

Entry Preemption by Domestic Leaders and Home-Bias Patterns: Theory and Empirics*

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Abstract

I study a mechanism which acts through the strategic use of demand-enhancing investments. Relative to a non-strategic benchmark, domestic leaders follow an overinvestment strategy which preempts the entry of importers and increases their domestic sales. This generates greater concentration and home-bias patterns at both the firm and aggregate level. The results are robust to the type of competition (i.e., prices or quantities) and whether investments trigger increases or decreases in prices. For the analysis, I propose a novel framework that allows for firm heterogeneity, strategic interactions, multiple choices, and extensive-margin adjustments. Estimating the model for the Danish manufacturing, I show that, in industries with lower substitutability, the strategic effect on concentration and domestic intensity is greater for consumer goods and lower regarding concentration for producer goods.

Keywords: domestic leaders, demand-enhancing investments, preemption, concentration, home bias.

JEL codes: F12, F14, L13, L6.

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1 Introduction

In recent years, a surge of studies have raised concerns about the increase in concentration and markups in some developed economies. The underlying causes which lead to these outcomes, nonetheless, are not unique and might entail radically different policy interventions. Thus, as emphasized by [Berry et al. \(2019\)](#), there is a need for more research on how markets function in the modern economy and, as a corollary, on the determinants of concentration. At the same time, other studies have remarked on home-bias phenomena in open economies. Specifically, at an aggregate level, industries exhibit a tendency towards consumption of domestic goods while, at the firm level, the share of a firm's exports relative to its total sales tends to be less than its domestic portion. This phenomenon has been considered by [Obstfeld and Rogoff \(2001\)](#) as one of the six major puzzles in International Macroeconomics, and delving into the sources of trade barriers remains a key challenge ([Bernard et al., 2012](#); [Chaney, 2014](#)).

In this paper, I study theoretically and quantify a mechanism in which greater concentration and home-bias patterns arise endogenously by the strategic behavior of domestic firms. I conceive an environment in which firms make entry choices and, conditional on being active, decide on prices and sunk expenditures which enhance demand as in [Sutton \(1991,1998\)](#). These investments cover outlays on a wide range of variables, which all share the property of boosting the firm's quantity demanded and possibly entailing different effects on prices. For instance, while quality upgrades could increase prices, investments to reach low-valuation consumers (e.g., introducing inferior-quality products or improving the distribution network) might decrease them.

The mechanism I posit has some key elements. First, following a vast literature in International Business, I acknowledge that domestic firms have an advantage over those which serve the market without being established where the buyer is. Furthermore, domestic leaders (henceforth, DLs), defined as the firms with the greatest market share, are in a position to exploit these advantages strategically: given their size in the market, they are capable of shaping market conditions. In addition, I rely on an empirical fact that I identify in my dataset ([Stylized Fact 1](#) of this paper), where in a typical manufacturing industry, DLs coexist with numerous firms that have insignificant domestic market shares. In my model, all these elements are assembled in such a way that DLs sink investments in their home market and force the negligible firms in the market (either domestic or

non-domestic) to condition their decisions on these expenditures.

To isolate the effects stemming from strategic motives to invest, I compare the situation against a non-strategic benchmark, where rivals make decisions without conditioning on DLs' investments.¹ Relative to that baseline, I show that DLs invest more heavily at home, triggering a reallocation of domestic market shares towards them. This determines the existence of two home-bias patterns. First, the expansion of DLs in their home market is, in part, at the expense of the least-productive importers. This crowding out of foreign competition creates an aggregate bias towards sales by domestic firms. In addition, the greater domestic revenues of DLs increase their domestic intensity, defined as the firm's share of domestic sales relative to its total revenues. This last result is in line with an empirical regularity that I identify in my dataset, presented as **Stylized Fact 2** in this paper, where DLs display a greater domestic bias relative to firms with negligible market shares.

After presenting some remarks on the strategic exploitation of domestic advantages (**Section 2**) and stating some empirical regularities (**Section 3**), I proceed to study the mechanism theoretically in **Section 4** and **Section 5**. Mainly, this attends to the usual concern that results coming from oligopoly models can be subject to a lack of robustness. When strategic interactions are incorporated, models might become quite sensitive to specific details and have multiple equilibria. The concern turns to be especially acute for the mechanism under study since, in typical two-stage oligopoly models with one incumbent and one entrant, it has been shown that an overinvestment or underinvestment pattern can emerge depending on whether the competition is à la Cournot or Bertrand.²

In my model, the overinvestment pattern arises irrespective of whether the competition is on quantities or prices or, more generally, on strategic substitutes or complements. It is also independent of whether investments provide incentives to the firm to increase or decrease its prices, and even robust to an alternative where firms undertake cost-reducing investments that are country specific. The key for this is that, unlike previous studies, I incorporate the existence of an unbounded pool of entrants which are ready to enter if they anticipate positive profits.³ This feature makes underinvestment strategies to accommo-

¹The approach is standard in Game Theory to measure the strategic value of a strategy. In that field, the scenario in which rivals condition on the action under analysis is known as the closed-loop equilibrium, while the benchmark where rivals do not condition on it is referred to as the open-loop equilibrium (Fudenberg and Tirole, 1991).

²See, for instance, Tirole (1988).

³A similar intuition, but where investments by domestic leaders are compared relative to investments

date entry unprofitable. The reason is that any excess of unexploited profits would trigger entry and, therefore, undermine the original attempt to keep rents high. The mechanism posited underscores tacit competition as an important determinant of market decisions and ascribes a preemptive motive to the overinvestment strategy: DLs find it optimal to expand sales that would otherwise be made by rival firms. This logic accords well with the common idea in business of why big firms launch new products and persistently overhaul goods, which can be summarized through the famous phrase by Steve Jobs that “if you don’t cannibalize yourself, someone else will” (Isaacson, 2011).⁴

The incorporation of nonzero measure firms, heterogeneity, multiple choices, and endogenous entry might introduce complex strategic interactions that turn the search for a solution into an unwieldy task. This calls for building up a framework that incorporates these features parsimoniously to, ultimately, be able to take the model to the data. I accomplish this by adding two features to my setup. First, I consider a demand system that allows me to describe the strategic interactions between firms through the theory of Aggregative Games. This technique, recently put forward by Acemoglu and Jensen (2013), is particularly well suited for setups with firm heterogeneity and multidimensional strategies. Remarkably, it covers augmented versions of standard demand systems, including those to tackle empirical questions.⁵ The second feature of the model comes from the empirical fact mentioned above (Stylized Fact 1), which indicates that DLs coexist with a myriad of firms that have trivial market shares. Based on this, I build a setup where a fixed number of firms that are non-negligible in their home market are embedded into a monopolistic competition model as in Melitz (2003).⁶ Remarkably, the model constitutes a strict generalization of Melitz (2003) that collapses to it when either the number of DLs

by entrants, is present in Etro (2006).

⁴Formal evidence of the use of demand-enhancing investments to preempt entry can be found in Boulding and Christen (2009). They consider product lines as a choice variable. Paton (2008) does the same for advertising as a decision choice by conducting an anonymous questionnaire to more than 800 advertising managers of medium and large size UK-based firms. In the survey, a considerable portion of managers view advertising as a strategic tool to be deployed in response to entry. Also, advertising is used more intensively by large firms and, consistent with my model, firms that are dominant in their markets are significantly more likely to perceive advertising as a strategic instrument.

⁵For instance, it encompasses (possibly asymmetric) augmented versions of the CES, Logit, affine translations of these two, a linear demand, and a translog demand. In case that firms are multiproduct, it also encompasses the nested versions of the CES and Logit with each group defined by own firm’s varieties.

⁶Sutton (1991) shows that when there are demand-enhancing investments, market concentration is bounded away from zero. Thus, there is always at least one firm that is large and there can be coexistence of leaders with firms having trivial market shares. Also, it is worth remarking Shimomura and Thisse (2012) and Parenti (2018) who, under homogeneity of firms within groups, consider a market structure with coexistence of small and large firms.

is assumed to be zero or their effect on the market conditions is trivial.

Regarding the empirical side, with the aim of taking the model to the data, in [Section 6](#) I turn the general framework into a structural model.⁷ The quantitative approach is widely applicable since it only requires information on DLs' market shares and the determination of two parameters, with one of them being the elasticity of substitution. In particular, information related to other firms is not needed, while information on prices is only necessary for estimating parameters.

To study the mechanism, I draw on information from manufacturing industries in Denmark. In [Section 7](#), I describe the datasets at my disposal and remark on two features of the data which make them suitable for the analysis. First, the information on domestic firms is presented at the firm-product level. This allows me to allocate each firm-product to a properly defined market and, hence, obtain the firm's market share for each industry where the firm is active. Second, the information on international transactions is also presented at the firm-product level and encompasses imports by both manufacturing and non-manufacturing firms. In this way, I am able to account for a proper measure of import competition for each industry.

The results point out that strategic gains can be important determinants of the market structure regarding concentration and firm's domestic intensity. In addition, they also reveal a stark heterogeneity of outcomes. Nevertheless, arguably, magnitudes of estimations coming from structural models should be interpreted with some caveats. The reason lies in the simplifications and assumptions about unobservables to which they are subject. On the other hand, they constitute a powerful tool for comparing the relative importance of either two mechanisms within a group or, as in this paper, the same mechanism across different groups.

To study how strategic motives to invest impact industries differently, I begin by performing an analysis of the theoretical predictions. For the range of values observed in the data, this indicates that the magnitude of outcomes depends mainly on the distributions of market shares and domestic intensity. Specifically, industries are more impacted when DLs command more market share and have sales well diversified across markets.⁸ Regard-

⁷The main reason to resort to a structural model is based on [Sutton \(1996\)](#). He argues that strategic asymmetries of the nature I study pose a limitation in any empirical analysis. Except for what he denominates "happy accidents" (i.e., natural experiments), which are rarely if ever observed, strategic asymmetries should be considered as unobservables.

⁸From a theoretical point of view, while variations in market share and domestic intensity are positive, these increases are non-monotone. Nonetheless, when market shares are not disproportionately large and

ing the former, this is because when DLs have a low presence in the industry, they are less capable of influencing the market conditions and, hence, gaining market share strategically. As for domestic intensity, if a firm sells almost exclusively at home, any increase in domestic revenue has a trivial impact on the relative proportions of sales between domestic and export markets.

Based on this, I make use of an empirical pattern that I identify in my dataset (**Stylized Fact 3** of this paper). The result provides me with some guidance for identifying the industries where the deployment of strategic moves would have a more pronounced impact. It states that the distributions of market shares and domestic intensity differ in industries according to their good substitutability and the final user of the good (i.e., consumers or producers).⁹ Specifically, in consumer goods, a greater substitution is associated with lower concentration and greater domestic intensity. On the other hand, for producer goods, industries with greater substitution are more concentrated, displaying statistically insignificant differences in terms of domestic intensity.

Guided by this empirical fact, I corroborate that the outcomes in the structural model differ by good substitutability and the final user of the good. As expected based on the analysis outlined above, the results establish that, for consumer goods, increases in concentration and domestic intensity are greater in industries with lower substitutability. As for producer goods, there are greater concentration effects in industries with a higher substitutability, and a statistically insignificant relation with domestic intensity.

Related Literature and Contributions. My paper contributes to different strands of the literature. First, it touches upon studies which build up structural models incorporating large firms as key players in international economies.¹⁰ In general, these papers suppose either a fixed number of firms or resort to ad hoc assumptions to model entry.¹¹ Moreover, at a theoretical level, they obtain predictions under specific functional forms, such as a CES demand. My contribution in this regard is developing a framework to study theoretically and quantify phenomena with non-negligible firms. The setup is based on em-

DLs have sales skewed to their home market, as it happens in the Danish data, the model behaves as described.

⁹Substitution is measured through the elasticity of substitution from Soderbery (2015), which is estimated following the procedure by Broda and Weinstein (2006) but corrected for small-sample biases. In addition, the classification by final user of the good corresponds to the BEC classification.

¹⁰For some recent literature, see, for instance, Atkeson and Burstein (2008), Eaton et al. (2012), Edmond et al. (2015), and Gaubert and Itskhoki (2018).

¹¹For instance, Eaton et al. (2012) and Gaubert and Itskhoki (2018) assume that the number of firms is a random variable.

pirical regularities and underscores Aggregative Games as a fruitful tool for incorporating strategic interactions into empirical models. It can also be turned into a structural model that incorporates strategic behavior while allowing for firm heterogeneity, extensive-margin adjustments, and multidimensional strategies. In addition, the demand system allows for different functional forms, while heterogeneity of large firms is not restricted to a specific distribution. Remarkably, the approach constitutes a strict generalization of a model à la Melitz (2003) such that, if all firms in an industry had negligible market shares, it would generate the same results. Due to this, it improves upon structural estimations under this variant of monopolistic competition where only atomistic firms are allowed.

My paper is also related to a growing literature that studies the impact on the economy of the so-called *superstar firms*.¹² In particular, there has been an ongoing debate about the welfare consequences regarding the rise of concentration in the USA. In line with Autor et al. (2017), my model underlines that reallocation of market share towards more productive firms does not necessarily make consumers worse off. This holds even when the outcomes depict a situation with more concentration, less international trade, and possibly higher markups. Although general results are not possible to obtain theoretically, the strategic behavior makes consumers better off under the assumptions of an augmented CES (as in my structural model) and investments on non-price variables which are desirable for the consumers (e.g., quality). This echoes the core intuition of market contestability by Baumol et al. (1982): even when a concentrated market is observed, the threat of entry, in opposition to actual entry, might discipline the incumbents in such a way that desirable welfare outcomes emerge. Thus, my model highlights *tacit* competition as a potential welfare-improving channel.

Finally, my paper relates to a literature that deals with home-bias patterns. After McCallum's (1995) border puzzle and Treffer's (1995) mystery of missing trade, the great bulk of studies with open economies have assumed the existence of trade costs as parameters that capture frictions between countries. Some other studies have generated the phenomenon endogenously by resorting to explanations based on the supply nature or preferences.¹³ By considering the preemptive strategies pursued by DLs, I focus on a

¹²See, for instance, Autor et al. (2017), De Loecker and Eeckhout (2017), Gutiérrez and Philippon (2017), and De Loecker and Eeckhout (2018).

¹³Explanations based on the supply side can be found in, among others, Hillberry and Hummels (2002), Yi (2010), and Chaney (2014). Regarding reasons related to the demand side, there is a vast literature resorting to a taste for national goods which goes back to at least Armington (1969). Also, Caron et al. (2014) propose a mechanism based on nonhomotheticities of preferences.

cause which, to the best of my knowledge, has not been explored before.

2 On Domestic Advantages and Strategic Moves

In a setting where firms are non-negligible in their own industry, the incorporation of non-price choices expands the sources of strategic interactions and the possibilities that firms have to achieve a better position in the market. As my results hinge on this line of reasoning, I expand upon it in this section.

The formalization that the competition between firms is broader than the mere choice of prices has been at the roots of the Industrial Organization literature since its inception. In particular, [Schelling \(1960\)](#) argues that, in any game, once that it is recognized that agents behave in a strategic way regarding their actions, it should also be acknowledged that they behave strategically concerning the game itself. That is, if agents have the opportunity, they do not take the rules of the game as given and make moves that alter the original situation with the aim of achieving a better outcome. [Schelling \(1960\)](#) refers to these actions as *strategic moves*. The idea has been applied to oligopolies in order to explain non-price decisions made by incumbents with the aim of endogenously creating market conditions favorable to them. As [Porter \(1998\)](#) points out, “successful firms not only respond to their environment but also attempt to influence it in their favor”.

In tradable industries, domestic firms, defined as those which have established operations in the market to serve, enjoy certain potential advantages by being located where the buyer is. It allows them to collect more and better information regarding the local environment, react quicker to changes in market conditions, establish and maintain relations more easily with local intermediaries, and get a better grasp of customer tastes. In addition, the superior knowledge of the market might improve the effectiveness of firms’ strategies if they are tailored to the idiosyncrasies of the country.¹⁴

In my setting, domestic firms are endowed with a strategic asymmetry which is materialized through sunk expenditures.¹⁵ By their sunk nature, these investments, once incurred, cannot be recovered. Consequently, domestic firms that have the necessary size

¹⁴In the International Business literature, the fact that foreign firms are at a disadvantage is known as “liability of foreignness”. There is a vast literature on the subject which includes both theoretical and empirical studies. For a summary of the arguments see, for instance, [Porter \(1980, pp. 281-287; 2011, Chapter 3\)](#).

¹⁵The sunk nature of investments plays an important role. Otherwise, if costs were fixed but not sunk, the decisions could be reverted and the interaction would be better described by a setting with simultaneous decisions. In terms of [Schelling \(1960\)](#), the sunk nature of costs turns decisions into *commitments*.

in the industry to influence market conditions are capable of committing resources in such a way that competitors, and in particular smaller firms (either local or from abroad), condition their choices on these investments.

Different variables have been considered in the literature as endogenous sunk expenditures which enhance demand.¹⁶ By definition, they include any expenditure which boosts firm demand in subsequent stages of the market. Thus, for instance, they include advertisement, investments to broaden the line of goods, adaptation of products to consumers' tastes, and investments on distribution channels that make the product more widely available.

Some remarks are in order in relation to this paper. First, the mechanism posited in my model does not aim at explaining why a firm is an industry leader in the first place. Rather, the focus is on how, *conditional on being an important player in the industry, a DL deploys strategies to improve its position in the market.*

Second, I classify a firm as domestic if it has production activities within the country, irrespective of their ownership or whether part of their home sales comes from imports. This follows because the advantages I mentioned are acquired by “being in the market”. On the other hand, pure importers (i.e., those that are not residing where the buyer is) face additional difficulties of the type indicated above. This might prevent them from succeeding in the market to the extent that they can get the presence necessary to emerge as a player capable of setting the rules of that market. One way to overcome these hurdles is establishing operations in the host country.¹⁷ The idea has been central in the theories of multinational enterprises since, at least, the seminal works of Hymer (1960) and Dunning (1977). Thus, *the strategic gains also relate to the benefits that mature multinational enterprises might reap from doing foreign direct investment.*

Third, formally, the strategic motive to invest is captured in my model by an early choice that generates a first-mover advantage. Nonetheless, the framework should be conceived in terms of what rivals know and condition on when they choose a strategy, rather than the timing itself. The intuition is the same as in the Prisoner's Dilemma, or

¹⁶Firms might also have the possibility of sinking investments that affect marginal costs. My analysis applies to this case if the cost advantages are country specific (see [Appendix E.4](#)). For instance, this arises if a firm invests on distribution networks which reduce the marginal costs in that specific market.

¹⁷For an empirical study on the effects of foreign direct investment on profitability, see [Coşar et al. \(2018\)](#) for the car industry. This paper is particularly noteworthy given the richness of the data. They provide evidence that establishing local assembly plants benefit firms more through the increases in the demand that it entails rather than from savings on the cost side.

more generally the definition of static games: *assumptions on timing are not necessarily about the time at which a player makes a choice, but about whether others observe and condition on it.* Due to this, DLs are not necessarily the firms which have entered first to the market, but rather those which have succeeded in the industry and have achieved enough size so that their actions shape the conditions of the market.

In relation to this last point, it is worth remarking that *the mechanism works through the conditioning of the least-productive firms (either domestic or non-domestic) on the DLs' investments.* In particular, given a market structure where DLs and small firms coexist, these firms correspond to those with negligible market shares. A corollary of this is that I allow for the existence of large non-domestic firms which are not preempted, or more generally affected, by the DLs' overinvestment. In fact, in my framework, this possibility is accounted for explicitly.

3 Empirical Facts

In this section, I list some empirical facts that guide my research questions and methodology. They draw on information from Danish manufacturing sectors for the year 2005. A more detailed description of the data and measures used is included in [Section 7.1](#) and [Appendix C.1](#). Also, in [Appendix D](#), I characterize DLs in terms of their features.

Throughout the paper, I refer to a **sector** as a 2-digit industry (according to the NACE rev. 1.1 classification) and reserve the term **industry** when it is defined at the 4-digit level. In addition, a DL in an industry is defined empirically as a firm with production activity in Denmark and a domestic market share greater than 3% (measured in sales values).

Stylized Fact 1. *Concentration in industries is widespread, even accounting for import competition. Moreover, in a typical industry, a few domestic leaders coexist with a myriad of firms with negligible market shares.*¹⁸

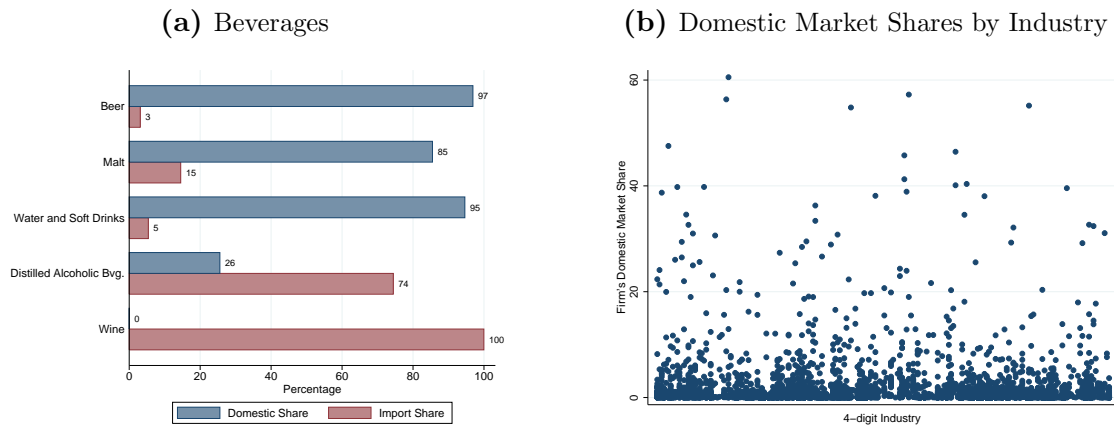
Small open economies like Denmark are usually characterized by large shares of imports out of total values. However, at the industry level, this features a pattern of specialization and, thus, it does not preclude the existence of concentration. [Figure 1a](#) illustrates this by making use of several industries belonging to the beverage sector. Considering industries

¹⁸Coexistence of large and small firms at the country level has been obtained for several countries, including the USA ([Axtell, 2001](#)) and different European countries ([Fujiwara et al., 2004](#)). At the industry level, see for instance [Hottman et al. \(2016\)](#) for the case of the USA.

dominated by domestic firms and subject to import competition, a market structure where a group of firms with negligible market shares coexist with DLs is the norm.

This can be appreciated in [Figure 1b](#), which displays a scatter plot of domestic firms' market shares accounting for import competition. Each vertical line corresponds to a different industry, with the vertical axis indicating the domestic market share of each firm belonging to that industry. Quantitatively, industries comprising firms with trivial market shares and subject to import competition represent more than 80% of the total manufacturing value. In addition, more than 82% of this value corresponds to industries in which there is coexistence with DLs. A corollary of this is that a standard monopolistic-competition market structure would only be appropriate for 18% of the manufacturing value.

Figure 1. *Market Share of Domestic Firms (Import Corrected)*



Note: Market shares measured in terms of sales value of the industry (including imports). In [Figure 1b](#), each vertical line represents a different 4-digit industry. Each dot indicates the domestic market share of a firm in that industry.

Stylized Fact 2. *In industries with coexistence of domestic leaders and firms with negligible market shares, there is a home bias at the firm level measured through domestic intensity (i.e., a firm's domestic sales share relative to its total sales). Furthermore, the bias is more pronounced for domestic leaders.*¹⁹

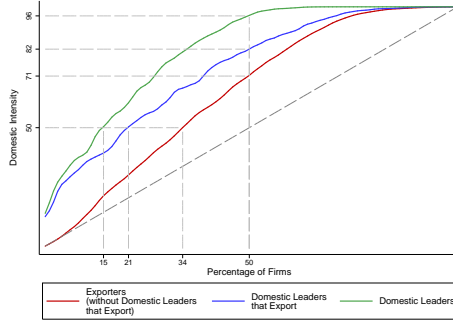
[Figure 2](#) provides evidence of Stylized Fact 2. The correlation observed between domestic intensity and domestic market share can be generated by different factors. The most immediate reason is that firms which allocate more resources to the domestic market achieve a better performance there. On the other hand, in this paper, I focus on a reason

¹⁹Home bias at the firm level has been documented for the USA ([Bernard et al., 2012](#)) and several European countries ([Mayer and Ottaviano, 2008](#)).

that is more subtle and where the relation is inverted: conditional on having a significant presence in the market, firms behave strategically and skew resources to the domestic market, thus increasing their presence in their home market.

Figure 2. *Relation between Domestic Market Share and Domestic Intensity of Firms*

(a) Cumulative Distribution of Firm's Domestic Intensity



(b) Regressions

	Firm's Domestic Intensity			
	(1)	(2)	(3)	(4)
Dom. Market Share	0.223** (0.113)	1.120*** (0.225)		
DL				18.817*** (1.533)
Size			-10.439*** (2.495)	-16.386*** (2.487)
Industry FE	Yes	Yes		
Sector FE			Yes	Yes
Sample Unit	Firm-Ind	Exp-Ind	Exp-Sect	Exp-Sect
Observations	5,236	2,141	1,903	1,903
R-squared	0.145	0.120	0.106	0.180

Note: Domestic leader in an industry defined as a Danish firm that has a domestic market share greater than 3%. Domestic intensity defined as the ratio between firm's domestic sales and its own total sales. In Figure 2a, information is at firm-industry level. Firms ordered from the left to the right starting with the firms with lowest domestic intensity. In Figure 2b, Exp-Ind indicates that the sample takes firm-industry as unit of observation and it is restricted to exporters. In Columns (3) and (4), observations are at the firm level. Each firm is assigned to the sector in which it obtains its greatest revenue. Exp-Sect indicates that only exporters are considered. DL is a dummy variable that takes 1 if the firm has a market share greater than 3% in at least one industry of the sector. Size is a dummy variable that takes 1 if the number of employees is greater than 250.

In Column (3) of Figure 2b, I also show that the fact that DLs have a greater domestic intensity does not contradict previous studies (e.g., Mayer and Ottaviano, 2008) that indicate a positive correlation between export intensity and size. In the Danish data too, greater size in terms of employment is associated with lower domestic intensity and, hence, greater export intensity.

Stylized Fact 3. *Consider sectors that include industries with both consumer and producer goods, and industries with coexistence of domestic leaders and firms with negligible market shares. Then, the relation of concentration and domestic intensity with the substitutability of the goods depends on the final user of the good.*

For consumer goods, the greater the substitutability, the lower the concentration and the greater the domestic intensity. For producer goods, the greater the substitutability, the greater the concentration and no statistically significant relation with domestic intensity.

