International Trade[1]

Lecture Note 4: Successful Firms: Appeal vs Efficiency

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[1]The notes are still preliminary and in beta. Please, if you find any typo or mistake, send it to malfaro@ualberta.ca.

Contents

Notation

This is a derivation

This is some comment

This is a comment on advanced topics that are not part of the course (you can ignore it without loss of continuity regarding the text)

- The symbol ":=" means "by definition".
- Vectors are denoted by bold lowercase letters (for instance, $\mathbf{x}$) and matrices by bold capital letters (for instance, $\mathbf{X}$).
- The set of nonnegative real numbers is denoted by $\mathbb{R}_+ := [0, \infty)$
- The set of of positive real numbers is denoted by $\mathbb{R}_{++} := (0, \infty)$
- The Cartesian product is denoted by $X_1 \times X_2 \times ... \times X_N$. If each set comprises the nonnegative real numbers, we use the notation $\mathbb{R}_+^N$.
- To differentiate between the verb "maximize" and the operator "maximum", I denote the former with "max" and the latter with "sup" (i.e., supremum). The same caveat applies to "minimize" and "minimum", where I use "min" and "inf", with the latter indicating infimum.
- "iff" means "if and only if"
- $\exp(x)$ is the function e^x.
- Random variables are denoted with a bar below. For instance, $\underline{x}$.

These notes contain hyperlinks in blue and red text. If you are using Adobe Acrobat Reader, you can click on the link and easily navigate back by pressing Alt+Left Arrow.

1 Introduction

In this note, we develop a formal framework to identify several types of successful firms we can find in practice. Briefly, the sources of success can be due to demand or supply aspects: a firm could supply a good with high appeal for consumers (either because of quality or other features) or be highly efficient (thereby achieving low prices).

The purpose of this note is to provide a framework for understanding empirical studies/ This includes in particular those analyzing the characteristics of successful firms, including the so-called "superstar firms." One key feature being studied is whether their size can be explained by demand or supply factors.

Given that our aim is understanding how firms make pricing decisions, we keep the model as simple as possible. In particular, we analyze one industry in isolation with only one operating firm. How restrictive is this assumption for the analysis? We usually tend to associate monopoly with the existence of one firm in the market. However, this is quite a narrow definition.[1] A **monopoly** is any firm that is able to increase its price without losing its demand completely. Technically, we say that a monopoly does not face an infinitely elastic demand. Therefore, we can refer to a firm as a monopoly even when there are numerous firms in the industry. A typical example is Apple in the cell phones industry.

Despite the simplicity of the model, the insights presented are useful for the behavior of firms in more complex market structures. This includes the two models of imperfect competition that we will study later in the course, namely monopolistic competition and oligopoly with differentiated goods.

1.1 Setup

We focus on an industry with a single firm supplying one good. This firm has constant marginal costs c and no fixed costs. Formally, its cost function is $C(q) := cq$, where q is the quantity produced by the firm.

[1] The existence of only one firm is, in fact, not sufficient to have a monopoly. If barriers to entry are really low, the tacit competition a firm faces could determine that the industry is competitive. Same caveat applies to the case of an oligopoly with a small number of firms. For instance, in the case of Bertrand competition with homogeneous products and same marginal costs, even if there are two firms, a competitive outcome would emerge. In the literature, the Contestability Theory shows that the number of firms does not necessarily predict how tough the competitive environment is.

The total demand for the good is a function $q(p;\alpha) \in \mathcal{C}^2$ (i.e., twice continuously differentiable) where $p \in [\underline{p}, \overline{p}]$ is the good's price and $\alpha \in \mathbb{R}_+$ a parameter. We assume that $\frac{\partial q(p;\alpha)}{\partial p} < 0$. and suppose that the features of the good are exogenously given.

We refer to α as the **good's appeal**. This parameter captures that consumers usually make consumption decisions looking not only at the price, but also other non-price aspects of the good (e.g., quality, after-sale services). Below, we will state assumptions consistent with this interpretation for α (for instance, that a greater α increases the demand). Notice we are defining α as a real number rather than a vector. This is a simplification indicating that all the aspects of a good are encompassed through a single measure α. This has the goal of keeping the model parsimonious. Alternatively, we could break down each of the tangible and intangible characteristics of the good into several parameters. For our purposes, this is not necessary.

1.2 Basic Assumptions

In this part, we state some basic assumptions about the relation of the quantity demanded with p and with α. We will do it in terms of elasticities. Later, we will add some additional assumptions to have unambiguous comparative statics.

Regarding price, we have already stated that $\frac{\partial q(p;\alpha)}{\partial p} < 0$, and so $\varepsilon_p(p;\alpha) > 0$ for all $(p;\alpha)$. The price elasticity provides information about how sensitive the quantity demanded is when the price increases. This provides valuable information, since it quantifies a crucial trade off that any firm faces: higher prices increase revenue ceteris paribus, but it also triggers decreases in quantities that lower its revenues.

