choke price and entry. First, consider Figures 9a and 9b. They refer to a specific variation in trade costs, allowing for the index of heterogeneity to change. Figure 9a presents the impact on the choke price, while Figure 9b depicts the effect on the measure of MIEs. They reveal that, in terms of adjustments, there is an inverse relation between these two variables. When MIEs become less heterogeneous (i.e., l_{me} is lower and, so, the levels of productivity are less dispersed), the intensity of adjustment in terms of the choke price is diminished. Consequently, the competitive environment is less affected and the model adjusts more intensively in terms of the extensive margin.

Consider now Figures 9c and 9d. They capture the outcomes when a vector of trade-costs variations is considered. The different lines correspond to different degrees of heterogeneity measured through l_{me}. From them, we can infer two conclusions. First, by comparing the curves in each figure, we can appreciate that they follow the opposite ordering. This is consistent with the relationships depicted in Figures 9a and 9b. Second, when the MIEs start to have a low degree of heterogeneity, the impact on the mass of MIEs becomes quite pronounced. In particular, the change is greater for variations between $l_{me} = 0.8$ and $l_{me} = 2.0$, relative to the variations between $l_{me} = 15$ and $l_{me} = 20$. If we shrink the length of the uniform distribution until $l_{me} \to 0$, the adjustment completely takes place through the extensive margin. However, this case cannot be depicted given the problems of numerical convergence that arise when we try to simulate it. For this reason, next we resort to productivity distributions that exhibit mass points.

C.1.2 Productivity Distributions With Mass Points

In this part, we modify the productivity distribution for MIEs, while other aspects remain as in Appendix C.1.1. We suppose that the set of inactive firms can be partitioned into several non-zero measure groups where, within each of them, firms share the same productivity. Formally, this is reflected by assuming that firms within each subset obtain productivity draws from a degenerate distribution.

This type of distribution determines that, depending on the magnitude of the shock to trade costs, MIEs can be homogeneous or heterogeneous. On the one hand, MIEs are homogeneous when increases in trade costs are small enough (or the subset of the most productive firms among MIEs is big enough) so that all of the adjustment takes place within one particular group. On the other hand, MIEs are heterogeneous when the same shock induces entry of firms coming from more than one group of inactive firms. Thus, *this distribution allows us to reflect in a unified way how the process of adjustment occurs when MIEs are homogeneous or heterogeneous.*

For the numerical exercise, we define several distributions of marginal costs distinguished by a parameter c_m. This parameter indicates the difference of marginal costs between subsets of inactive firms, such that a greater value of c_m corresponds to a scenario with more heterogeneity across firms. Due to this, in this experiment, c_m becomes the index of heterogeneity.

Unlike the exercise considered previously, the simulations we present here avoid any convergence issues. In this way, we are able to show that the relation between heterogeneity of MIEs and the impact on the choke price collapses to a zero effect when MIEs are homogeneous (i.e., when $c_m = 0$).

Outline of the Algorithm.

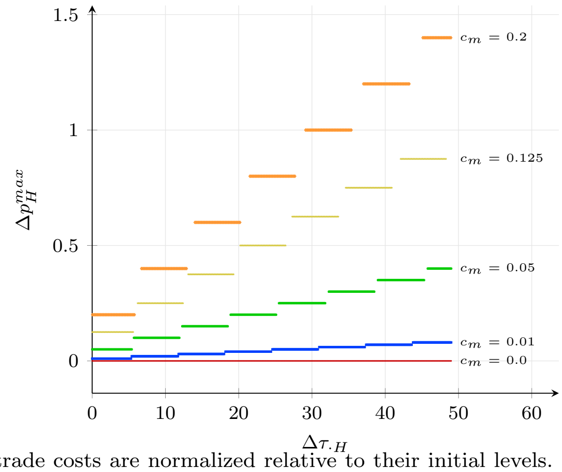
[1] Break down the distribution of domestic firms into always-active firms and the rest of the firms. Set a function that returns the trade costs which make all the firms in the group with a Pareto distribution serve the market, while the rest of the firms inactive.