Stylized Fact 3 follows from Table 1 and is used as a lens to interpret some of the empirical results I obtain through the structural model. It provides information on how the distributions of market shares and domestic intensity vary across industries in relation to good substitutability σ . This is measured through the elasticity of substitution from

Soderbery (2015), which is estimated following the procedure by Broda and Weinstein (2006) but corrected for small-sample biases.

Table 1. *Substitutability and Patterns of Concentration and Domestic Intensity*

	Total Market Share					Firm's Domestic Intensity		
	DLs (1)	Top 3 DLs (2)	Non-Top 3 DLs (3)	DNLs (4)	Imports (5)	(6)	(7)	(8)
$\ln \sigma$ (consumers)	-37.87** (16.57)	-31.71** (14.82)	-6.161 (5.481)	4.456 (5.415)	33.42** (15.11)	19.15** (8.358)	21.98** (8.413)	26.82** (10.48)
$\ln \sigma$ (producers)	27.21** (12.00)	18.17** (7.550)	9.048 (7.344)	10.70 (9.506)	-37.91* (19.93)	6.462 (9.615)	5.152 (12.51)	14.88 (42.19)
Sector- FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector-Rank FE								
Sample Unit	Industry	Industry	Industry	Industry	Industry	Firm-Ind	Firm-Ind	Exp-Ind
Observations	39	39	39	39	39	170	139	81
R-squared	0.309	0.294	0.297	0.391	0.494	0.280	0.356	0.307

Note: Domestic Leader in an industry defined as a Danish firm with a domestic market share greater than 3%. The term CR stands for concentration ratio, while DLs and DNLs refer to domestic leaders and domestic non-leaders, respectively. Industries with producer and consumer goods defined according to the BEC classification, with producer goods encompassing intermediate goods and capital. The variable σ corresponds to the elasticity of substitution from Soderbery (2015). Rank refers to the position in the market according to the domestic market share of the firm. Firm-Ind indicates that the observations are at the firm-industry level, while Exp-ind indicates that the sample is restricted to exporters. All the results come from regressions of the type $y = FEs + \alpha \ln \sigma + \beta \times \mathbb{1}(user)$. By including all the interaction terms through the fixed effects FEs, the α estimated is identical to that obtained through a separate regression with the sample restricted to one type of final user. Heteroskedastic-robust standard errors used.

Column (1) shows that, controlling for sectors, a lower σ is associated with greater concentration for consumer goods, while the opposite pattern arises for producer goods. Columns (2) to (5) provide evidence that this is driven by a greater presence of the top DLs, with a reduction in the import penetration as a counterpart, rather than a decline of market shares of other domestic firms (either non-leaders or non-top DLs).

Likewise, Column (6) shows that a lower σ is negatively related with the firm's domestic intensity in consumers goods. On the other hand, σ has a statistically insignificant relation with domestic intensity in the case of producer goods.

While the outcome for consumer goods in terms of domestic intensity might arise because a lower σ is associated with more concentration and, hence, by Stylized Fact 2 a greater domestic intensity, in Columns (7) and (8) I show the result holds even after controlling for firms rank by market share. In addition, in order to show the robustness of the results, the regression in Column (8) restricts the sample to exporters and controls for some additional variables (import penetration of the industry as a proxy of tradability, and domestic market share).

4 Setup

In this section, I outline the framework used for the theoretical analysis which, additionally, forms the basis for the structural model utilized in Section 6. Throughout the paper, any

subscript ij refers to i as the country of origin and j as the destination. All the proofs are relegated to [Appendix A](#).

I consider a setup with competition à la Bertrand and sunk investments that are demand enhancing. It can be shown that all the results also hold under competition in quantities ([Appendix E.3](#)) and cost-reducing investments ([Appendix E.4](#)). In fact, this can be easily grasped through the baseline setup, since I do not make any assumption on whether prices are strategic complements or substitutes, and I allow for investments which increase or reduce prices.

There is a world economy with a discrete set of countries \mathcal{C} and I model an industry in isolation which is composed of horizontally differentiated varieties. I define $\bar{\Omega} \subseteq \mathbb{R}$ as the set of potential conceivable varieties and suppose that each of them can only be produced by a single firm.²⁰ Due to this, I refer to a firm ω or variety ω indistinctly. Besides, it is supposed that the set $\bar{\Omega}$ can be partitioned into a countable set $\bar{\mathcal{B}}$ and a real interval $\bar{\mathcal{S}}$, where the symbols \mathcal{B} and \mathcal{S} are mnemonics for big and small.

For each $i \in \mathcal{C}$, I endow $\bar{\Omega}$ with measures $(\mu_i)_{i \in \mathcal{C}}$ where μ_i indicates the size of each firm in i . Each μ_i partitions $\bar{\Omega}$ into two classes of sets, $(\bar{\mathcal{B}}_{ki})_{k \in \mathcal{C}}$ and $(\bar{\mathcal{S}}_{ki})_{k \in \mathcal{C}}$, where $\bar{\mathcal{B}}_{ki}$ comprises the firms from k which are capable of affecting the aggregate conditions of the industry in i , and $\bar{\mathcal{S}}_{ki}$ those which cannot. Formally, I capture this by defining μ_i as a mixed measure which corresponds to the Lebesgue measure and the counting measure when it is restricted to, respectively, measurable subsets of $\cup_{k \in \mathcal{C}} \bar{\mathcal{S}}_{ki}$ and $\cup_{k \in \mathcal{C}} \bar{\mathcal{B}}_{ki}$.²¹

According to the size in its home market, I define a firm located in i with $\omega \in \bar{\mathcal{S}} \cap \bar{\mathcal{S}}_{ii}$ as a domestic non-leader (DNL) and a firm $\omega \in \bar{\mathcal{B}} \cap \bar{\mathcal{B}}_{ii}$ as a DL. I suppose that if a firm from i is such that $\omega \in \bar{\mathcal{S}}$ then $\omega \in \cap_{k \in \mathcal{C}} \bar{\mathcal{S}}_{ik}$, so that it is negligible everywhere. As for DLs, I do not impose any restriction on this matter.

Finally, I denote by Ω_{ji} the subset of varieties from j sold in i , with $\Omega_i := \cup_{k \in \mathcal{C}} \Omega_{ki}$ being the total varieties available in i . Likewise, I define $\Omega_{ji}^{\mathcal{S}} := \bar{\mathcal{S}}_{ji} \cap \Omega_{ji}$ and $\Omega_{ji}^{\mathcal{B}} := \bar{\mathcal{B}}_{ji} \cap \Omega_{ji}$ as, respectively, the subsets of varieties of DNLs and DLs from j which are available in i .

²⁰Multiproduct firms can be easily incorporated into this setup. See [Appendix E.5](#). Besides, in terms of the baseline framework, demand-enhancing investments can be considered as a composite variable comprising any variable that boosts the firm's own demand. In particular, they could encompass product line as a choice variable. In that case, a pair of quantities-prices for a firm would constitute an average across all its varieties.

²¹Specifically, $\mu_i : \Sigma \rightarrow \mathbb{R} \cup \{\infty\}$, where Σ is the collection of Borel sets on $\bar{\Omega}$, and it satisfies $\mu_i(\cdot) := \lambda[\cdot \cap (\cup_{k \in \mathcal{C}} \bar{\mathcal{S}}_{ki})] + \#[\cdot \cap (\cup_{k \in \mathcal{C}} \bar{\mathcal{B}}_{ki})]$ where λ is the Lebesgue measure and $\#$ the counting measure.

4.1 Supply Side

The supply side of the model can be understood as an augmented version of Melitz (2003) that incorporates non-prices choices and has an embedded set of firms that affect aggregate conditions. In fact, in order to establish a direct link to Melitz (2003), I allow for firm-specific functional forms for DLs but make some symmetry assumptions for DNLs. None of them are actually required for the results.²²

In each country $i \in \mathcal{C}$, there is an unbounded pool of potential entrants that are ex-ante identical and do not know their productivity. They consider paying a sunk entry cost F_i^S to receive a productivity draw φ and an assignation of a unique variety $\omega \in \bar{\mathcal{S}}$. Productivity draws come from a continuous random variable that has a non-negative support and a cdf G_i^S . The measure of DNLs in i that pay F_i^S is denoted by M_i^E .

In addition, there is an exogenous number of DLs, with each having assigned a unique variety $\omega \in \bar{\mathcal{B}} \cap \bar{\mathcal{B}}_{ii}$ and productivity φ_ω . These firms are active in their domestic market and their productivity is common knowledge across the world.

The technology of production determines constant marginal costs $c(\varphi, \tau_{ij})$, where τ_{ij} represents a trade cost that any firm in i incurs when it sells to j . The function c is smooth and satisfies $\frac{\partial c(\varphi, \tau_{ij})}{\partial \varphi} < 0$ and $\frac{\partial c(\varphi, \tau_{ij})}{\partial \tau_{ij}} > 0$. I adopt the convention that firms do not incur in any trade cost to sell in the domestic market and make the usual assumption that all firms which are active in at least one country also serve their domestic market. Also, throughout the paper, I focus on equilibria where there is a subset of DNLs that are active and some of them export.²³

DLs and the mass M_i^E of DNLs have the option of not selling in country $j \in \mathcal{C}$, or doing so and incurring an overhead fixed cost $f_{ij}^\omega \geq 0$.²⁴ In particular for DNLs, the fixed cost is the same and equal to f_{ij}^S . I denote by M_{ij} the measure of active DNLs from i selling in j .

At the market stage, each firm from $i \in \mathcal{C}$ makes two choices in $j \in \mathcal{C}$: it decides on prices $p_{ij}^\omega \in P$ and investments $z_{ij}^\omega \in Z$, where P and Z are real non-negative compact intervals with $0 \in Z$. A level of investments z_{ij}^ω entail sunk expenditures $f_z^\omega(z_{ij}^\omega)$ where

²²Basically, the only difference that would arise by dispensing with this assumption is that the firm's decision to whether serve a market would not be characterized through a survival productivity cutoff. However, a specific characterization of this decision is inconsequential for the results of this paper.

²³The latter is supported by the Danish data, where around 48% of the DNLs are exporters.

²⁴I assume that, if there is an infinite choke price, then $f_{ij}^\omega > 0$. The possibility that $f_{ij}^\omega = 0$ covers the case where the choke price is finite and generates extensive-margin adjustments.

f_z is a smooth weakly convex function for $z > 0$ with $f_z^\omega(0) = 0$. I assume that for DNLs this function is symmetric and denote it by f_z^S . To account for the possibility of not serving market j , I augment firms' choices by adding an element $\bar{\mathbf{x}} := (\bar{p}, \bar{z})$ that represents inaction in j , where $\bar{z} = 0$, and $\bar{p} \in \mathbb{R}_{++} \cup \{\infty\}$ is greater than or equal to the choke price.

Finally, I denote the strategy of a firm ω from i in j by $\mathbf{x}_{ij}^\omega := (p_{ij}^\omega, z_{ij}^\omega)$ and its space by $X_{ij}^\omega := P \times Z$. A profile of strategies in j for active firms from i is $\mathbf{x}_{ij} := (\mathbf{x}_{ij}^\omega)_{\omega \in \Omega_{ij}}$ where $\mathbf{x}_{ij} \in X_{ij} := \times_{\omega \in \Omega_{ij}} X_{ij}^\omega$.

4.2 Demand Side

One of the main challenges in frameworks with heterogeneity, nonzero measure firms, multiple choices, and entry/exit of firms is the proliferation of dimensions to which the model may be subject to. Without additional structure the problem would become unwieldy. This is a key matter given that my ultimate goal is turning the theoretical model into a structural one to deal with the empirical side.

Attending to this, I work with a demand system that enables me to describe the strategic interactions through a real-valued function that aggregates the strategies of all the firms. This feature of the demand makes it possible to use the tools of Aggregative Games, which I exploit throughout the paper.²⁵

Following [Acemoglu and Jensen \(2013\)](#), I distinguish between \mathcal{A}_i and \mathbb{A}_i . The former is called an *aggregator* and it is a function of all the strategies chosen by firms. The latter is an *aggregate* and corresponds to a specific value of the aggregator's range. Examples of aggregators are the price index in the CES demand or the choke price in a linear demand. Consistent with assumptions I establish below, it can be interpreted as a measure of how tough the competitive environment is.

Without any additional structure on the aggregator, its derivatives could depend on its composition. This would affect the characterization of optimal strategies that are obtained through first-order conditions. For this reason, Aggregative Games impose the condition that the aggregator is strongly separable, thus ensuring that the aggregate is a single sufficient statistic for the derivatives too. The formal definition of an aggregator is as follows. Throughout this paper, any integral is Lebesgue.

²⁵For a survey on Aggregative Games, see [Jensen \(2018\)](#).

Definition 1. An **aggregator** for country i is a smooth function $\mathcal{A}_i : \times_{k \in \mathcal{C}} X_{ki} \rightarrow \mathbb{R}_+$ such that there is a strictly monotone function $H_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and smooth strictly monotone component-wise functions $h_{ki}^\omega : X_{ki}^\omega \rightarrow \mathbb{R}_+$ for each $k \in \mathcal{C}$ and $\omega \in \bar{\Omega}$ such that

$$\mathcal{A}_i [(\mathbf{x}_{ki})_{k \in \mathcal{C}}] := H_i \left\{ \sum_{k \in \mathcal{C}} \left[\int_{\omega \in \Omega_{ki}} h_{ki}^\omega (\mathbf{x}_{ki}^\omega) d\mu_i(\omega) \right] \right\}.$$

An **aggregate** for country i is defined as a value $\mathbb{A}_i \in \text{range } \mathcal{A}_i$.

Notice that, by interpreting the integral as Lebesgue and given the definition of μ_i , the integral allows for a simplified notation. For instance,

$$\int_{\omega \in \Omega_{ii}} h_{ii}^\omega (\mathbf{x}_{ii}^\omega) d\mu_i(\omega) = \int_{\omega \in \Omega_{ii}^S} h_{ii}^\omega (\mathbf{x}_{ii}^\omega) d\omega + \sum_{\omega \in \Omega_{ii}^B} h_{ii}^\omega (\mathbf{x}_{ii}^\omega).$$

In words, an aggregator is any function of firms' strategies such that, after a monotone transformation, it has an additive form. Since it accepts monotone transformations, it is not uniquely defined and determines a class of functions. Using the definition of an aggregator, the demand system is defined as follows.

Definition 2. The **aggregate demand** of a variety ω produced by a firm from i and sold in j is a smooth real-valued function $Q_{ij}^\omega [\mathbf{x}_{ij}^\omega, \mathcal{A}_j [(\mathbf{x}_{kj})_{k \in \mathcal{C}}]]$ where the aggregator is as in Definition 1.

Demands satisfying this definition encompass several standard cases, including those used for empirical analysis.²⁶ Moreover, the demand system is quite flexible, since I do not impose any restriction on the nature of the choke price (i.e., finite or infinite) while demand parameters for DLs are allowed to be firm dependent. For instance, with a CES demand, it allows for firm-specific elasticities of substitution.

Next, I formalize the demand side of the model by making use of the definitions established above. In line with the situation I intend to capture, I also incorporate some monotonicity assumptions to reflect that investments are demand enhancing and that the aggregator can be interpreted as a measure of toughness of competition. Specifically, regarding the latter, the greater the aggregate, the lower the firm's demand.

²⁶For instance, for the case of demands depending only on prices, it covers demands from a discrete choice model as in McFadden (1973) (e.g., Multinomial Logit) or from a discrete-continuous choice as in Nocke and Schutz (2018), a linear demand, the translog demand, demands derived from an additive indirect utility as in Bertolotti and Etro (2015), and affine translations of the CES as in Arkolakis et al. (2019). See Appendix E.1 for a description of these examples. In Appendix E.5, I also show that the model can be easily extended to cover multiproduct firms with nested demands, such as the nested CES or nested Logit, where groups are defined by varieties produced by the same firm.

Assumption DEM. *The aggregate demand of variety ω is as in Definition 2 and for any $i, j \in \mathcal{C}$ satisfies that $\frac{\partial Q_{ij}^\omega}{\partial A_j} < 0$, $\frac{\partial Q_{ij}^\omega}{\partial p_{ij}^\omega} < 0$, and $\frac{\partial Q_{ij}^\omega}{\partial z_{ij}^\omega} > 0$. Moreover, regarding the aggregator, it is supposed that $H' > 0$, $\frac{\partial h_{ij}^\omega}{\partial p_{ij}^\omega} < 0$ and $\frac{\partial h_{ij}^\omega}{\partial z_{ij}^\omega} > 0$.*

Assumption DEM is silent regarding whether prices are strategic complements or substitutes. In addition, notice that I do not specify the sign of the cross derivative of demand with respect to own prices and investments. Thus, investments can provide each DL with incentives to increase or decrease its prices.

While not necessary for the results, I also suppose that for each DNL from i the functions Q_{ij}^ω and h_{ij}^ω are the same. I denote them by Q_{ij}^S and h_{ij}^S . This is to be consistent with the goal of making clear that the model can be understood as an extension of Melitz (2003). In particular, the assumption implies that, as in the Melitz model, profitability of DNLs depends exclusively on differences in productivity rather than demand.

5 Theoretical Analysis

Following Fudenberg and Tirole (1991), strategic motives of choices, as well as the gains stemming from it, can be isolated by a comparison of the outcomes in the so-called open-loop and closed-loop equilibrium. These concepts refer to the equilibria of two different game structures.

Applied to my model, the closed-loop equilibrium corresponds to a situation in which rival firms condition their decisions on the investments made by DLs. Regarding the open-loop equilibrium, it acts as a benchmark in which competitors do not observe the investments made by DLs. Thus, by definition, rivals cannot condition on these choices, and DLs do not behave strategically in this respect.

I add two technical assumptions which are necessary for comparative statics. I suppose that the profit functions in each optimization problem are strictly pseudo-concave in the own strategy of the firm.²⁷ Moreover, I assume that profits functions satisfy Inada-type conditions at the boundaries. These two assumptions ensure that firms' optimal choices are interior and that first-order conditions are not only necessary but also sufficient for a

²⁷Specifically, in terms of the functions I define below, I assume that each term of the sums in (1) and (2) are strictly pseudo-concave. Pseudo-concavity is similar to but somewhat stronger than quasi-concavity. Essentially, the difference lies in the behavior at points where the derivative vanishes. At those points, quasi-concavity cannot distinguish between a saddle point and an optimum. This implies that the first-order conditions are not sufficient to identify an optimum. On the other hand, pseudo-concavity of a function f holds iff f is quasi-concave and $\nabla f(x^*) = 0$ implies that x^* is a global maximizer. For further details, see, for instance, Takayama (1993).

global optimum.

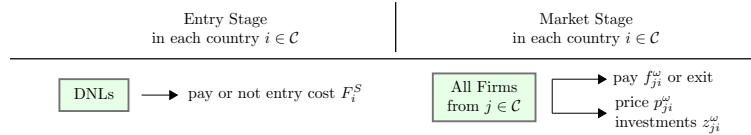
While other assumptions are necessary to get existence and uniqueness of equilibrium, I simply assume that they hold. The reason to do this follows [Milgrom and Shannon's \(1994\)](#) approach of only stating assumptions that are necessary to get definite results for comparative statics, thus not confounding them with those that are necessary to have a well-behaved problem.²⁸

Next, I describe and solve the model under each scenario. In both cases, a backward-induction procedure is used. After this, I compare their solutions and state the main results.

5.1 Simultaneous Case

The timing of the simultaneous case is presented in [Figure 3](#). First, each DNL of $i \in \mathcal{C}$ decides whether to pay the sunk entry cost F_i^S . If it does so, a unique variety ω is assigned to it along with a draw of productivity φ . After this, the market stage in each country takes place. At this stage, each firm ω from $j \in \mathcal{C}$ decides whether to pay f_{ji}^ω to serve i . If it does so, it chooses prices p_{ji}^ω and investments z_{ji}^ω .

Figure 3. *Timing of the Simultaneous Case*



At the market stage, the mass of DNLs in the world, denoted by $\mathbf{M}^E := (M_i^E)_{i \in \mathcal{C}}$, is given. Given other firms' strategies, a firm ω from $i \in \mathcal{C}$ (irrespective of whether it is a leader or not) chooses prices and investments for each market by solving the following optimization problem:

$$\max_{(\mathbf{x}_{ij}^\omega)_{j \in \mathcal{C}}} \pi_i^\omega = \sum_{j \in \mathcal{C}} \mathbb{1}_{(\mathbf{x}_{ij}^\omega \neq \bar{\mathbf{x}})} \left[\pi_{ij}^\omega(\mathbf{x}_{ij}^\omega, \mathcal{A}_j[(\mathbf{x}_{kj})_{k \in \mathcal{C}}]; \varphi_\omega) - f_{ij}^\omega \right], \quad (1)$$

where $\pi_{ij}^\omega(\mathbf{x}_{ij}^\omega, \mathcal{A}_j; \varphi_\omega) := Q_{ij}^\omega(\mathbf{x}_{ij}^\omega, \mathcal{A}_j) [p_{ij}^\omega - c(\varphi_\omega, \tau_{ij})] - f_z^\omega(z_{ij}^\omega)$.

If the firm is active in country $j \in \mathcal{C}$, its optimal decisions given rivals' strategies are

²⁸I consider existence and uniqueness of the equilibrium in [Appendix E.2](#). Essentially, since the market structure can be interpreted as an extension of the Melitz model with an exogenous number of large firms, the conditions are similar to those in Melitz.

characterized implicitly by

$$\frac{\partial \pi_{ij}^\omega(\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial p_{ij}^\omega} + \mathbb{1}_{(\omega: \mu_j(\{\omega\}) > 0)} \frac{\partial \pi_{ij}^\omega(\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial \mathbb{A}_j} \frac{\partial \mathcal{A}_j[(\mathbf{x}_{kj})_{k \in \mathcal{C}}]}{\partial p_{ij}^\omega} = 0 \quad (\text{PRICE})$$

$$\frac{\partial \pi_{ij}^\omega(\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial z_{ij}^\omega} + \mathbb{1}_{(\omega: \mu_j(\{\omega\}) > 0)} \frac{\partial \pi_{ij}^\omega(\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial \mathbb{A}_j} \frac{\partial \mathcal{A}_j[(\mathbf{x}_{kj})_{k \in \mathcal{C}}]}{\partial z_{ij}^\omega} = 0. \quad (z\text{-SIM})$$

In any aggregative game, optimal strategies can be characterized in terms of the so-called *backward-response functions* rather than best-response functions. They express each firm's optimal strategy as a function of the aggregate, thus including not only other firms' strategies but, also, its own strategy.²⁹ I denote the implicit solutions of (PRICE) and (z-SIM) for $\omega \in \bar{\mathcal{B}}_{ij}$ by $\mathbf{x}_{ij}^\omega(\mathbb{A}_j, \varphi_\omega) := (p_{ij}^\omega(\mathbb{A}_j, \varphi_\omega), z_{ij}^\omega(\mathbb{A}_j, \varphi_\omega))$. For $\omega \in \bar{\mathcal{S}}_{ij}$, I denote them by using a superscript S instead of ω .

Given the optimal profit that a firm ω from i would get in j , it decides whether to pay f_{ij}^ω and serve it. If it does not, it chooses $\bar{\mathbf{x}}$ and avoids paying f_{ij}^ω . In particular for DNLs, given that their profits are strictly increasing in productivity, the decision can be characterized by a survival productivity cutoff $\varphi_{ij}(\mathbb{A}_j)$. This is the solution to

$$\pi_{ij}^S[\mathbb{A}_j, \varphi_{ij}(\mathbb{A}_j)] = f_{ij}^S, \quad (\text{ZCP})$$

where π_{ij}^S is the optimal gross profits of DNLs.

Given optimal strategies at the market stage, all that remains to be specified are the conditions for a Nash equilibrium at the market stage and the free-entry conditions of DNLs. For the former, I exploit the aggregative game structure of the model. To do this, I introduce the concept of *an aggregate backward-response function* for country i , which is defined as

$$\Gamma_i(\mathbb{A}_i, \mathbf{M}^E) := H_i \left\{ \sum_{k \in \mathcal{C}} \left[M_k^E \int_{\varphi_{ki}(\mathbb{A}_i)}^{\bar{\varphi}_k} h_{ki}^S[\mathbf{x}_{ki}^S(\mathbb{A}_i, \varphi)] dG_k^S(\varphi) + \int_{\omega \in \Omega_{ki}^B} h_{ki}^\omega[\mathbf{x}_{ki}^\omega(\mathbb{A}_i, \varphi_\omega)] d\mu_i(\omega) \right] \right\}.$$

A Nash equilibrium at the market stage in i requires that \mathbb{A}_i is a fixed point of Γ_i . This determines that the optimal decisions of the agents self-generate the value \mathbb{A}_i . Formally, the equilibrium condition is

$$\Gamma_i(\mathbb{A}_i, \mathbf{M}^E) = \mathbb{A}_i. \quad (\text{NE})$$

²⁹The property follows because of the additive separability of the aggregator, which determines that the terms $\frac{\partial \mathcal{A}_j[(\mathbf{x}_{kj})_{k=1}^C]}{\partial p_{ij}^\omega}$ and $\frac{\partial \mathcal{A}_j[(\mathbf{x}_{kj})_{k=1}^C]}{\partial z_{ij}^\omega}$ can be expressed as functions of j 's aggregate. At the intuitive level, this implies that, conditional on knowing the value of the aggregate, the composition of the aggregator is irrelevant.

Furthermore, the following free-entry condition in each $i \in \mathcal{C}$ has to hold:

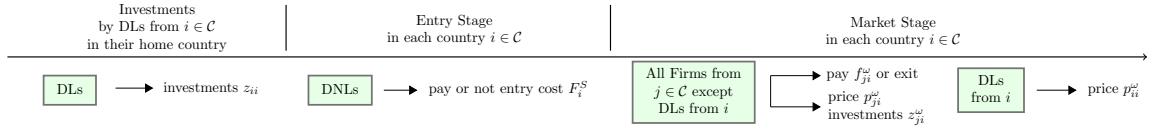
$$\sum_{j \in \mathcal{C}} \int_{\varphi_{ij}(\mathbb{A}_j)}^{\bar{\varphi}_i} \{ \pi_{ij}^S(\mathbb{A}_j, \varphi) - f_{ij}^S \} dG_i^S(\varphi) = F_i^S. \quad (\text{FE})$$

An equilibrium for the simultaneous scenario is given by $\mathbf{M}_{\text{sim}}^E := (M_i^{E*})_{i \in \mathcal{C}}$ and $(\mathbb{A}_i^*)_{i \in \mathcal{C}}$ that satisfy conditions (NE) and (FE) for each $i, j \in \mathcal{C}$. Given these values, the equilibrium strategies are determined too.

5.2 Sequential Case

The timing of the sequential game is presented in [Figure 4](#). It is the same as the simultaneous case except for the fact that DLs make their domestic investments decisions at the beginning of the game.

