Trade Liberalizations with Granular Firms

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Abstract

We quantify the effects of trade liberalization accounting for a fact identified with rich firm-product Danish data: the bulk of manufacturing revenue comes from industries where domestic leaders coexist with a competitive fringe. Our framework features firm-level heterogeneity, extensive-margin adjustments, and allows for a subset of non-negligible firms that behave strategically and have positive profits which are passed back to consumers. In this setup, since profits are affected positively by new export opportunities but negatively by tougher competition, gains of trade are not guaranteed. Estimating the model establishes that a trade liberalization increases Danish income and, hence, welfare. Nonetheless, the predicted gains are substantially lower than if we characterize firms as in the standard monopolistic competition framework.

Keywords: granularity, large firms, gains of trade, small-economy assumption.

JEL codes: F12, F10, L13.

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1 Introduction

What is the market structure of a typical industry? How does trade liberalization quantitatively affect economies when that market structure is accounted for? By providing answers to these questions, in this paper we underscore the role of large firms in the determination of market outcomes. In particular, we highlight the presence of what we denominate domestic leaders (henceforth, DLs): firms that command the greatest domestic market shares of their industries (thus having market power at its home market) and whose domestic and export revenues constitute a great bulk of each industry's income.

Our goal is twofold. The first one is studying the mechanisms operating in a trade liberalization under the presence of DLs. The second one is to compare welfare gains relative to the standard monopolistic competition setting as in Melitz (2003). Estimating the model for Denmark, our main conclusion is that the gains of trade predicted by our setup are substantially lower.

We begin the analysis by presenting some empirical facts that guide our approach. By drawing on rich Danish data at the firm-product level, we allocate each good sold by a domestic firm and import to a specific industry. This allows us to obtain domestic market shares at the firm-industry level, which capture the competitive environment in each industry accurately and, hence, a firm's market power.

The analysis reveals that the bulk of manufacturing revenue comes from industries that are neither purely monopolistic nor purely oligopolistic. Rather, they display coexistence of a few firms with great domestic market shares, and numerous firms with insignificant market shares.[1] While the former correspond to DLs, we define the latter as domestic non-leaders (DNLs). In particular, among the subset of industries with presence of DNLs, 96% of the revenue is generated by industries where there is at least one DL. Additionally, the results point out that DLs as a group generate a great part of the total income (specifically 52%), with virtually every DL contributing with at least 3% of the industry revenues in which it operates.

In Section 3, guided by this empirical fact, we set a framework that accounts for this market structure in a tractable way. It rests on the idea that firms are different in nature.

[1] A similar pattern is observed by Bronnenberg et al. (2011) and Hottman et al. (2016) for consumer-packaged-good industries in the US, and Gaubert and Itskhoki (2018) for manufacturing industries in France. Also, there is an extensive literature regarding Zipf's law, which refers to estimations of the shape parameter of a Pareto distribution that are close to one (see, for instance, Axtell 2001 and di Giovanni and Levchenko 2013). Those papers show a coexistence of large and small firms, although they take as measure of firm size variables such as employment and total revenue. Rather, we use domestic market share at the firm-industry level, which captures a firm's market power and, thus, the market structure of an industry.

Regarding DLs, it has been documented that firm leadership is persistent over time.[2] Bronnenberg et al. (2009) even present evidence that, in the US, many of the current leading brands were originated as early as the late nineteenth century.

As for DNLs, several patterns have been established in the literature. Decker et al. (2014) show that, in the US, a typical small firm starts its operations at a small scale and faces high probability of exit. Furthermore, conditional on surviving, the overwhelming majority of these businesses remain small throughout their life cycle. Thus, overall, firms start small, and either exit or stay in the market operating at a low scale.[3]

While it is likely that exogenous differences in entrepreneurial skills play a role in the emergence of this pattern, there is also evidence that other factors are at play. For instance, it has been documented that small entrepreneurs do not have as a goal to grow big and, even when this is the case, DLs usually engage in strategic behavior to halt their growth.[4]

Based on this evidence, we set a model with coexistence of DNLs and DLs whose features capture these regularities. The former are modeled as in Melitz (2003) and, thus, we conceive them as new entrepreneurs venturing into markets to explore their possibilities in the industry. Their fate is such that either they do not succeed and exit the market or, if they survive, they remain negligible in terms of size and profits. Among them, the most successful ones can even export, although at a low scale too. On the other hand, an exogenous number of DLs, which are non-negligible and potentially heterogeneous, are present in each industry. These firms know their productivity, are always active in the domestic market, and might also be exporters.[5]

In Section 4, we consider a world economy with two symmetric countries. The goal is to establish outcomes and operating mechanisms in our model within a standard setting. The conclusions regarding the mechanisms at play are as follows. First, as in Melitz, a trade liberalization decreases the domestic price index, making competition tougher in the

[2]For instance, see Sutton (2007) for several industries from Japan, and Bronnenberg et al. (2009; 2011) for the USA.

[3]These patterns have been documented several times for the US since, at least, Dunne et al. (1988) and have been observed in other countries (see, for instance, Schoar 2010 and La Porta and Shleifer 2008).

[4]Hurst and Pugsley (2011) provide information referred to the US about the expectations that small firms have when they start their operations: the majority of firm owners report that, actually, they do no expect or have as a goal to grow big. This suggests that, in part, being small does not necessarily mean that the firm failed in its attempt to become big. Likewise, D'Aveni (2002) presents several cases which reveal that, when small firms start to loom large, DLs make strategic moves to hinder their chances to succeed in the market. Regarding this, see also Kwoka and White (2001).

[5]To illustrate what kind of market structure we envision, the beer industry can be used as an example. This industry encompasses a group of DLs, with different brands accruing top market shares in each country (e.g., Budweiser in the USA, Molson in Canada, Heineken in the Netherlands, Carlsberg in some Scandinavian countries). Simultaneously, these companies coexist with a group of DNLs comprising small local firms (e.g., small brewpubs and microbreweries) producing at a low scale and earning small profits. Even though these firms are negligible when each is taken in isolation, they are non-trivial considered as a whole.

market. This induces the exit of the least productive firms and reduces the markups set by DLs. Since the price index is lower, consumers benefit from trade through this channel. On the other hand and in contrast to Melitz, DLs do not pay an entry cost to know their productivity and, hence, have positive profits. Taking this into account, a trade liberalization has an ambiguous impact on the country's income, once that there are countervailing effects affecting DLs' profits. For one thing, there are new export opportunities that increase the profits of DLs that export. For another, trade induces greater competition in the market, making DLs garner lower profits. Overall, gains of trade are not guaranteed, and it is an empirical matter to establish whether this occurs.