[2] Set a vector of trade costs greater than the value obtained in the previous step. Create a function that calculates the equilibrium for each vector of trade costs and distribution of inactive firms, where the distributions differ according to the value of c_m. The calculation of the equilibrium for a specific variation of trade costs involves two steps. First, order the groups of inactive firms from the most to the least productive. Start by assuming that the equilibrium is given by entry of the first group. Calculate the measure of firms belonging to that group which would restore the equilibrium. If the measure is lower than the actual measure of potential firms within that group, the outcome constitutes an equilibrium. If the measure is greater, then consider an equilibrium with firms belonging to the second group. Iterate until there is convergence.

In Figure 10, we include only equilibrium values in which there is a positive measure of firms having zero profits.[9] The different lines in the graph correspond to different magnitudes of c_m.

[9] In other words, since we want to focus on the channels arising in standard models of monopolistic competition, we consider equilibria where the MIEs profits channel is inactive.

Figure 10. *Unilateral Liberalizations in a Small Economy: Monopolistic Competition with Mass Points*



Note: The choke price and trade costs are normalized relative to their initial levels. Only points where zero profits hold are considered.

Consider one of the lines with $c_m > 0$. An increase in inward trade costs leads to a shortage of supply. Under the presence of mass points, the model begins to adjust by entry of firms with the same productivity, among the group of the most-productive inactive ones. This occurs without any variation in the choke price, explaining the horizontal portions of the line.

When the variation in the measure of firms of that group is not capable of restoring the equilibrium, the choke price has to rise in order to further increase the quantity supplied. This is achieved through both active firms increasing their quantity supplied and the entry of additional firms. In particular, the latter occurs via entry from the second group of most-productive inactive firms. In terms of the graph, this is reflected by each line exhibiting a stepped pattern.

The graph also shows the implications of Proposition B.1: the lower the heterogeneity of the MIEs (i.e., the lower c_m), the lower the impact on the choke price. Furthermore, in the limit with $c_m = 0$, firms become homogeneous and all of the adjustment is through the mass of MIEs, with no impact on market conditions.

C.2 Cournot Competition

Here, we provide further details of the numerical illustration in Section 6.1 corresponding to the Cournot model. We describe the algorithm to compute the equilibrium and present some additional figures.

The search for the equilibrium exploits the monotonicity of profits in $p_H^{\max *}$ (by Lemma A.14), the monotonicity of $p_H^{\max *}$ in N_H (by Lemma A.16), and the order of profits (by Lemma A.17). The algorithm applied to a domestic country H is as follows.

Outline of the Algorithm.

[1] Define the marginal costs (inclusive of trade costs) for all the potential firms across the world. Order the firms from the lowest to the greatest cost to serve market H.

[2] Establish a function for H as in (NE) that returns the total quantities in H for a given a number of firms N.

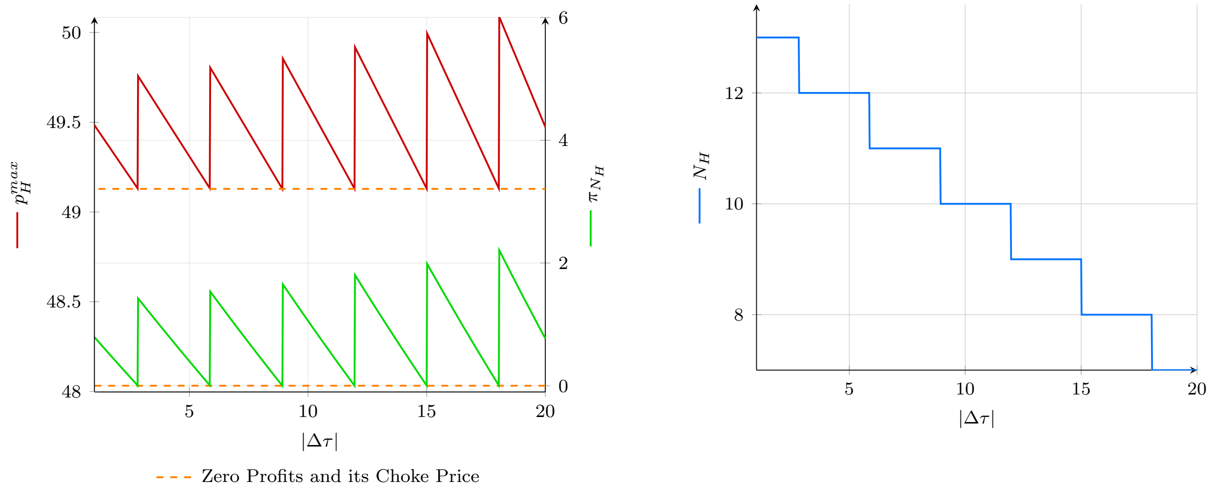
[3] Given an initial number of firms N_0, such that the least-productive firm has positive profits, then iterate (i.e., set $N_{j+1} = N_j + 1$) until the last entrant obtains negative profits. The equilibrium number of firms, N^*, is given by $N^* = N_k - 1$, if this condition is triggered after k iterations. Alternatively, if the least-productive firm has negative profits with N_0 firms, then iterate (i.e., set $N_{j+1} = N_j - 1$) until the last entrant obtains positive profits, so that $N^* = N_k$.