Some terminology is in order. We say that the **demand is inelastic** when $\varepsilon_p(p;\alpha) < 1$, while $\varepsilon_p(p;\alpha) > 1$ defines an **elastic demand**. For the baseline model, we assume that $\varepsilon_p(p;\alpha) > 1$ for all $(p;\alpha)$. As a corollary, we rule out demands that are inelastic for all levels of prices.[2]

We have referred to α as the appeal of the good. But we have to actually state conditions that capture this to give meaning to the label. There are two salient features that demand appeal should fulfill to receive such denomination. First, it ought to determine

[2] This is an assumption that is stronger than what we need. We could allow for demands that are price inelastic for some combinations of prices.

that **a higher appeal results a greater quantity demanded**. This is captured by the assumption $\frac{\partial q(p;\alpha)}{\partial \alpha} > 0$, or $\varepsilon_\alpha := \frac{\partial \ln q(p;\alpha)}{\partial \ln \alpha} > 0$ in terms of elasticities. It states that, keeping the prices fixed, greater values of α increase the quantity demanded. Notice we remain agnostic about the channel through which appeal increases the demand. This could take place either because old consumers demand more quantities or because the firm starts selling to new customers.

The second feature of appeal is given by its relation with price. We could imagine that a more attractive product not only increases the total quantities sold, but that it also affects how sensitive consumers are to higher prices. For instance, consider you want to buy the latest iPhone, which is faster, has more RAM, and better resolution. It is reasonable to assume that you will be less likely to buy the new iPhone if Apple charges a higher price relative to its older version, but less so relative to an iPhone without improved features.

This aspect is reflected into the model by assuming that a greater α is associated with a lower $\varepsilon_p(p;\alpha)$. Expressed in words, **when a good has more appeal, the demand becomes less price elastic** (equivalently, more price inelastic). Formally, $\frac{\partial \varepsilon_p(p;\alpha)}{\partial \alpha} \leq 0$. Notice we allow for the possibility that it does not affect the price at all. This could occur if greater appeal captures a firm improving its distribution channels, and in this way it reaches consumers with the same valuation as the old ones. In that case, the quantities demanded would increase, but the sensitivity of consumers to prices would not be affected, since the pool of new consumers is the same as the original consumer base.

Assumption 1.1. *Summing up, we assume:*

- $\varepsilon_p > 1$
- $\varepsilon_\alpha(p;\alpha) > 0$ *and* $\frac{\partial \varepsilon_p(p;\alpha)}{\partial \alpha} \leq 0$.

We will actually need some more assumptions in relation with $\frac{\partial \varepsilon_p(p;\alpha)}{\partial p}$. They will be added later, since they require further results and explanations.

1.3 The Optimization Problem

The optimization problem of the monopoly consists in choosing the price of its good with the goal of maximizing profits. Although the firm also has to choose the quantity supplied, this is completely determined by the demand function $q(p;\alpha)$ once the prices have been chosen. In other words, the firm chooses the price, and then quantities are determined by the fact that supply equals demand in equilibrium.

Remark *The firm could alternatively choose quantities, and let the price be determined by the condition that supply equals demand. This requires making use of the inverse demand $p(q;\alpha)$ when the firm optimizes, rather than the direct demand $q(p;\alpha)$. However, apart from this detail,* **both optimization problems provide exactly the same result**. *It is only when strategic interactions are incorporated that we need to distinguish between both cases. This is relevant if we analyze oligopolies, giving rise to the Cournot and Bertrand models.*

Formally, the optimization problem is

$$\max_{p\in[\underline{p},\overline{p}]} \pi(p;\alpha,c) := q(p;\alpha)(p-c)$$

There are different conditions guaranteeing that the problem is well-defined, i.e., that a solution exists, is unique, and interior. Nonetheless, we will only state conditions that are necessary to get unambiguous results when we perform comparative statics. On the contrary, we proceed by assuming that a solution exists, is unique, and interior.

In recent decades, comparative statics has had some revival. One message of this new literature is that a lot of assumptions usually made to perform comparative statics are in fact not necessary.[3] For this reason, it has been common since then to only establish a minimal set of assumptions to get unambiguous comparative statics results. The goal is to make a clear distinction between assumptions made to have a well-behaved problem and those related to a comparative–static analysis.

Actually, the conditions for existence, uniqueness and no boundaries solutions for the problem at hand can be characterized in a relative simple way. Since the price domain is compact and π is continuous, an optimal price exists. Moreover, by using Inada conditions and assuming that profits are strictly quasiconcave, then a solution is interior and unique.

However, these are only sufficient conditions. For instance, it is not uncommon to have profits that are not strictly quasiconcave, but the solution is unique anyway. Furthermore, sometimes we are able to predict how a parameter affects a solution, even if the solution is not unique.

[3] For instance, it has been shown that we can predict how a parameter affects a solution, even when the solution is not unique.

As you can perceive from these remarks, there are a lot of details to take into account. However, a lot of them are not related or are necessary to provide an answer to what makes a firm successful. This is in last instance what we are interested in.