In Section 5, in line with our empirical analysis, we consider a small economy in the spirit of Demidova and Rodríguez-Clare (2009; 2013). This implies that the country is negligible for the domestic conditions of foreign countries and, so, it affects neither the price indices nor the incomes of them. Also, in this setting, DLs have market power in their domestic market but, even if they generate sizable export revenues for their home country, they are negligible for any specific foreign market.[6]

Additionally, we incorporate to the setup the existence of an outside sector, with a Cobb Douglas upper-tier utility function, which pins down wages. This enables us to focus on the impact of DLs' profits on income and welfare, and make the analysis robust to some variants of the setup. In particular, it allows us to subsequently quantify the effects of a trade liberalization with asymmetries between countries, without requiring further information relative to the symmetric-countries scenario.

In Section 6, we conduct a quantitative assessment of the effects of a trade liberalization on welfare. In addition to the information necessary to estimate gains of trade in Melitz, our approach only requires of information on domestic market shares and revenue shares of top Danish firms. Through different empirical exercises, we examine whether a trade liberalization creates gains and the relative importance of its determinants (i.e., reductions in price index versus income variations). Furthermore, we elaborate on how our results differ to those in a standard monopolistic competition setting.

Our main findings are twofold. First, the effect of a trade liberalization on profits is positive, which ensures the existence of gains of trade. Nonetheless, this increase is almost null. The reason is that the greater profits of DLs due to new export opportunities are almost

[6] With a CES demand and a single-sector economy, Demidova and Rodríguez-Clare (2013) show that H is a small economy when it has a population size that is negligible relative to the rest of the world. While this rationalization does not apply when there is an outside sector pinning down wages as we assume, Alfaro (2019) provides an alternative justification in which each country takes H as part of a continuum of trading partners. In that case, the same implications of a small-country assumption hold and they can be extended to environments with large firms.

entirely offset by the tougher competition that these firms face. As a corollary, the gains of trade are primarily driven by the reductions in the price index.

Second, when we compare our estimations of gains of trade with those in Melitz, we find that they are considerably lower. Specifically, welfare increases around 35% and 23% less in the small-economy and the symmetric-countries model, respectively.

Primarily, there are two possible sources for these differences in welfare. First, while aggregate profits are zero in Melitz, in our model DLs garner profits that are passed back to consumers. This affects welfare directly through variations income but, also, indirectly through the impact of income on the price index. Nonetheless, the magnitude of this is small and, even, predicts that our model would generate greater gains relative to Melitz.

Instead, the second source explains the discrepancy of welfare between models. This is given by the impact of a trade liberalization on the price index. To understand this, notice that Melitz corresponds to a special case of our model where all firms are treated as DNLs. Thus, after we substitute in optimal variables and acknowledge for the almost null variation in income, both our model and Melitz identify the price index through the zero-expected-profits conditions.

Nonetheless, the calibrations of DNLs in each framework are different. This is because, in Melitz, firms consider the whole productivity distribution when they make entry decisions, which includes the range of productivity corresponding to DLs. Once that we incorporate this, the data indicate that the average export intensity of all firms in Melitz is greater than the DNLs in our model, due to the existence of a positive correlation between domestic market shares and a firm's exports.

This fact implies that the decrease in the price index is greater in Melitz. Specifically, this occurs because the greater export intensity in Melitz makes new export opportunities have a greater impact on expected profits and, as a corollary, the decrease in the price index has to be more pronounced to restore zero expected profits. Intuitively, the mechanism acts through a more intense entry of firms induced by the better export opportunities which, in turn, creates more pronounced tougher competitive conditions.

Related Literature and Contributions. The first contribution of our paper relates to empirical models that incorporate non-negligible firms in international economies.[7] In general, these papers assume standard oligopoly models, with the exceptions of Eaton et al. (2012) and Gaubert and Itskhoki (2018) who innovate by assuming that the total number of firms is a random variable. Instead, our approach regarding market structure is different,

[7]For some recent studies, see, for instance, Atkeson and Burstein (2008), Eaton et al. (2012), Edmond et al. (2015), Gaubert and Itskhoki (2018), and Bernard et al. (2018). Also, di Giovanni and Levchenko (2013) consider a fat-tailed distribution, although their approach treats firms as negligible.

and so are the research questions, which include how DLs affect outcomes relative to a setup where all firms are deemed negligible.

Methodologically, our paper is more in line with Shimomura and Thisse (2012) and Parenti (2018). Both study the consequences of assuming a market structure with coexistence of large and small firms, with the latter modeled as a continuum. The first pair of authors do it for a closed economy, while the other for an open economy. Since their focus is on theoretical aspects, both assume that firms are homogeneous within groups, making their frameworks not suitable for an empirical analysis. Instead, our framework incorporates heterogeneity of firms, which makes it possible to estimate the model.

Finally, our paper also speaks to a vast literature that studies gains of trade empirically.[8] Our contribution in this matter is twofold. First, we provide a structural approach to estimating these gains while accounting for large firms parsimoniously. Additionally, we underline how estimation procedures can tractably incorporate asymmetries between countries when the country under study is small.

2 Empirical Facts

In this section, we present empirical evidence regarding the market structure of Denmark's highest revenue industries. Our analysis draws on information for Danish manufacturing in 2005, with identical patterns observed for other years. Given that the information is part of the official statistics of the country, they are of high quality. Further description of data and measures used is included in Section 6.1.

We make use of two datasets, which can be easily merged through a unique firm identifier. The first one constitutes the main source for Prodcom statistics and contains data about production value of manufacturing firms. The information encompasses 3,517 firms and is collected ensuring that at least 90% of the total production value in each 4-digit NACE industry is covered. The second dataset has information on exports and imports, and is collected by Danish Customs. Importantly, trade flows by both manufacturing and non-manufacturing firms are included, with almost the universe of transactions covered, allowing us to obtain an accurate measure of import competition and export revenues.[9]

Overall, the data provide us with information on total turnover, exports, and imports presented at the firm-product level. In particular, the information on goods is disaggregated

[8]For some of the most recent papers, see Simonovska and Waugh (2014), Bertoletti et al. (2018), Feenstra (2018), Gaubert and Itskhoki (2018), Arkolakis et al. (2019), and Fally (2019).

[9]The coverage is 95% for imports and 97% for exports with a EU trading partner, while the universe of transactions is covered for trade with non-EU countries.

at the 8-digit level according to the Combined Nomenclature (CN), whose first 6 digits are identical to the Harmonized System classification.

In order to define industries, we exploit the fact that the data is highly disaggregated. This allows us to overcome restrictions imposed by, for instance, datasets based on the firms' balance sheets declared to tax agencies, where each firm's revenue is allocated to its main industry. Thus, we assemble the data such that each good at the CN 8-digit level is allocated to a 4-digit NACE industry, leaving us with 203 industries out of 5,212 goods at the 8-digit CN level in the sample.