Figure 4. *Timing of the Sequential Case*



Given the structure of the game, the sequential scenario takes the simultaneous game as a class of subgames for each vector of domestic investments. Therefore, due to the backward-induction procedure, the solution is the same as for the simultaneous case up to the domestic investments decisions. Given this, it only rests to characterize the optimal domestic investments.

While DLs are capable of influencing the aggregate conditions of the market, the system of equations (NE) and (FE) are separable. Specifically, $(\mathbb{A}_i)_{i \in \mathcal{C}}$ is determined by (FE) and independently of both $(M_i^E)_{i \in \mathcal{C}}$ and domestic investments. As a result, the simultaneous and sequential games share the same equilibrium aggregates $(\mathbb{A}_i^*)_{i \in \mathcal{C}}$.³⁰

Combining this result with the optimal domestic price of a DL ω , which is given by (PRICE) with solution $p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)$, the problem of a DL ω from i at the first stage is

$$\max_{z_{ii}^\omega} \pi_{ii}^\omega [p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega), z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega]. \quad (2)$$

Thus, domestic investments of ω are characterized by the following first-order condition:

$$\frac{\partial \pi_{ii}^\omega [p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega), z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega]}{\partial z_{ii}^\omega} + \frac{\partial \pi_{ii}^\omega [p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega), z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega]}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)}{\partial z_{ii}^\omega} = 0. \quad (z\text{-SEQ})$$

³⁰For a formal argument of this result, as well as derivations for the sequential case, see [Appendix A.1](#).

The sum in (z -SEQ) comprises two terms. The first one is analogous to the first term in (z -SIM). It represents the non-strategic portion of investments that a DL takes into account in both scenarios. The second one is different from the second term in (z -SIM) and it captures the preemption motive to invest by DLs.

The second terms differ because of the asymmetry of timing in DLs' choice of investments. On the one hand, in the simultaneous scenario investments are decided by each DL considering its effect on the aggregate. This implies that each choice is made by incorporating that greater investments increase the aggregate and, hence, decrease its profits through that channel. On the other hand, in the sequential scenario, investments do not affect the aggregate, which is determined by the free-entry conditions of DNLs in the world. This fact constitutes the key mechanism of the model through which the results are generated. I proceed to its explanation.

To illustrate this idea, suppose that $\frac{\partial p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)}{\partial z_{ii}^\omega} = 0$, so that the second term in (z -SEQ) is zero. In that scenario, if a DL increases its investments, it makes competition tougher and reduces DNLs' profits. Given free entry, this triggers the exit of DNLs until zero expected profits are restored. Ultimately, both effects perfectly offset and the aggregate does not vary. However, by preempting entry, the DL is able to expand its presence in the market which, otherwise, would have been captured by DNLs.

In the general case where $\frac{\partial p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)}{\partial z_{ii}^\omega} \neq 0$, the second term in (z -SEQ) reflects that a DL also takes into account the indirect effect of investments on its incentives to choose prices. Consequently, relative to the simultaneous case, there is overinvestment as long as this effect is not positive and so pronounced that greater investments increase prices to the extent that competition is lessened.

5.3 Results

Consistent with the argument outlined above, I add two assumptions which rule out scenarios where greater investments lead to increases in prices so large that make competition less tough or decrease the firm's revenues. Remarkably, none of the results require further assumptions in case investments reduce prices. In addition, I allow for decreases in quantities as long as revenues do not fall.

To formalize the assumptions, I define two types of demand elasticities. The first one is given by the demand elasticity when the DL ignores its effect on the aggregate. I use a

superscript “mc” to denote this case, as a mnemonic that it is the elasticity prevailing in monopolistic competition. Formally, $\varepsilon_{ii}^{p,mc}(\mathbf{x}_{ii}^\omega; \mathbb{A}_i) := -\frac{\partial \ln Q_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i)}{\partial \ln p_{ii}^\omega}$ and $\varepsilon_{ii}^{z,mc}(\mathbf{x}_{ii}^\omega; \mathbb{A}_i) := \frac{\partial \ln Q_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i)}{\partial \ln z_{ii}^\omega}$. In addition, I define elasticities which incorporate the influence of the DL on the aggregate. Formally, $\varepsilon_{ii}^p(\mathbf{x}_{ii}^\omega, \mathbb{A}_i) := -\frac{d \ln Q_{ii}^\omega[\mathbf{x}_{ii}^\omega, \mathcal{A}_i(\cdot)]}{d \ln p_{ii}^\omega}$ and $\varepsilon_{ii}^z(\mathbf{x}_{ii}^\omega, \mathbb{A}_i) := \frac{d \ln Q_{ii}^\omega[\mathbf{x}_{ii}^\omega, \mathcal{A}_i(\cdot)]}{d \ln z_{ii}^\omega}$. When the elasticity is evaluated at the optimal pricing, I use $(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega)$ as argument of the function.

Assumption 5a. $\frac{\partial \ln p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega)}{\partial \ln z_{ii}^\omega} < \frac{\varepsilon_{ii}^{z,mc}(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega) - \varepsilon_{ii}^z(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega)}{\varepsilon_{ii}^{p,mc}(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega) - \varepsilon_{ii}^p(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega)}$, where \mathbb{A}_i^* is the equilibrium aggregate.

Assumption 5b. $\frac{\partial \ln p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega)}{\partial \ln z_{ii}^\omega} < \min \left\{ \frac{\varepsilon_{ii}^{z,mc}(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega) - \varepsilon_{ii}^z(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega)}{\varepsilon_{ii}^{p,mc}(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega) - \varepsilon_{ii}^p(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega)}, \frac{\varepsilon_{ii}^{z,mc}(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega)}{\varepsilon_{ii}^{p,mc}(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega) - 1} \right\}$, where \mathbb{A}_i^* is the equilibrium aggregate.

Some comments are in order. First, **Assumption 5b** implies **Assumption 5a**. Second, since $\varepsilon_{ii}^{p,mc} > 1$ in any interior optimal solution for prices and, also, $\varepsilon_{ii}^p < \varepsilon_{ii}^{p,mc}$, and $\varepsilon_{ii}^z < \varepsilon_{ii}^{z,mc}$, both **Assumption 5a** and **Assumption 5b** automatically hold when $\frac{\partial p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i^*, \varphi_\omega)}{\partial z_{ii}^\omega} \leq 0$. Thus, these assumptions are only relevant for situations where investments have a positive impact on prices.

Assumption 5a establishes that the direct effect of z_{ii} on \mathbb{A}_i dominates the indirect effects of z_{ii} on \mathbb{A}_i through prices. This ensures that greater investments by a DL generate a tougher competitive environment. Regarding **Assumption 5b**, it also ensures this and, in addition, rules out scenarios where the increases in prices due to greater investments lower the firm’s revenues.

Next, I present the results that emerge from the comparison between the simultaneous and sequential scenarios. For the propositions, I denote by $R_{ii}(\omega)$ the domestic revenues of ω , and denote with superscripts “sim” and “seq” the equilibrium values in each scenario.

Main Propositions

Given a DL from $i \in \mathcal{C}$ producing variety ω and serving its home market:

Proposition 5.1 $\pi_i^{seq}(\omega) \geq \pi_i^{sim}(\omega)$, with strict inequality if **Assumption 5a** holds,

Proposition 5.2 if **Assumption 5a** holds at the simultaneous equilibrium, then $z_{ii}^{seq}(\omega) > z_{ii}^{sim}(\omega)$,

Proposition 5.3 if **Assumption 5b** holds, then $R_{ii}^{seq}(\omega) > R_{ii}^{sim}(\omega)$, and

Proposition 5.4 if countries are symmetric and **Assumption 5b** holds, then there is a

home bias in aggregate sales in i at the sequential equilibrium.

In words, [Proposition 5.1](#) indicates that each DL gets greater profits in the sequential scenario. [Proposition 5.2](#) dictates that there is overinvestment by each DL, and, by [Proposition 5.3](#), this allows each of them to get greater domestic revenues. As a corollary, in the sequential equilibrium, DLs have bigger domestic market shares and greater domestic intensities. Finally, [Proposition 5.4](#) shows that the reallocation of market shares towards DLs is not exclusively at the expense of DNLs from the same country. Instead, it necessarily entails a decrease of import shares, determining a home bias at the industry level.

6 Taking the Model to the Data

In this section, I conduct a quantitative analysis of the mechanism under study. To do this, I turn the general framework into a structural model. Conditional on some given values for parameters, the approach allows for a quantification of the results by just knowing the market shares of DLs. In particular, information on DNLs and non-domestic firms is not needed, while information on prices is only used for estimating parameters.

Given that the derivations are algebraically intensive, I relegate all the details to [Appendix B](#) and only present the elements necessary for understanding the approach.

6.1 Functional Forms

To take the model to the data, it is necessary to make some choices on functional forms. Regarding the demand side, I suppose that country i 's demand system is derived from a representative consumer with a two-tier utility. I denote by \mathcal{N} the set of differentiated industries with indices prices $(\mathbb{P}_i^n)_{n \in \mathcal{N}}$ and indices quantities $(\mathbb{Q}_i^n)_{n \in \mathcal{N}}$. Also, I suppose the existence of a homogeneous good 0 with unit price and quantities \mathbb{Q}_i^0 . The upper tier is given by

$$\max_{(\mathbb{Q}_i^n)_{n \in \mathcal{N}}, \mathbb{Q}_i^0} U_i := \sum_{n \in \mathcal{N}} E_i^n \ln \mathbb{Q}_i^n + \mathbb{Q}_i^0 \text{ subject to } \sum_{n \in \mathcal{N}} \mathbb{P}_i^n \mathbb{Q}_i^n + \mathbb{Q}_i^0 = Y_i,$$

where Y_i is i 's total income and $E_i^n \in \mathbb{R}_{++}$. I suppose that income is high enough so that there is positive consumption of both goods. The solution to the optimization problem determines that $\mathbb{P}_i^n \mathbb{Q}_i^n = E_i^n$ for any $n \in \mathcal{N}$. Hence, the parameter E_i^n represents the expenditure allocated to industry n .

From now on, I omit industry superscripts and consider one specific industry. For the lower tier, I resort to an augmented CES incorporating a demand shifter. This can be interpreted as a composite variable that encompasses all the demand-enhancing investments of a DL.³¹ Formally, the lower-tier utility function for the industry is,

$$\mathbb{Q}_i := \left\{ \sum_{k \in \mathcal{C}} \int_{\omega \in \Omega_{ki}} \left[(z_{ki}^\omega)^{\frac{\delta}{\sigma-1}} Q_{ki}^\omega \right]^{\frac{\sigma-1}{\sigma}} d\mu_i(\omega) \right\}^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ and $\delta < 1$. Given expenditure E_i , routine calculations establish that the optimal demand in i of a variety ω from j is

$$Q_{ji}(\mathbf{x}_{ji}^\omega, \mathbb{P}_i) := E_i(\mathbb{P}_i)^{\sigma-1} (z_{ji}^\omega)^\delta (p_{ji}^\omega)^{-\sigma}, \quad (3)$$

where $\mathcal{P}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}] := \left\{ \sum_{k \in \mathcal{C}} \int_{\omega \in \Omega_{ki}} (z_{ki}^\omega)^\delta (p_{ki}^\omega)^{1-\sigma} d\mu_i(\omega) \right\}^{\frac{1}{1-\sigma}}$ and $\mathbb{P}_i \in \text{range } \mathcal{P}_i$. I denote the domestic sales in i of a firm ω by $R_{ii}(\omega)$ and its export values by $\sum_{k \in \mathcal{C} \setminus \{i\}} R_{ik}(\omega)$.

Notice that (3) is consistent with a demand as in **Assumption DEM** by setting $\mathbb{A}_i := (\mathbb{P}_i)^{1-\sigma}$ and $h_{ji}^\omega(\mathbf{x}_{ji}^\omega) := (z_{ji}^\omega)^\delta (p_{ji}^\omega)^{1-\sigma}$. Using these definitions, the market share of a variety ω can be expressed as

$$s_{ji}(\mathbf{x}_{ji}^\omega, \mathbb{A}_i) = \frac{(z_{ji}^\omega)^\delta (p_{ji}^\omega)^{1-\sigma}}{\mathbb{A}_i} = \frac{(z_{ji}^\omega)^\delta (p_{ji}^\omega)^{1-\sigma}}{\sum_{k \in \mathcal{C}} \int_{\omega \in \Omega_{ki}} (z_{ki}^\omega)^\delta (p_{ki}^\omega)^{1-\sigma} d\mu_i(\omega)}. \quad (4)$$

Intuitively, the numerator of (4) can be interpreted as a measure of how attractive variety ω is by its features (i.e., price and the non-price feature). Thus, given that the denominator is the sum of attractiveness of varieties in the market, the firm's market share is determined by how attractive variety ω is relative to the other options.

Regarding the costs of DLs, I suppose that $f_z(z_{ij}^\omega) := \bar{f}_z z_{ij}^\omega$ where $\bar{f}_z > 0$. Moreover, the empirical results do not require specifying a functional form for $c_{ji}(\varphi_\omega, \tau_{ji})$ or a characterization of any variable related to either DNLs or non-domestic firms.

Some comments on the specific assumptions of the structural model are in order. First, in **Appendix B.3**, I prove that **Assumption 5b** (and, hence, **Assumption 5a**) holds for any values of the parameters. Thus, **Propositions 5.1 - 5.4** hold.

Second, in this model, greater investments determine higher prices and markups. However, the consumer is not worse off. To see this, notice that, given that investments enter the utility function, it is implicitly supposed that they encompass valuable features for the

³¹In **Appendix B.1**, I outline different properties of an augmented CES which made me incline towards its use, relative to other tractable alternatives as the Logit.

consumer. Also, with an augmented CES, the aggregate is a single sufficient statistic for industry welfare. Therefore, since the aggregate's value is the same under the simultaneous and sequential equilibrium, welfare measured at the industry level is the same in both scenarios. In addition, the reallocation of market shares towards DLs in the sequential scenario increases aggregate profits. If they are passed back to the consumer, there would be greater consumption of the homogeneous good and, so, the consumer would be better off. For more on welfare, see [Appendix E.6](#).

6.2 Counterfactual and Outcomes

After solving the model under each scenario, the optimal solutions of prices and investments can be expressed in terms of market shares.³² This allows me to present the effect of overinvestment on each industry through the following outcomes. First, given market shares $s_{ii}^{\text{sim}}(\omega)$ and $s_{ii}^{\text{seq}}(\omega)$ for a firm ω , strategic gains are expressed through differences in the domestic market share under each scenario. Moreover, given $s_{ii}^{\text{sim}}(\omega)$ and industry expenditures E_i , the revenues on the domestic market in the simultaneous case can be recovered. Thus, with the information on domestic sales in each scenario along with the export value $\sum_{k \in C \setminus \{i\}} R_{ik}(\omega)$ (which does not vary between scenarios), I can measure the domestic intensity of each firm for both scenarios.

While $s_{ii}^{\text{seq}}(\omega)$, E_i , and $\sum_{k \in C \setminus \{i\}} R_{ik}(\omega)$ are obtained from the data, it is necessary to recover $s_{ii}^{\text{sim}}(\omega)$. I outline how to obtain $s_{ii}^{\text{sim}}(\omega)$, given (σ, δ) and $s_{ii}^{\text{seq}}(\omega)$. The approach exploits that market shares are determined structurally through (4), and that \mathbb{A}_i^* is the same in the sequential and simultaneous cases. Expressing market shares in relative terms, I obtain

$$\frac{s_{ii}^{\text{seq}}(\omega)}{s_{ii}^{\text{sim}}(\omega)} = \frac{[p_{ii}^\omega(s_{ii}^{\text{seq}}(\omega))]^{1-\sigma} [z_{ii}^{\text{seq}}(s_{ii}^{\text{seq}}(\omega))]^\delta}{[p_{ii}^\omega(s_{ii}^{\text{sim}}(\omega))]^{1-\sigma} [z_{ii}^{\text{sim}}(s_{ii}^{\text{sim}}(\omega))]^\delta}, \quad (5)$$

where prices and investments correspond to the optimal solutions. Once these solutions are incorporated into (4), $s_{ii}^{\text{sim}}(\omega)$ is retrieved by knowledge of (σ, δ) and $s_{ii}^{\text{seq}}(\omega)$.³³ Intuitively, $s_{ii}^{\text{sim}}(\omega)$ is recovered by using how price and investment decisions vary structurally between the scenarios, along with the impact on market shares that these changes entail.

³²Specifically, they are given by $p_{ii}^\omega(s_{ii}^\omega) = \frac{\sigma}{\sigma-1} \left(1 + \frac{1}{\sigma} \frac{s_{ii}^\omega}{1-s_{ii}^\omega}\right) c_{ii}(\varphi_\omega)$, $z_{ii}^{\text{sim}}(s_{ii}^\omega) := \frac{\delta s_{ii}^\omega (1-s_{ii}^\omega)}{f_z \varepsilon(s_{ii}^\omega)}$, and $z_{ii}^{\text{seq}}(s_{ii}^\omega) := \frac{\delta s_{ii}^\omega (1-s_{ii}^\omega)}{f_z \varepsilon(s_{ii}^\omega)} \left(\frac{\sigma}{\sigma - s_{ii}^\omega \varepsilon(s_{ii}^\omega)} \right)$.

³³In [Appendix B.4](#), I show that, given (σ, δ) and $s_{ii}^{\text{seq}}(\omega)$, there exists a solution to (5) in terms of $s_{ii}^{\text{sim}}(\omega)$ and this is unique.

Some remarks are in order. First, although market shares in (4) depend on $c(\varphi_\omega)$ and \bar{f}_z , their values cancel out in (5). The reason is that the marginal costs of producing and investing do not change between scenarios. Second, (5) dictates that gains in market shares can be estimated without knowing which firms the shares are reallocated from. While this information affects the home-bias impact at the aggregate level, an upper bound of this is still obtained by the total concentration increase accrued by DLs.

7 Empirical Analysis

For the empirics, I draw on information from Denmark provided by Statistics Denmark. The datasets used are part of the country's sources for official statistics. Several features of the data make it appropriate for conducting the analysis. First, the information on domestic firms' sales and imports are disaggregated at the 8-digit product level, allowing me to allocate each product to a properly-defined market. In addition, the information of imports encompasses both manufacturing and non-manufacturing firms, thus providing me with an accurate measure of import competition for each industry.

I start by describing the information included in the datasets. After this, I proceed to construct the data analogs of the model concepts. Finally, I perform the quantitative analysis. Alternative specifications that check the robustness of the results are included in [Appendix F](#).

7.1 Data Description

I make use of two datasets. One provides information about the production of manufacturing firms. The other contains international transactions by both manufacturing and non-manufacturing firms. Both datasets have information reported at the year-firm-product level and can be easily merged through a unique firm identifier. I take 2005 as the baseline year.³⁴

The first dataset contains information about physical production in manufacturing industries and constitutes the source for Danish Prodcom statistics. Any unit with at least ten employees that lists manufacturing as its main activity is included. Overall, at least 90% of the total production value in each NACE (revision 1.1) 4-digit industry is

³⁴In [Appendix F.1](#), I recalculate the results for all years between 2001 and 2007 and show that the average results are similar.

covered.³⁵ Products are defined in terms of the Combined Nomenclature at the 8-digit level (hereafter, CN8), with information on values and quantities at the firm-product level.³⁶

The second dataset is collected by Danish customs and reports imports (CIF values) and exports (FOB values) at the CN8 firm-product level. It covers firms belonging to the production dataset and also non-manufacturing firms. Overall, the trade flows recorded for EU countries are 95% for imports and 97% for exports, while the universe of transactions is covered for non-EU countries.

7.2 Definitions and Dataset

With the aim of defining market shares, it is necessary to establish the bounds of each market, how sales are calculated, and some criteria to identify domestic firms and the imports that compete with domestic products in the industry.

Regarding the definition of a market, I assemble the firm-product information at the CN8 level such that it is defined as all the goods belonging to the same 4-digit NACE industry. I keep referring to a sector as a 2-digit NACE industry. Moreover, I classify a firm as domestic when it reports positive production in Denmark. The definition is applied on an industry-by-industry basis and, so, some firms could be considered domestic in one industry but not in another.

Market shares are defined relative to total sales value in the market. This comprises sales by domestic firms and imports. Regarding the former, I take as a baseline case the total turnover reported in the dataset of production.³⁷ Results with alternative definitions are presented in [Appendix F.2](#).³⁸ Moreover, I consider two types of imports as foreign competition in an industry. The first type includes imports made by firms that do not

³⁵NACE is the standard industry classification used in the EU. It is similar to the NAICS system for North American countries, or the older SIC used in the USA.

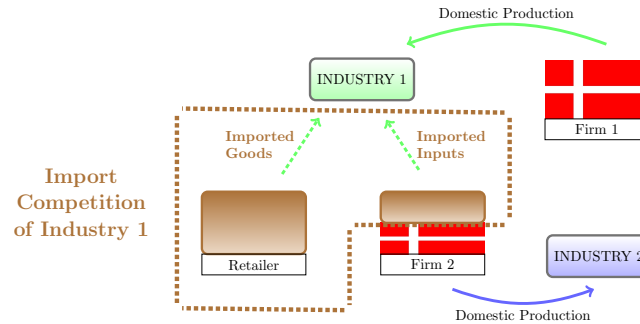
³⁶The Combined Nomenclature is the nomenclature used by EU countries to report trade data. Their first six digits coincide with the Harmonized System (HS) nomenclature.

³⁷Since firms report their total value of production without a breakdown between domestic sales and exports, I compute the former as the difference between their total production value and its exports.

³⁸Total turnover is not defined as production on the physical territory of Denmark. Instead, it is by the economic ownership of goods sold and produced by Danish firms. Specifically, it includes sales of own goods (either produced, processed or assembled by the firm), goods produced by a subcontractor established abroad (if the firm owns the inputs of the subcontracted firm), and resales of goods bought from other domestic firms and sold with any processing. Essentially, it excludes sales of goods imported which are produced by foreign firms not owned by the Danish firm. Taking total production as the baseline sales is a conservative assumption to not either double count some of the imports, mistakenly consider expenditure on inputs as outputs, or overestimate the market share of some firms which mainly act as retailers selling several brands. In [Appendix F.2](#), I consider different definitions of firm imports as part of the total supply and show that results are essentially the same.

engage in any production activity in Denmark. For instance, they comprise imports made by retailers that are competing directly with sales by domestic firms. The second type includes imports made by domestic firms not producing in that industry. These imports represent foreign competition since they are inputs that the firm could have bought from a domestic firm. **Figure 5** illustrates the classification.

Figure 5. *Classification by Industry into Domestic Firms and Import Competition*



Note: The classification of a firm as domestic or part of the import competition is conducted at the industry level. Thus, a firm can be domestic in one industry but represent import competition in another. In the figure, I illustrate this by considering two firms. Firm 1 and 2 are domestic in industries 1 and 2, respectively, since each sells a good that belongs to that industry. In industry 1, the import competition encompasses imports of industry 1's goods by non-manufacturing firms (e.g., retailers) and manufacturing firms that do not produce goods belonging to industry 1 (e.g., Firm 2).

I consider a Danish firm-industry as a DL if it has a domestic market share greater than 3%. Remarkably, given that the structural model determines that a DL with negligible market shares behaves as a DNL, the cutoff only affects the presentation of average results, but not the impact at the industry level.

Consistent with the theoretical model, I only consider industries which include a subset of firms having negligible market shares, and drop those that exhibit no import competition.³⁹ After this, I end up with 107 industries out of a total of 203. The sample of industries covers around 80% of the total value of the manufacturing. At the firm-industry level and in terms of Danish firms, it encompasses 331 DLs and 5,350 DNLs. Furthermore, since 92 industries have at least one DL, it reveals that the standard monopolistic-competition market structure with negligible firms is valid for only 15 out of 107 industries.

7.3 Determination of Parameters

To obtain the counterfactual in [Section 6.2](#), I have assumed that σ and δ are given. Now I proceed to their determination. For σ , I make use of the estimates provided by [Soderbery \(2015\)](#). His approach is based on [Broda and Weinstein \(2006\)](#) and improves upon it to

³⁹The procedure, as well as additional results, are presented in [Appendix C.1](#).

account for small-sample biases. I aggregate them at the industry level through product-expenditure weights.

Regarding δ , I choose a value such that the model can fit as close as possible the dispersion within industry of DLs' market shares, not explained by either prices or common shocks to all DLs in the industry. To do this, I express (4) in logarithms such that the domestic market share of a DL producing variety ω in industry n is

$$\ln s_n(\omega) = (1 - \sigma_n) \ln p_n(\omega) + \delta \ln z_n(\omega) - \ln \mathbb{A}_n + \varepsilon_n(\omega), \quad (6)$$

where $\varepsilon_n(\omega)$ is an error term. Substituting the optimal investments predicted in the sequential model into (6) and working out the expression, I obtain a regression equation which I use to estimate δ . Using data of unit values as prices and treating \mathbb{A}_n as an industry fixed effect, I obtain $\delta = 0.872$. For further details, see [Appendix C.2](#).

7.4 Results

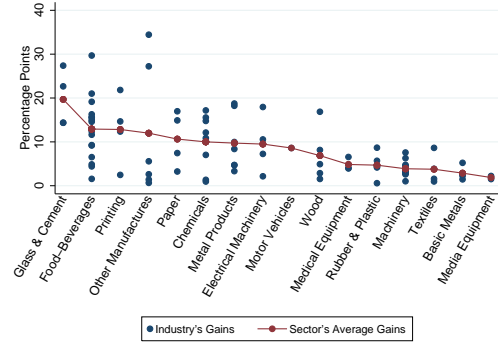
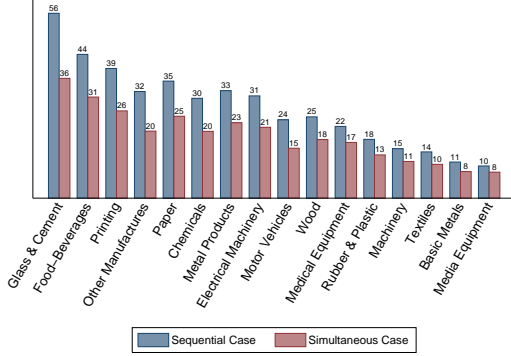
[Table 2](#) and [Figure 6](#) present the results coming from a comparison of the sequential and simultaneous cases.