We characterize the solution by the first-order condition:

$$p^* = \frac{\varepsilon_p (p^*; \alpha)}{\varepsilon_p (p^*; \alpha) - 1} c. \tag{PRICE}$$

The first-order condition is
$\frac{\mathrm{d}\pi}{\mathrm{d}p} = \frac{\partial Q(p;\alpha)}{\partial p} (p-c) + Q(p,\alpha) = 0 \Rightarrow \frac{\partial Q(p;\alpha)}{\partial p} \frac{1}{Q(p,\alpha)} = -\frac{1}{(p-c)}$
By multiplying both sides by p, then $-\frac{\partial Q(p;\alpha)}{\partial p} \frac{p}{Q(p,\alpha)} = \frac{p}{(p-c)}$.
Since $\varepsilon(p;\alpha) := -\frac{\partial Q(p;\alpha)}{\partial p} \frac{p}{Q(p,\alpha)}$ then $\varepsilon(p;\alpha) = \frac{p}{(p-c)}$ which by working it out becomes $p = \frac{\varepsilon(p;\alpha)}{\varepsilon(p;\alpha)-1} c$.

Since we have assumed that $\varepsilon_p (p; \alpha) > 1$ for all $(p; \alpha)$, then (PRICE) determines that $p^* > c$. Notice that (PRICE) provides only an implicit characterization for optimal prices p^*. This is because the price elasticity also depends on prices. We denote the implicit value p^* that satisfies (PRICE) by $p^* (\alpha, c)$.

Once that optimal prices are pinned down, optimal profits are

$$\pi^* (\alpha, c) := Q [p^* (\alpha, c), \alpha] [p^* (\alpha, c) - c].$$

1.4 A Digression: The Inelastic Demand Case (OPTIONAL)

There is a still a solution if $\varepsilon (p; \alpha) < 1$ for any p. However, this is not given by (PRICE), and the solution would actually lie on the boundary. This case is particularly interesting since it rationalizes why markets with an inelastic good could require regulations from the governments.

A demand is inelastic when a firm can increases the price of its good, without consumers decreasing their consumption much. For instance, consider the extreme case where $\varepsilon (p; \alpha) = 0$. Then, the consumers would keep demanding the same quantity, no matter the price set by a firm. Some examples of inelastic goods are necessity goods (electricity, water) and critical medicine (v.gr. oncological medication, insulin). As for the latter, competition could keep prices low, despite its inelastic nature. On the contrary, public utilities tend to be supplied by only one firm, given the infrastructure required. In this context, a natural monopoly could arise, and hence a firm would charge a price as high as possible.

Let's first provide some intuition for the solution. Then we show formally that increases in price make the firm garner higher profits. Firm's profit is given by the difference between revenues $R(p;\alpha) := pq(p;\alpha)$ and costs $C(p;\alpha) := cq(p;\alpha)$. Revenues can be increased by either increasing the price or the quantities. If the firm chooses to do by increasing its prices, it faces the issue that quantities demanded become lower. Nonetheless, when demand is inelastic, quantities demanded decrease in less than 1% following an increase in 1% of prices. Thus, overall, increases in prices end up determining higher revenues. At the same time, the decrease in quantities reduces costs: since the firm has to produce less, the cost $C(p;\alpha)$ becomes lower. All this provides the intuition that the **best strategy for a monopoly when it sells an inelastic product is charging a price as high as possible**.

Let's now show this formally. First, equation (PRICE) cannot be a maximum. In fact, that solution determines the minimum profit. which you can see since (PRICE) is giving a solution entailing prices that are lower than costs. Thus, profits would be necessarily negative: every time the firm sells one unit, it would get less revenue than the cost of production. In summary, the first-order condition does not provide us with a solution. Since the first-order condition is a necessary condition for an interior solution and it can be shown that a solution exists, the solution must lie at the boundary.

How can we detect a corner solution? One clue is determining whether an increase or decrease of the price entails greater profits for any p. Formally, this requires showing that the first derivative is monotone. When this is the case, the optimal choice would require increasing or decreasing the choice variable as much as possible.

To use the fact that $\varepsilon_p(p;\alpha) < 1$, it will be easier to analyze the behavior of $\frac{\partial \ln \pi(p,\alpha)}{\partial \ln p}$. Notice this is justified because $\frac{\partial \ln \pi(p,\alpha)}{\partial \ln p} > 0$ iff $\frac{\partial \pi(p,\alpha)}{\partial p} > 0$, since profits and prices are positive variables and $\frac{\partial \ln \pi(p,\alpha)}{\partial \ln p} = \frac{\partial \pi(p,\alpha)}{\partial p}\frac{p}{\pi(p,\alpha)}$.