For each Danish firm-industry, we compute its domestic market share.[10] This is defined relative to the total expenditure of the industry, which is given by the sum of domestic sales and imports. Since Denmark is a small highly open economy, accounting for imports is crucial in order to obtain market shares that reflect a firm's market power.

To classify industries according to their market structure, in each of them we split firms into DNLs and DLs, using a domestic market of 3% as threshold. Based on this classification, we begin by identifying industries that include a pool of DNLs. This guarantees the existence of several domestic firms with negligible market shares serving the market, which can be considered as a necessary condition (although not sufficient) to have a monopolistic competition market structure.

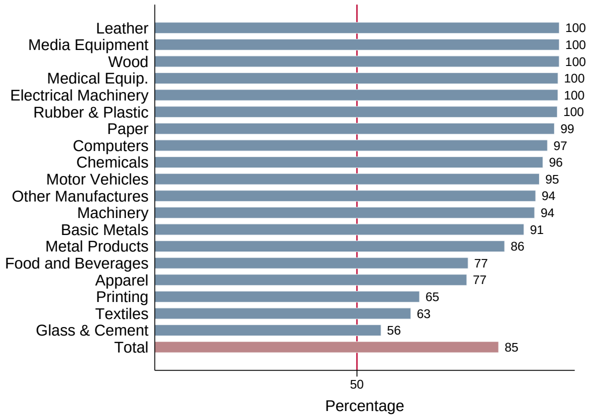
After applying the procedure, it is determined that 107 industries out of 203 include a subset of DNLs, where each has at least 12 firms, with an average number of 57 and a maximum above 330.[11] While this encompasses a little bit more than half of the industries, they account for 85% of the income generated by the manufacturing sector, where income is defined as the sum of domestic sales and exports by domestic firms. In Figure 1a, we present the proportion of income accounted for these industries. The results are aggregated at the 2-digit industry level using industry-revenue weights, with the last bar applying to the whole manufacturing sector.

[10] We consider a firm Danish if it has production activities in Denmark, which is defined according to whether it is included in the Danish Prodcom dataset. Instead, any firm that has no production activities in Denmark but imports goods is considered part of the import competition.

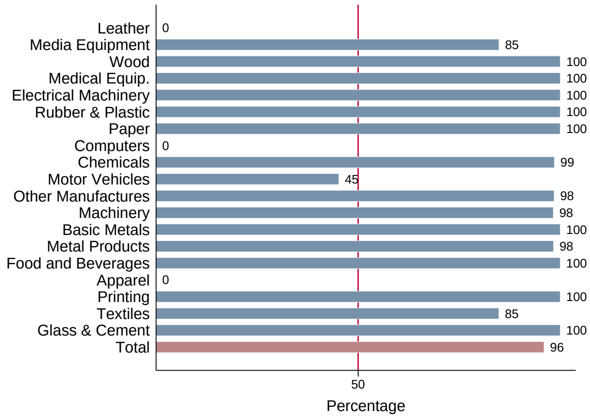
[11] The criteria we use to identify these industries are the following. First, we check that there are at least 10 firms in the market, and that the 10 firms or 20% of the firms with the lowest market share do not accumulate more than 6% of total market share. In addition, to account for markets with international trade, we only consider industries that are subject to some import competition, such that at least 4% of market share corresponds to imports.

Figure 1. *Proportion of Income According to the Industry's Market Structure*

(a) Industries with DNLs



(b) Industries with DNLs and DLs



Note: In both figures, the results for the total and each sector are calculated using industry-revenue weights. In Figure 1a, the proportion of income for each sector is relative to the total revenue of the industries in the sector. In Figure 1b, this proportion is relative to the subset of industries with DNLs.

Furthermore, among the industries that include a set of DNLs, we inquire upon whether they fit a pure monopolistic competition market structure. This is the case when there are no DLs in the market. The results indicate that, in 92 out of the 107 industries, in addition to DNLs there is at least one DL serving the market. Remarkably, relative to the industries with a pool of DNLs, industries displaying this feature generate 96% of the total income.[12] The results are presented in Figure 1b.[13]

Delving into the role of DLs in industries with coexistence of DLs and DNLs, Figure 2 provides information on how revenues are generated. In Figure 2a, we inquire upon the share of industry revenue that is generated by each DL. The table indicates the percentage of DLs that generate at least a certain proportion of total revenue in each industry. In the case of our baseline definition of DLs (i.e., with a threshold of 3% of domestic market), virtually every firm (98.79% of them) generates at least 3% of the industry revenue. When we use a 5% of domestic market share to define a DL, the minimum revenue generated by each firm is greater, determining that almost all of them has an income representing at least a 5% of the total industry revenue.

[12] Alternatively, if we define DLs as firms with a market share greater than 5%, they encompass 81 industries and generate 90% of the income. Also, measuring the relative importance of each industry in terms of expenditure delivers similar conclusions. The numbers, nonetheless, are a little bit lower since some of the industries are almost completely dominated by imports. Specifically, industries having DNLs cover around 82% of the manufacturing expenditures and, among these industries, 86% is generated by industries with coexistence of DNLs and DLs.

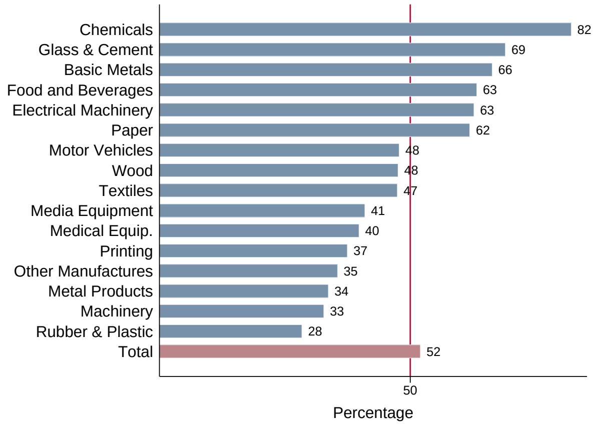
[13] While the figure indicates that Leather, Computers, and Apparel do not include industries with DLs, this has no major impact regarding total revenue. The reason is that these sectors are almost completely served by imports, rather than DNLs.

Figure 2. *Revenue Generated by DLs in Industries with Coexistence of DNLs and DLs*

(a) % of DLs that Generate a Minimum Industry-Revenue Share

Threshold of Market Share defining a DL	Minimum Industry-Revenue Share Generated by each DL 3%	5%
3%	98.79	83.99
5%	100	99.46

(b) % of Revenue Generated by all DLs as Group



Finally, regarding the importance of DLs as a group in terms of revenue, Figure 2b provides information about the percentage of industry revenue generated by all of them as a group. The results are aggregated using industry-revenue weights and reveal that more than 50% of the total revenue is due to domestic sales and exports by DLs.