Table 2. *Estimated Impact of the Strategic Behavior - Differences between the Simultaneous and Sequential Scenario*

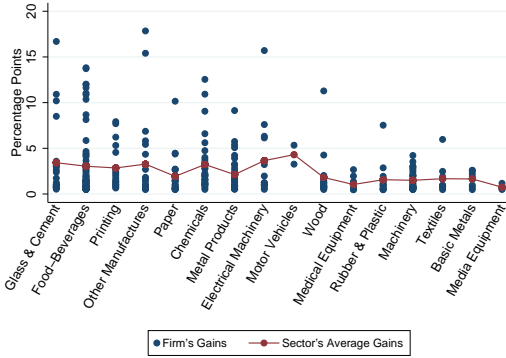
	Avg. Per Firm			Avg. Ind.	Aggregate Manufacture	
	Market Share Points	Domestic Intensity Points	Domestic Sales Increase	Domestic Concentration Points	Domestic Intensity Points	DLs Sales Increase
Glass & Cement	3.4	3.9	41.8	19.7	9.5	50.1
Food & Beverages	3.0	4.8	35.4	12.9	6.8	41.9
Printing	2.9	3.4	41.6	12.8	1.5	47.0
Other Manufactures	3.3	6.6	41.2	12.0	9.9	53.2
Paper	1.9	3.8	35.0	10.6	3.3	41.6
Chemicals	3.2	6.0	42.5	10.0	10.3	55.6
Metal Products	2.1	3.9	35.8	9.7	2.8	44.2
Electrical Machinery	3.7	5.2	38.6	9.5	6.0	41.5
Motor Vehicles	4.3	.	56.1	8.6	.	56.9
Wood	1.8	4.7	31.4	6.9	5.3	33.6
Medical Equipment	1.0	5.6	26.8	4.8	6.1	28.9
Rubber & Plastic	1.6	4.2	30.0	4.7	5.9	31.8
Machinery	1.5	4.2	31.3	3.9	7.3	34.7
Textiles	1.7	3.4	31.0	3.8	6.8	31.5
Basic Metals	1.6	5.4	33.1	2.9	6.7	37.8
Media Equipment	0.7	4.4	23.2	1.9	5.2	23.7
Sectors Average	2.4	4.6	35.9	8.4	6.2	40.9
All Industries	2.5	5.2	35.9	9.0	8.3	43.1

Note: Only industries with coexistence of DNLs, DLs, and importers considered. Results in terms of points expressed in percentage-points differences between the sequential and simultaneous scenarios. Results in terms of increases expressed in percentage increases relative to the simultaneous case. Market shares based on total sales of the industry and account for import competition. At the industry level, domestic concentration defined as the increases in market shares accrued by all DLs. Domestic intensity defined as firm's domestic sales relative to its own total sales, with only exporters considered for calculations. Entries with "." reflect that all the firms in the sector serve the Danish market exclusively. At the manufacture level, domestic intensity and DLs sales measured by taking the total of manufacturing.

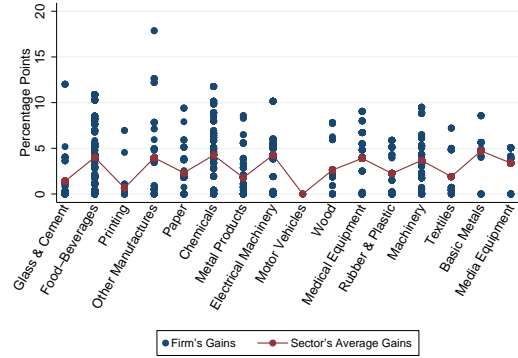
Figure 6. *Sequential vs. Simultaneous Scenarios*
 (a) Domestic Concentration - Levels (b) Domestic Concentration - Points Increase by Industry



(c) Firm's Domestic Market Share - Points Increase per Firm



(d) Firm's Domestic Intensity - Points Increase per Firm



Note: Figure 6a expressed in market-share levels. In the rest of the figures, outcomes are percentage-points differences between the sequential and simultaneous case. Figures with firm's domestic intensity excludes firms that sell exclusively in the domestic market. Market shares measured in terms of total sales value of the industry and account for import competition.

The first conclusion is that there is pronounced heterogeneity in concentration outcomes across sectors (Figure 6a) and industries (Figure 6b). Regarding firms, the market shares of the top 3 DLs, which are respectively around 14%, 7%, and 6% in the sequential scenario, would instead have been 9%, 5%, and 5% in the simultaneous scenario. Nonetheless, Figure 6c reveals that these gains can be substantially greater for some of the firms. In terms of domestic intensity, the top DL has increases of around 4% and 6%, respectively, depending on whether all firms or just exporters are considered. For the same cases, both the second and third top DLs have increases of around 2% and 4%, on average. From Figure 6d, it is clear that there is also pronounced heterogeneity in terms of domestic intensity.

Arguably, the magnitude of estimates coming from structural models should be interpreted with caution, given the simplifications and assumptions made about unobservables. On the other hand, the approach is a powerful tool for comparing the relative importance of a mechanism across different groups of observations. Showing how outcomes differ by

groups of industries is of relevance for this paper, given the stark differences in the results obtained across them.

At a theoretical level, by [Propositions 5.1 - 5.4](#), market-share gains and increases in domestic intensity are always positive. Nevertheless, these results are silent about their magnitudes. In the structural model, it can be shown that the strength of the effects display non-monotonicities in relation to the market shares and domestic intensity observed. However, as demonstrated via simulations in [Appendix B.5](#), for the range of values in the data, the sign of the relations can be predicted. This is because, on average, firms have market shares that are not disproportionately large, while home-bias is pronounced. Both facts also determine that differences in σ have a small direct impact on the results. Thus, the magnitude of concentration gains is mainly determined by the level of concentration observed, while increases in domestic intensity are primarily determined by both market shares and domestic intensity observed.

This provides me with some guidance for identifying the industries where the impact of strategic investments should be more pronounced. First, regarding concentration, if DLs accumulate a small market share, they will have a small influence on the aggregate conditions of the market. Thus, they will obtain only a small strategic gain of market shares. Additionally, in industries where firms, on average, sell almost exclusively in their home market, domestic-intensity increases will be less affected by greater domestic revenues. Based on this intuition, the correlation patterns in [Stylized Fact 3](#) are informative. They point out that lower values of σ are associated with greater concentration and lower domestic intensity in industries with consumer goods, and lower concentration for producer goods. Thus, I proceed to analyze the outcomes in terms of good substitutability.

Classifying industries in terms of low and high σ (relative to the whole manufacturing sector), the results indicate that strategic gains arise in 65 and 27 industries, respectively.⁴⁰ As for the magnitude of outcomes in relation with σ , regressions that pool all the observations suggest that there is no relation. However, this unmasks heterogeneity across industries, with a pattern emerging once that I classify them according to their

⁴⁰Similar results arise by breaking down industries into differentiated and homogeneous goods as in [Rauch \(1999\)](#). This determines that 68 industries are differentiated and 24 homogeneous. Nonetheless, unlike σ , they do not constitute a good predictor of the level of concentration. In addition, the classification by [Rauch \(1999\)](#) should be interpreted with some caution depending on the analysis carried out. As [Broda and Weinstein \(2006\)](#) indicate, the classification, although correlated with σ , does not imply that when a good is labeled as homogeneous it is perceived as a perfect substitute. They provide tea as an example, which is considered as a homogeneous good by [Rauch \(1999\)](#), but it surely is perceived to be quite differentiated by consumers (the same remark applies to, for instance, beer).

final user (i.e., producer vs consumer). To make the comparison meaningful, I discard sectors where all the industries are classified exclusively as either comprising producer or consumer goods.⁴¹ This determines that there are, respectively, 24 and 20 industries with producer and consumer goods displaying strategic gains. The results are presented in [Table 3](#).

Table 3. *Substitutability and Final Users (consumers vs producers)*

	Concentration Increase						Domestic Intensity Increase			
	BW-S σ			σ constant			BW-S σ		σ constant	
	DLs (1)	Top 3 (2)	Non-Top 3 (3)	DLs (4)	Top 3 (5)	Non-Top 3 (6)	(7)	(8)	(9)	(10)
ln σ (consumers)	-16.16** (7.405)	-14.79** (6.997)	-1.369 (1.192)	-12.66** (6.925)	-11.38* (6.039)	-1.283 (1.139)	-2.886** (1.150)	-2.861* (1.442)	-3.053*** (1.099)	-2.308* (1.345)
ln σ (producers)	7.970** (3.615)	5.581* (3.258)	2.120 (1.515)	9.903** (3.680)	7.726** (3.089)	2.177 (1.524)	-1.113 (1.084)	0.041 (3.387)	-1.090 (1.365)	-1.024 (4.154)
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		
Sector-Rank FE									Yes	Yes
Sample Unit	Industry	Industry	Industry	Industry	Industry	Industry	Firm-Ind	Exp-Ind	Firm-Ind	Exp-Ind
Observations	39	39	39	39	39	39	170	104	139	81
R-squared	0.305	0.292	0.306	0.301	0.283	0.308	0.251	0.211	0.367	0.271

Note: Domestic Leaders (DLs) in an industry defined as Danish firms with a domestic market share greater than 3%. Industries with producer and consumer goods defined according to the BEC classification, with producer goods encompassing intermediate goods and capital. The variable BW-S σ corresponds to the elasticity of substitution from [Soderbery \(2015\)](#). Rank refers to the position in the market according to the domestic market share of the firm. Firm-Ind indicates that the observations are at the firm-industry level, while Exp-ind indicates that the sample is restricted to firms that export. All the results come from regressions of the type $y = FEs + \alpha \ln \sigma + \beta \times \mathbb{1}(user)$. By including all the interaction terms through the fixed effects FEs, the α estimated is identical to that obtained through a separate regression with the sample restricted to one type of final user. Heteroskedastic-robust standard errors used.

The parameter estimates in Columns (1) to (3) indicate that a lower σ is associated with greater gains of market shares for consumer goods. The opposite result is obtained for producer goods, where greater increases in concentration are found in industries with greater values of σ . Moreover, while this holds for the concentration by the top 3 DLs, there are no statistically significant differences for the non-top DLs.

Since the outcomes are calculated using σ , and a greater σ determines a smaller impact on the gains of market share, we may suspect that this is driving the results. This is especially the case for consumer goods where the sign of the results are consistent with an explanation along those lines. I provide evidence that this is not the case and, instead, it responds to the pattern of concentration of consumer and producer goods implied by [Stylized Fact 3](#). To corroborate this, in Columns (4) to (6), I present estimates obtained by assuming σ is constant across industries. The results are consistent with Columns (1) to (3).

In addition, for consumer goods, Columns (7) and (8) establish that the domestic-intensity increases are more pronounced for industries with lower values of σ . This is

⁴¹I have also discarded the machinery sector since it has 11 industries with producer goods and just one with consumer goods.

irrespective of whether all DLs or just those that export are considered. On the contrary, there seems to be no relation between σ and domestic-intensity increases for the case of producer goods. In Columns (9) and (10), to isolate the role of the domestic-intensity distribution, I replicate the results by keeping σ constant across industries and controlling for the rank position by market share. This corroborates that the distribution of domestic intensity indicated in **Stylized Fact 3** is important in the determination of these outcomes. Moreover, the results also indicate that the same qualitative outcomes are obtained irrespective of whether the sample is restricted to exporters. In addition, it can be shown that the results hold by including market shares as a control variable.

8 Conclusions

In this paper, I studied a mechanism in which domestic leaders engage in strategic behavior to preempt entry and gain a better position in their home market. In order to analyze the phenomenon, I built up a framework which tractably incorporates strategic interactions and an endogenous number of heterogeneous firms making multiple choices. Given the flexibility of the setup, I was able to conduct the study at both the theoretical and empirical level.

The model combines the technical advantages of two different approaches. First, I based my approach on an empirical fact that I identified in my data, where in a typical industry there is coexistence of leaders and firms with trivial market shares. Thus, I exploited the parsimonious approach of Melitz (2003) to model the behavior of negligible firms and account for extensive-margin adjustments. Second, I resorted to the tools of Aggregative Games to tractably incorporate the existence of heterogeneous firms behaving strategically and deciding on multidimensional strategies. This turns a potentially complex multidimensional problem into a unidimensional one.

At the theoretical, I established that, relative to a non-strategic benchmark, domestic leaders overinvest on demand-enhancing instruments at their home market, resulting in a crowding out of rival firms and, in particular, of importers. This strategy causes a reallocation of market share towards domestic leaders which generates greater concentration and home-bias patterns at both the firm and industry level. I showed that the outcomes are robust to the type of competition at the market stage (prices or quantities), the effect of investments on prices (increases or decreases), and even the nature of investments (i.e.,

cost-reducing or demand-enhancing).

Turning the theoretical model into a structural one, I conducted an empirical study of the mechanism by making use of disaggregated Danish manufacturing data. I showed that, in industries with consumer goods, a lower substitutability is associated with greater increases of concentration and domestic intensity. On the contrary, for producer goods, industries with lower substitutability have lower concentration gains and display statistically insignificant differences in terms of domestic intensity. I provided evidence that the main determinants of these results are the distributions of market shares and domestic intensity in relation to substitutability.

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Online Appendix - not for publication

A Derivations and Proofs

In this appendix, I provide derivations of some results included in [Section 5](#) regarding the sequential scenario. In particular, I present a formal justification for the fact that the aggregate is the same in the simultaneous and sequential scenarios. After this, I present proofs for [Propositions 5.1 - 5.4](#).

A.1 Derivations of the Sequential Scenario

Denote by $\mathbf{z}_{ii} := (z_{ii}^\omega)_{\omega \in \Omega_{ii}^B}$ and $\mathbf{z}_{\text{dom}} := (\mathbf{z}_{ii})_{i \in \mathcal{C}}$ the vector of domestic investments by DLs from i and by DLs around the world, respectively. As argued in the main part of the paper, the simultaneous game with a given \mathbf{z}_{dom} defines a class of subgames for the sequential scenario. Furthermore, since markets are segmented, \mathbf{z}_{dom} can only affect market i through \mathbf{z}_{ii} . Using these facts, next I characterize the solution of this subgame in $i \in \mathcal{C}$ for a given \mathbf{z}_{ii} .

Regarding optimal choices, notice that, conditional on \mathbb{A}_i , the profits of both DNLs and non-domestic firms in i do not depend on \mathbf{z}_{ii} . Thus, the optimal decisions of these firms are the same as in the simultaneous scenario: their prices, investments, and survival productivity cutoffs are still characterized by [\(PRICE\)](#), [\(z-SIM\)](#), and [\(ZCP\)](#), respectively. On the other hand, \mathbf{z}_{ii} affects the pricing of DLs from i . The optimal price of a firm ω is still characterized by [\(PRICE\)](#) but it determines a solution $p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega)$.

With the optimal choices determined and conditional on a given \mathbf{z}_{ii} , the equilibrium of the subgame can be obtained as in the simultaneous scenario. To do this, let's define the aggregate backward-response function of country i :

$$\Gamma_i(\mathbb{A}_i, \mathbf{M}^E, \mathbf{z}_{ii}) := H_i \left\{ \sum_{k \in \mathcal{C}} M_k^E \int_{\varphi_{ki}(\mathbb{A}_i)}^{\bar{\varphi}_k} h_{ki}^S[\mathbf{x}_{ki}^S(\mathbb{A}_i, \varphi)] dG_k^S(\varphi) + \sum_{j \in \mathcal{C} \setminus \{i\}} \int_{\omega \in \Omega_{ji}^B} h_{ji}^\omega[\mathbf{x}_{ji}^\omega(\mathbb{A}_i, \varphi_\omega)] d\mu_i(\omega) + \sum_{\omega \in \Omega_{ii}^B} h_{ii}^\omega[p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega), z_{ii}^\omega] \right\}. \quad (7)$$

Thus, the Nash equilibrium in i requires that \mathbb{A}_i constitutes a fixed point of (7). Formally,

$$\Gamma(\mathbb{A}_i, \mathbf{M}^E, \mathbf{z}_{ii}) = \mathbb{A}_i. \quad (\text{NE-seq})$$

Finally, since \mathbf{z}_{ii} does not directly affect the profits of DNLs conditional on \mathbb{A}_i , the free-entry condition for each i remains identical as in the simultaneous case and given by (FE).

From all this, we conclude that an equilibrium of the subgame for a given \mathbf{z}_{ii} is obtained through values $\mathbf{M}_{\text{seq}}^E := (M_i^{E*}(\mathbf{z}_{ii}))_{i \in \mathcal{C}}$ and $(\mathbb{A}_i^{\text{seq}}(\mathbf{z}_{ii}))_{i \in \mathcal{C}}$ such that (FE) and (NE-seq) for each $i \in \mathcal{C}$ hold. Once that these values are pinned down, the optimal strategies can also be obtained for a given \mathbf{z}_{dom} .

Next, I establish some properties of the subgame equilibrium with the goal of showing that \mathbf{z}_{ii} cannot affect the equilibrium aggregates but only the mass of DNLs. To keep track of these features, I state them as properties. They follow by simple inspection of the equilibrium conditions.

Subgame Equilibrium Property P1. *Conditional on \mathbb{A}_i , the optimal choices made by DNLs and foreign firms in i are independent of \mathbf{M}^E , \mathbf{z}_{dom} and $(\mathbb{A}_j)_{j \neq i}$.*

It is worth remarking that **Property P1** does not imply that optimal choices are independent of any variable chosen by a rival firm. Rather, it states that, if any variable has an effect on them, it has to be indirectly and through \mathbb{A}_i .

Property P1 characterizes the decisions in i for all firms except DLs from i . For those, the following property holds.

Subgame Equilibrium Property P2. *The optimal price at home of a DL ω from i is completely determined by \mathbb{A}_i and z_{ii}^ω . This implies that, conditional on \mathbb{A}_i and z_{ii}^ω , its domestic pricing decision is independent of \mathbf{M}^E , $(\mathbb{A}_j)_{j \neq i}$, $(\mathbf{z}_{jj})_{j \neq i}$ and $(z_{ii}^{\omega'})_{\omega' \neq \omega}$.*

Property P2 establishes that, regarding information of the industry conditions, the investment decisions by DLs are made by taking \mathbb{A}_i as a single sufficient statistic. Its composition is irrelevant. Thus, conditional on \mathbb{A}_i , the values of $(z_{ii}^{\omega'})_{\omega' \neq \omega}$ or \mathbf{M}^E do not convey any valuable information to the firm. Consequently, by establishing how $(\mathbb{A}_i^{\text{seq}})_{i \in \mathcal{C}}$ is identified, the investment decisions by DLs are also characterized.

To inquire upon how $(\mathbb{A}_i^{\text{seq}})_{i \in \mathcal{C}}$ is determined, by inspection of the equilibrium conditions (FE) and (NE-seq), it can be noticed that \mathbf{M}^E does not influence directly the expected

profits of any DNL and, hence, (FE). Consequently, the system (FE) for each $i \in \mathcal{C}$ completely pins down the equilibrium aggregates. Thus, the following property holds.

Subgame Equilibrium Property P3. *The system (FE) for each $i \in \mathcal{C}$ pins down the equilibrium value $(\mathbb{A}_i^{seq})_{i \in \mathcal{C}}$ independently of \mathbf{M}^E . Furthermore, given $(\mathbb{A}_i^{seq})_{i \in \mathcal{C}}$, \mathbf{M}_{seq}^E is determined by the system of equations (NE-seq) for each $i \in \mathcal{C}$.*

Property P3 has a simple interpretation. In equilibrium, the free-entry conditions of DNLs around the world completely determine the equilibrium aggregates. Given these values, the masses of DNLs adjust such that there is no excess of supply in equilibrium, making (NE-seq) hold in each country. Finally, all the properties stated lead to the following conclusion.

Subgame Equilibrium Property P4. *\mathbf{z}_{dom} cannot affect $(\mathbb{A}_i^{seq})_{i \in \mathcal{C}}$. It only affects \mathbf{M}_{seq}^E .*

Property P4 provides a formal justification for the claim stated in the main part of the paper: the simultaneous and sequential games share the same equilibrium aggregates $(\mathbb{A}_i^*)_{i \in \mathcal{C}}$. Variations in \mathbf{z}_{ii} only affect the mass of DNLs.

A.2 Proofs of Section 5.3

Proof of Proposition 5.1. We know that, under both scenarios, the same $(\mathbb{A}_k^*)_{k \in \mathcal{C}}$ holds and DLs choose a level of investment from the same choice set Z . Thus, if in the sequential game DLs chose the same level of investments as in the simultaneous case, they would achieve the same profits. Hence, by a revealed-preference argument, $\pi_i^{seq}(\omega) \geq \pi_i^{sim}(\omega)$. Furthermore, by the strictly-pseudo concavity and Assumption 5a, it can be shown that the level of investments in each scenario differ (this is shown formally in the proof of Proposition 5.2). Thus, by the strictly-pseudo concavity, this fact implies that $\pi_i^{seq}(\omega) > \pi_i^{sim}(\omega)$. ■

For the subsequent proofs, I make use of the following lemma.

Lemma A.1. *For a given \mathbb{A}_i^* , condition $\frac{d\mathcal{A}_i[p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)]}{dz_{ii}^\omega} > 0$ holds if and only if $\frac{\partial \ln p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)}{\partial \ln z_{ii}^\omega} < \frac{\varepsilon_{ii}^{z,mc}(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega) - \varepsilon_{ii}^z(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)}{\varepsilon_{ii}^{p,mc}(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega) - \varepsilon_{ii}^p(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)}$.*

Proof of Lemma A.1. The statement can be equivalently rephrased as $\text{sgn}\left\{\frac{d\mathcal{A}_i[\cdot]}{dz_{ii}^\omega}\right\} = \text{sgn}\left\{\frac{\varepsilon_{ii}^{z,mc}[\cdot] - \varepsilon_{ii}^z[\cdot]}{\varepsilon_{ii}^{p,mc}[\cdot] - \varepsilon_{ii}^p[\cdot]} - \frac{\partial \ln p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)}{\partial \ln z_{ii}^\omega}\right\}$. By definition, $\frac{\varepsilon_{ii}^{z,mc}[\cdot] - \varepsilon_{ii}^z[\cdot]}{\varepsilon_{ii}^{p,mc}[\cdot] - \varepsilon_{ii}^p[\cdot]} = \frac{-\frac{\partial \ln Q_{ii}^\omega[\cdot]}{\partial \ln \mathbb{A}_i} - \frac{\partial \ln \mathcal{A}_i[\cdot]}{\partial \ln z_{ii}^\omega}}{\frac{\partial \ln Q_{ii}^\omega[\cdot]}{\partial \ln \mathbb{A}_i} - \frac{\partial \ln \mathcal{A}_i[\cdot]}{\partial \ln p_{ii}^\omega}}$ or, what

is same, $\frac{\varepsilon_{ii}^{z,mc}[\cdot] - \varepsilon_{ii}^z[\cdot]}{\varepsilon_{ii}^{p,mc}[\cdot] - \varepsilon_{ii}^p[\cdot]} = \frac{-\frac{\partial \ln \mathcal{A}_i[\cdot]}{\partial \ln z_{ii}^\omega}}{\frac{\partial \ln \mathcal{A}_i[\cdot]}{\partial \ln p_{ii}^\omega}}$. Moreover, $\frac{d \ln \mathcal{A}_i[\cdot]}{d \ln z_{ii}^\omega} = \frac{\partial \ln \mathcal{A}_i[\cdot]}{\partial \ln z_{ii}^\omega} + \frac{\partial \ln \mathcal{A}_j[\cdot]}{\partial \ln p_{ii}^\omega} \frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega}$, so that $\frac{d \ln \mathcal{A}_j[\cdot]}{d \ln z_{ii}^\omega} > 0$ iff $\frac{\partial \ln \mathcal{A}_j[\cdot]}{\partial \ln z_{ii}^\omega} + \frac{\partial \ln \mathcal{A}_j[\cdot]}{\partial \ln p_{ii}^\omega} \frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega} > 0$, which holds iff $\frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega} < \frac{\frac{\partial \ln \mathcal{A}_i[\cdot]}{\partial \ln z_{ii}^\omega}}{\frac{\partial \ln \mathcal{A}_i[\cdot]}{\partial \ln p_{ii}^\omega}}$. ■

Proof of Proposition 5.2. Consider the decisions in the home market by a DL ω from i with productivity φ_ω . The marginal profits of domestic investments in the simultaneous and sequential case are given, respectively, by

$$\gamma_\omega^{\text{sim}}(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega) := \frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial z_{ii}^\omega} + \frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial \mathbb{A}_i} \frac{\partial \mathcal{A}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}]}{\partial z_{ii}^\omega} \quad (8a)$$

$$\gamma_\omega^{\text{seq}}(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega) := \frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial z_{ii}^\omega} + \frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega)}{\partial z_{ii}^\omega}. \quad (8b)$$

Independently of whether we consider the simultaneous or sequential case, optimal prices are characterized by (PRICE). This implies that $\frac{\partial \pi_{ii}^\omega(\cdot)}{\partial p_{ii}^\omega} = -\frac{\partial \pi_{ii}^\omega(\cdot)}{\partial \mathbb{A}_i} \frac{\partial \mathcal{A}_i[\cdot]}{\partial p_{ii}^\omega}$ and, so, we can reexpress (8b) as

$$\gamma_\omega^{\text{seq}}(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega) := \frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial z_{ii}^\omega} - \frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial \mathbb{A}_i} \frac{\partial \mathcal{A}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}]}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega)}{\partial z_{ii}^\omega}.$$

Let \mathbb{A}_i^* be the equilibrium aggregate under the simultaneous and sequential case, and $z_{ii}^{\text{sim}}(\omega)$ the optimal investments of the firm ω in the simultaneous scenario. Also, denote the domestic prices in the simultaneous scenario by $p_{ii}^{\text{sim}}(\omega) := p_{ii}^\omega[z_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*, \varphi_\omega]$.