We will show that $\frac{\partial \ln \pi(p,\alpha)}{\partial \ln p} > 0$ for any $p > c$. Thus, the proof requires that the inequality holds irrespective of the price we are evaluating the derivative. Applying logs to the profit function, $\ln \pi(p;\alpha) = \ln Q(p;\alpha) + \ln(p-c)$, and

$$\frac{\partial \ln \pi(p,\alpha)}{\partial \ln p} = \underbrace{\frac{\partial \ln Q(p,\alpha)}{\partial \ln p}}_{=-\varepsilon_p(p;\alpha)} + \frac{\partial \ln(p-c)}{\partial \ln p}.$$

Notice $\frac{\partial \ln(p-c)}{\partial \ln p} = \frac{\partial \ln(p-c)}{\partial p} p$ because $\mathrm{d} \ln p = \frac{1}{p} \mathrm{d}p$. Then $\frac{\partial \ln(p-c)}{\partial \ln p} = \frac{p}{p-c}$, and so

$$\frac{\partial \ln \pi (p, \alpha)}{\partial \ln p} = -\underbrace{\varepsilon_p (p; \alpha)}_{<1} + \underbrace{\frac{p}{p-c}}_{>1},$$

$$> 0.$$

The sign of each term follows because $\varepsilon_p (p; \alpha) < 1$ by assumption, and firm always sells its good to a price $p > c$ and hence $\frac{p}{p-c} > 1$. Since $\frac{\partial \ln \pi(p,\alpha)}{\partial \ln p} > 0$ for any $p > c$, then $p^* (\alpha, c) := \overline{p}$ for any (α, c) if $p \in \left[\underline{p}, \overline{p}\right]$. In the limit, if $\overline{p}$ is a big number, the firm ends up selling a quantity close to zero.

1.5 About Markups

Let's go back to the baseline case where $\varepsilon_p (p; \alpha) > 1$ for any $(p; \alpha)$. The optimal price is implicitly characterized by equation (PRICE).

We introduce an important concept for the analysis, which is known as the **markup**. This is denoted by μ and defined by

$$\mu (p; \alpha) := \frac{\varepsilon_p (p; \alpha)}{\varepsilon_p (p; \alpha) - 1}.$$

The name follows because when μ is evaluated at the price p^*, we can reexpress equation (PRICE) by:

$$p^* = \mu (p^*; \alpha) c \tag{PRICE-1}$$

implying that $\frac{p^*}{c} = \mu (p^*; \alpha)$.

Markups give information about the revenue got y a firm from selling one unit of the good (i.e. p) relative to the cost of producing that unit (i.e. c). Put it simple, it provides a ratio of revenues over costs per unit sold.

A first conclusion we can obtain by using markups is regarding consumer welfare. In perfect competition, prices equal marginal costs, and so $\mu = \frac{p}{c} = 1$. For this reason, markups tend to be used to measure the monopoly power a firm has relative to perfect competition. This also reveals why consumers do not benefit from a monopoly relative to perfect competition: a firm prefers to restrict the quantities to both benefit from higher prices and avoid production costs. Furthermore, the markup provides information about the extent at which this strategy is pursued.

2 Comparative Statics (CS)

CS identifies how a model's endogenous variables are affected by changes in its parameters. In our model, the endogenous variable used as choice decision is price, and there are two parameters, c and α. Once we determine how prices are affected by these parameters, we can also inquire upon the effect on other endogenous variables such as quantities and markups.

For now, let's focus on how α and c affect the optimal price $p^*(\alpha, c)$. We will perform a CS analysis by varying one parameter at a time. Our goal is to establish the signs of $\frac{\partial p^*(\alpha,c)}{\partial c}$ and $\frac{\partial p^*(\alpha,c)}{\partial \alpha}$.

2.1 Some Additional Assumptions

Before doing a CS analysis, we need to add some assumptions to obtain definite results. These assumptions are related to the impact of prices on the price elasticity and on markups.

We begin by showing that the sign of the effect of prices on the price elasticity coincides with the negative effect of prices on markups. To observe this, we know that $\mu(p;\alpha) := \frac{\varepsilon_p(p;\alpha)}{\varepsilon_p(p;\alpha)-1}$. Taking μ as a function of ε_p, the relation between both terms is:

$$\frac{\partial \mu(\varepsilon_p)}{\partial \varepsilon_p} = \frac{-1}{(\varepsilon_p - 1)^2}.$$

This indicates that if the price elasticity decreases then the markup increases. As a corollary, if any parameter or variable decreases the price elasticity, the markup will increase. Overall, markups are determined by whether the parameter or the variable makes the demand more inelastic (higher markup) or more elastic (lower markups). As a result, the assumptions we make about how ε_p is impacted when p or α varies determine completely how markups are affected.

So far, we have only supposed that $\frac{\partial \varepsilon_p(p;\alpha)}{\partial \alpha} \leq 0$. Therefore,

$$\frac{\partial \mu(p^*;\alpha)}{\partial \alpha} = \frac{-1}{(\varepsilon_p - 1)^2} \frac{\partial \varepsilon_p(p^*;\alpha)}{\partial \alpha} \geq 0.$$

Thus, **increases in appeal determine a higher markup,** because it makes demand more inelastic.