Overall, the information established in Figure 1 and Figure 2 suggests that accounting for the idiosyncratic features of granular firms can be of first-order relevance for the performance of an industry and, in particular, regarding its income generation.

3 General Framework and Equilibrium

In this section, we outline the general framework utilized to model a differentiated industry. In subsequent sections, we add some specific assumptions depending on the scenario under analysis. Anticipating this, we set the model allowing for the possibility that there is either a single industry or that, in addition to this, there is an outside sector consisting of a homogeneous good.

Throughout the paper, any subscript ij refers to i as the origin country and j as the destination country. Also, all the derivations and proofs of this paper are relegated to Appendix A.

3.1 Characterization of Firms

Consider a set of countries $\mathcal{C}$ comprising a potential arbitrary number of them. In each country $i \in \mathcal{C}$, there is a set of firms $\overline{\Omega}_i$ that can potentially serve a country with a unique variety. Due to this, we refer indistinctly to either a firm ω or a variety ω.

We rule out the possibility of the so-called Ford effects, i.e., we suppose that no firm makes choices taking into account its influence on income or wages of any country. This entails that, as in Shimomura and Thisse (2012), firms are income and wage takers. For this reason, even when we deal with frameworks with either one or two sectors, we should conceive them as representative sectors composed of a continuum of industries as in Neary (2016). Thus, firms can be large in their industry but are always small for the country. Our empirical approach is consistent with this view, where we derive results for a sector constructed as an average across industries.

In contrast, we incorporate the possibility that a firm can be non-negligible for the industry that serves. We accomplish this by endowing $\cup_{k\in\mathcal{C}}\overline{\Omega}_k$ with a measure μ that indicates the size of a firm. Formally, μ is such that $\mu(\{\omega\}) > 0$ if ω is non-negligible, and $\mu(\{\omega\}) = 0$ if it is negligible. We suppose that the measure μ partitions $\overline{\Omega}_k$ for each $k \in \mathcal{C}$ into a finite set $\overline{\mathscr{L}}_k$ and a real interval $\overline{\mathcal{N}}_k$, whose letters are mnemonics for large and negligible. We refer to them as DLs and DNLs, respectively. In the general model, DLs are treated as non-negligible in any country while, in the small-economy setting we explore, they have market power domestically but are negligible for any specific foreign market.

In order to incorporate μ parsimoniously, we characterize this measure in a more specific way. However, since there are some technical considerations that are necessary to do this, we relegate the formal description to Section 3.3.

Finally, in terms of notation, we denote by Ω_{ji} the subset of varieties from $j \in \mathcal{C}$ sold in $i \in \mathcal{C}$, with $\Omega_i := \cup_{k\in\mathcal{C}}\Omega_{ki}$ being the total varieties available in i. Likewise, $\Omega_{ji}^{\text{DNL}} := \overline{\mathcal{N}}_j \cap \Omega_{ji}$ and $\Omega_{ji}^{\text{DL}} := \overline{\mathscr{L}}_j \cap \Omega_{ji}$ are, respectively, the subsets of varieties available in i produced by DNLs and DLs from j.

3.2 Supply Side

The supply side of the model constitutes an augmented version of Melitz (2003), with an embedded set of firms that can affect industry conditions. In each country $i \in \mathcal{C}$, there is a mass of identical agents L_i that are immobile across countries. Labor is the only production factor and each agent offers a unit of labor inelastically. In addition, these agents are the owners of their own country's firms and get the same fraction of profits.

Throughout the description of the setup, we take $i, j \in \mathcal{C}$ and focus on the characterization of firms from i. Regarding DNLs, they are ex-ante identical and do not know their productivity. Besides, they consider whether to pay a sunk entry cost, which consists of F_i units of home workers, to receive a productivity draw φ and an assignation of a unique variety $\omega \in \overline{\mathcal{N}}_i$. Regarding productivity draws, they come from a continuous random variable that has non-negative support $\left[\underline{\varphi}_i, \overline{\varphi}_i\right]$ and a cdf G_i. We denote the measure of DNLs that pay the entry cost by M_i^E.

As for DLs, there is an exogenous number of them, with each having assigned a unique variety $\omega \in \overline{\mathscr{L}}_i$ and productivity φ_ω. We suppose that $\varphi_\omega > \overline{\varphi}_i$ for any $\omega \in \overline{\mathscr{L}}_i$, so that any DL is more productive than the most productive DNL. Furthermore, we assume that DLs are always active in their domestic market and that their costs are common knowledge across the world.

Concerning costs, a DNL with productivity φ that serves j from i has constant marginal costs $c\left(\varphi, \tau_{ij}, w_i\right) := \frac{w_i}{\varphi}\tau_{ij}$ where τ_{ij} is a trade cost with $\tau_{ii} := 1$. As for a DL ω, we suppose that its marginal costs, denoted by c_{ij}^ω, are given by the same function $c\left(\varphi_\omega, \tau_{ij}^\omega, w_i\right)$, where $\tau_{ij}^\omega := \tau^\omega \tau_{ij}$ if $j \neq i$, and $\tau_{ii}^\omega := 1$. Since trade costs of DLs are firm specific, a DL can have greater domestic revenues than a less productive firm without implying that its exports revenues are greater too. In fact, by allowing for $\tau^\omega = \infty$, it is possible that some DLs do not export at all.[14] Primarily, the incorporation of firm-specific trade costs is because, in the representative industry we construct with Danish data, the second DL with greatest domestic market share exports less than any of the other top four DLs.

DLs from i and the mass M_i^E of DNLs have the option of not selling in j, or doing so by paying an overhead fixed cost f_{ij}, where $f_{ij} > 0$ is expressed in units of home workers.[15] At the market stage, firms compete à la Bertrand. Thus, each firm makes prices choices $p_{ij}^\omega \in P$ with $P := \left[\underline{p}, \overline{p}\right] \cup \{\infty\}$, where $p_{ij}^\omega = \infty$ captures not serving j. Also, we denote by $\mathbf{p}_{ij} := \left(p_{ij}^\omega\right)_{\omega \in \Omega_{ij}}$ the vector of prices set in j by all active firms from i, and by M_{ij} the measure of active DNLs from i selling in j.

3.3 Demand Side

Preferences are represented by a two-tier utility function, with an upper tier that is Cobb Douglas between the homogeneous and differentiated good. Consequently, the expenditure in country $j \in \mathcal{C}$ on the differentiated good is given by $E_j := \beta_j Y_j$ where $\beta_j \in (0, 1]$ and Y_j is the total income in j. This representation covers both a single-sector economy, in which case $\beta_j = 1$, and a situation with an outside sector, so that $\beta_j < 1$.

Concerning the differentiated sector, irrespective of whether a firm ω from $i \in \mathcal{C}$ is a DL

[14] This is a common pattern in some specific Danish industries. In fact, around 30% of the DLs serve the domestic market exclusively.