To simplify the analysis and ensure that foreign firms are always active, the results are presented under the assumption that these firms constitute the set of most-productive ones in the country. Thus, the analysis resembles a closed economy where the firms with lowest marginal costs are always active.

Figure 11 and Figure 12 illustrate how variations in trade costs impact entry for the case of homogeneous and heterogeneous last entrants, respectively. The explanations of the graphs are the same as those outlined in Section 6.1.

Figure 11. *Unilateral Liberalizations in a Small Economy: Cournot with Homogeneous Last Entrants*

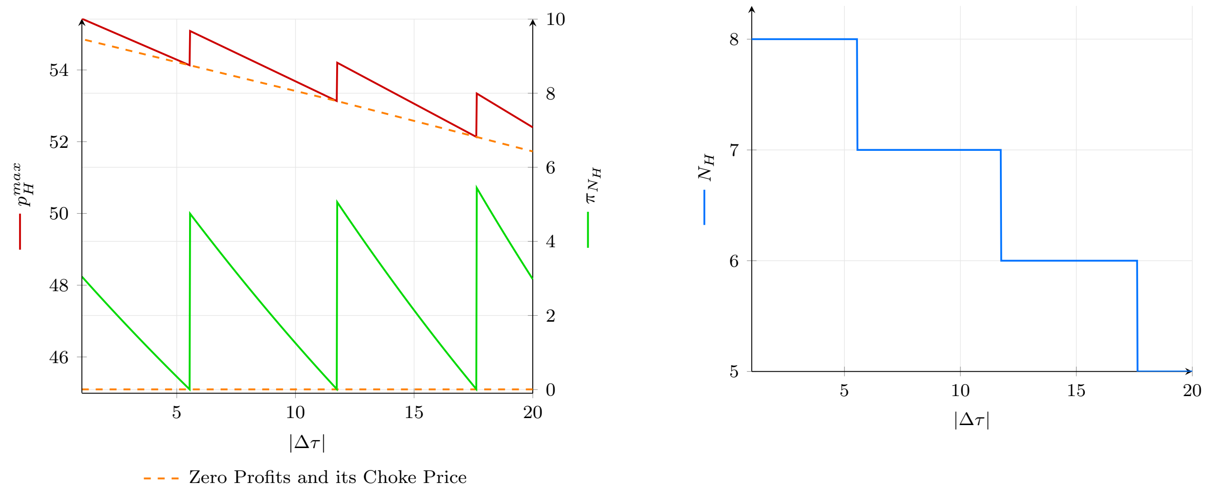
(a) Impact on Choke Price **(b)** Impact on Entry



Note: π_{N_H} refers to the profits of the last entrant, and N_H refers to the total number of active firms in the market.

Figure 12. *Unilateral Liberalizations in a Small Economy: Cournot with Heterogeneous Last Entrants*

(a) Impact on Choke Price **(b)** Impact on Entry



Note: π_{N_H} refers to the profits of the last entrant, and N_H refers to the total number of active firms in the market.

D Iceberg Trade costs

In the main part of the paper, we claimed that iceberg trade costs generate the same qualitative results as the baseline case with additive trade costs. Next, we show this by establishing that Propositions 3.1, 3.2, and 3.3

hold under iceberg trade costs.