Evaluating (8a) at the simultaneous equilibrium, we get $\gamma_\omega^{\text{sim}}[\mathbf{x}_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*; \varphi_\omega] = 0$. Given the pseudo-concavity of the profits function, if we show that $\gamma_\omega^{\text{seq}}[\mathbf{x}_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*; \varphi_\omega] > 0$, then the result follows. Defining

$$\Delta_\omega^{\text{sim}} := \Delta_\omega[\mathbf{x}_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*; \varphi_\omega] := \gamma_\omega^{\text{seq}}[\mathbf{x}_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*; \varphi_\omega] - \gamma_\omega^{\text{sim}}[\mathbf{x}_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*; \varphi_\omega],$$

we need to show that $\Delta_\omega^{\text{sim}} > 0$. Then,

$$\Delta_\omega^{\text{sim}} = -\frac{\partial \pi_{ii}^\omega(\cdot)}{\partial \mathbb{A}_i} \left[\frac{\partial \mathcal{A}_i(\mathbb{A}_i^*)}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega[z_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*, \varphi_\omega]}{\partial z_{ii}^\omega} + \frac{\partial \mathcal{A}_i(\mathbb{A}_i^*)}{\partial z_{ii}^\omega} \right],$$

where $\frac{\partial \mathcal{A}_i(\mathbb{A}_i^*)}{\partial z_{ii}^\omega}$ is shorthand notation for the fact that I am evaluating the derivative at \mathbb{A}_i^* . Given $\frac{\partial \pi_{ii}^\omega(\cdot)}{\partial \mathbb{A}_i} < 0$, Assumption 5a, and Lemma A.1, the term in brackets is positive and, so, the result follows.⁴² ■

Proof of Proposition 5.3. For a given \mathbb{A}_i^* , the revenues of a DL ω as a function of its investments are $R_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega) := R_{ii}^\omega[p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega), z_{ii}^\omega, \mathbb{A}_i^*]$ which, in turn, are given by the product $Q_{ii}^\omega[p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega), z_{ii}^\omega, \mathbb{A}_i^*] p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)$. Let the domestic investments of a

⁴²The fact that $\Delta_\omega^{\text{sim}} > 0$ also proves formally the fact used in the proof of Proposition 5.1 that the solutions of domestic investments in each scenario are not the same.

firm ω denoted by z_ω^{sim} and z_ω^{seq} in each equilibrium, respectively. Since Assumption 5b implies Assumption 5a, by Proposition 5.2 $z_\omega^{\text{sim}} < z_\omega^{\text{seq}}$. By the Fundamental Theorem of Calculus, we know that $R_{ii}^\omega(z_\omega^{\text{seq}}; \mathbb{A}_i^*, \varphi_\omega) - R_{ii}^\omega(z_\omega^{\text{sim}}; \mathbb{A}_i^*, \varphi_\omega) = \int_{z_\omega^{\text{sim}}}^{z_\omega^{\text{seq}}} \frac{\partial R_{ii}^\omega(z; \mathbb{A}_i^*, \varphi_\omega)}{\partial z} dz$.⁴³ If we show that $\frac{\partial R_{ii}^\omega(z; \mathbb{A}_i^*, \varphi_\omega)}{\partial z_{ii}} > 0$ for $z \in (z_\omega^{\text{sim}}, z_\omega^{\text{seq}})$, then the result follows. The effect of investments on revenues for a given \mathbb{A}_i^* are

$$\begin{aligned} \frac{d \ln R_{ii}^\omega[\cdot]}{d \ln z_{ii}^\omega} &= \frac{\partial \ln Q_{ii}^\omega[\cdot]}{\partial \ln z_{ii}^\omega} + \frac{\partial \ln Q_{ii}^\omega[\cdot]}{\partial \ln p_{ii}^\omega} \frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega} + \frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega} \\ &= \varepsilon_{ii}^{z,mc}(\cdot) - [\varepsilon_{ii}^{p,mc}(\cdot) - 1] \frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega} \end{aligned}$$

Therefore, $\frac{d \ln R_{ii}^\omega[\cdot]}{d \ln z_{ii}^\omega} > 0$ iff $\frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega} < \frac{\varepsilon_{ii}^{z,mc}(\cdot)}{\varepsilon_{ii}^{p,mc}(\cdot) - 1}$ which holds by Assumption 5b. ■

Proof of Proposition 5.4. By Assumption 5b, Proposition 5.3 holds, determining that in the sequential case the domestic sales of each DL are greater relative to the simultaneous scenario. In order to show that the relative aggregate sales by domestic firms increase with respect to imports, we need to show that the increment in sales of DLs from i is not exactly offset by a reduction of sales by DNLs from i .

To be more specific, let country $i \in \mathcal{C}$ be the home economy under analysis. We know that the same \mathbb{A}_i^* holds in the simultaneous and sequential cases and it is determined independently of \mathbf{M}^E and \mathbf{z}_{dom} (Property P3). Moreover, variations of \mathbf{z}_{dom} affect only condition (NE-seq) of i and the masses of incumbents (Property P4). As a result, the proof requires to show that the increase of domestic sales by DLs in i is not exactly offset by a reduction of M_i^E . In this way, we rule out that the increase of revenues by DLs have as counterpart a reduction of revenues coming exclusively through the exit of DNLs from the same country.

Given the symmetry of the countries assumed, there is a symmetric equilibrium with the same \mathbb{A}^* and M^E for each country as equilibrium values. By differentiating (NE-seq) for i , we can show that increases of z_{ii}^ω for each $\omega \in \bar{\mathcal{B}}_{ii}$ create a reduction of M^E and, hence, of each M_k^E with $k \in \mathcal{C}$. Therefore, overinvestment entails exit of foreign firms too. Formally,

$$(-1) \sum_{\omega \in \Omega_{ii}^B} \frac{dh_{ii}^\omega[p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega), z_{ii}^\omega]}{dz_{ii}^\omega} = \frac{\partial M^E}{\partial z_{ii}^\omega} \sum_{k \in \mathcal{C}} \int_{\varphi_{ki}(\mathbb{A}_i)}^{\bar{\varphi}_k} h_{ki}^S[\mathbf{x}_{ki}^S(\mathbb{A}_i, \varphi)] dG_k^S(\varphi).$$

By Assumption 5b and Lemma A.1, the LHS is positive. Also, $h_{ki}^S \geq 0$ and positive for

⁴³Given the assumptions that functions are smooth and domestic investments belong to a real compact set, the theorem can be applied since $\frac{\partial R_{ii}^\omega(z; \mathbb{A}_i^*, \varphi_\omega)}{\partial z_{ii}}$ is Riemann integrable.

at least one foreign country since we are ruling out autarky as an equilibrium. Hence, $\frac{\partial M^E}{\partial z_{ji}^\omega} < 0$, and the result follows. ■

B The Structural Model

In this appendix, I begin by providing some reasons for why I chose an augmented CES as demand system for the structural model, relative to other alternatives as the Logit ([Appendix B.1](#)). After this, I establish formal derivations of the solutions in the simultaneous and sequential scenarios ([Appendix B.2.1](#) and [Appendix B.2.2](#), respectively) for the structural model. In [Appendix B.3](#), I show that [Propositions 5.1 - 5.4](#) hold in the structural model for any values of the parameters. In [Appendix B.4](#), I establish that, given values of σ and δ and knowledge of a firm's market share in the sequential scenario, the market share of a firm in the simultaneous scenario can be recovered. In addition, I prove that this value always exists and is unique. Finally, in [Appendix B.5](#), through simulations, I conduct an analysis of the determinations of the effects' magnitudes. In particular, I establish results for the range of values holding in the data.

B.1 On the Choice of an Augmented CES blas

Next, I begin by stating different properties of an augmented CES. Then, I proceed to compare this demand system with the Logit.

Given some expenditure E_i for the industry, recall the demand in i of a variety ω from j , which is given by

$$Q_{ji}(\mathbf{x}_{ji}^\omega, \mathbb{P}_i) := E_i (\mathbb{P}_i)^{\sigma-1} (z_{ji}^\omega)^\delta (p_{ji}^\omega)^{-\sigma}, \quad (\text{CES})$$

where $\mathcal{P}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}] := \left\{ \sum_{k \in \mathcal{C}} \left[\int_{\omega \in \Omega_{ki}} h(\mathbf{x}_{ki}^\omega) d\mu_i(\omega) \right] \right\}^{\frac{1}{1-\sigma}}$, $\mathbb{P}_i \in \text{range } \mathcal{P}_i$, and $h(\mathbf{x}_{ji}^\omega) := (z_{ji}^\omega)^\delta (p_{ji}^\omega)^{1-\sigma}$.

While the aggregator is not uniquely defined, one convenient choice, following the main part of the paper, is $\mathbb{A}_i := \mathbb{P}^{\sigma-1}$. This allows for expressing [\(CES\)](#) in terms of market shares and, thus, I am able to provide an interpretation through the attraction models of the marketing literature. In particular, the interpretation I present follows the axiomatic derivation of market shares by [Bell et al. \(1975\)](#).

The expenditure on variety ω with [\(CES\)](#) is $R_{ji}(\mathbf{x}_{ji}^\omega, \mathbb{A}_i) := E_i \frac{h(\mathbf{x}_{ji}^\omega)}{\mathbb{A}_i}$. Thus, the market

share of ω in terms of value is defined by $s_{ji}(\mathbf{x}_{ji}^\omega, \mathbb{A}_i) := \frac{R_{ji}(\mathbf{x}_{ji}^\omega, \mathbb{A}_i)}{E_i}$ and given by

$$s_{ji}(\mathbf{x}_{ji}^\omega, \mathbb{A}_i) = \frac{h(\mathbf{x}_{ji}^\omega)}{\mathbb{A}_i} = \frac{h(\mathbf{x}_{ji}^\omega)}{\sum_{k \in \mathcal{C}} \int_{\omega \in \Omega_{ki}} h(\mathbf{x}_{ki}^\omega) d\mu_i(\omega)}. \quad (9)$$

The function h can be interpreted as a mapping that assigns a value to the bundle of characteristics (including the price) for variety ω . Thus, interpreting \mathbb{A}_i as the total “attractiveness” of the industry good, market shares are allocated by the attractiveness of ω relative to the attractiveness of the rest of varieties available in the industry.

The functional form of h gives a specific role to the parameters $1 - \sigma$ and δ : they are the elasticities of h with respect to prices and investments, respectively. As a corollary, for a given denominator and in percentage terms, these parameters indicate how each feature impacts the attractiveness of the firm’s variety and, hence, its market share.

Once that market shares are expressed as in (9), it is also possible to see that other functional forms for h could have been chosen. In fact, in the marketing literature, it is common to take (9) as a primitive of the model and establish specific properties through the choice of h . As for this paper, the choice of h determines different relations between the investments and the prices. For instance, by choosing $h_\omega(p_\omega, z_\omega) := \beta_p(p_\omega)^{1-\sigma} + \beta_z(z_\omega)^\delta$, it would have implicated that variations in investments do not affect prices in equilibrium. I chose a homogeneous function h since it allows me to express optimal choices and profits in terms of market shares. Thus, a direct link between the model and the data can be established.

Another additional property that the demand (CES) has is that the choices of DNLs have an equivalent characterization to those of DLs. Specifically, terms like price elasticities or the influence of a DL on the aggregate collapse to those of a DNL if the DL has a zero market share. This property becomes relevant for the empirical analysis since, in this way, the market-share cutoff to define a DL has a minor impact on the aggregate results.

How does the augmented CES compare to other standard demands for conducting an empirical analysis, such as the Multinomial Logit? The main difference between them is that, while the Multinomial Logit displays similar properties to the CES, market shares are defined in terms of quantities instead of revenues. This feature could present a distorted picture of how much market power a firm has in the market. Quantity-based market shares cannot distinguish between two firms that are selling the same amount of a good but one of them is able to charge higher prices. This explains my inclination towards the use of a

CES specification.

B.2 Simultaneous and Sequential Solutions

Next, I derive the solution for the simultaneous and sequential scenarios. I focus on the derivations that are necessary to conduct the analysis. Thus, I concentrate on obtaining the optimal choices of DLs and expressing the solutions in the way I use them to conduct the empirical analysis.

B.2.1 Simultaneous Solution

I use s_{ii}^ω to denote the domestic market share of a DL in i producing variety ω with productivity φ_ω . To keep the notation simple, I do not emphasize the distinction of s_{ii}^ω as a value or a function.

It can be shown that $\frac{\partial \ln h(\mathbf{x}_{ii}^\omega)}{\partial \ln z_{ii}^\omega} = \delta$, $\frac{\partial \ln h(\mathbf{x}_{ii}^\omega)}{\partial \ln p_{ii}^\omega} = \sigma - 1$, and $\frac{\partial \ln \mathcal{P}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}]}{\partial \ln h_{ii}^\omega} = \frac{s_{ii}^\omega}{1-\sigma}$ where $h_{ii}^\omega \in \text{range } h(\mathbf{x}_{ii}^\omega)$. These calculations are used to determine that $\frac{\partial \ln \mathcal{P}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}]}{\partial \ln z_{ii}^\omega} = \frac{\delta}{1-\sigma} s_{ii}^\omega$, $\frac{\partial \ln \mathcal{P}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}]}{\partial \ln p_{ii}^\omega} = s_{ii}^\omega$. Also, $\frac{\partial \ln s_{ii}^\omega}{\partial \ln z_{ii}^\omega} = \delta$ and $\frac{\partial \ln s_{ii}^\omega}{\partial \ln p_{ii}^\omega} = 1 - \sigma$. Using these results, I can determine the optimal prices chosen by a DL for a given $(s_{ii}^\omega, \varphi_\omega)$.

The first-order condition for prices requires the optimal price to satisfy $p_{ii}^\omega = \mu_{ii}^\omega c_{ii}^\omega$, where $\mu_{ii}^\omega := \frac{\varepsilon_{ii}^\omega}{\varepsilon_{ii}^\omega - 1}$ is the markup, and ε_{ii}^ω is the price elasticity of demand. For the demand under analysis, $\varepsilon_{ii}^\omega = \sigma - s_{ii}^\omega(\sigma - 1)$. Therefore, after working out the markup expression, the optimal price as a function of the market share is

$$p_{ii}(s_{ii}^\omega, \varphi_\omega) = \frac{\sigma}{\sigma - 1} \left(1 + \frac{1}{\sigma} \frac{s_{ii}^\omega}{1 - s_{ii}^\omega} \right) c_{ii}(\varphi_\omega). \quad (10)$$

Notice that in (10) it is not necessary to indicate whether the optimal price corresponds to a DL or DNL. The reason is that the price set by a DNL with productivity φ_ω is the limit case of a DL with productivity φ_ω and zero market share. Thus, when a DL has a negligible market share, the solution converges to the monopolistic competition case.

Regarding the optimal domestic investments of DLs, they are determined by the following optimization problem:

$$\begin{aligned} \max_{z_{ii}^\omega} \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{P}_i; \varphi_\omega) &:= E_i(\mathbb{P}_i)^{\sigma-1} (z_{ii}^\omega)^\delta (p_{ii}^\omega)^{-\sigma} [p_{ii}^\omega - c_{ii}(\varphi_\omega)] - \bar{f}_z z_{ii}^\omega \\ \text{subject to } p_{ii}^\omega &= p_{ii}(z_{ii}^\omega; \mathbb{P}_i, \varphi_\omega), \end{aligned}$$

where $p_{ii}(z_{ii}^\omega; \mathbb{P}_i, \varphi_\omega)$ is the implicit solution to (10) in terms of prices. The first-order

condition of this problem is

$$\frac{d\pi_{ii}^\omega[\cdot]}{dz_{ii}^\omega} := \frac{\partial\pi_{ii}^\omega}{\partial z_{ii}^\omega} + \frac{\partial\pi_{ii}^\omega}{\partial \mathbb{P}_i} \frac{\partial \mathbb{P}_i}{\partial z_{ii}^\omega} + \left(\frac{\partial\pi_{ii}^\omega}{\partial p_{ii}^\omega} + \frac{\partial\pi_{ii}^\omega}{\partial \mathbb{P}_i} \frac{\partial \mathbb{P}_i}{\partial p_{ii}^\omega} \right) \frac{\partial p_{ii}^\omega}{\partial z_{ii}^\omega} = 0. \quad (11)$$

Since the DL is choosing optimal prices, $\left(\frac{\partial\pi_{ii}^\omega}{\partial p_{ii}^\omega} + \frac{\partial\pi_{ii}^\omega}{\partial \mathbb{P}_i} \frac{\partial \mathbb{P}_i}{\partial p_{ii}^\omega} \right) = 0$. Furthermore,

$$\begin{aligned} \frac{\partial\pi_{ii}^\omega[\cdot]}{\partial z_{ii}^\omega} &:= \frac{\partial\pi_{ii}^\omega}{\partial Q_{ii}^\omega} \frac{\partial \ln Q_{ii}^\omega}{\partial \ln z_{ii}^\omega} \frac{Q_{ii}^\omega}{z_{ii}^\omega} - \bar{f}_z \\ &= [p_{ii}^\omega - c_{ii}(\varphi_\omega)] \delta \frac{Q_{ii}^\omega}{z_{ii}^\omega} - \bar{f}_z \\ &= \frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)} \frac{\delta}{z_{ii}^\omega} - \bar{f}_z, \end{aligned}$$

where the last line uses the fact that, at optimal prices, $Q_{ii}^\omega [p_{ii}^\omega - c_{ii}(\varphi_\omega)] = \frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)}$. Regarding the term $\frac{\partial\pi_{ii}^\omega}{\partial \mathbb{P}_i} \frac{\partial \mathbb{P}_i}{\partial z_{ii}^\omega}$, using the same fact and that $\frac{\partial \ln \mathbb{P}_i}{\partial \ln z_{ii}^\omega} = \frac{\delta}{1-\sigma} s_{ii}^\omega$,

$$\begin{aligned} \frac{\partial\pi_{ii}^\omega}{\partial \mathbb{P}_i} \frac{\partial \mathbb{P}_i}{\partial z_{ii}^\omega} &= \left(\frac{\partial\pi_{ii}^\omega}{\partial \ln Q_{ii}^\omega} \frac{\partial \ln Q_{ii}^\omega}{\partial \ln \mathbb{P}_i} \right) \left(\frac{\delta}{1-\sigma} \frac{s_{ii}^\omega}{z_{ii}^\omega} \right) \\ &= (-1) \frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)} \delta \frac{s_{ii}^\omega}{z_{ii}^\omega}. \end{aligned}$$

Finally, by making use of all these results, I can reexpress (11) as

$$\frac{d\pi_{ii}^\omega}{dz_{ii}^\omega} := \frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)} \frac{\delta}{z_{ii}^\omega} (1 - s_{ii}^\omega) - \bar{f}_z = 0.$$

Working out the expression, I get the DL's optimal expenditure on domestic investments as a function of its market share

$$\bar{f}_z z_{ii}^{\text{sim}}(s_{ii}^\omega) := \frac{\delta s_{ii}^\omega (1 - s_{ii}^\omega)}{\varepsilon(s_{ii}^\omega)},$$

with optimal domestic profits given by

$$\pi_{ii}^{\text{sim}}(s_{ii}^\omega) := \frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)} [1 - \delta (1 - s_{ii}^\omega)].$$

B.2.2 Sequential Solution

I use the same notation as in the simultaneous case. Consider a DL ω from i . Its optimal price can be characterized in the same way as in the simultaneous case. In addition, when a DL makes investments decisions, by Property P4, the equilibrium value of the price index \mathbb{P}_i^* is determined by the free-entry conditions of DNLs. Thus, it is not impacted by DLs' investments and it can be treated as exogenous. Domestic investments by DLs are determined by the following optimization problem:

$$\max_{z_{ii}^\omega} \pi_{ii}(\mathbf{x}_{ii}^\omega, \mathbb{P}_i; \varphi_\omega) \text{ subject to } \begin{cases} \mathbb{P}_i = \mathbb{P}_i^* \\ p_{ii}^\omega = p_{ii}(z_{ii}^\omega, \mathbb{P}_i, \varphi_\omega) \end{cases}.$$

Introducing the two restrictions into the objective function, the optimization problem becomes

$$\max_{z_{ii}^\omega} \pi_{ii}(z_{ii}^\omega; \mathbb{P}_i^*, \varphi_\omega), z_{ii}^\omega; \mathbb{P}_i^*, \varphi_\omega := Q_{ii}[p_{ii}(z_{ii}^\omega; \mathbb{P}_i^*, \varphi_\omega), z_{ii}^\omega; \mathbb{P}_i^*][p_{ii}(z_{ii}^\omega; \mathbb{P}_i^*, \varphi_\omega) - c_{ii}(\varphi_\omega)] - \bar{f}_z z_{ii}^\omega.$$

The first-order condition of this problem is

$$\frac{d\pi_{ii}^\omega}{dz_{ii}^\omega} := \frac{\partial \pi_{ii}^\omega}{\partial z_{ii}^\omega} + \frac{\partial \pi_{ii}^\omega}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega}{\partial z_{ii}^\omega} + \frac{\partial \pi_{ii}^\omega}{\partial \mathbb{P}_i} \underbrace{\frac{d\mathcal{P}_i^*}{dz_{ii}^\omega}}_{=0 \text{ by property (P4)}} = 0,$$

where \mathcal{P}_i^* is the aggregator corresponding to the aggregate \mathbb{P}_i^* . Next, I determine expressions for each term. I have already shown that $\frac{\partial \pi_{ii}^\omega}{\partial z_{ii}^\omega} = \frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)} \frac{\delta}{z_{ii}^\omega} - \bar{f}_z$. Besides, regarding $\frac{\partial \pi_{ii}^\omega}{\partial p_{ii}^\omega}$, by the first-order condition of prices,

$$\begin{aligned} \frac{\partial \pi_{ii}^\omega}{\partial p_{ii}^\omega} &= -\frac{\partial \pi_{ii}^\omega}{\partial \mathbb{P}_i} \frac{\partial \mathcal{P}_i^*}{\partial p_{ii}^\omega} \\ &= -\left(\frac{\partial \pi_{ii}^\omega}{\partial \ln Q_{ii}^\omega} \frac{\partial \ln Q_{ii}^\omega}{\partial \ln \mathbb{P}_i} \right) \frac{\partial \ln \mathcal{P}_i^*}{\partial \ln p_{ii}^\omega} \\ &= -\left(\frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)} (\sigma - 1) \right) s_{ii}^\omega. \end{aligned}$$

Regarding $\frac{\partial p_{ii}(z_{ii}^\omega, \mathbb{P}_i^*, \varphi_\omega)}{\partial z_{ii}^\omega}$, using (10),

$$\begin{aligned} \frac{\partial p_{ii}(s_{ii}^\omega, \varphi_\omega)}{\partial z_{ii}^\omega} &= \frac{\partial p_{ii}^\omega}{\partial s_{ii}^\omega} \frac{\partial s_{ii}^\omega}{\partial z_{ii}^\omega} + \frac{\partial p_{ii}^\omega}{\partial s_{ii}^\omega} \frac{\partial s_{ii}^\omega}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega}{\partial z_{ii}^\omega} \\ &= \frac{\frac{\partial p_{ii}^\omega}{\partial s_{ii}^\omega} \frac{\partial s_{ii}^\omega}{\partial z_{ii}^\omega}}{1 - \frac{\partial p_{ii}^\omega}{\partial s_{ii}^\omega} \frac{\partial s_{ii}^\omega}{\partial p_{ii}^\omega}}. \end{aligned}$$

Given $\frac{\partial p_{ii}^\omega}{\partial s_{ii}^\omega} = \frac{1}{\varepsilon(s_{ii}^\omega)} \frac{s_{ii}^\omega}{1-s_{ii}^\omega}$, $\frac{\partial \ln s_{ii}^\omega}{\partial \ln z_{ii}^\omega} = \delta$, and $\frac{\partial \ln s_{ii}^\omega}{\partial \ln p_{ii}^\omega} = 1 - \sigma$, I obtain

$$\frac{\partial \ln p_{ii}(s_{ii}^\omega, \varphi_\omega)}{\partial \ln z_{ii}^\omega} = \frac{\delta s_{ii}^\omega}{\sigma - s_{ii}^\omega \varepsilon(s_{ii}^\omega)}.$$

All this determines that

$$\frac{d\pi_{ii}^\omega[\cdot]}{dz_{ii}^\omega} := \frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)} \frac{\delta}{z_{ii}^\omega} (1 - s_{ii}^\omega) \left(\frac{\sigma}{\sigma - s_{ii}^\omega \varepsilon(s_{ii}^\omega)} \right) - \bar{f}_z = 0.$$

Thus, DL's optimal expenditure on domestic investments is

$$\bar{f}_z z_{ii}^{\text{seq}}(s_{ii}^\omega) := \frac{\delta s_{ii}^\omega (1 - s_{ii}^\omega)}{\varepsilon(s_{ii}^\omega)} \left(\frac{\sigma}{\sigma - s_{ii}^\omega \varepsilon(s_{ii}^\omega)} \right).$$

Likewise, optimal domestic profits are

$$\pi_{ii}^{\text{seq}}(s_{ii}^\omega) := \frac{s_{ii}^\omega}{\varepsilon(s_{ii}^\omega)} \left[1 - \delta (1 - s_{ii}^\omega) \left(\frac{\sigma}{\sigma - s_{ii}^\omega \varepsilon(s_{ii}^\omega)} \right) \right].$$

B.3 Checking Assumptions 5a and 5b

For Propositions 5.1 - 5.4 to hold in the structural model, we need that Assumption 5a and Assumption 5b are satisfied. In particular, since Assumption 5b implies Assumption 5a, we need to check that

$$\frac{\partial \ln p_{ii}^\omega(z_{ii}^\omega, \mathbb{P}_i^*, \varphi_\omega)}{\partial \ln z_{ii}^\omega} < \frac{\varepsilon_{ii}^{z,mc}(\cdot)}{\varepsilon_{ii}^{p,mc}(\cdot) - 1} \text{ and } \frac{\partial \ln p_{ii}^\omega(z_{ii}^\omega, \mathbb{P}_i^*, \varphi_\omega)}{\partial \ln z_{ii}^\omega} < \frac{\varepsilon_{ii}^{z,mc}(\cdot) - \varepsilon_{ii}^z(\cdot)}{\varepsilon_{ii}^{p,mc}(\cdot) - \varepsilon_{ii}^p(\cdot)}.$$

I begin by showing that both inequalities are in fact equivalent in the structural model.

First, all the elasticities can be expressed in terms of market shares: $\varepsilon_{ii}^p(s_{ii}^\omega) := \sigma - s_{ii}^\omega(\sigma - 1)$, $\varepsilon_{ii}^{p,mc} := \sigma$, $\varepsilon_{ii}^{z,mc} = \delta$, and $\varepsilon_{ii}^{z,mc}(s_{ii}^\omega) = \delta(1 - s_{ii}^\omega)$. This determines that

$$\frac{\varepsilon_{ii}^{z,mc}(\cdot)}{\varepsilon_{ii}^{p,mc}(\cdot) - 1} = \frac{\delta}{\sigma - 1} \text{ and } \frac{\varepsilon_{ii}^{z,mc}(\cdot) - \varepsilon_{ii}^z(\cdot)}{\varepsilon_{ii}^{p,mc}(\cdot) - \varepsilon_{ii}^p(\cdot)} = \frac{\delta}{\sigma - 1}.$$

Thus, we need to show that $\frac{\partial \ln p_{ii}^\omega(z_{ii}^\omega; \mathbb{P}_i^*, \varphi_\omega)}{\partial \ln z_{ii}^\omega} < \frac{\delta}{\sigma - 1}$. Reexpressed in terms of market shares,

$$\frac{\partial \ln p_{ii}^\omega(s_{ii}^\omega, \varphi_\omega)}{\partial \ln z_{ii}^\omega} = \frac{\delta s_{ii}^\omega}{\sigma - s_{ii}^\omega \varepsilon_{ii}^p(s_{ii}^\omega)},$$

so we need to check that $\frac{\delta s_{ii}^\omega}{\sigma - s_{ii}^\omega \varepsilon_{ii}^p(s_{ii}^\omega)} < \frac{\delta}{\sigma - 1}$. It can be shown that this inequality holds by using the definition of $\varepsilon_{ii}^p(s_{ii}^\omega)$.