[15] Notice that, since DLs are always active in their domestic market, we are implicitly supposing that the gross profits that they obtain in their home market are greater than their overhead costs.

or a DNL, its demand Q_{ij}^{ω} derives from a CES sub-utility function and is given by

$$Q_{ij}\left(p_{ij}^{\omega}, \mathbb{P}_j, E_j\right) := E_j \left(\mathbb{P}_j\right)^{\sigma-1} \left(p_{ij}^{\omega}\right)^{-\sigma},$$

where $\mathbb{P}_j$ is the price index of the industry in j and $\sigma > 1$. Since firms can attain different masses, captured by the measure μ, the expression of the price index differs from a typical model of monopolistic competition. Specifically, the price-index function in j is given by

$$\mathbb{P}_j\left[\left(\mathbf{p}_{kj}\right)_{k\in\mathcal{C}}\right] := \left[\sum_{k\in\mathcal{C}} \int_{\omega\in\Omega_{kj}} \left(p_{kj}^{\omega}\right)^{1-\sigma} \mathrm{d}\mu\left(\omega\right)\right]^{\frac{1}{1-\sigma}}. \tag{1}$$

As previously mentioned, we need further characterization of μ to parsimoniously incorporate it into the model. We define μ such that the price index of a standard oligopoly and of a monopolistic model arise as special cases. This means that, when the set of DNLs is empty, the price-index function can be expressed by sums while, if the set of DLs is empty, it is expressed by an integral. At a formal level, this is accomplished by supposing that the integral in (1) is Lebesgue and $\mu\left(\cdot\right) := \ell\left[\cdot \cap \left(\cup_{k\in\mathcal{C}}\overline{\mathcal{N}}_k\right)\right] + \#\left[\cdot \cap \left(\cup_{k\in\mathcal{C}}\overline{\mathscr{L}}_k\right)\right]$. where ℓ is the Lebesgue measure and $\#$ the counting measure.

This measure is convenient as it allows for a unified treatment of sums and integrals. To demonstrate the implications as clearly as possible, consider first the case of a closed economy. This allows us to omit country indices. With the definition of μ stated, (1) becomes

$$\mathbb{P}\left(\mathbf{p}\right) = \left[\int_{\omega\in\Omega} \left(p^{\omega}\right)^{1-\sigma} \mathrm{d}\mu\left(\omega\right)\right]^{\frac{1}{1-\sigma}} = \left[\int_{\omega\in\Omega^{\mathrm{DNL}}} \left(p^{\omega}\right)^{1-\sigma} \mathrm{d}\omega + \sum_{\omega\in\Omega^{\mathrm{DL}}} \left(p^{\omega}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

In particular, if all firms in the country are negligible, so that $\overline{\mathscr{L}} = \emptyset$, (1) collapses to the case of monopolistic competition,

$$\mathbb{P}\left(\mathbf{p}\right) = \left[\int_{\omega\in\Omega} \left(p^{\omega}\right)^{1-\sigma} \mathrm{d}\omega\right]^{\frac{1}{1-\sigma}},$$

while, if all firms are non-negligible, so that $\overline{\mathcal{N}} = \emptyset$, then (1) is expressed as in a typical oligopoly model,

$$\mathbb{P}\left(\mathbf{p}\right) = \left[\sum_{\omega\in\Omega} \left(p^{\omega}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

In the case of our model with open economies, (1) becomes

$$\mathbb{P}_j\left[\left(\mathbf{p}_{kj}\right)_{k\in\mathcal{C}}\right] = \left\{\sum_{k\in\mathcal{C}} \left[\int_{\omega\in\Omega_{kj}^{\mathrm{DNL}}} \left(p_{kj}^{\omega}\right)^{1-\sigma} \mathrm{d}\omega + \sum_{\omega\in\Omega_{kj}^{\mathrm{DL}}} \left(p_{kj}^{\omega}\right)^{1-\sigma}\right]\right\}^{\frac{1}{1-\sigma}}.$$

Using this characterization of demand and price index, let the market share of a firm ω

from $i \in \mathcal{C}$ in $j \in \mathcal{C}$ be s_{ij}^{ω} and given by the following function:

$$s\left(p_{ij}^{\omega}, \mathbb{P}_j\left[(\mathbf{p}_{kj})_{k\in\mathcal{C}}\right]\right) := \frac{\left(p_{ij}^{\omega}\right)^{1-\sigma}}{\sum_{k\in\mathcal{C}}\int_{\omega\in\Omega_{kj}}\left(p_{kj}^{\omega}\right)^{1-\sigma}\,\mathrm{d}\mu\left(\omega\right)},$$

or, expressed as a function of $\mathbb{P}_j$ rather than $(\mathbf{p}_{kj})_{k\in\mathcal{C}}$,

$$s\left(p_{ij}^{\omega}, \mathbb{P}_j\right) := \frac{\left(p_{ij}^{\omega}\right)^{1-\sigma}}{\mathbb{P}_j^{1-\sigma}}. \tag{2}$$

Finally, we characterize the perceived price elasticity of demand. The fact that some firms can influence industry conditions establishes that this elasticity differs according to the type of firm. Formally, the price elasticity of demand in $j \in \mathcal{C}$ of a firm ω from $i \in \mathcal{C}$ is denoted by $\varepsilon_{ij}^{\omega} := \left|\frac{\mathrm{d}\ln Q_{ij}^{\omega}}{\mathrm{d}\ln p_{ij}^{\omega}}\right|$, and for a DL ω is

$$\varepsilon\left(s_{ij}^{\omega}\right) := \sigma + s_{ij}^{\omega}\left(1-\sigma\right),$$

while, for a DNL ω, it is $\varepsilon_{ij}^{\omega} = \sigma$.

3.4 Equilibrium Characterization

Consistent with the fact that our goal is to reflect a market structure with coexistence of both types of firms, we consider equilibria where, in each country, there is always a subset of DNLs that are active. In addition, we add some assumptions. First, in line with the Danish data, we suppose that the parameters of the model are such that some DNLs export. Furthermore, we assume that any DNL that exports also finds it profitable to serve its domestic market.

In Section 3.4.1 we characterize the equilibrium of the differentiated industry. Specifically, the equilibrium conditions are stated for given expenditures and wages in each country, $(E_i, w_i)_{i\in\mathcal{C}}$. After this, in Section 3.4.2, we close the model in general equilibrium.