Now, let's consider how variations in prices affect the price elasticity and, hence, markups. Formally,

$$\frac{\partial \mu\left(p^{*};\alpha\right)}{\partial p}=\frac{-1}{\left(\varepsilon_{p}-1\right)^{2}}\frac{\partial \varepsilon_{p}\left(p^{*};\alpha\right)}{\partial p},$$

determining that there is a negative relation between $\frac{\partial \mu(p^{*};\alpha)}{\partial p}$ and $\frac{\partial \varepsilon_{p}(p^{*};\alpha)}{\partial p}$. It is not obvious what the sign $\frac{\partial \varepsilon_{p}(p^{*};\alpha)}{\partial p}$ should have. Consistent with the results we want to get below, suppose that

$$\frac{\partial \varepsilon_{p}\left(p^{*};\alpha\right)}{\partial p}<0,$$

which implies that

$$\frac{\partial \mu\left(p^{*};\alpha\right)}{\partial p}>0.$$

Thus, **firms charging a higher price will set a higher markup**. Equivalently, firms charging a lower price set a lower markup. One way to justify this is to think about income-constrained consumers. Presumably, richer people are less sensitive to increases in prices. Following an increase in price, poor people might not afford the good, making rich customers be the only relevant portion of the demand. In this scenario, the price elasticity of the aggregate demand would be lower and determine that increases in price allow the firm to raise its markup.

The second assumption we make is that, even though $\frac{\partial \mu(p^{*};\alpha)}{\partial p}>0$, the effect is such that:

$$1-\frac{\partial \ln \mu\left(p;\alpha\right)}{\partial \ln p}>0 \tag{1}$$

What is the justification for Assumption (1)? It is necessary to get a specific characterization of firms we are interested in. However, it could also be justified in other grounds: Assumption (1) at the optimal price p^{*} is necessary to ensure both the second-order condition and uniqueness of the equilibrium.

Summary of the Asssumptions

- $\varepsilon_{p}\left(p;\alpha\right)>1$ for any $\left(p;\alpha\right)$ (elastic demand at any point)

- α is appeal:
  - $\varepsilon_\alpha(p;\alpha) > 0$ (increases of α boost demand)
  - $\frac{\partial \varepsilon_p(p;\alpha)}{\partial \alpha} \leq 0$ (greater α makes demand more inelastic/less elastic)
- $\frac{\partial \varepsilon_p(p^*;\alpha)}{\partial p} < 0$ which implies $\frac{\partial \ln \mu(p;\alpha)}{\partial \ln p} > 0$ (for definite CS)
- $1 - \frac{\partial \ln \mu(p;\alpha)}{\partial \ln p} > 0$ (for definite CS)

2.2 Variations in c

Let's first consider the case where the parameter of interest is c. Formally, $\mathrm{d}c \neq 0$ and $\mathrm{d}\alpha = 0$. Allowing for all the endogenous variable to react to this change, we can differentiate equation (PRICE-1) for $\mathrm{d}p^* \neq 0$ and $\mathrm{d}c \neq 0$ and obtain:

$$\frac{\partial p^*(\alpha, c)}{\partial c} = \frac{\mu(p^*;\alpha)}{1 - \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}} > 0 \tag{PRICE-c}$$

where we have used Assumption (1) to determine that $\frac{\partial p^*(\alpha,c)}{\partial c} > 0$. From this we conclude that **more efficient firms (lower c) charge lower prices.**

The FOC is $p^* = \mu(p^*;\alpha)c$ and differentiating it with $\mathrm{d}p^* \neq 0$ and $\mathrm{d}c \neq 0$:

$$\left[1 - \frac{\partial \mu(p^*;\alpha)}{\partial p} c\right] \mathrm{d}p^* = \mu(p^*;\alpha)\,\mathrm{d}c$$

which implies that $\frac{\partial p^*(\alpha,c)}{\partial c} = \frac{\mu(p^*;\alpha)}{1 - \frac{\partial \mu(p^*;\alpha)}{\partial p} c}$.

I want to show that $\frac{\partial \mu(p^*;\alpha)}{\partial p} c = \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}$. Starting from $\frac{\partial \mu(p^*;\alpha)}{\partial p} c$, we know by equation (PRICE-1) that $p^* = \mu(p^*;\alpha)c$ and so we can substitute c for $\frac{p^*}{\mu(p^*;\alpha)}$, implying that $\frac{\partial \mu(p^*;\alpha)}{\partial p} c = \frac{\partial \mu(p^*;\alpha)}{\partial p} \frac{p^*}{\mu(p^*;\alpha)}$. Then, since $\frac{\partial \mu(p^*;\alpha)}{\partial p} \frac{p^*}{\mu(p^*;\alpha)} = \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}$, we have that $\frac{\partial p^*(\alpha,c)}{\partial c} = \frac{\mu(p^*;\alpha)}{1 - \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}}$. Notice this is positive since Assumption (1) is $1 - \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p} > 0$.

Once we have determined the effect of variations in c on prices, we can determine the effect of c on quantities and markups. Optimal quantities are given by $q^*[p(\alpha,c);\alpha]$. Thus,

$$\frac{\mathrm{d}q^*[p^*(\alpha,c);\alpha]}{\mathrm{d}c} = \underbrace{\frac{\partial q(p^*;\alpha)}{\partial p}}_{-} \underbrace{\frac{\partial p^*(\alpha,c)}{\partial c}}_{+} < 0 \tag{QUANT-c}$$

The result is intuitive. **Less efficient firms (firms with greater marginal costs) set a higher price, and sell less as a consequence**.