Consider the framework of monopolistic competition outlined in Section 2. With iceberg trade costs, the definition of c_{ij}^{τ} becomes $c_{ij}^{\tau} := \tau_{ij}c$ with $\tau_{ii} := 1$. The proofs exploit that we have expressed several conditions and results for additive trade costs in terms of c_{ij}^{τ}. This implies that the optimal prices, quantities, markups, and profits are still valid. In addition, the equations that determine the two possible marginal-cost cutoffs, i.e. (ZCP) and (ZCP2), become, respectively,

$$c_{ij}^{*}\left(p_j^{\max};\tau_{ij}\right) := \frac{p_j^{\max} - \xi_{ij}}{\tau_{ij}}, \tag{37}$$

$$\frac{\left(p_i^{\max *} - c_{ii}^{*}\right)^2}{4\gamma_i} + \sum_{k\in\mathcal{F}} \frac{\left(p_k^{\max *} - c_{ii}^{*}\tau_{ik}\right)^2}{4\gamma_k} = F_i^E + f_{ii} + \sum_{k\in\mathcal{F}} f_{ik}. \tag{38}$$

Notice, also, that $\mathbb{C}_{ij}^{\tau *}$ is defined in the same way. Therefore, the equilibrium at the market stage is given by (MS) for H but with Φ_{ij} defined through $\mathbb{C}_{ij}^{\tau *}$ in terms of iceberg trade costs. This also implies that Lemma A.3 applies.

Next, we proceed to state some lemmas, which are the equivalent of Lemma A.4 and Lemma A.5. for iceberg trade costs.

Lemma D.1. *$p_{ij}\left(p_j^{\max *};c_{ij}^{\tau}\right)$, $q_{ij}\left(p_j^{\max *};c_{ij}^{\tau}\right)$, $m_{ij}\left(p_j^{\max *};c_{ij}^{\tau}\right)$, $\mu_{ij}\left(p_j^{\max *},c;\tau_{ij}\right)$, and $c_{ij}^{*}\left(p_j^{\max *};\tau_{ij}\right)$ are increasing in $p_j^{\max *}$.*

Proof of Lemma D.1. Taking derivatives and working out the expressions: $\frac{\partial p_{ij}(\cdot)}{\partial p_j^{\max}} = \frac{1}{2}$, $\frac{\partial q_{ij}(\cdot)}{\partial p_j^{\max}} = \frac{1}{2\gamma_j}$, $\frac{\partial m_{ij}(\cdot)}{\partial p_j^{\max}} = \frac{1}{2c_{ij}^{\tau}}$ and $\frac{\partial \mu_{ij}(\cdot)}{\partial p_j^{\max}} = \frac{1}{2}$. Moreover, if c_{ij}^{*} is given by (37) then $\frac{\partial c_{ij}^{*}(\cdot)}{\partial p_j^{\max}} = \frac{1}{\tau_{ij}}$. In case c_{ij}^{*} is given by (38), then $\frac{\partial c_{ij}^{*}}{\partial p_j^{\max *}} = \left(\frac{p_j^{\max *}-c_{ii}^{*}\tau_{ij}}{2\gamma_j}\right)\left(\frac{p_i^{\max *}-c_{ii}^{*}}{\gamma_i} + \sum_{k\in\mathcal{F}}\frac{p_k^{\max *}-c_{ii}^{*}\tau_{ik}}{2\gamma_k}\right)^{-1} > 0$. ■

Lemma D.2. *$c_{ij}^{*}\left(p_j^{\max *};\tau_{ij}\right)$ is decreasing in τ_{ij}.*

Proof of Lemma D.2. If c_{ij}^{*} is given by (37) then $\frac{\partial c_{ij}^{*}}{\partial \tau_{ij}} = -\frac{p_j^{\max *}-\xi_{ij}}{(\tau_{ij})^2}$. In case c_{ij}^{*} is given by (38), then $\frac{\partial c_{ii}^{*}(\cdot)}{\partial \tau_{ij}} = c_{ii}^{*}\frac{(p_k^{\max *}-c_{ii}^{*}\tau_{ik}^{*})}{2\gamma_k}\left(\frac{p_i^{\max *}-c_{ii}^{*}}{2\gamma_i} + \sum_{k\in\mathcal{F}}\frac{\tau_{ik}(p_k^{\max *}-c_{ii}^{*}\tau_{ik}^{*})}{2\gamma_k}\right)^{-1}$. ■

Finally, we show that a similar result as in Lemma A.6 holds.