3.4.1 Industry-Equilibrium Characterization

Consider a DL ω from $i \in \mathcal{C}$ that is active in $j \in \mathcal{C}$. Its optimal prices in j are obtained by solving the following optimization problem:

$$\max_{p_{ij}^{\omega}} \pi_{ij}^{\omega} = E_j\left(\mathbb{P}_j\left[(\mathbf{p}_{kj})_{k\in\mathcal{C}}\right]\right)^{\sigma-1}\left(p_{ij}^{\omega}\right)^{-\sigma}\left(p_{ij}^{\omega} - c_{ij}^{\omega}\right) - w_i f_{ij}. \tag{3}$$

Routine calculations determine that optimal prices are given by

$$p_{ij}^{\omega} = m\left(s_{ij}^{\omega}\right)c_{ij}^{\omega}, \tag{4}$$

where $m\left(s_{ij}^{\omega}\right) := \frac{\varepsilon\left(s_{ij}^{\omega}\right)}{\varepsilon\left(s_{ij}^{\omega}\right)-1}$ is the firm's markup. Making use of (2), (4) determines an implicit solution for prices, denoted $p\left(\mathbb{P}_j, \varphi_{\omega}; \tau_{ij}\right)$.

The optimal market share in j of this DL is

$$s\left(\mathbb{P}_j, \varphi_\omega; \tau_{ij}\right) := \left(\frac{p\left(\mathbb{P}_j, \varphi_\omega; \tau_{ij}\right)}{\mathbb{P}_j}\right)^{1-\sigma}, \tag{5}$$

which determines that its optimal revenues in j, R_{ij}^{ω}, are a function

$$R\left(\mathbb{P}_j, E_j, \varphi_\omega; \tau_{ij}\right) := E_j s\left(\mathbb{P}_j, \varphi_\omega; \tau_{ij}\right),$$

and its optimal profits in j are

$$\pi_{ij}^{\omega}\left(\mathbb{P}_j, E_j, \varphi_\omega; \tau_{ij}\right) := \frac{R\left(\mathbb{P}_j, E_j, \varphi_\omega; \tau_{ij}\right)}{\varepsilon\left[s\left(\mathbb{P}_j, \varphi_\omega; \tau_{ij}\right)\right]} - w_i f_{ij}.$$

As for a DNL ω with productivity φ, the same optimization problem (3) has to be solved. Since its price elasticity is σ, (4) determines a solution $p^{\mathrm{DNL}}\left(\varphi; \tau_{ij}\right)$. Moreover, its profits in j are

$$\pi_{ij}^{\mathrm{DNL}}\left(\mathbb{P}_j, E_j, \varphi; \tau_{ij}\right) := \frac{R\left(\mathbb{P}_j, E_j, \varphi; \tau_{ij}\right)}{\sigma} - w_i f_{ij}.$$

By setting π_{ij}^{DNL} to zero, we obtain the survival productivity cutoff in j of DNLs from i:

$$\varphi_{ij}^{\mathrm{DNL}}\left(\mathbb{P}_j, E_j; \tau_{ij}\right) = \frac{\sigma \tau_{ij} w_i}{(\sigma-1)\,\mathbb{P}_j}\left(\frac{\sigma w_i f_{ij}}{E_j}\right)^{\frac{1}{\sigma-1}}.$$

Substituting the survival productivity cutoff into optimal profits, it is determined that the expected profits in j of a DNL from i are

$$\pi_{ij}^{\mathbb{E},\mathrm{DNL}}\left(\mathbb{P}_j, E_j; \tau_{ij}\right) := \int_{\varphi_{ij}^{\mathrm{DNL}}\left(\mathbb{P}_j, E_j; \tau_{ij}\right)}^{\overline{\varphi}_i} \left[\pi_{ij}^{\mathrm{DNL}}\left(\mathbb{P}_j, E_j, \varphi; \tau_{ij}\right) - w_i f_{ij}\right] \mathrm{d}G_i(\varphi),$$

while the total revenues in j of DNLs as a group are denoted R_{ij}^{DNL} and given by

$$R_{ij}^{\mathrm{DNL}}\left(\mathbb{P}_j, E_j, M_i^E; \tau_{ij}\right) := M_i^E \int_{\varphi_{ij}^{\mathrm{DNL}}\left(\mathbb{P}_j, E_j; \tau_{ij}\right)}^{\overline{\varphi}_i} R\left(\mathbb{P}_j, E_j, \varphi; \tau_{ij}\right) \mathrm{d}G_i(\varphi).$$

To keep notation simple, let $\mathbf{P} := \left(\mathbb{P}_k\right)_{k\in\mathcal{C}}$, $\mathbf{E} := \left(E_k\right)_{k\in\mathcal{C}}$, and $\mathbf{M}^E := \left(M_k^E\right)_{k\in\mathcal{C}}$. In addition, define the inward and outward trade costs of i by, respectively, $\boldsymbol{\tau}_i^{\mathrm{IMP}} := \left(\tau_{ki}\right)_{k\in\mathcal{C}}$ and $\boldsymbol{\tau}_i^{\mathrm{EXP}} := \left(\tau_{ik}\right)_{k\in\mathcal{C}}$.

Once the survival productivity cutoff of each country is substituted into optimal total profits, the free-entry condition in each country $i \in \mathcal{C}$ can be expressed as

$$\pi_i^{\mathbb{E},\mathrm{DNL}}\left(\mathbf{P}, \mathbf{E}; \boldsymbol{\tau}_i^{\mathrm{EXP}}\right) := \sum_{k\in\mathcal{C}} \pi_{ik}^{\mathbb{E},\mathrm{DNL}}\left(\mathbb{P}_k, E_k; \tau_{ik}\right) = w_i F_i. \tag{FE}$$

Finally, there is market clearing in i when (1) for i is consistent with the firms' optimal decisions. Specifically, let

$$\widetilde{\mathbb{P}}_i^{\text{DNL}}\left(\mathbb{P}_i, E_i, \mathbf{M}^E; \boldsymbol{\tau}_i^{\text{IMP}}\right) := \sum_{k \in \mathcal{C}} M_k^E \int_{\varphi_{ki}^{\text{DNL}}(\mathbb{P}_i, E_i; \tau_{ki})}^{\overline{\varphi}_k} \left[\frac{\sigma}{\sigma - 1} \frac{w_k}{\varphi} \tau_{ki}\right]^{\sigma - 1} \mathrm{d}G_k(\varphi),$$

$$\widetilde{\mathbb{P}}_i^{\text{DL}}\left(\mathbb{P}_i, E_i, \boldsymbol{\omega}_{\cdot i}^{\text{DL}}; \boldsymbol{\tau}_i^{\text{IMP}}\right) := \sum_{k \in \mathcal{C}} \sum_{\omega \in \Omega_{ki}^{\text{DL}}} \left[m\left(\mathbb{P}_i, \varphi_\omega; \tau_{ki}\right) \frac{w_k}{\varphi_\omega} \tau_{ki}^\omega\right]^{\sigma - 1},$$