As far as markups go, the optimal value is given by $\mu\left[p^*(\alpha, c);\alpha\right]$ and so

$$\frac{\mathrm{d}\mu^*\left[p^*(\alpha, c);\alpha\right]}{\mathrm{d}c} = \underbrace{\frac{\partial\mu(p^*;\alpha)}{\partial p}}_{+}\underbrace{\frac{\partial p^*(\alpha, c)}{\partial c}}_{+} > 0 \tag{MK-c}$$

Thus, **more efficient firms charge a lower markup**.

Overall, we have determined **that less productive firms charge higher prices, sell less, and charge higher markups**. Equivalently, **more productive firms charge lower prices, sell more quantity, and charge lower markups**.

2.3 Variations in α

Let's consider variations in α. Differentiating equation (PRICE-1) for $\mathrm{d}p^* \neq 0$ and $\mathrm{d}\alpha \neq 0$, we obtain

$$\frac{\partial \ln p^*(\alpha, c)}{\partial \ln \alpha} = \frac{\frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln \alpha}}{1 - \frac{\partial \ln \mu(p^*;\alpha)}{\partial \ln p}} \geq 0 \tag{PRICE-α}$$

Since $p^*, \alpha > 0$, we have that $\frac{\partial \ln p^*(\alpha,c)}{\partial \ln \alpha} \geq 0$ iff $\frac{\partial p^*(\alpha,c)}{\partial \alpha} \geq 0$. Thus, a greater appeal makes the firm charge a higher (or the same) price .

The FOC is $p^* = \mu(p^*;\alpha)\,c$ and differentiating it where $\mathrm{d}p^* \neq 0$ and $\mathrm{d}\alpha \neq 0$:

$$\left[1 - \frac{\partial\mu(p^*;\alpha)}{\partial p}c\right]\mathrm{d}p^* = \frac{\partial\mu(p^*;\alpha)}{\partial\alpha}c\,\mathrm{d}\alpha$$

which implies that $\frac{\partial p^*(\alpha,c)}{\partial\alpha} = \frac{\frac{\partial\mu(p^*;\alpha)}{\partial\alpha}c}{1-\frac{\partial\mu(p^*;\alpha)}{\partial p}c}$. Regarding the denominator, we have already shown in the derivation of (PRICE-c) that $\frac{\partial\mu(p^*;\alpha)}{\partial p}c = \frac{\partial \ln\mu(p^*;\alpha)}{\partial \ln p}$. Concerning the numerator, using that $c = \frac{p^*}{\mu(p^*;\alpha)}$, then $\frac{\partial\mu(p^*;\alpha)}{\partial\alpha}c = \frac{\partial\mu(p^*;\alpha)}{\partial\alpha}\frac{p^*}{\mu(p^*;\alpha)}$ which equals $\frac{\partial \ln\mu(p^*;\alpha)}{\partial\alpha}p^*$.
With all these results, we have shown that

$$\frac{\partial p^*(\alpha, c)}{\partial\alpha} = \frac{\frac{\partial \ln\mu(p^*;\alpha)}{\partial\alpha}p^*}{1 - \frac{\partial \ln\mu(p^*;\alpha)}{\partial \ln p}}$$

Multiplying both sides by α, then $\frac{\partial p^*(\alpha,c)}{\partial\alpha}\alpha = \frac{\frac{\partial \ln\mu(p^*;\alpha)}{\partial\alpha}\alpha p^*}{1-\frac{\partial \ln\mu(p^*;\alpha)}{\partial \ln p}}$. Dividing both sides by p^* $\frac{\partial p^*(\alpha,c)}{\partial\alpha}\frac{\alpha}{p^*} = \frac{\frac{\partial \ln\mu(p^*;\alpha)}{\partial\alpha}\alpha}{1-\frac{\partial \ln\mu(p^*;\alpha)}{\partial \ln p}}$.
Since $\frac{\partial \ln\mu(p^*;\alpha)}{\partial\alpha}\alpha = \frac{\partial \ln\mu(p^*;\alpha)}{\partial \ln\alpha}$ and $\frac{\partial p^*(\alpha,c)}{\partial\alpha}\frac{\alpha}{p^*} = \frac{\partial \ln p^*(\alpha,c)}{\partial \ln\alpha}$, the result follows.

To understand why prices are increasing in appeal, keep in mind that a firm faces a more inelastic demand when it sells a product with higher appeal. Hence, the firm might increase its price, without the quantities sold being heavily affected.

Regarding optimal quantities $q\left[p^*\left(\alpha,c\right);\alpha\right]$:

$$\frac{\mathrm{d}q\left[p^*\left(\alpha,c\right);\alpha\right]}{\mathrm{d}\alpha}=\underbrace{\frac{\partial q\left(p^*;\alpha\right)}{\partial\alpha}}_{+}+\underbrace{\frac{\partial q\left(p^*;\alpha\right)}{\partial p}}_{-}\underbrace{\frac{\partial p\left(\alpha,c\right)}{\partial\alpha}}_{+\text{ or }0}\gtreqless 0 \tag{QUANT-α}$$

and so the effect is ambiguous.