Lemma D.3. *Suppose that the least-productive firms from i that are active in j belong to $\mathcal{E}$. Then, at the market stage of either the non-degenerate or degenerate variant of monopolistic competition, $\frac{\partial \Phi_{ij}^{\theta *}}{\partial p_j^{\max *}} > 0$ and $\frac{\partial \Phi_{ij}^{\theta *}}{\partial \tau_{ij}} < 0$ for $\theta \in \{\mathcal{E},\mathcal{I}\}$.*

Proof of Lemma D.3. At the market stage, $M_i^{\mathcal{I}}$ and $M_i^{\mathcal{E}}$ are given. We begin by establishing some additional calculations regarding $\mathbb{C}_{ij}^{\tau,\theta *} := \int_{\underline{c}_i^{\theta}}^{c_{ij}^{\theta *}} c_{ij}^{\tau}\, \mathrm{d}G_i^{\theta}(c)$. If $c_{ij}^{\theta *} = \overline{c}_i^{\theta}$, then all firms in θ are active and, so, $\frac{\partial \mathbb{C}_{ij}^{\tau,\theta *}}{\partial p_j^{\max *}} = 0$ and $\frac{\partial \mathbb{C}_{ij}^{\tau,\theta *}}{\partial \tau_{ij}} = \int_{\underline{c}_i^{\theta}}^{\overline{c}_i^{\theta}} \frac{\partial c_{ij}^{\tau}}{\partial \tau_{ij}}\, \mathrm{d}G_i^{\theta}(c) = \frac{\mathbb{C}_{ij}^{\tau,\theta *}}{\tau_{ij}} > 0$. If $c_{ij}^{\theta *} = c_{ij}^{*}$, then $\frac{\partial \mathbb{C}_{ij}^{\tau,\theta *}}{\partial p_j^{\max *}} = \left(c_{ij}^{*}\tau_{ij}\right) g_{ij}^{\theta *}\frac{\partial c_{ij}^{*}}{\partial p_j^{\max *}}$ and $\frac{\partial \mathbb{C}_{ij}^{\tau,\theta *}}{\partial \tau_{ij}} = \left(c_{ij}^{*}\tau_{ij}\right) g_{ij}^{\theta *}\frac{\partial c_{ij}^{*}}{\partial \tau_{ij}} + \frac{\mathbb{C}_{ij}^{\tau,\theta *}}{\tau_{ij}}$.

As for $\Phi_{ij}^{\theta *}$, in the non-degenerate variant, if $c_{ij}^{\theta *} = \overline{c}_i^{\theta}$ then $\Phi_{ij}^{\theta *} := M_i^{\theta}\left(p_j^{\max *} - \mathbb{C}_{ij}^{\tau,\theta *}\right)$ and, so, $\frac{\partial \Phi_{ij}^{\theta *}}{\partial p_j^{\max *}} = M_i^{\theta}$ and $\frac{\partial \Phi_{ij}^{\theta *}}{\partial \tau_{ij}} = -M_i^{\theta}\frac{\mathbb{C}_{ij}^{\tau,\theta *}}{\tau_{ij}}$. The same result is obtained for the degenerate variant by substituting M_i^{θ} with $\overline{M}_i^{\theta}$.