where $m\left(\mathbb{P}_i, \varphi_\omega; \tau_{ki}\right)$ is DL ω's markup evaluated at its optimal market shares, and the incorporation of $\boldsymbol{\omega}_{\cdot i}^{\text{DL}} := \left(\omega \in \cup_{k \in \mathcal{C}} \Omega_{ki}^{\text{DL}}\right)$ into $\widetilde{\mathbb{P}}_i^{\text{DL}}$ reflects that the price of each DL from k active in i depends on $(\varphi_\omega, \tau_{ki}^\omega)$. Using these definitions, the condition for i is

$$\mathbb{P}_i^{1-\sigma} = \widetilde{\mathbb{P}}_i^{\text{DNL}}\left(\mathbb{P}_i, E_i, \mathbf{M}^E; \boldsymbol{\tau}_i^{\text{IMP}}\right) + \widetilde{\mathbb{P}}_i^{\text{DL}}\left(\mathbb{P}_i, E_i, \boldsymbol{\omega}_{\cdot i}^{\text{DL}}; \boldsymbol{\tau}_i^{\text{IMP}}\right), \tag{MS}$$

where "MS" refers to the fact that (MS) constitutes the equilibrium condition at the market stage.

Given $(E_i, w_i)_{i \in \mathcal{C}}$, the industry equilibrium can be characterized through a vector $\left(\mathbb{P}_i^*, M_i^{E*}\right)_{i \in \mathcal{C}}$ that satisfies the systems of equations (FE) and (MS) for each $i \in \mathcal{C}$. Once that we identify those values, the rest of the industry equilibrium variables can be determined.

One important feature of the system of equilibrium conditions, which we exploit throughout the paper, is that it is separable: $(\mathbb{P}_i^*)_{i \in \mathcal{C}}$ is identified by the system (FE), and independently of (MS).

3.4.2 General Equilibrium

With the equilibrium characterization of the differentiated industry, we proceed to embed it in general equilibrium. In that way, we are able to determine $(E_i, Y_i, w_i)_{i \in \mathcal{C}}$.

In all the scenarios considered in subsequent sections, $(w_i)_{i \in \mathcal{C}}$ can be treated as a parameter in equilibrium. This responds to the following. In the first scenario we study, it is supposed that the differentiated industry constitutes the only sector in the economy. Thus, joint with symmetry assumptions regarding countries and trade shocks, wages are equal in each country and can be taken as the numéraire. In the second scenario, we concentrate on the case of a small country, which determines that wages in the rest of the world are not affected by trade shocks in the home economy. In addition, domestic wages are pinned down by the incorporation of an outside sector.

Given this, only optimal incomes and expenditures remain to be determined. As for income, define the total profits of DLs in $i \in \mathcal{C}$ by

$$\Pi_i^{\text{DL}}\left[\mathbf{P}, \mathbf{E}, \left(\omega \in \overline{\mathscr{L}}_i\right); \boldsymbol{\tau}_i^{\text{EXP}}\right] := \sum_{\omega \in \overline{\mathscr{L}}_i} \pi_i^\omega\left(\mathbf{P}, \mathbf{E}; \boldsymbol{\tau}_i^{\text{EXP}}\right), \tag{PROF}$$

where $\pi_i^\omega := \sum_{k\in\mathcal{C}} \pi_{ik}^\omega (\mathbb{P}_k, E_k; \tau_{ik})$ and the dependence on $\left(\omega \in \overline{\mathscr{L}}_i\right)$ responds to the heterogeneity of DLs.

Making use of (PROF), total income in i is determined by

$$Y_i = L_i w_i + \Pi_i^{\text{DL}} \left[\mathbf{P}, \mathbf{E}, \left(\omega \in \overline{\mathscr{L}}_i\right); \boldsymbol{\tau}_i^{\text{EXP}}\right]. \tag{INC}$$

Likewise, total expenditures in i are given by $E_i = \beta_i Y_i$ which, by substituting (INC) in, establishes that

$$E_i = \beta_i \left[L_i w_i + \Pi_i^{\text{DL}} \left(\mathbf{P}, \mathbf{E}, \left(\omega \in \overline{\mathscr{L}}_i\right); \boldsymbol{\tau}_i^{\text{EXP}}\right)\right]. \tag{EXD}$$

By solving the system of equations (EXD), we obtain $E_i(\mathbf{P})$ for each $i \in \mathcal{C}$ and thereby, using (INC), $Y_i(\mathbf{P})$ for each $i \in \mathcal{C}$.

In the different scenarios that we consider, the system of industry equilibrium conditions is separable, with the price index in each country determined exclusively by the system (FE). Both facts determine that, in the analysis of the next sections, the impact of trade liberalizations on price index, income, and welfare can be determined by the systems (FE), (INC), and (EXD), without needing to solve for (MS) or pin down $\mathbf{M}^{E*}$. Thus, it is not necessary to fully solve the model if the goal is to compute the effects of a trade liberalization on these variables.

4 Trade Liberalization between Symmetric Countries

In this section, we consider a trade liberalization with two large symmetric countries and a single sector composed of a differentiated good. We proceed by, first, adding specific assumptions relative to the framework in Section 3. After this, we set the equilibrium conditions that are necessary to study the impact of a trade liberalization on the home economy.

4.1 Setup

Relative to the general framework, we suppose that the set of countries is $\mathcal{C} := \{H, F\}$ and that there is only one sector, which is differentiated and characterized as in Section 3. Regarding (EXD), this implies that $\beta_i = 1$ holds for each $i \in \mathcal{C}$. Thus, income and expenditure of the sector are equal in equilibrium.

In addition, countries are identical, which entails the following. First, for each country $i, j \in \mathcal{C}$ with $i \neq j$, $\tau_{ij} =: \tau$ and $f_{ij} =: f_X$. In addition, $L_i =: L$, $F_i =: F$, $f_{ii} =: f_D$, and $G_i =: G$. As for DLs, for each $\omega \in \overline{\mathscr{L}}_H$ with $(\varphi_\omega, \tau_{HF}^\omega)$ there is a firm $\omega \in \overline{\mathscr{L}}_F$ with the same productivity and trade costs. The experiment under study is given by a small proportional reduction in τ. Moreover, all the results are expressed in elasticity terms.

4.2 Results

In general terms, the equilibrium is characterized as in Section 3.4. Moreover, by taking wages of H as the numéraire and given the symmetry of equilibrium and trade shock under consideration, there are unitary wages in F. Thus, trade is always balanced. Following standard notation and exploiting symmetry, we replace countries subscripts ii and ij for $i \neq j$ by D and X, respectively.

The equilibrium is described by a vector $\left(\mathbb{P}^*, Y^*, E^*, M^{E*}\right)$ that holds in both countries. We exploit the fact that $(\mathbb{P}^*, Y^*, E^*)$ can be identified without solving for (MS) or determining M^{E*}.

