International Trade[1]

Lecture Note 1: Gains of Trade in Neoclassical Models

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[1]The notes are still preliminary and in beta. Please, if you find any typo or mistake, send it to malfaro@ualberta.ca.

Contents

Notation

This is a derivation

This is some comment

This is a comment on advanced topics that are not part of the course (you can ignore it without loss of continuity regarding the text)

- The symbol ":=" means "by definition".
- I denote vectors by bold lowercase letters (for instance, $\mathbf{x}$) and matrices by bold capital letters (for instance, $\mathbf{X}$).
- To differentiate between the verb "maximize" and the operator "maximum", I denote the former with "max" and the latter with "sup" (i.e., supremum). The same caveat applies to "minimize" and "minimum", where I use "min" and "inf", with the latter indicating infimum.
- "iff" means "if and only if"
- $\exp(x)$ is the function e^x.
- Random variables are denoted with a bar below. For instance, $\underline{x}$.

These notes contain hyperlinks in blue and red text. If you are using Adobe Acrobat Reader, you can click on the link and easily navigate back by pressing Alt+Left Arrow.

1 Introduction

The first set of models that we will study belong to the so-called **Neoclassical models**. They are are among the oldest explanations for why countries engage in international trade. All Neoclassical models share a motive to trade based on *differences between countries*, and in particular regarding the supply side. Furthermore, they all predict that countries mutually benefit from trade.

In the following, we outline the common structure of these models. Subsequently, we show that all of them guarantee positive gains of trade in each country. Remarkably, the proof provided applies to all Neoclassical models, regardless of the specific feature in which countries differ.

2 Neoclassical Models

The concept of Comparative Advantages (CAs) is fundamental for Neoclassical models, as they allow us to analyze differences between countries. Following Dixit and Norman (1980), CAs are identified by comparing each country's relative price in autarky.[1] This price plays a pivotal role, as it reflects a country's opportunity cost in a world where perfect competition prevails.

The implications of perfect competition are twofold. First, **goods are homogeneous**. This means that the identity of the country/firm producing the good is irrelevant to consumers: all goods possess identical observable and unobservable features. A corollary of this is that price is the only aspect in which goods could differ and affect a consumer's decision. The second implication is that prices are solely determined by marginal costs.[2] As a result, each country's opportunity costs can be compared to ascertain which country produces a good at its lowest cost.

Neoclassical models highlight the role of imports in a country, giving a specific interpretation for exports in comparison. This view is perfectly summarized by the following two quotes from **Milton Friedman**.

[1]Recall that autarky means a closed economy.

[2]Perfect competition should not be conceived as a realistic assumption, but a way to inquire upon whether trade *might* entail benefits for each country. Reality is always more complex, and there are a lot of frictions that could be preventing potential gains of trade to be realized.

> "*Exports are the cost of trade, imports the return from trade, not the other way around.*"[3]

> "*Let's suppose that everything is cheaper in Japan. The Japanese sellers would be paid for them in dollars. What would they do with the dollars? Nothing for them to buy in the United States. If they would be willing to burn them up or to bury them in the Pacific Ocean, ah, that would be wonderful. After all, there is no product we can produce more cheaply than green pieces of paper.*"[4]

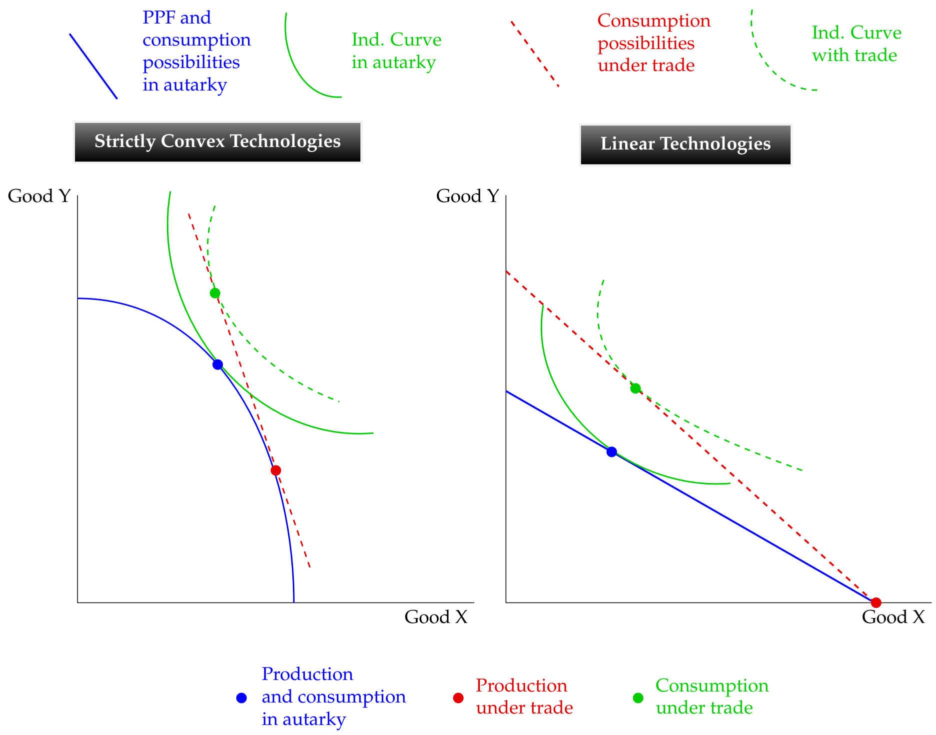
3 Gains From Trade in Neoclassical Models

Showing positive gains of trade in Neoclassical models only requires assuming perfect competition and differences in countries. Other model aspects are irrelevant for this purpose. This includes, for instance, each model's source of CAs (e.g., differences in technology, factor endowments) or the number of goods and countries. These features only become significant if our aim is to derive further predictions, such as patterns of production and trade.