Consider now $c_{ij}^{\theta *} = c_{ij}^{*}$. For the non-degenerate variant, $\Phi_{ij}^{\theta *} := M_i^{\theta}\left(G_{ij}^{\theta *}p_j^{\max *} - \mathbb{C}_{ij}^{\tau,\theta *}\right)$. Thus, $\frac{\partial \Phi_{ij}^{\theta *}}{\partial p_j^{\max *}} = M_i^{\theta}\left(G_{ij}^{\theta *} + p_j^{\max *}g_{ij}^{\theta *}\frac{\partial c_{ij}^{*}}{\partial p_j^{\max *}} - \frac{\partial \mathbb{C}_{ij}^{\tau,\theta *}}{\partial p_j^{\max *}}\right)$, which can be reexpressed as $\frac{\partial \Phi_{ij}^{\theta *}}{\partial p_j^{\max *}} = M_i^{\theta}\left[G_{ij}^{\theta *} + g_{ij}^{\theta *}\frac{\partial c_{ij}^{*}}{\partial p_j^{\max *}}\left(p_j^{\max *} - c_{ij}^{*}\tau_{ij}\right)\right] > 0$. Moreover, $\frac{\partial \Phi_{ij}^{\theta *}}{\partial \tau_{ij}} = M_i^{\theta}\left(g_{ij}^{\theta *}p_j^{\max *}\frac{\partial c_{ij}^{*}}{\partial \tau_{ij}} - \frac{\partial \mathbb{C}_{ij}^{\tau,\theta *}}{\partial \tau_{ij}}\right)$, which can be reexpressed as $\frac{\partial \Phi_{ij}^{\theta *}}{\partial \tau_{ij}} = M_i^{\theta}\left(g_{ij}^{\theta *}\frac{\partial c_{ij}^{*}}{\partial \tau_{ij}}\left(p_j^{\max *} - c_{ij}^{*}\tau_{ij}\right) - \frac{\mathbb{C}_{ij}^{\tau,\theta *}}{\tau_{ij}}\right) < 0$.

Regarding the degenerate variant, the same result holds when either (ZCP) or (ZCP2) is satisfied if M_i^{θ} is substituted by $\overline{M}_i^{\theta}$. ■

Next, we show that, by assuming that trade costs are bounded so that before and after the trade shock the MIEs belong to $\mathcal{E}$, then all the propositions in the main part of the paper hold. As a corollary, all the applications entail the same results.

Proposition D.4

Suppose the existence of iceberg trade costs. Then, Propositions 3.1, 3.2, and 3.3 hold.

Proof of Proposition D.4. Regarding Propositions 3.1 and 3.2, the proofs follow verbatim by applying Lemmas D.1 and D.3 instead of Lemmas A.4 and A.6, respectively. For Proposition 3.3, in addition we need to use Lemma D.2 instead of Lemma A.5. ■

E Existence and Uniqueness of the Equilibrium

In this appendix, we sketch some arguments to have uniqueness of the equilibrium. We also outline arguments for the existence of an equilibrium.

Consider a country H. Irrespective of the setup variant under consideration, the set of choke prices is compact. To see this, by definition, $p_H^{\max}(\mathbb{Q}_j) := \alpha_H - \eta_H \mathbb{Q}_H$. Thus, $p_H^{\max} \in \left[\underline{p}_H^{\max}, \alpha_H\right]$. Moreover, since $\underline{p}_H^{\max}$ is nonnegative, we can suppose that $\underline{p}_H^{\max} := 0$.

Consider the degenerate variant. For country H, a solution requires that we find a $p_H^{\max *}$ such that condition (MS) holds. When $p_H^{\max} = \alpha_H$, then $\sum_{j \in \mathcal{C}} \Phi_{jH}(p_H^{\max}; \tau_{jH}) > 2\beta_H(\alpha_H - p_H^{\max}) = 0$, and, when $p_H^{\max} = \underline{p}_H^{\max *}$, we can assume that α_H is high enough such that $\sum_{j \in \mathcal{C}} \Phi_{jH}(p_H^{\max}; \tau_{jH}) + 2\beta_H p_H^{\max} < 2\beta_H \alpha_H$. Thus, since $\Phi_{jH}(\cdot; \tau_{jH})$ is continuous, a solution exists by the Intermediate Value Theorem. Moreover, given that $\frac{\partial \Phi_{jH}^*}{\partial p_H^{\max *}} > 0$, the solution is unique. Once that $p_H^{\max *}$ is determined, the rest of the equilibrium variables can be identified.