B.4 Counterfactual: Existence, Uniqueness and Computation

I proceed to show how, for given values of σ and δ , the market shares of the simultaneous scenario can be recovered. I exploit that market shares can be expressed as in equation (9), and the fact that the same equilibrium aggregate, \mathbb{A}_i^* , holds in the sequential and simultaneous scenario. Thus, the quotient of market shares in each scenario of the DL ω is,

$$\frac{s_{ii}^{\text{seq}}(\omega)}{s_{ii}^{\text{sim}}(\omega)} = \frac{[p_{ii}(s_{ii}^{\text{seq}}(\omega), \varphi_\omega)]^{1-\sigma} [z_{ii}^{\text{seq}}(s_{ii}^{\text{seq}}(\omega))]^\delta}{[p_{ii}(s_{ii}^{\text{sim}}(\omega), \varphi_\omega)]^{1-\sigma} [z_{ii}^{\text{sim}}(s_{ii}^{\text{sim}}(\omega))]^\delta}. \quad (12)$$

Using the interpretation of market shares from Appendix B.1, (12) indicates that, since \mathbb{A}_i^* is the same in both scenarios, differences in market shares are explained by how the strategic overinvestment impacts the attractiveness of the variety. This takes place by the direct impact of investments as well as its indirect effect on prices through markups.

By substituting in the optimal solutions for the optimal prices and the investments in each scenario, I obtain

$$\xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}}) = \xi_{ii}^{\text{seq}}(s_{ii}^{\text{seq}}), \quad (13)$$

where

$$\begin{aligned}\xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}}) &:= (s_{ii}^{\text{sim}})^{\delta-1} \left(\frac{\varepsilon^p(s_{ii}^{\text{sim}})}{\varepsilon^p(s_{ii}^{\text{sim}}) - 1} \right)^{1-\sigma} \left(\frac{(1 - s_{ii}^{\text{sim}})}{\varepsilon(s_{ii}^{\text{sim}})} \right)^\delta, \\ \xi_{ii}^{\text{seq}}(s_{ii}^{\text{seq}}) &:= (s_{ii}^{\text{seq}})^{\delta-1} \left[\frac{\varepsilon^p(s_{ii}^{\text{seq}})}{\varepsilon^p(s_{ii}^{\text{seq}}) - 1} \right]^{1-\sigma} \left[\frac{(1 - s_{ii}^{\text{seq}})}{\varepsilon^p(s_{ii}^{\text{seq}})} \left(\frac{\sigma}{\sigma - s_{ii}^{\text{seq}} \varepsilon^p(s_{ii}^{\text{seq}})} \right) \right]^\delta.\end{aligned}$$

From (13) s_{ii}^{sim} can be determined for a given s_{ii}^{seq} . Since (13) does not have a closed-form solution, it has to be determined numerically.

Next I show that the solution in (13) exists and is unique. I show this by proving that

$$\lim_{s_{ii}^{\text{sim}} \rightarrow 0} \xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}}) = \infty, \quad \lim_{s_{ii}^{\text{sim}} \rightarrow 1} \xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}}) = 0, \quad \text{and} \quad \frac{d\xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}})}{ds_{ii}^{\text{sim}}} < 0.$$

First,

$$\lim_{s_{ii}^{\text{sim}} \rightarrow 0} \xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}}) = \underbrace{(s_{ii}^{\text{sim}})^{\delta-1}}_{\rightarrow \infty} \underbrace{\left(\frac{\varepsilon^p(s_{ii}^{\text{sim}})}{\varepsilon^p(s_{ii}^{\text{sim}}) - 1} \right)^{1-\sigma} \left(\frac{1 - s_{ii}^{\text{sim}}}{\varepsilon^p(s_{ii}^{\text{sim}})} \right)^\delta}_{\in \mathbb{R}_{++}} = \infty,$$

where we have used the fact that $\delta < 1$.

Regarding the case $s_{ii}^{\text{sim}} \rightarrow 1$, it requires that we rearrange some of the terms and use that the price elasticity can be expressed as $\varepsilon^p(s_{ii}^{\text{sim}}) - 1 = (1 - s_{ii}^{\text{sim}})(\sigma - 1)$, yielding

$$\lim_{s_{ii}^{\text{sim}} \rightarrow 1} \xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}}) = \underbrace{(s_{ii}^{\text{sim}})^{\delta-1}}_{\rightarrow 1} \underbrace{[\varepsilon^p(s_{ii}^{\text{sim}})]^{1-\sigma-\delta}}_{\in \mathbb{R}_{++}} \underbrace{(\sigma - 1)^{\sigma-1} (1 - s_{ii}^{\text{sim}})^{\sigma-1+\delta}}_{\rightarrow 0} = 0.$$

Finally, we show that ξ_{ii}^{sim} is strictly decreasing. Applying logs and rearranging some of the terms, it is determined that

$$\frac{d \ln \xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}})}{ds_{ii}^{\text{sim}}} = \frac{(\sigma - 1)^2}{\varepsilon^p(s_{ii}^{\text{sim}})} - \frac{(\sigma - 1)^2}{\varepsilon^p(s_{ii}^{\text{sim}}) - 1} + \frac{\delta(\sigma - 1)}{\varepsilon^p(s_{ii}^{\text{sim}})} - \frac{\delta}{1 - s_{ii}^{\text{sim}}} - \frac{1 - \delta}{s_{ii}^{\text{sim}}}.$$

The difference between the first and second term of the RHS is negative, since trivially $\varepsilon^p(s_{ii}^{\text{sim}}) > \varepsilon^p(s_{ii}^{\text{sim}}) - 1$. Moreover, by using that $\varepsilon^p(s_{ii}^{\text{sim}}) - 1 = (1 - s_{ii}^{\text{sim}})(\sigma - 1)$, we can reexpress the fourth term, $\frac{\delta}{1 - s_{ii}^{\text{sim}}}$, as $\frac{\delta(\sigma - 1)}{\varepsilon^p(s_{ii}^{\text{sim}}) - 1}$. Therefore, the difference between the third and fourth terms of the RHS is negative too. Given that the fifth term of the RHS is positive and enters as a subtraction, I conclude that $\frac{d \ln \xi_{ii}^{\text{sim}}(s_{ii}^{\text{sim}})}{ds_{ii}^{\text{sim}}} < 0$.

B.5 Magnitude of the Strategic Gains

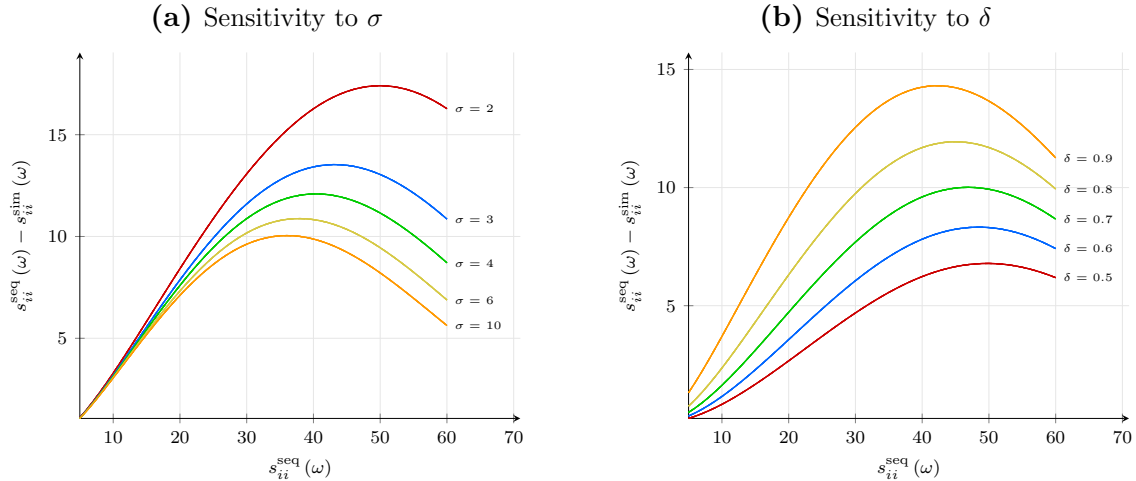
In this appendix, I illustrate through simulations how different variables affect the magnitude of effects in the structural model. In all of the examples, I consider a DL ω from some country $i \in \mathcal{C}$.

The conclusions of the analysis are twofold. First, given the range of values of the Danish data, the gains of market shares are mainly determined by the market share observed.

In addition, regarding the increases in domestic intensity and also for the values in the Danish data, they are affected by both the market shares and domestic intensity observed in the data. Regarding differences in σ across industries, they have a reduced impact on the results.

I begin by inquiring upon the determinants of gains in domestic market shares through the use of [Figure 7](#). The figures show the relation between the market share that we would observe in the data (horizontal axis) and the gains associated with it (vertical axis). Gains are expressed as percentage-point increases of market share in the sequential case relative to the simultaneous scenario. [Figure 7a](#) shows how gains vary for different values of σ , while [Figure 7b](#) does the same for δ .

Figure 7. *Market Share Gains*



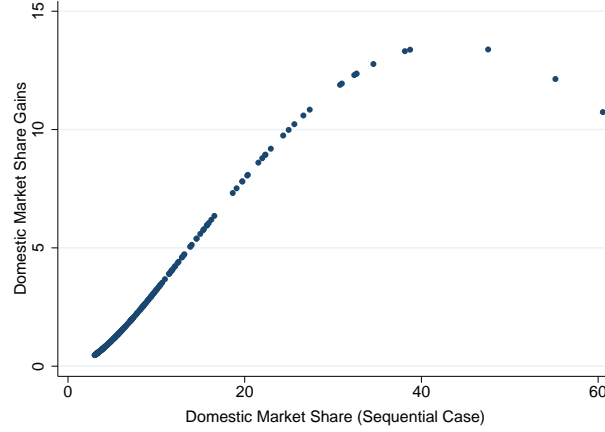
Note: In [Figure 7a](#), $\delta = 0.872$. In [Figure 7b](#), $\sigma = 3$.

The first conclusion obtained from both graphs is that, for a given value of market share, the lower the substitutability (i.e., low σ values) and the greater the effectiveness of demand-enhancing investments (i.e., high δ values), the greater the market-share gains. Additionally, [Figure 7a](#) reveals that differences in σ can potentially lead to a dispersion in gains, but only for firms with a large market share. However, since only a few firms have a market share exceeding 35% in the Danish data, differences in σ have a negligible impact on the gain in market share. Evidence of this is that, only 5 out of 331 DLs have a market share greater than 35%, and the average value of the top DLs is around 14%.

Furthermore, it can also be concluded that, even though market-share gains are necessarily positive, they are not monotone. For low values of market share, the gains are increasing while, after a certain threshold, they start to decrease. As a consequence, the

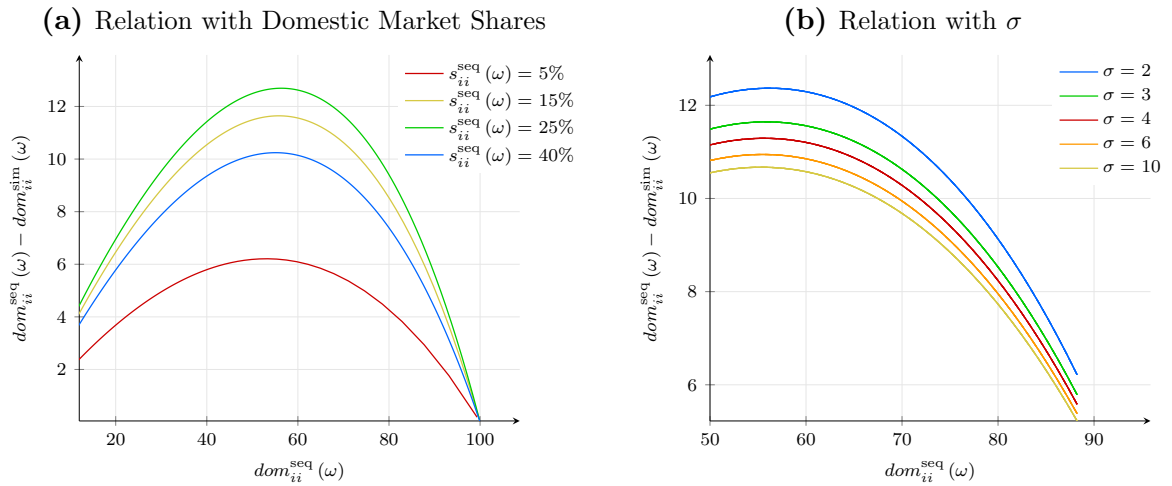
top leader in each industry is not necessarily the one with the greatest gains. Nonetheless, for the range of values observed in the Danish dataset, the overwhelming majority of firms have market shares that place them on the increasing part of the curve. This can be observed in Figure 8, which reproduces the market-share gains for the Danish data for a constant σ .

Figure 8. *Market Share Gains for the Danish Data*



Regarding domestic intensity, I use Figure 9 to show how increases in domestic intensity depend on domestic market shares and domestic intensity observed in the data.

Figure 9. *Domestic Intensity*



(c) Constant Sigma

	Increases in Firm's Domestic Intensity (1)
Domestic Market Share	0.100*** (0.382)
Domestic Intensity	-0.039*** (0.011)
Industry FE	Yes
Sample Unit	Exp-Ind
Observations	160
R-squared	0.531

Note: In [Figure 9a](#) and [Figure 9b](#), $\delta = 0.872$ and, along each curve, domestic intensity varies due to changes in the value of the firm's exports. In [Figure 9a](#), $\sigma = 3$. In [Figure 9b](#), $s_{ii}^{\text{seq}}(\omega) = 10\%$.

In both [Figure 9a](#) and [Figure 9b](#), δ and the total expenditure in the domestic market E are held constant. Along the horizontal axis, the domestic intensity that we would observe in the data varies by assigning different values to the firm's exports. This information, together with $s_{ii}^{\text{seq}}(\omega)$ can be used to calculate the domestic-intensity variation. The procedure is as follows. First, given $s_{ii}^{\text{seq}}(\omega)$, $s_{ii}^{\text{sim}}(\omega)$ is determined by equation (13). With the values of the domestic market share in each scenario, a firm's domestic revenue in each scenario is calculated through $s_{ii}^{\text{sim}}(\omega) \times E$ and $s_{ii}^{\text{seq}}(\omega) \times E$, respectively. Likewise, given a firm's exports and with the domestic revenues calculated, the domestic intensity in each scenario can also be calculated. While in [Figure 9a](#) this exercise is repeated for different values of $s_{ii}^{\text{seq}}(\omega)$, the results in [Figure 9b](#) assume a specific value of $s_{ii}^{\text{seq}}(\omega)$.

Consider one of the curves in [Figure 9a](#) so that $s_{ii}^{\text{seq}}(\omega)$ is kept fixed. By this, it can be appreciated that variations in domestic intensity are always positive but non-monotone in relation with the domestic intensity that we observe in the data. Nonetheless, given the existence of a home bias at the firm level, the increases in domestic intensity mainly move along the decreasing part of the curve. Furthermore, by comparing the curves for different market-share values, we can see that increases in domestic intensity have a non-monotone relation with domestic market shares. While increases for domestic market shares lower than 25% predict greater increases of domestic intensity, the opposite occurs if we move from a market share of 25% to one of 40%. Since for the average industry firms do not have disproportionately large market shares, in general a greater domestic market share observed determines a bigger domestic-intensity increase.

In [Figure 9b](#), I also show that, for some given domestic market share observed in the data, increases in σ result in smaller increases in the domestic intensity. Nonetheless, since on average the home bias is quite pronounced in the data, the impact of σ is quite small.

C Sample Determination and Estimation of δ

In [Appendix C.1](#), I provide information about the data at my disposal and describe the procedure used to select a sample of industries with coexistence of DNLs, DLs, and importers. Then, in [Appendix C.2](#) I describe the estimation process for δ .

C.1 Sample Determination

[Table 4](#) contains descriptive statistics of the entire sample at my disposal. It contains information on Danish manufacturing in 2005, comprising 203 industries and 3,686 firms, where I define a sector as a 2-digit industry and an industry at the 4-digit level according to the NACE rev. 1.1 classification.

Table 4. *Descriptive Statistics*

	% value	# industries	# firms	# exporters	% exporters	import share
Food & Beverages	14.4	33	317	224	71	41
Chemicals	10.6	17	125	110	88	60
Machinery	10.2	22	598	375	63	61
Metal Products	6.6	13	534	192	36	33
Motor Vehicles	6.5	3	66	41	62	91
Electrical/Machinery	4.7	7	131	75	57	45
Printing	4.5	7	296	89	30	11
Media & Equipment	4.3	3	60	40	67	85
Basic Metals	4.3	12	52	33	63	86
Rubber & Plastic	4.2	7	234	163	70	55
Other Manufactures	4.2	12	310	207	67	51
Computers	3.9	2	19	15	79	95
Wood	3.4	6	183	61	33	46
Glass & Cement	3.4	23	159	62	39	32
Apparel	3.0	6	42	38	90	92
Paper	2.9	6	120	76	63	58
Medical Equipment	2.8	4	132	94	71	56
Other Transports	2.8	8	27	18	67	73
Textiles	2.1	10	98	71	72	79
Leather	0.9	2	8	8	100	97
Per Avg Sector	5	10	175	99	64	62
All	100	203	3686	2091	57	58

Note: % value relative to total sales, % exporters relative to its own sector, and import shares relative to total sales.

Next, I provide details about the process of selecting a set of industries consistent with the theoretical model, which I outlined in [Section 7.2](#). This is accomplished by dropping industries according to the following criteria. First, to ensure that there is a set of DNLs, I remove industries with a low number of firms, where the cutoff is set at 10. Moreover, I exclude industries where either the subset of 20% of domestic firms with lowest market share or the 10 firms with lowest market share accumulate more than 6% of market share. Second, to ensure that there is import competition, I only keep industries where importers have at least 4% of the total market share.

After removing industries that do not meet this criteria, 107 industries remain in the sample. These industries have DNLs and importers, but not necessarily DLs. Considering only industries with coexistence of DNLs, DLs, and importers, there are 92 out of the 107 industries.

Regarding the percentage of total industries covered relative to total manufacturing, I present the results in Table 5. Specifically, Table 5a indicates the percentage of industries covered per sector among the 107 industries with the coexistence of DNLs and importers, while Table 5b presents percentage of the 92 industries where DNLs, importers, and DLs coexist relative to the 107 industries.

Table 5. *Final Dataset*

(a) Industries with DNLs and Import Competition

	% value	% industries
Media Equipment	100	100
Leather	100	100
Wood	99	83
Medical Equipment	97	75
Electrical Machinery	95	71
Computers	94	50
Apparel	94	50
Chemicals	93	71
Rubber & Plastic	91	71
Machinery	90	59
Other Manufactures	89	58
Paper	89	67
Metal Products	88	62
Basic Metals	87	33
Textiles	80	60
Food & Beverages	77	49
Printing	67	57
Glass & Cement	44	17
Motor Vehicles	29	66
Total	80.4	52.7

(b) Industries with DNLs, Import Competition, and DLs

	% value	% industries
Media Equipment	58	67
Leather	0	0
Wood	100	100
Medical Equipment	100	100
Electrical Machinery	100	100
Computers	0	0
Apparel	0	0
Chemicals	80	83
Rubber & Plastic	100	100
Machinery	93	92
Other Manufactures	85	86
Paper	100	100
Metal Products	95	88
Basic Metals	100	100
Textiles	69	67
Food & Beverages	100	100
Printing	100	100
Glass & Cement	100	100
Motor Vehicles	49	50
Total	82.3	86.0

Note: In both tables, market shares are import corrected and relative to sales. Value corresponds to the total sales value of the sector. For Table 5a, industries with DNLs and import competition comprise those where the 10 firms or 20% of firms with lowest market share accumulate less than 6% of the market share, and have at least 4% of import share. Percentages are relative to all the manufacturing industries. For Table 5b, industries with DNLs, import competition, and DLs is the subset of industries with DNLs and import competition which have at least one domestic firm that has a market share greater than 3%. Percentages are relative to the industries with DNLs and import competition.

C.2 Estimation of δ

Here, I describe the procedure to obtain δ . Equation (9), expressed in logarithms, determines that the market share of a Danish DL producing variety ω in the industry n is,

$$\ln s_n(\omega) = (1 - \sigma_n) \ln p_n(\omega) + \delta \ln z_n(\omega) - \ln A_n. \quad (14)$$

Regarding each term of equation (14), $\ln A_n$ is treated as an industry fixed-effect. Moreover, for p_n I use information on unit values from the Prodcum dataset, while σ_n is the

elasticity of substitution by [Soderbery \(2015\)](#) aggregated at the industry level by expenditure weights. Since the term $z_n(\omega)$ is unobservable, I use the structural solution under the sequential scenario. Adding an error term, this implies that δ is obtained from a regression based on the following equation,

$$\ln \tilde{s}_n(\omega) = \delta \ln z_n(\omega) - \ln \mathbb{A}_n + \varepsilon_n(\omega), \quad (15)$$

where $\ln \tilde{s}_n(\omega) := \ln s_n(\omega) - (1 - \sigma_n) \ln p_n(\omega)$. Thus, δ is set as the value which fits the dispersion of market shares within industries not explained by prices or common shocks to all firms in the industry. Incorporating that some of the variables determining investments are industry specific, equation (14) can be equivalently expressed in the following way,

$$\ln \tilde{s}_n(\omega) = \Lambda_n + \delta \ln \xi_n(\omega) + \varepsilon_n(\omega). \quad (16)$$

where $\xi_n(\omega) := \left[\frac{s_n(\omega)(1-s_n(\omega))}{\varepsilon[s_n(\omega)][\sigma_n - s_n(\omega)\varepsilon[s_n(\omega)]]} \right]$ and $\Lambda_n := \delta \ln \left(\frac{\delta \sigma_n}{f_z} \right) - \ln \mathbb{A}_n$. I perform the regression by using equation (16).

As I have mentioned, for the estimation of δ , I need information on prices. Equation (16) is at the firm-industry level, while the information on prices is at the CN8 level, which is more disaggregated. For this reason, firm prices are calculated as a weighted average of firm prices at the CN8 level, with weights given by the contribution of each CN8 product to the firm's revenue.

As is well known, unit values constitute an extremely noisy measure of prices. As the estimation of δ is based on a small number of observations given by the set of DLs, measurement error of particular observations makes the problem more severe. Thus, this problem needs to be addressed. Moreover, in the Danish data, some additional issues arise since, as it happens in the Prodcom datasets of some European countries, firms are not obliged to report quantities.⁴⁴ Thus, the data include missing values and some of the quantities are reported using different units of measure.

To reduce the noise of the estimation, I clean the data in several ways following standard procedures as in, for instance, [Amiti and Khandelwal \(2013\)](#). Using the logarithm of unit values as prices, I perform the following steps:

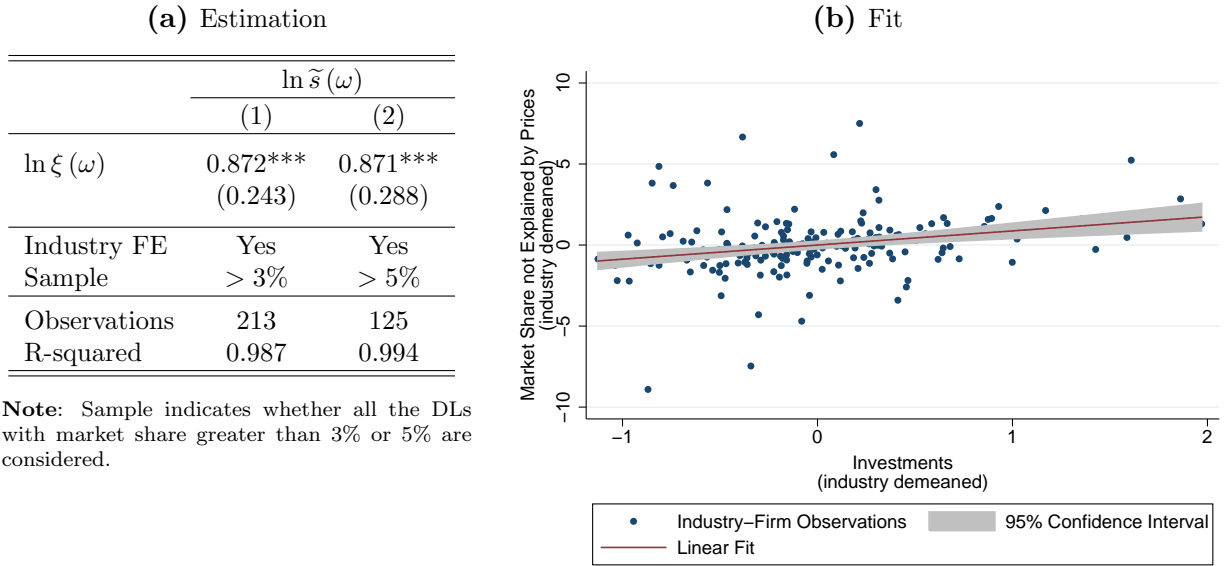
- By CN8 product, I drop those prices within the category that fall below the 5 percentile or above the 95 percentile.

⁴⁴Whenever possible, these quantities are approximated by statistical agencies using imputations based on reports of the same good from other production units in the same quarter. Otherwise, no value is reported.

- By firm-CN8 product, I remove prices which are 150% greater or 66% lower than the previous or subsequent year relative to the reference year.
- I remove industries where at least one DL does not report quantities.
- I drop industries where at least one CN8 is expressed in non-comparable units.⁴⁵

After this process, I end up with 65 industries out of the 92 industries with coexistence of DNLs, DLs, and importers. The information covers all the 16 sectors of the original sample and encompasses 213 firms. The results of the fit are presented in Figure 10.

Figure 10. δ Estimation and Fit



D Empirical Features of Domestic Leaders

In this appendix, I elaborate on some empirical regularities regarding DLs. Specifically, I present some correlations of DLs with different features at the firm level.

The information comes from an additional dataset to those used in the main part of the paper which provides accounting information at the firm level. Since the data to conduct the empirical analysis is at the firm-industry level, while this dataset is at the firm level, some of the definitions need to be adapted. First, I assign each firm to the sector from which it obtains its greatest revenue. In addition, I define a DL as a firm that in at least

⁴⁵Within industries, and even for a same product, some of the quantities reported are in different units of measure. For the cases in which units are expressed in different but comparable units, I express them in a same unit. For instance, if some CN8 is expressed in kilograms and other CN8 in tons, I express both in kilograms.

one industry belonging to its sector has a market share greater than 3%. The results are presented in [Table 6](#).