Intuitively, gains of trade expands a country's consumption choices following trade liberalization. Graphically, this can be observed by the expansion of the possibilities of consumption under trade (red dashed line) relative to the consumption possibilities in autarky (blue solid line).

[3] Quote extracted from this article.

[4] Quote from "*Free Trade: Producer vs. Consumer*", lecture delivered at the Alfred M. Landon Lecture at Kansas State University 1978 April 27. The video is available here.



3.1 Intuition using Pareto Optimality

There are two ways to prove gains of trade in each country. The first one is grounded in Pareto optimality, while the other relies on duality. Next, we focus on the former.

In addition to perfect competition, we introduce two assumptions. First, we consider that demands allow for a representative consumer with non-satiated preferences. Second, we suppose that countries cannot run a trade deficit, so that no country finances a foreign country's consumption. Notice we can rule out a trade surplus as an equilibrium outcome, since consumers are non-satiated and therefore always want to consume as much as they can. Overall, the second assumption can be stated as balanced trade in equilibrium, meaning that the value of exports and imports are equal.

According to the Fundamental Theorems of Welfare, any allocation under perfect competition is Pareto efficient. Moreover, this allocation maximizes an agent's utility under balanced trade and subject to the economy's production constraints (i.e., resources and technology). Determining the gains of trade requires comparing a closed economy (autarky) with the same economy under trade (either free or restricted). The proof follows by interpreting autarky as a specific consumption allocation satisfying trade balance, with exports and imports being zero. In other words, autarky is feasible under trade, and hence the optimal allocation under trade cannot make an agent worse by a revealed-preference argument.

3.2 Formal Proof Using Duality

We now provide a formal proof for the existence of gains of trade, based on consumer duality. With this goal, let's keep assuming the existence of one representative agent. Moreover, using superscripts NT and T for "not-trading" and "trading", define the following:

- $\mathbf{q} := (q_1, q_2, ..., q_m)$ the vector of net outputs produced by the firms
- $\mathbf{p} := (p_1, p_2, ..., p_n)$ the vector of goods prices
- $\mathbf{c} := (c_1, c_2, ..., c_n)$ the vector of goods consumptions
- $\mathbf{v}$ the vector of inputs
- y consumer's income
- $e(\mathbf{p}, u)$ the expenditure function to achieve a level of utility u
- $\mathbf{p}^{NT}$ and $\mathbf{p}^{T}$ the vector of prices under autarky and trade, respectively.
- $\mathbf{c}^{NT}$ and $\mathbf{c}^{T}$ the consumption allocations under autarky and trade, respectively.

The proof makes use of an agent's minimum-expenditure function, and the fact

$$
\begin{aligned}
e\left(\mathbf{p}^{T}, u^{NT}\right) &\leq \mathbf{p}^{T} \cdot \mathbf{c}^{NT} \\
&= \mathbf{p}^{T} \cdot \mathbf{q}^{NT} \\
&\leq r\left(\mathbf{p}^{T}, \mathbf{v}\right) \\
&= e\left(\mathbf{p}^{T}, u^{T}\right),
\end{aligned}
$$

which implies that $e\left(\mathbf{p}^{T}, u^{NT}\right) \leq e\left(\mathbf{p}^{T}, u^{T}\right)$. Since e is increasing in the level of utility, it follows that $u^{T} \geq u^{NT}$.

To explain the result, let's inspect the role of each line. By definition, the minimum-expenditure function is the value of the least expensive bundle giving utility u^{NT}. Since $\mathbf{c}^{NT}$ allows the consumer to achieve the utility u^{NT}, the first line stipulates that the expenditure $\mathbf{p}^{T} \cdot \mathbf{c}^{NT}$ is either a solution (in which case the first line holds with equality) or a feasible bundle (since it provides utility u^{NT}). Thus, $\mathbf{p}^{T} \cdot \mathbf{c}^{NT}$ has to be strictly lower by definition of a solution, implying that consumers can derive the same utility as in autarky more economically.

The second line reflects that demand equals supply in equilibrium, so that $\mathbf{q}^{NT} = \mathbf{c}^{NT}$. As for the third line, quantities maximize revenues in perfect competition. Thus, $r\left(\mathbf{p}^{T}, \mathbf{v}\right)$ is higher than any other feasible revenue, in particular of that arising when $\mathbf{q}^{NT}$ is produced. The consequence is that the output value under trade is greater than when autarky quantities are produced.

Finally, the last line stipulates that the revenue under trade equals the country's expenditure, since trade is balanced. Thus, the maximum revenue under trade equals the minimum expenditure to get u^{T}.