For the non-degenerate variant, uniqueness requires that there is a unique $p_H^{\max *}$ that satisfies (FE-ND) for H, and a unique $M_H^{\mathcal{E}*}$ that satisfies (MS) for H given the optimal $p_H^{\max *}$. Regarding the former condition, the expected domestic profits of a firm belonging to $\mathcal{E}$ are $\widetilde{\pi}_{HH}^{\mathcal{E}}(p_H^{\max}) := \int_{\underline{c}_H^{\mathcal{E}}}^{p_H^{\max} - \xi_{HH}} \left[\frac{(p_H^{\max} - c)^2}{4\gamma_H} - f_{HH}\right] \mathrm{d}G_H(c)$ and satisfy that $\frac{\partial \widetilde{\pi}_{HH}^{\mathcal{E}}(p_H^{\max})}{\partial p_H^{\max}} = \int_{\underline{c}_H}^{p_H^{\max} - \xi_{HH}} \left(\frac{p_H^{\max} - c}{2\gamma_H}\right) \mathrm{d}G_H(c) > 0$. Given the monotonicity of the expected profits, if the equilibrium exists, $p_H^{\max *}$ is unique. For existence of $p_H^{\max *}$, typical arguments can be applied. Since $\widetilde{\pi}_{HH}^{\mathcal{E}}$ is continuous, we can suppose parameters values such that $\widetilde{\pi}_{HH}^{\mathcal{E}}\left(\underline{p}_H^{\max}\right) < F_H^E$ and $\widetilde{\pi}_{HH}^{\mathcal{E}}(\alpha_H) > F_H^E$. Then, the result would follow by applying the Intermediate Value Theorem. In addition, given the value $p_H^{\max *}$ that satisfies (FE-ND) and $\overline{M}_H^{\mathcal{E}}$ is monotone (MS), we know that there is a unique $M_H^{\mathcal{E}*}$ that satisfies (MS). Existence of this value would be obtained by defining conditions on the parameters such that we can always apply the Intermediate Value Theorem to (MS).

Regarding the Cournot model, the equilibrium at the market stage requires us to find a $\mathbb{Q}_H^*$ such that (NE) holds for a given number of active firms from each country $j \in \mathcal{C}$. Let $\mathcal{F}_H(\mathbb{Q}_H; \boldsymbol{\tau}_{\cdot H}) := \sum_{j \in \mathcal{C}} \sum_{\omega \in \Omega_{jH}} \frac{\alpha_H - c_\omega - \tau_{jH} - \eta_H \mathbb{Q}_H}{2\gamma_H + \eta_H}$. At $\mathbb{Q}_H = 0$, we have that $\mathcal{F}_H(0; \boldsymbol{\tau}_{\cdot H}) > 0$. Moreover, we can always define a $\overline{\mathbb{Q}}_H$ such that $\mathcal{F}_H\left(\overline{\mathbb{Q}}_H; \boldsymbol{\tau}_{\cdot H}\right) < \overline{\mathbb{Q}}_H$ (for instance, we can accomplish this by assuming that α_H is large enough). Then, the $\mathbb{Q}_H^*$ that constitutes a fixed point of $\mathcal{F}$ would exist. Moreover, $\frac{\partial \mathcal{F}_H(\mathbb{Q}_H; \boldsymbol{\tau}_{\cdot H})}{\partial \mathbb{Q}_H} = \sum_{j \in \mathcal{C}} \sum_{\omega \in \Omega_{jH}} \frac{-\eta_H}{2\gamma_H + \eta_H}$ and so $\frac{\partial \mathcal{F}_H(\mathbb{Q}_H; \boldsymbol{\tau}_{\cdot H})}{\partial \mathbb{Q}_H} < 0$, which implies that the solution is unique, since $\frac{\partial \mathcal{F}_H(\mathbb{Q}_H; \boldsymbol{\tau}_{\cdot H})}{\partial \mathbb{Q}_H} - 1 < 0$. It remains to show that the number of firms from $j \in \mathcal{C}$ that are serving H is unique. By applying Lemma A.16 under $\boldsymbol{\tau}''_{\cdot H} = \boldsymbol{\tau}'_{\cdot H}$, we know that $\mathbb{Q}$ is strictly increasing in the number of firms. In turn, profits are strictly increasing in $\mathbb{Q}$. Hence, given that the number of firms is determined by condition (FE-C), the equation has at most one solution. If we assume that α_H is big enough, so that when there is only one active firm this has positive profits, then the solution would exist.