The intuition behind is the following. When there is increase in the appeal of the good, there are two effects working simultaneously. First, there is a direct effect, where the mere fact of selling a product with more appeal increases the demand for the good. However, there is also an indirect effect if appeal turns the demand more inelastic: the firm would have incentives to increase its price, thus reducing its demand. Overall, depending on which effect dominates, total demand can increase or decrease. Notice that if appeal does not affect price elasticity, then the quantity demanded would be necessarily greater. For future references, we define the two possibilities.

$$\text{Case I of (QUANT-}\alpha\text{): }\frac{\mathrm{d}q\left[p^*\left(\alpha,c\right);\alpha\right]}{\mathrm{d}\alpha}>0$$
$$\text{Case II of (QUANT-}\alpha\text{): }\frac{\mathrm{d}q\left[p^*\left(\alpha,c\right);\alpha\right]}{\mathrm{d}\alpha}<0$$

Concerning the effects on markups:

$$\frac{\mathrm{d}\mu\left[p^*\left(\alpha,c\right);\alpha\right]}{\mathrm{d}\alpha}=\underbrace{\frac{\partial \mu\left(p^*;\alpha\right)}{\partial\alpha}}_{+}+\underbrace{\frac{\partial \mu\left(p^*;\alpha\right)}{\partial p}}_{+}\underbrace{\frac{\partial p^*\left(\alpha,c\right)}{\partial\alpha}}_{+}>0 \tag{MK $-$ α}$$

Intuitively, we have shown that there is a one-to-one relation between the sign of μ and of ε_p. Since a greater appeal turns the demand more inelastic directly through both α and indirectly through p^* (increases in prices make the demand more inelastic), then the firm increases the markup.

3 What Makes A Firm Successful?

All the analysis performed so far had the ultimate goal of characterizing successful firms. A successful firm is defined as having high profits. To identify when this occurs, we first need to distinguish between the different type of firms we can conceive within the model. We define the types of firms according to the specific vector (α, c) a firm can have.

We begin by showing formally that greater appeal (greater α) or more efficiency (lower c) makes the firm earn higher profits:

$$\frac{\partial \pi^* (\alpha, c)}{\partial \alpha} = \frac{\partial Q (p^*; \alpha)}{\partial \alpha} > 0$$
$$\frac{\partial \pi^* (\alpha, c)}{\partial c} = -Q (p^*; \alpha) < 0$$

We know that optimal profits are $\pi^* (\alpha, c) := Q [p^* (\alpha, c), \alpha] [p^* (\alpha, c) - c]$. Therefore, for a change in α

$$\begin{aligned}
\frac{\mathrm{d}\pi^* (\alpha, c)}{\mathrm{d}\alpha} &= \frac{\partial \pi^* (\alpha, c)}{\partial \alpha} + \underbrace{\frac{\partial \pi^* (\alpha, c)}{\partial p}}_{=0 \text{ by the FOC}} \frac{\partial p^* (\alpha, c)}{\partial \alpha} \\
&= \frac{\partial \pi^* (\alpha, c)}{\partial \alpha} \\
&= \frac{\partial Q (p^*; \alpha)}{\partial \alpha} > 0
\end{aligned}$$

For a change in c:

$$\begin{aligned}
\frac{\mathrm{d}\pi^* (\alpha, c)}{\mathrm{d}c} &= \frac{\partial \pi^* (\alpha, c)}{\partial c} + \underbrace{\frac{\partial \pi^* (\alpha, c)}{\partial c}}_{=0 \text{ by the FOC}} \frac{\partial p^* (\alpha, c)}{\partial c} \\
&= \frac{\partial \pi^* (\alpha, c)}{\partial c} \\
&= -Q (p^*; \alpha) < 0
\end{aligned}$$

Notice also that we have treated as c and α as a parameters. In general, the appeal and productivity of each firm is, in part, decided by each company. This does *not* mean that the conclusions of the model are not general enough. We could conceive the assumption that c and α parameters as a shortcut for a result generated within a model. For instance, c and α could be an outcome of in a model where we assume that firms have different abilities to differentiate product or reduce costs. Let's define φ^α and φ^c these skills, respectively. Each firm will have a value $(\varphi^c, \varphi^\alpha)$ and it will make choices $c^* (\varphi^c, \varphi^\alpha)$ and $\alpha^* (\varphi^c, \varphi^\alpha)$.
If there is a distribution of $(\varphi^c, \varphi^\alpha)$ across firms belonging to the industry, there will be firms making different choices and so, in equilibrium there will be different combinations (c^*, α^*).
By treating (c, α) those values as parameters and asking how variations in them affect firms' decisions, we are in fact asking how firms with a greater φ^c (lower c) or greater φ^α (higher α) make their choices.

According to Michael Porter, a famous academic specialized in business, there are three strategies that firms can pursue to be successful. He refers to them as Generic Competitive Strategies, and comprise the following:

[1] **Overall cost leadership**

Examples: Walmart, Costco and Aldi (retailers), Ryanair and EasyJet (airlines), Ikea (furniture), H&M (apparel).