Table 6. *DLs' Features*

	Size (1)	Exporter (2)	Importer (3)	R/L (4)	K/L (5)	Wages (6)
DL (dependent variable)	0.517*** (0.0359)	0.0938*** (0.0114)	0.0984*** (0.00973)	0.126*** (0.0122)	0.0677*** (0.106)	0.0947*** (0.0122)
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample Unit	Firm-Sector	Firm-Sector	Firm-Sector	Firm-Sector	Firm-Sector	Firm-Sector
Observations	3,100	3,100	3,100	3,100	3,100	3,100
R-squared	0.200	0.062	0.054	0.078	0.053	0.062

Note: Each column provides the result of a regression where the dependent variable is DL and each variable indicated in Columns (1) to (6) is the independent variable. Each firm is assigned to the sector in which it obtains its greatest revenue. DL is a dummy variable that takes 1 if the firm has a market share greater than 3% in at least one industry of the sector. Market share is measured in terms of total sales value of the industry and account for import competition. Size is a dummy variable that takes 1 if the number of employees is greater than 250. Exporter and Importer are dummy variables. *Rev/L* is the revenue per employee, *K/L* is the capital per employee, and Wages is total wages per employee. The three variables are expressed in logs. Heteroskedastic-robust errors used.

As it can be observed, all the variables included correlate positively with being a DL. Thus, DLs have a greater size by employment, are more likely to export and import, have greater revenue-productivity, are more capital intensive, and pay higher wages.

E Additional Theoretical Results

In this appendix, I present additional results and robustness checks regarding the theoretical model. In [Appendix E.1](#), I begin by presenting examples of demand systems that can be expressed through an additive separable aggregator. In [Appendix E.2](#), I outline conditions for existence and uniqueness of the solution. In [Appendix E.3](#), I show that the model is robust to assuming Cournot competition, while in [Appendix E.4](#) I do the same for cost-reducing investments. In particular, I show that all the propositions of the baseline model hold. In [Appendix E.5](#), I consider multiproduct firms. Finally, in [Appendix E.6](#), I make some remarks on welfare. In particular, I show conditions such that the aggregate is a single sufficient statistic for welfare.

E.1 Demand Systems: Examples

In the main text, I have argued that some standard demand systems can be expressed through an aggregator which is additive separable. Here, I present some examples. To keep it simple, I consider demands that only depend on prices. They can be extended in

different ways to incorporate non-price choices.

Suppose a continuum of varieties with mass Ω . Any Greek letter represents a positive parameter and I use a subscript ω to remark on the fact that the demand system considered in the paper allows for firm heterogeneity. Let the demand be Q_ω with price p_ω and E income.

- **Demands from a discrete choice model** (McFadden, 1973): $Q_\omega := \frac{h_\omega(p_\omega)}{\mathbb{A}}$ with $\mathbb{A} := H \left(\int_{\omega'} h_{\omega'}(p_{\omega'}) d\omega' \right)$.
 - Multinomial Logit demand: $h_\omega(p_\omega) := \exp(\alpha_\omega - \beta_\omega p_\omega)$
 - CES without income effects: $h_\omega(p_\omega) := \alpha_\omega (p_\omega)^{-\beta_\omega}$.
- **Demands from discrete-continuous choices model** as in Nocke and Schutz (2018): $Q_\omega := \frac{\partial h_\omega(p_\omega)/\partial p_\omega}{\mathbb{A}}$ with $\mathbb{A} := H \left(\int_{\omega'} h_{\omega'}(p_{\omega'}) d\omega' \right)$. It includes the Logit and the CES (without income effects) as special cases.
- **Constant expenditure demand systems** (Vives, 2001): $Q_\omega := \frac{E}{p_\omega} \frac{h_\omega(p_\omega)}{\mathbb{A}}$ with $\mathbb{A} := H \left(\int_{\omega'} h_{\omega'}(p_{\omega'}) d\omega' \right)$. It includes:
 - CES: $h_\omega(p_\omega) := \alpha_\omega (p_\omega)^{-\beta_\omega}$.
 - Exponential demand: by defining $h_\omega(p_\omega) := \exp(\alpha_\omega - \beta_\omega p_\omega)$.
- **Demands from an additively separable indirect utility** as in Bertoletti and Etro (2015): given an indirect utility $V[(p_{\omega'})_{\omega' \in \Omega}, E] := \int_{\omega'} v_{\omega'} \left(\frac{p_{\omega'}}{E} \right) d\omega'$, the demands are , $Q_\omega := \frac{v'_\omega \left(\frac{p_\omega}{E} \right)}{\mathbb{A}}$ with $\mathbb{A} := \int_{\omega'} v'_{\omega'} \left(\frac{p_{\omega'}}{E} \right) \frac{p_{\omega'}}{E} d\omega'$.
- **Linear demand**: $Q_\omega := \alpha_\omega - \beta_\omega p_\omega + \mathbb{A}$ with $\mathbb{A} := \int_{\omega \in \Omega} \gamma_\omega p_\omega d\omega$.
- **Translog functional form**: $Q_\omega := \frac{E}{p_\omega} [\mathbb{A} - \ln(p_\omega)]$ where $\mathbb{A} := \int_{\omega'} \gamma_\omega \ln p_{\omega'} d\omega'$.
- **An affine translation of the CES** as in Arkolakis et al. (2019): $Q_\omega := \left(\frac{\mathbb{A}}{p_\omega} \right)^{\sigma_\omega} - \alpha_\omega$ with $\mathbb{A} := H \left(\int_{\omega'} h_{\omega'}(p_{\omega'}) d\omega' \right)$.

E.2 On the Existence and Uniqueness of the Equilibrium

I outline some remarks on existence and uniqueness of the equilibrium. Conditional on optimal strategies, the equilibrium conditions are, essentially, the same as in the monopolistic competition model by Melitz (2003). This follows because the equilibrium aggregates are pinned down by the free-entry conditions of DNLs. Thus, they are independent of DLs. In addition, given the optimal aggregates, the measure of DNLs incumbents is determined by the market-stage equilibrium. Due to this, the equilibrium at this stage is similar to the one of monopolistic competition but embedding DLs which, for a given equilibrium aggregate, act as shifters.

Next, I present some arguments to see that standard procedures of existence and uniqueness can be applied. Given that the strategy space of each DL is the Cartesian product of compact sets, then the strategy set is compact. In addition, given the pseudo-concavity of profits, and under proper Inada conditions, optimal prices and investments would exist, be interior and single valued.

In both the simultaneous and sequential case, the aggregate is determined by condition (FE). Applying Berge's maximum theorem, if the profits function is continuously differentiable $n + 1$ times, then the value function $\pi_{ij}^S(\mathbb{A}, \varphi)$ is continuously differentiable n times. Joint with the fact that costs functions are smooth and monotone in productivity, then $\varphi_{ij}(\mathbb{A}_j)$ is single valued and continuously differentiable. Let $\mathbf{A} := (\mathbb{A}_k)_{k \in \mathcal{C}}$ and $\tilde{\pi}_i^S(\mathbf{A}) := \sum_{j \in \mathcal{C}} \int_{\varphi_{ij}(\mathbb{A}_j)}^{\bar{\varphi}_i} \{\pi_{ij}^S(\mathbb{A}_j, \varphi) - f_{ij}^S\} dG_i^S(\varphi)$. If there exists an integrable function g such that $\left| \frac{\partial \pi_{ij}^S}{\partial \mathbb{A}_j} \right| \leq g_{ij}$ for each $i, j \in \mathcal{C}$, we can apply Leibniz rule and, so, $\tilde{\pi}(\mathbf{A})$ is continuously differentiable too. I show below that $\mathbb{A}_i \in [\underline{\mathbb{A}}_i, \bar{\mathbb{A}}_i]$, and, by assumption, π_{ij}^S is smooth for each $i, j \in \mathcal{C}$. Then, assuming that the Jacobian of $(\tilde{\pi}_i^S(\mathbf{A}))_{i \in \mathcal{C}}$ satisfies conditions for global univalence (for instance, Gale-Nikaido's Theorem), the equilibrium \mathbf{A}^* is unique.

Given \mathbb{A}_i^* for each $i \in \mathcal{C}$, $\mathbb{A}_i^* \in \text{range } \mathcal{A}_i$ is well defined. To see this, notice that, when the number of DLs is zero, by the fact that $h_k^S(\infty) = 0$ and applying Lyapunov's Convexity Theorem, Γ_i is compact and convex for each $i \in \mathcal{C}$. Moreover, using that H_i^{-1} exists for each $i \in \mathcal{C}$ because H_i is strictly monotone, $(H_i^{-1}[\Gamma_i(\mathbb{A}_i^*, \mathbf{M}^E)])_{i \in \mathcal{C}}$ is linear in \mathbf{M}^E . Therefore, we can apply standard results of linear algebra (e.g., full rank of the multiplying matrix and conditions on eigenvalues) to ensure that there is a unique positive solution \mathbf{M}^{E*} . Now, consider the case where the number of DLs is different from zero.

Given \mathbf{A} , the inclusion of DLs acts as a linear translation of $H_i^{-1}(\Gamma_i)$. Therefore, if these terms are not big enough such that all the DNLs of a country disappear (that is, if the solution to the system is not such that $M_j^E < 0$ for some $j \in \mathcal{C}$), then the necessary conditions are still the same.

E.3 Cournot Competition

In this part, I consider a setup with Cournot competition at the market stage and show that an overinvestment strategy also arises in this scenario. The setup of the model is the same as in [Section 4](#) but with some modifications regarding the supply and demand side. First, the strategy of a firm ω from i in j becomes $\mathbf{x}_{ij}^\omega := (Q_{ij}^\omega, z_{ij}^\omega)$. Moreover, the demand side is described through an inverse demand function which for a variety ω from i in j is given by

$$p_{ij}^\omega [\mathbf{x}_{ij}^\omega, \mathcal{A}_j [(\mathbf{x}_{kj})_{k \in \mathcal{C}}]] ,$$

where \mathcal{A}_j is an aggregator defined as in [Definition 1](#). I assume that $\frac{\partial p_{ij}^\omega}{\partial \mathbb{A}_j} < 0$, $\frac{\partial p_{ij}^\omega}{\partial Q_{ij}^\omega} > 0$, and $\frac{\partial p_{ij}^\omega}{\partial z_{ij}^\omega} > 0$. Moreover, regarding the aggregator, it is supposed that $H' > 0$, $\frac{\partial h_{ij}^\omega}{\partial Q_{ij}^\omega} > 0$ and $\frac{\partial h_{ij}^\omega}{\partial z_{ij}^\omega} > 0$.

In this framework, a firm ω from i that is active in j has profits in j given by

$$\pi_{ij}^\omega (\mathbf{x}_{ij}^\omega, (\mathbf{x}_{kj})_{k \in \mathcal{C}}; \varphi_\omega) := Q_{ij}^\omega [p_{ij}^\omega (\mathbf{x}_{ij}^\omega, \mathcal{A}_j [(\mathbf{x}_{kj})_{k \in \mathcal{C}}]) - c_{ij}(\varphi_\omega)] - f_z^\omega (z_{ij}^\omega) .$$

In the simultaneous case, an active firm ω from i chooses optimal quantities and investments in j by

$$\begin{aligned} \frac{\partial \pi_{ij}^\omega (\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial Q_{ij}^\omega} + \mathbb{1}_{(\omega: \mu_j(\{\omega\}) > 0)} \frac{\partial \pi_{ij}^\omega (\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial \mathbb{A}_j} \frac{\partial \mathcal{A}_j [(\mathbf{x}_{kj})_{k \in \mathcal{C}}]}{\partial Q_{ij}^\omega} &= 0. \quad (\text{QTY}) \\ \gamma_\omega^{\text{sim}} (\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega) &:= \frac{\partial \pi_{ij}^\omega (\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial z_{ij}^\omega} + \mathbb{1}_{(\omega: \mu_j(\{\omega\}) > 0)} \frac{\partial \pi_{ij}^\omega (\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial \mathbb{A}_j} \frac{\partial \mathcal{A}_j [(\mathbf{x}_{kj})_{k \in \mathcal{C}}]}{\partial z_{ij}^\omega} = 0. \quad (z\text{-SIM2}) \end{aligned}$$

The rest of equilibrium conditions are the same as those in Bertrand competition. Specifically, accounting for an inverse demand, similar equations to [\(ZCP\)](#), [\(NE\)](#), and [\(FE\)](#) have to hold.

As far as the sequential scenario goes, by similar properties as [Appendix A.1](#), for each $i \in \mathcal{C}$, \mathbb{A}_i^* is the equilibrium aggregate in both scenarios. Let $Q_{ii}^\omega (z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)$ be the optimal quantities chosen by a DL. The problem of a DL ω from i at the first stage is

$$\max_{z_{ii}^\omega} \pi_{ii}^\omega [Q_{ii}^\omega (z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega), z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega] ,$$

and the first-order condition is

$$\gamma_{\omega}^{\text{seq}}(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i; \varphi_{\omega}) := \frac{\partial \pi_{ii}^{\omega}(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i; \varphi_{\omega})}{\partial z_{ii}^{\omega}} - \frac{\partial \pi_{ii}^{\omega}(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i; \varphi_{\omega})}{\partial \mathbb{A}_i} \frac{\partial \mathcal{A}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}]}{\partial Q_{ii}^{\omega}} \frac{\partial Q_{ii}^{\omega}(z_{ii}^{\omega}, \mathbb{A}_i, \varphi_{\omega})}{\partial z_{ii}^{\omega}} = 0. \quad (17)$$

With the characterization of each scenario, I now proceed to add some assumptions that are necessary for the proofs. They are analogous to Assumptions 5a and 5b but defined for quantity elasticities of the inverse demand. I begin by adding some definitions. Let the elasticities of the inverse demand that would hold in monopolistic competition be $\varepsilon_{ii}^{Q,mc}(\mathbf{x}_{ii}^{\omega}; \mathbb{A}_i) := -\frac{\partial \ln p_{ii}^{\omega}(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i)}{\partial \ln Q_{ii}^{\omega}}$ and $\varepsilon_{ii}^{z,mc}(\mathbf{x}_{ii}^{\omega}; \mathbb{A}_i) := \frac{\partial \ln p_{ii}^{\omega}(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i)}{\partial \ln z_{ii}^{\omega}}$. In addition, define the elasticities that incorporate the DL's influence on the aggregate by $\varepsilon_{ii}^Q(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i) := -\frac{\partial \ln p_{ii}^{\omega}[\mathbf{x}_{ii}^{\omega}, \mathcal{A}_i[\cdot]]}{\partial \ln Q_{ii}^{\omega}}$ and $\varepsilon_{ii}^z(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i) := \frac{\partial \ln p_{ii}^{\omega}[\mathbf{x}_{ii}^{\omega}, \mathcal{A}_i[\cdot]]}{\partial \ln z_{ii}^{\omega}}$. When the elasticity is evaluated at the optimal quantities, I use $(z_{ii}^{\omega}, \mathbb{A}_i, \varphi_{\omega})$ as argument of the function. The assumptions are the following.

Assumption Ea. $\frac{\partial \ln Q_{ii}^{\omega}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}{\partial \ln z_{ii}^{\omega}} > \frac{\varepsilon_{ii}^{z,mc}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega}) - \varepsilon_{ii}^z(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}{\varepsilon_{ii}^{Q,mc}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega}) - \varepsilon_{ii}^Q(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}$, where \mathbb{A}_i^* is the equilibrium aggregate.

Assumption Eb. $\frac{\partial \ln Q_{ii}^{\omega}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}{\partial \ln z_{ii}^{\omega}} > \min \left\{ \frac{\varepsilon_{ii}^{z,mc}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega}) - \varepsilon_{ii}^z(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}{\varepsilon_{ii}^{Q,mc}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega}) - \varepsilon_{ii}^P(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}, \frac{\varepsilon_{ii}^{z,mc}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}{\varepsilon_{ii}^{Q,mc}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega}) - 1} \right\}$, where \mathbb{A}_i^* is the equilibrium aggregate.

Notice that, if $\frac{\partial \ln Q_{ii}^{\omega}(\cdot)}{\partial \ln z_{ii}^{\omega}} > 0$, then Ea holds automatically. Next, I proceed to show that through replacing Assumption 5a and Assumption 5b by Assumption Ea and Assumption Eb, then Propositions 5.1 - 5.4 hold under this setup too.

Proof of Proposition 5.1. It follows verbatim the proof under Bertrand competition.

■

Proof of Proposition 5.2. For the proof, a result similar to Lemma A.1 holds. Specifically, for a given \mathbb{A}_i^* , $\frac{\partial \mathcal{A}_i[Q_{ii}^{\omega}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega}), z_{ii}^{\omega}, \mathbf{x}_{ii}^{\omega}]}{\partial z_{ii}^{\omega}} > 0$ holds iff $\frac{\partial \ln Q_{ii}^{\omega}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}{\partial \ln z_{ii}^{\omega}} > \frac{\varepsilon_{ii}^{z,mc}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega}) - \varepsilon_{ii}^z(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}{\varepsilon_{ii}^{P,mc}(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega}) - \varepsilon_{ii}^P(z_{ii}^{\omega}, \mathbb{A}_i^*, \varphi_{\omega})}$. Then, proceeding in the same fashion as for Bertrand competition, it can be shown that overinvestment arises if

$$\Delta_{\omega}^{\text{sim}} := -\frac{\partial \pi_{ii}^{\omega}(\cdot)}{\partial \mathbb{A}_i} \left[\frac{\partial \mathcal{A}_i[(\mathbf{x}_{ki}^{\text{sim}})_{k \in \mathcal{C}}]}{\partial Q_{ii}^{\omega}} \frac{\partial Q_{ii}^{\omega}(\cdot)}{\partial z_{ii}^{\omega}} + \frac{\partial \mathcal{A}_i[(\mathbf{x}_{ki}^{\text{sim}})_{k \in \mathcal{C}}]}{\partial z_{ii}^{\omega}} \right] > 0$$

where $\Delta_{\omega}^{\text{sim}} := \gamma_{\omega}^{\text{seq}}[\mathbf{x}_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*; \varphi_{\omega}] - \gamma_{\omega}^{\text{sim}}[\mathbf{x}_{ii}^{\text{sim}}(\omega), \mathbb{A}_i^*; \varphi_{\omega}]$. Given $\frac{\partial \pi_{ii}^{\omega}(\cdot)}{\partial \mathbb{A}_i} < 0$ and that the term in brackets is $\frac{\partial \mathcal{A}_i[(\mathbf{x}_{ki}^{\text{sim}})_{k \in \mathcal{C}}]}{\partial z_{ii}^{\omega}}$ which is positive by Assumption Ea, the result follows.

■

Proof of Proposition 5.3. It follows verbatim the proof under Bertrand competition with the difference that

$$\begin{aligned} \frac{d \ln R_{ii}^\omega [\cdot]}{d \ln z_{ii}^\omega} &= \frac{\partial \ln p_{ii}^\omega [\cdot]}{\partial \ln z_{ii}^\omega} + \frac{\partial \ln p_{ii}^\omega [\cdot]}{\partial \ln Q_{ii}^\omega} \frac{\partial \ln Q_{ii}^\omega (\cdot)}{\partial \ln z_{ii}^\omega} + \frac{\partial \ln Q_{ii}^\omega (\cdot)}{\partial \ln z_{ii}^\omega} \\ &= \varepsilon_{ii}^{z,mc} (\cdot) - \left[\varepsilon_{ii}^{Q,mc} (\cdot) - 1 \right] \frac{\partial \ln Q_{ii}^\omega (\cdot)}{\partial \ln z_{ii}^\omega}. \end{aligned}$$

Therefore, $\frac{d \ln R_{ii}^\omega [\cdot]}{d \ln z_{ii}^\omega} > 0$ iff $\frac{\partial \ln Q_{ii}^\omega (\cdot)}{\partial \ln z_{ii}^\omega} > \frac{\varepsilon_{ii}^{z,mc} (\cdot)}{\varepsilon_{ii}^{Q,mc} (\cdot) - 1}$ which holds by Assumption Eb. ■

Proof of Proposition 5.4. It follows verbatim the proof under Bertrand competition. ■

E.4 Cost-Reducing Investments

I modify the baseline framework in order to account for sunk investments that reduce marginal costs. The setup of the model is the same as in Section 4 but with some modifications regarding the supply and demand side.

Regarding the former, I suppose that marginal costs of a firm ω located in i to sell in j are $c(z_{ij}^\omega, \varphi_\omega, \tau_{ij})$, with $\frac{\partial c(z_{ij}^\omega, \varphi_\omega, \tau_{ij})}{\partial z_{ij}} < 0$. As for the demand side, the demand function is as in Assumption DEM but where the aggregator and the demand are functions of prices exclusively. Formally, given $\mathbf{p}_{kj} := (p_{kj}^\omega)_{\omega \in \Omega_{kj}}$, the demand of a firm ω from i in j is

$$Q_{ij}^\omega [p_{ij}^\omega, \mathcal{A}_j [(\mathbf{p}_{kj})_{k \in \mathcal{C}}]] ,$$

with aggregator defined by

$$\mathcal{A}_j [(\mathbf{p}_{kj})_{k \in \mathcal{C}}] := H_j \left\{ \sum_{k \in \mathcal{C}} \left[\int_{\omega \in \Omega_{kj}} h_{kj}^\omega (p_{kj}^\omega) d\mu_j (\omega) \right] \right\}.$$

I begin by analyzing the simultaneous solution. Given other firms' strategies, a firm ω from i solves the same optimization problem (1) but where profits from j are

$$\pi_{ij}^\omega (\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega) := Q_{ij}^\omega (p_{ij}^\omega, \mathbb{A}_j) [p_{ij}^\omega - c(z_{ij}^\omega, \varphi_\omega, \tau_{ij})] - f_z^\omega (z_{ij}^\omega).$$

Optimal prices are still characterized by (PRICE) and, irrespective if the firm is a DL or a DNL, the first-order condition for investments in j is given by

$$\gamma_\omega^{\text{sim}} (\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega) := \frac{\partial \pi_{ij}^\omega (\mathbf{x}_{ij}^\omega, \mathbb{A}_j; \varphi_\omega)}{\partial z_{ij}^\omega} = 0. \quad (z\text{-SIM3})$$

The rest of equilibrium conditions are the same as in the case of demand-enhancing investments. Thus, accounting for the corresponding changes in the definitions of the demand and aggregator, (ZCP), (NE), and (FE) have to hold,

As for the sequential scenario, similar properties to [Appendix A.1](#) establish that, for each $i \in \mathcal{C}$, \mathbb{A}_i^* is the equilibrium aggregate in both scenarios. Let the optimal price of a DL ω be $p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)$. The problem of a DL ω from i at the first stage is the same as in the main part of the paper and given by (2). The first-order condition for that problem is

$$\frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial z_{ii}^\omega} - \frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial \mathbb{A}_i} \frac{\partial \mathcal{A}_i[(\mathbf{p}_{ki})_{k \in \mathcal{C}}]}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega)}{\partial z_{ii}^\omega} = 0. \quad (18)$$

The characterization of optimal prices establishes that $\frac{\partial \pi_{ii}^\omega(\cdot)}{\partial p_{ii}^\omega} = -\frac{\partial \pi_{ii}^\omega(\cdot)}{\partial \mathbb{A}_i} \frac{\partial \mathcal{A}_i[\cdot]}{\partial p_{ii}^\omega}$ has to hold in equilibrium. Hence, (18) can be reexpressed as

$$\gamma_\omega^{\text{seq}}(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega) := \frac{\partial \pi_{ii}^\omega[p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega), z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega]}{\partial z_{ii}^\omega} + \frac{\partial \pi_{ii}^\omega[p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega), z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega]}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega(z_{ii}^\omega; \mathbb{A}_i^*, \varphi_\omega)}{\partial z_{ii}^\omega} = 0. \quad (z\text{-SEQ3})$$

With the characterization of the equilibrium conditions for both scenarios, now I proceed to show that [Propositions 5.1 - 5.4](#) hold under this setup too. Remarkably, no additional assumptions such as [5a](#) or [5b](#) are required. At an intuitive level, this follows because cost-reducing investments resemble the case of demand-enhancing investments when they reduce prices. In that case, both assumptions automatically hold.

Proof of [Proposition 5.1](#). It follows verbatim the proof under demand-enhancing investments and Bertrand competition. ■

Proof of [Proposition 5.2](#). It follows verbatim the proof under demand-enhancing investments and Bertrand competition except that, in this case,

$$\Delta_\omega^{\text{sim}} = -\frac{\partial \pi_{ii}^\omega(\mathbf{x}_{ii}^\omega, \mathbb{A}_i; \varphi_\omega)}{\partial \mathbb{A}_i} \frac{\partial \mathcal{A}_i[(\mathbf{x}_{ki})_{k \in \mathcal{C}}]}{\partial p_{ii}^\omega} \frac{\partial p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega)}{\partial z_{ii}^\omega}.$$

The product of the first two terms is positive. So, if I show that the third term is negative, the result follows. I do this by showing that π_{ii}^ω is supermodular in $(p_{ii}^\omega, z_{ii}^\omega)$ when the firm is active (i.e., when p_{ij}^ω is such that $p_{ij}^\omega > c$ and $Q_{ij}^\omega(p_{ij}^\omega, \mathbb{A}_j) > 0$). After some algebra,⁴⁶ we get

$$\frac{\partial^2 \pi_{ii}^\omega}{\partial p_{ii}^\omega \partial z_{ii}^\omega} = \underbrace{Q_{ii}^\omega(p_{ii}^\omega, \mathbb{A}_i) [p_{ii}^\omega - c(z_{ii}^\omega, \varphi_\omega)]}_{+} \underbrace{\frac{\frac{\partial c(z_{ii}^\omega, \varphi_\omega)}{\partial z_{ii}^\omega}}{p_{ii}^\omega - c(z_{ii}^\omega, \varphi_\omega)}}_{-} \underbrace{\left(-\frac{\partial \ln Q_{ii}^\omega(p_{ii}^\omega, \mathbb{A}_i)}{\partial p_{ii}^\omega} \right)}_{+} < 0,$$

so that $\frac{\partial p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega)}{\partial z_{ii}^\omega} < 0$ and the result follows. ■

Proof of [Proposition 5.3](#). It follows verbatim the proof under Bertrand competition

⁴⁶The result follows more easily by reexpressing domestic profits as $\exp\{\ln Q_{ii}^\omega(p_{ii}^\omega, \mathbb{A}_i) + \ln[p_{ii}^\omega - c(z_{ii}^\omega, \varphi_\omega)]\} - f_z^\omega(z_{ii}^\omega)$.

with the difference that

$$\begin{aligned} \frac{d \ln R_{ii}^\omega[\cdot]}{d \ln z_{ii}^\omega} &= \frac{\partial \ln Q_{ii}^\omega[\cdot]}{\partial \ln p_{ii}^\omega} \frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega} + \frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega} \\ &= -[\varepsilon_{ii}^{p,mc}(\cdot) - 1] \frac{\partial \ln p_{ii}^\omega(\cdot)}{\partial \ln z_{ii}^\omega}. \end{aligned}$$

Since $\varepsilon_{ii}^{p,mc}(\cdot) > 1$ in any interior solution for prices, and the fact that $\frac{\partial p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega)}{\partial z_{ii}^\omega} < 0$, then $\frac{d \ln R_{ii}^\omega[\cdot]}{d \ln z_{ii}^\omega} > 0$. ■

Proof of Proposition 5.4. It follows verbatim the proof under demand-enhancing investments and Bertrand competition, with the difference that $\frac{dh_{ii}^\omega[p_{ii}^\omega(z_{ii}^\omega, \mathbb{A}_i, \varphi_\omega), z_{ii}^\omega]}{dz_{ii}^\omega} > 0$ without requiring any additional assumption analogous to 5a or 5b. ■

E.5 Multiproduct Firms

In the baseline model, I have supposed that firms are single product. Here, I consider the case of multiproduct firms.

In the main part of the paper I treat investments as a unidimensional variable. Due to this, investments can be considered as a composite variable comprising any investment that boost firm's own demand. Therefore, they could comprise the introduction of new varieties to the market. If that is the case, $\bar{\Omega}$ would become the set of potential firms, and quantities and prices of each firm would be an average of all its varieties.

Nonetheless, in case we want to explicitly consider multiproduct firms, we can proceed as follows. To keep it simple, I focus on a closed economy. Assume there is a set of firms \mathcal{F} where each firm $f \in \mathcal{F}$ can potentially produce a subset $\bar{\Omega}_f$ of varieties. In this scenario, each firm f makes choices regarding $\mathbf{p}_f := (p_\omega)_{\omega \in \bar{\Omega}_f}$ and $\mathbf{z}_f := (z_\omega)_{\omega \in \bar{\Omega}_f}$. I allow for the possibility of choosing $\mathbf{x}_\omega^f = \bar{\mathbf{x}}$ for a variety ω produced by firm f which represents inaction, so that not all the varieties are necessarily sold in the market. Also, assume that there is a continuum of varieties and denote $\mathbf{x}_\omega^f := (p_\omega, z_\omega)$ when $\omega \in \bar{\Omega}_f$ and $\mathbf{x}_f := (\mathbf{x}_\omega^f)_{\omega \in \bar{\Omega}_f}$. Finally, I define μ as the measure of the firm in the market. In this setup, an aggregator can be defined as

$$\mathcal{A}[(\mathbf{x}_f)_{f \in \mathcal{F}}] := H \left[\int_{f \in \mathcal{F}} \int_{\omega \in \bar{\Omega}_f} h_\omega(\mathbf{x}_\omega) d\omega d\mu(f) \right].$$

By the way in which I define the aggregator, it is quite general and allows for functions h_ω which are either variety or firm specific.

Under this setup, the demand of a variety ω produced by firm f is given by

$$Q_{\omega}(\mathbb{A}, \mathbf{x}_f).$$

Notice that, given the generality that I am allowing for the demand function, the framework allows for nested versions of the CES and Logit as demand, where groups are defined by varieties produced by the same firm.

E.6 Consumer Welfare

The theoretical analysis determines results which depict a situation with greater concentration, more intranational trade, and potentially higher markups. This might raise some concerns regarding whether consumers end up worse off. At a theoretical level, it is not possible to obtain general conclusions in relation to consumer welfare. The fact that the demand system depends on an aggregator that sums up the conditions of the whole market does not necessarily imply that the aggregator is also a sufficient statistic for welfare.⁴⁷ In addition, the general results of my framework have been obtained by considering the demand function as a primitive, without requiring integrability of the demand, i.e., that there exists a utility function from which the demands can be derived.

Nonetheless, in this appendix, I present a demand system that is integrable and depends on an aggregator which is additionally a single sufficient statistic for welfare.⁴⁸ While the assumptions for indirect utility functions to generate this property are somewhat stringent, the demand system encompasses standard cases, such as the augmented version of the CES that I use for my empirical setting and the Multinomial Logit. As I have stated in [Section 6.1](#), a demand with this feature allows us to determine the impact on welfare. Specifically, given that the aggregate's value is the same under the simultaneous and sequential equilibrium, welfare measured at the industry level is the same in both scenarios. Thus, ignoring any redistribution of income, the consumer would derive the same utility in each scenario. If, in addition, it is assumed that the increases of profits garnered by DLs are passed back to the consumer, then she would be actually better off in the sequential scenario.

⁴⁷An example of this is given by the linear demand. In that case, while the demand depends on only one aggregate (the average price of the sector), the indirect utility function depends on two aggregates: the variance and the average prices of the sector.

⁴⁸In a framework with absence of non-price characteristics of products, [Anderson et al. \(2016\)](#) and [Nocke and Schutz \(2018\)](#) provide conditions for obtaining demand systems where the aggregator is a single sufficient statistic for welfare.

To formalize this demand system, consider a representative consumer with income Y . I assume that there are two goods, one homogeneous and another differentiated, with the former having a unitary price. Moreover, the upper tier utility function is quasilinear in the homogeneous good and income is high enough so that there is a positive consumption of both goods. To keep matters simple, I consider a set Ω that comprises all the varieties of the differentiated good and ignore any partition between DNLs and DLs. Each variety $\omega \in \Omega$ is described by its price p_ω and a non-price variable z_ω which is desirable for the consumer. Denote $\mathbf{p} := (p_\omega)_{\omega \in \Omega}$ and $\mathbf{z} := (z_\omega)_{\omega \in \Omega}$.

To have that the aggregator is a sufficient statistic for welfare, we need to define an indirect utility function $V(\mathbf{p}, \mathbf{z}, Y)$ which can be expressed as a function $V(\mathbb{P}, Y)$, where \mathbb{P} is an aggregator for the differentiated sector. The use of \mathbb{P} , instead of \mathbb{A} , is to emphasize that the aggregator has welfare meaning. To accomplish this, define a smooth and monotone strongly separable function $H(\mathbb{P}) := H[\sum_{\omega \in \Omega} h_\omega(p_\omega, z_\omega)]$ such that the indirect utility function is

$$V(\mathbb{P}, Y) := Y + H(\mathbb{P}).$$

By Roy's identity,

$$q(\omega) = -\frac{\partial V(\mathbf{p}, \mathbf{z}, Y) / \partial p_\omega}{\partial V(\mathbf{p}, \mathbf{z}, Y) / \partial Y}.$$

Since $\frac{\partial V(\mathbf{p}, \mathbf{z}, Y)}{\partial Y} = 1$ and, by the strong separability of H , $\frac{\partial V(\mathbf{p}, \mathbf{z}, Y)}{\partial p_\omega} = H'(\mathbb{P}) \frac{\partial \mathbb{P}}{\partial p_\omega}$, and $\frac{\partial \mathbb{P}}{\partial p_\omega} = \frac{\partial h_\omega(p_\omega, z_\omega)}{\partial p_\omega}$, we get

$$q(\omega) = -H'(\mathbb{P}) \frac{\partial h_\omega(p_\omega, z_\omega)}{\partial p_\omega}.$$

And, thus, both the indirect utility function and the demand of each variety ω depends on the same aggregator \mathbb{P} .

To fix ideas, consider the case where H is the logarithmic function, so that⁴⁹

$$V(\mathbb{P}, Y) := Y + \beta \ln \left[\sum_{\omega \in \Omega} h_\omega(p_\omega, z_\omega) \right].$$

In that case, the demand of a variety ω is given by,

$$q(\omega) = \frac{-\frac{\partial h_\omega(p_\omega, z_\omega)}{\partial p_\omega}}{\sum_{\omega \in \Omega} h_\omega(p_\omega, z_\omega)}.$$

This functional form can generate some pervasive cases. In the Multinomial Logit, we have that $h_\omega(p_\omega, z_\omega) := \exp[\alpha + \beta_z z_\omega - \beta_p p_\omega]$ where $\alpha, \beta_z, \beta_p > 0$. In the main text, it is consid-

⁴⁹A demand system that depends only on prices and H is the logarithmic function is analyzed thoroughly by [Nocke and Schutz \(2018\)](#). They show that a demand with those properties can be generated by either a representative-consumer approach or by a discrete-continuous choice model.

ered an augmented CES by defining $h_\omega(p_\omega, z_\omega) := (p_\omega)^{1-\sigma} (z_\omega)^\delta$. Other options are possible, endowing the demand system with some flexibility. For instance, a non-homogeneous augmented version of a CES is possible by defining $h_\omega(p_\omega, z_\omega) := \beta \left[\alpha + (p_\omega)^{1-\sigma} (z_\omega)^\delta \right]^{\frac{1}{\gamma}}$. Also, it can be assumed that h_ω is such that the cross derivatives of the price and non-price variable are zero, $h_\omega(p_\omega, z_\omega) := \beta_p (p_\omega)^{1-\sigma} + \beta_z (z_\omega)^\delta$, which implies that variations in investments do not affect prices in equilibrium.

F Additional Quantitative Results

In this appendix, I present some robustness checks of the empirical results. First, I present the outcomes for all years between 2001 and 2007. After this, I recalculate the results with alternative calculations of domestic sales, where I consider imports of goods that are either produced or exported by the firm as part of its total supply.

The main conclusion that can be derived from all the cases considered is that, concerning the average of the economy, the effects due to strategic behavior are similar to those presented in the main text.

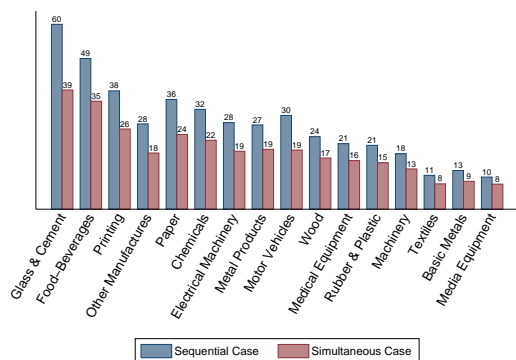
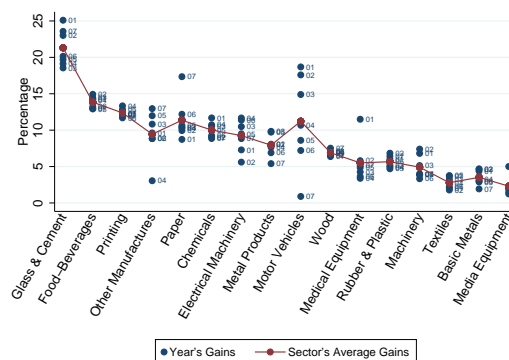
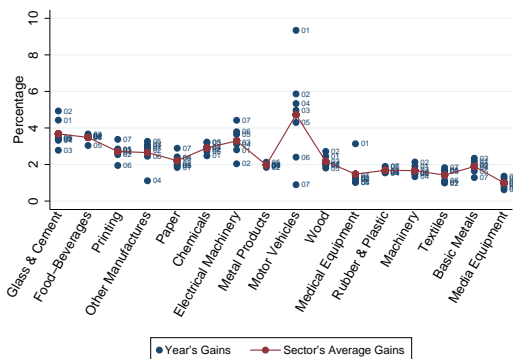
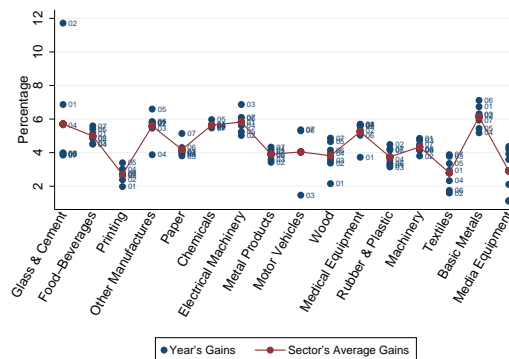
F.1 Years 2001-2007

I begin by recalculating the strategic gains for the years 2001-2007. For each year, I only include industries where DNLs, DLs, and importers coexist by following the same selection criteria as in the baseline case. The main conclusion is that, on average, the empirical outcomes are quite similar, although the gains can moderately vary across years for some industries.

Table 7. *Estimated Impact per Year of the Strategic Behavior - Differences between the Simultaneous and Sequential Scenario*

Year	Avg. Ind.	Avg. Per Firm			Avg. Per Sector	
	Domestic Concentration	Market Share	Domestic Intensity	Domestic Sales	Domestic Intensity	DLs Sales
	Points	Points	Points	Increase	Points	Increase
2001	10.9	3.1	4.4	36.2	6.5	43.4
2002	10.2	3.0	5.0	36.6	6.0	41.6
2003	10.0	3.0	4.7	38.0	7.0	43.3
2004	8.0	2.3	4.3	36.0	6.0	39.1
2005	8.4	2.4	4.6	35.9	6.2	40.9
2006	7.9	2.2	4.5	34.6	6.0	39.9
2007	8.3	2.3	4.8	35.0	6.2	40.8
Average	9.1	2.6	4.6	36.0	6.3	41.3

Note: Only industries with coexistence of DNLs, DLs, and importers considered. Results in terms of points are expressed in percentage-points differences between the sequential and simultaneous scenarios. Results in terms of increases are expressed in percentage increases relative to the simultaneous case. Market shares based on total sales of the industry and account for import competition. Domestic concentration defined as the increases in market shares accrued by all DLs. Domestic intensity defined as firm's domestic sales relative to its own total sales, with only exporters considered for calculations.

Figure 11. *Sequential vs. Simultaneous Cases - Years Average***(a)** Domestic Concentration - Levels**(b)** Domestic Concentration - Average Points Increase by Sector**(c)** Firm's Domestic Market Share - Average Points Increase by Sector**(d)** Firm's Domestic Intensity - Average Points Increase by Sector

Note: All the results compare the sequential and simultaneous scenarios. Only industries with coexistence of DNLs and DLs are considered. Market shares are relative to total industry expenditure and calculated accounting for import competition. Results in terms of points are expressed in percentage-points differences. Results in terms of increases are expressed in percentage variations relative to the simultaneous case. Domestic intensity defined as firm's domestic sales relative to its own total sales and only exporters considered. Entries with "." reflect that in the sector all firms serve only the domestic market.

F.2 Alternative Definitions of Domestic Sales

When a firm is classified as domestic, several definitions can be used to define its domestic sales. The goal is to obtain the value of its total supply in the market. This includes goods produced by itself (locally or abroad) and also goods which are produced by other firms (bought domestically or imported) with the aim of reselling. For the baseline calculations, I used the total turnover reported in the dataset of physical production. This includes sales of own goods (either produced, processed or assembled by the firm), goods produced by a subcontractor established abroad (if the firm owns the inputs of the subcontracted firm), and resales of goods bought from other domestic firms and sold with any processing. The only portion that is not covered is given by goods bought from firms established abroad which the firm does not own.

For this reason, I proceed to recalculate the empirical results by using two measures of sales that incorporate goods imported. Even though I have information on imports by DLs, they are not split into inputs and final goods. As a consequence, I need to take a stance on whether they are part of the total supply of the firm.

In the main part of the paper, by using total turnover, I adopt a conservative position that assumes all of the firm's imports are used as inputs.⁵⁰ Since the information at disposal is at the CN8 product level, this assumption means that if a firm imports a CN8 good and also produces it, this good has been assembled or reprocessed by it and, so, is included in the value of production that it reports. For the two alternatives I propose, I maintain the assumption that a firm's imports which do not belong to its industry are treated as an input. On the other hand, several options exist regarding imports of a good belonging to an industry for which the firm reports positive production.

As a first scenario, I incorporate the import of that CN8 good to the firm's total supply if a firm produces or exports it. In this case, the results are in [Table 8](#) and [Figure 12](#). As a second scenario, I incorporate to the firm's total supply any import of a CN8 product that belongs to its industry. The results are in [Table 9](#) and [Figure 13](#).

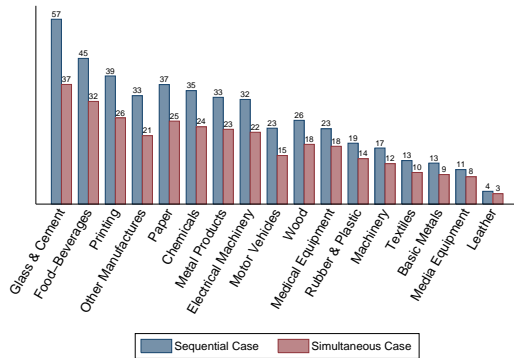
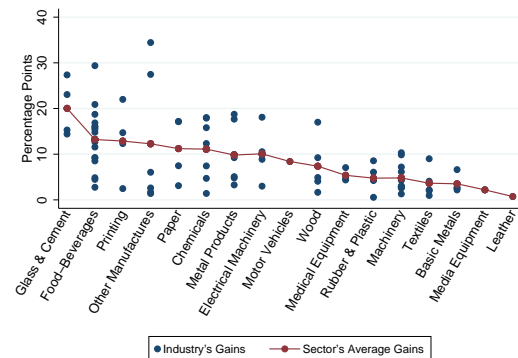
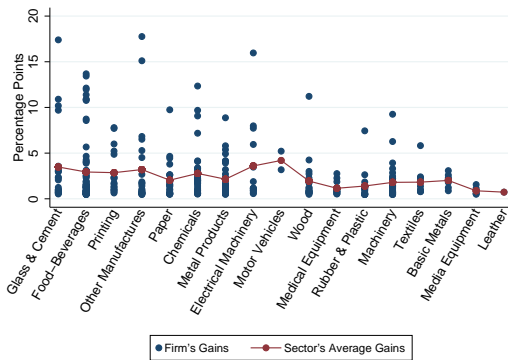
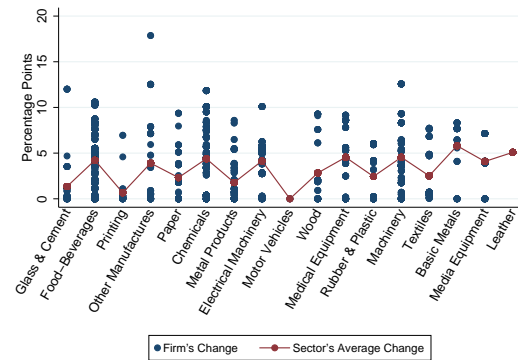
As it can be appreciated from both cases, the results do not differ substantially relative to the baseline outcomes.

⁵⁰In addition, it is a conservative position since, in this way, I do not overestimate the market power of firms which could be reporting low production because they are, essentially, retailers.

Table 8. *Estimated Impact of the Strategic Behavior - Differences between the Simultaneous and Sequential Scenario*

	Avg. Per Firm			Avg. Ind. Domestic Concentration Points	Aggregate Manufacture	
	Market Share Points	Domestic Intensity Points	Domestic Sales Increase		Domestic Intensity Points	DLs Sales Increase
Glass & Cement	3.5	3.7	41.6	20.0	9.5	50.2
Food & Beverages	2.9	4.9	35	13.2	7	41.5
Printing	2.9	3.4	41.8	12.9	1.5	47.0
Other Manufactures	3.2	6.8	40.9	12.3	9.8	52.8
Paper	2.0	3.6	36.2	11.2	3.4	41.8
Chemicals	2.8	5.8	39.7	11.1	10.0	53.0
Metal Products	2.1	3.9	36.1	9.8	2.8	44.3
Electrical Machinery	3.6	5.2	38.3	10.1	6.3	43.3
Motor Vehicles	4.2	.	55.4	8.4	.	56.3
Wood	1.9	5.3	32.6	7.4	5.4	35.5
Medical Equipment	1.2	5.7	28.3	5.4	6.7	30.6
Rubber & Plastic	1.4	4.6	28.4	4.8	5.8	31.1
Machinery	1.8	4.7	32.7	4.8	8.0	38.1
Textiles	1.8	4.2	33.4	3.7	7.1	33.3
Basic Metals	2.0	6.4	37.4	3.5	6.7	39.9
Media Equipment	0.9	5.1	24.8	2.2	5.7	27.2
Leather	0.7	5.1	22.8	0.7	5.1	22.8
Sectors Average	2.3	4.9	35.6	8.3	6.3	40.5

Note: Only industries with coexistence of DNLs, DLs, and importers considered. Results in terms of points expressed in percentage-points differences between the sequential and simultaneous scenarios. Results in terms of increases expressed in percentage increases relative to the simultaneous case. Market shares based on total sales of the industry and account for import competition. At the industry level, domestic concentration defined as the increases in market shares accrued by all DLs. Domestic intensity defined as firm's domestic sales relative to its own total sales, with only exporters considered for calculations. Entries with "." reflect that all the firms in the sector serve the Danish market exclusively. At the manufacture level, domestic intensity and DLs sales measured by taking the total of manufacturing.

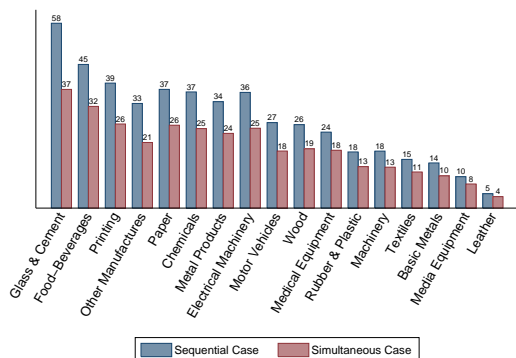
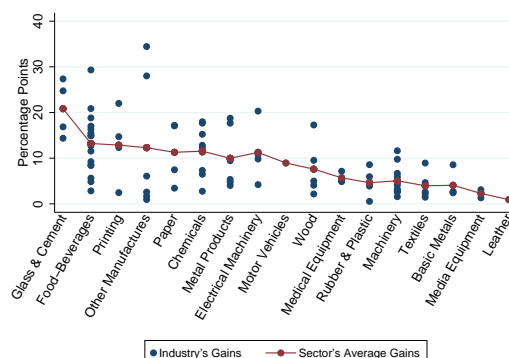
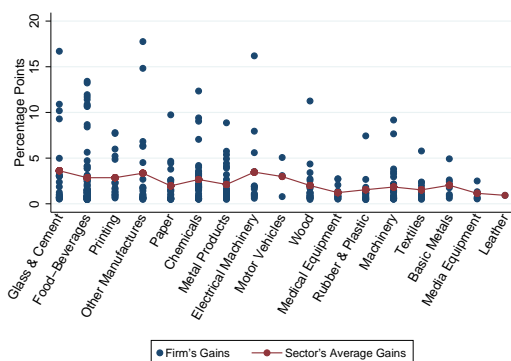
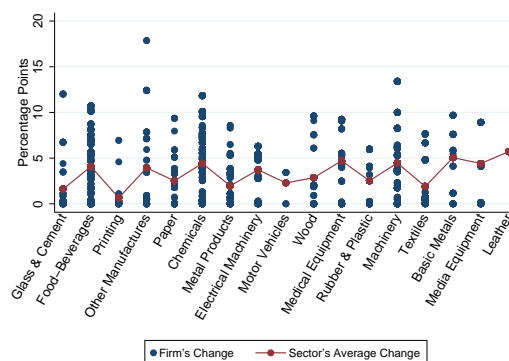
Figure 12. *Sequential vs. Simultaneous Scenarios***(a)** Domestic Concentration - Levels**(b)** Domestic Concentration - Points Increase by Industry**(c)** Firm's Domestic Market Share Gains - Points Increase per Firm**(d)** Firm's Domestic Intensity - Points Increase per Firm

Note: Figure 12a expressed in market-share levels. In the rest of the figures, outcomes are percentage-points differences between the sequential and simultaneous case. Figures with firm's domestic intensity excludes firms that sell exclusively in the domestic market. Market shares measured in terms of total sales value of the industry and account for import competition.

Table 9. *Estimated Impact of the Strategic Behavior - Differences between the Simultaneous and Sequential Scenario*

	Avg. Per Firm			Avg. Ind. Domestic Concentration Points	Aggregate Manufacture	
	Market Share Points	Domestic Intensity Points	Domestic Sales Increase		Domestic Intensity Points	DLs Sales Increase
Glass & Cement	3.6	4.1	43.1	20.8	9.7	51.3
Food & Beverages	2.9	4.9	34.7	13.2	6.9	41.0
Printing	2.9	3.4	41.8	12.9	1.5	47.0
Other Manufactures	3.4	7.0	42.2	12.3	9.8	54.3
Paper	2.0	3.6	35.3	11.3	3.5	40.2
Chemicals	2.7	5.6	38.8	11.5	10.0	52.7
Metal Products	2.1	3.9	35.8	10.0	2.8	44.0
Electrical/Machinery	3.5	4.7	38.3	11.3	8.5	44.6
Motor Vehicles	3.0	3.4	44.5	8.9	1.5	49.8
Wood	2.0	5.3	33.1	7.6	5.4	36.4
Medical Equipment	1.2	5.9	29.0	5.7	6.8	31.4
Rubber & Plastic	1.5	4.6	30.0	4.6	6.2	32.4
Machinery	1.8	4.8	32.8	5.1	8.0	38.3
Textiles	1.5	3.8	30.3	4.0	6.6	31.1
Basic Metals	2.0	5.7	36.4	4.1	6.8	41.2
Media Equipment	1.1	5.8	28.0	2.3	5.8	31.8
Leather	0.9	5.7	25.7	0.9	5.7	25.7
Sectors Average	2.2	4.8	35.3	8.6	6.2	40.8

Note: Only industries with coexistence of DNLs, DLs, and importers considered. Results in terms of points expressed in percentage-points differences between the sequential and simultaneous scenarios. Results in terms of increases expressed in percentage increases relative to the simultaneous case. Market shares based on total sales of the industry and account for import competition. At the industry level, domestic concentration defined as the increases in market shares accrued by all DLs. Domestic intensity defined as firm's domestic sales relative to its own total sales, with only exporters considered for calculations. Entries with "." reflect that all the firms in the sector serve the Danish market exclusively. At the manufacture level, domestic intensity and DLs sales measured by taking the total of manufacturing.

Figure 13. *Sequential vs. Simultaneous Scenarios***(a)** Domestic Concentration - Levels**(b)** Domestic Concentration - Points Increase by Industry**(c)** Firm's Domestic Market Share Gains - Points Increase per Firm**(d)** Firm's Domestic Intensity - Points Increase per Firm

Note: Figure 13a expressed in market-share levels. In the rest of the figures, outcomes are percentage-points differences between the sequential and simultaneous case. Figures with firm's domestic intensity excludes firms that sell exclusively in the domestic market. Market shares measured in terms of total sales value of the industry and account for import competition.