[2] **Differentiation**

Examples: Nike and Adidas (sport clothes) Coke and Pepsi (carbonated beverages), Duracell and Energizer (batteries) Bayer (pharmaceutical products), Apple (computers).

[3] **Focus**

Examples: Ferrari, BMW and Mercedes Benz (cars), Louis Vuitton and Gucci (apparel), Dom Pérignon (champagne), Rolex (clocks).

Next, we show how we can capture these categories in terms of α and c.

3.1 What are the Strategies that a Successful Firm Follows?

We have shown that firms which have lower c or greater α have higher profits. Moreover, we have shown the following results.

Summary of the Results

Variations in c

- $\frac{\partial \pi^*(\alpha,c)}{\partial c} < 0$
- $\frac{\partial p^*(\alpha,c)}{\partial c} > 0$
- $\frac{\mathrm{d}q^*[p^*(\alpha,c);\alpha]}{\mathrm{d}c} < 0$
- $\frac{\mathrm{d}\mu^*[p^*(\alpha,c);\alpha]}{\mathrm{d}c} > 0$

Variations in α

- $\frac{\partial \pi^*(\alpha,c)}{\partial \alpha} > 0$
- $\frac{\partial p^*(\alpha,c)}{\partial \alpha} > 0$
- $\frac{\mathrm{d}q[p^*(\alpha,c);\alpha]}{\mathrm{d}\alpha} \gtreqless 0$
  - Case I: $\frac{\mathrm{d}q[p^*(\alpha,c);\alpha]}{\mathrm{d}\alpha} > 0$
  - Case II: $\frac{\mathrm{d}q[p^*(\alpha,c);\alpha]}{\mathrm{d}\alpha} < 0$
- $\frac{\mathrm{d}\mu[p^*(\alpha,c);\alpha]}{\mathrm{d}\alpha} > 0$

With the information provided through the comparative statics analysis, we can now establish Porter's taxonomy:

[1] **Overall Cost Leadership**: type of firms with lower c, such that relative to other firms in the industry: high q^*, low p^* and low μ^*

[2] **Differentiation**: type of firms with high α and Case I of (QUANT-α), such that relative to other firms: high q^*, high p^* and high μ^*

[3] **Focus**: type of firms with high α and Case II of (QUANT-α), such that relative to other firms: low q^*, high p^* and high μ^*.

By using the definition of profits, we can also see how these strategies are reflected. There are two ways in which we can reexpress optimal profits. First,

$$\pi^* (\alpha, c) := \frac{R\left[p^* (\alpha, c), \alpha\right]}{\varepsilon_p \left[p^* (\alpha, c), \alpha\right]} \tag{PROFIT-1}$$

Let's indicate optimal variables without arguments and with a * as a superscript. Optimal profits are $\pi^* (\alpha, c) := Q^* (p^* - c)$. By the FOC, $p^* = \frac{\varepsilon^*}{\varepsilon^* - 1} c$ and so by subtracting $c = \frac{\varepsilon^* - 1}{\varepsilon^*} p^*$. Thus, optimal profits are $\pi^* (\alpha, c) := Q^* \left(p^* - \frac{\varepsilon^* - 1}{\varepsilon^*} p^*\right)$ or, just $\pi^* (\alpha, c) := Q^* p^* \left(1 - \frac{\varepsilon^* - 1}{\varepsilon^*}\right)$ which determines the result.

Moreover, by the mere definition of a profit function, we can divide and multiply by c and obtain:

$$\pi (p^*; \alpha, c) = \underbrace{cQ (p^*; \alpha)}_{=:(1)} \underbrace{[\mu (p^*; \alpha) - 1]}_{=:(2)} \tag{PROFIT-2}$$

where we have used the fact that $\frac{p^*}{c} = \mu (p^*; \alpha)$

Using (PROFIT-2), we can observe that one way to have high profits is by charging lower markups (low μ and so a small term (2)) and increasing the scale of production (high Q and so big term (1)). A firm that pursues this strategy would be characterized by being massive and cheap. For this to happen, the firm has to be extremely productive, in which case, it sells mainly to consumers that are price sensitive. This requires that demand is quite elastic, so that it can lower prices to an extent that the increase in quantities is substantial, thereby more than compensating for the low price charged. Thus, a cost leadership strategy would be profitable.

At the other extreme, (PROFIT-2) establishes that a firm could get high profits if it sells low quantities (and hence small term (1)) and charges high markups for each unit sold (high μ and so big term (2)). This type of firm focuses on a niche market composed of customers having a high purchasing power, along with high willing to pay for the distinctive features of the good. In terms of (PROFIT-1), the strategy would consist in increasing the appeal of the good so much that ε_p reduces significantly. Thus, the firm would charge a high price and, even if total revenues decrease, quantities sold would be low and the firm would save in production costs.

Finally, we can conceive successful firms that keep some balance between its sales and prices/markups charged. These firms set high prices relative to other firms, but their customer target is broader than a mere niche. This would require increasing the appeal of the good, without doing it to an extent that only few customers can afford it. In terms of Porter's taxonomy, this corresponds to the case of differentiation.