To identify the equilibrium, we proceed in the following way. First, we get a characterization of the partial effects of $Y(\mathbb{P};\tau)$ and $E(\mathbb{P};\tau)$. Then, by substituting $Y^* := Y(\mathbb{P}^*;\tau)$ and $E^* := E(\mathbb{P}^*;\tau)$ into (FE), we obtain the total effect of τ on $\mathbb{P}^*$. By making use of this, we are able to recover the total effect of τ on Y^* and E^*.

Regarding $Y(\mathbb{P};\tau)$, this is the implicit solution of (INC). Since income equals expenditure in equilibrium, the characterization of $Y(\mathbb{P};\tau)$ also describes $E(\mathbb{P};\tau)$. Notice that (INC) for i depends on the total profits of its DLs, which are given by (PROF). Incorporating the symmetry assumptions, (PROF) for $i \in \mathcal{C}$ becomes

$$\Pi_i^{\text{DL}} = \left[\sum_{\omega\in\Omega_D^{\text{DL}}}\left(\frac{R_D(\mathbb{P},E,\varphi_\omega)}{\varepsilon\left[s_D(\mathbb{P},\varphi_\omega)\right]} - f_D\right) + \sum_{\omega\in\Omega_X^{\text{DL}}}\left(\frac{R_X(\mathbb{P},E,\varphi_\omega;\tau)}{\varepsilon\left[s_X(\mathbb{P},\varphi_\omega;\tau)\right]} - f_X\right)\right].$$

By utilizing this expression, we can determine the partial effects $\frac{\partial \ln E}{\partial \ln \mathbb{P}}$ and $\frac{\partial \ln E}{\partial \ln \tau}$, which coincide, respectively, with $\frac{\partial \ln Y}{\partial \ln \mathbb{P}}$ and $\frac{\partial \ln Y}{\partial \ln \tau}$:

$$\frac{\partial \ln Y}{\partial \ln \mathbb{P}} = \frac{\partial \ln E}{\partial \ln \mathbb{P}} = (\sigma-1)\frac{\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega} + \sum_{\omega\in\Omega_X^{\text{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}}{Y - \sum_{\omega\in\Omega_D^{\text{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega} - \sum_{\omega\in\Omega_X^{\text{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}},$$

$$\frac{\partial \ln Y}{\partial \ln \tau} = \frac{\partial \ln E}{\partial \ln \tau} = (1-\sigma)\frac{\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}{Y - \sum_{\omega\in\Omega_D^{\text{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega} - \sum_{\omega\in\Omega_X^{\text{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}.$$

Substituting $E^* := E(\mathbb{P}^*;\tau)$ into (FE) and using the results for partial effects, we are able to obtain the total effect of τ on $\mathbb{P}^*$. Specifically, evaluating variables at equilibrium, (FE) for $i \in \mathcal{C}$ becomes

$$\pi_i^{\mathbb{E},\text{DNL}} := \int_{\varphi_D^*}^{\overline{\varphi}}\left[\frac{R_D(\mathbb{P}^*,E^*,\varphi)}{\sigma} - f_D\right]\mathrm{d}G(\varphi) + \int_{\varphi_X^*}^{\overline{\varphi}}\left[\frac{R_X(\mathbb{P}^*,E^*,\varphi;\tau)}{\sigma} - f_X\right]\mathrm{d}G(\varphi) = F, \tag{6}$$

where $\varphi_D^* := \varphi_D(\mathbb{P}^*, E^*)$ and $\varphi_X^* := \varphi_X(\mathbb{P}^*, E^*; \tau)$.

Differentiating (6), the total effect of τ on the price index is given by $\frac{\mathrm{d}\ln \mathbb{P}^*}{\mathrm{d}\ln \tau} = -\left(\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \tau}\right)\left(\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \mathbb{P}}\right)^{-1}$, with

$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \mathbb{P}} = \frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln \mathbb{P}} + \frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln E}\frac{\partial \ln E^*}{\partial \ln Y}\frac{\partial \ln Y^*}{\partial \ln \mathbb{P}},$$
$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \tau} = \frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln \tau} + \frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln E}\frac{\partial \ln E^*}{\partial \ln Y}\frac{\partial \ln Y^*}{\partial \ln \tau},$$

where $\frac{\partial \ln E^*}{\partial \ln Y} = 1$ since $E = Y$. Defining $r_{ij}^{\mathrm{DNL}} := \frac{R_{ij}^{\mathrm{DNL}}}{M_i^E}$, it can be shown that $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln \mathbb{P}} = \frac{\sigma-1}{\sigma}\left(r_D^{\mathrm{DNL}} + r_X^{\mathrm{DNL}}\right)$, $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln \tau} = \frac{1-\sigma}{\sigma} r_X^{\mathrm{DNL}}$, and $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln Y} = \frac{r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}}{\sigma}$.

With the purpose of expressing $\frac{\mathrm{d}\ln \mathbb{P}^*}{\mathrm{d}\ln \tau}$ in a way that allows us to estimate the model subsequently, we define the following variables. First, let $d^{\mathrm{DNL}} := \frac{R_D^{\mathrm{DNL}}}{R_D^{\mathrm{DNL}}+R_X^{\mathrm{DNL}}}$, which is the domestic intensity of DNLs, i.e., the domestic sales of DNLs relative to their total sales. Likewise, define the export intensity of DNLs by $x^{\mathrm{DNL}} := 1 - d^{\mathrm{DNL}}$. Finally, let $\widetilde{s}_D^{\omega} := \frac{R_D^{\omega}}{Y}$ and $\widetilde{s}_X^{\omega} := \frac{R_X^{\omega}}{Y}$ be the income share of DL ω relative to the sector's total income.

Notice that s_D^{ω} and s_X^{ω} are market shares, which correspond to the sales of ω in each market relative to the sector's expenditure. Besides, $\widetilde{s}_D^{\omega}$ and $\widetilde{s}_X^{\omega}$ measure the sales of ω in each market relative to the total income of the sector. In a one-sector economy, the total income and expenditure of the country are equal, determining that the terms s_D^{ω} and $\widetilde{s}_D^{\omega}$, as well as s_X^{ω} and $\widetilde{s}_X^{\omega}$, coincide. We distinguish between both terms based on the scenario that we explore later, where there is an outside sector and, so, $\widetilde{s}_D^{\omega}$ and s_D^{ω}, and $\widetilde{s}_X^{\omega}$ and s_X^{ω} do not necessarily have to be equal. Intuitively, income shares are used to account for the effects of profits on the economy, while market shares are utilized in different expressions to reflect market power (e.g., in the firm's price elasticity of demand).

Using these definitions, the total effect of τ on the price index is given by

$$\frac{\mathrm{d}\ln \mathbb{P}^*}{\mathrm{d}\ln \tau} = \frac{x^{\mathrm{DNL}} + \frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{\widetilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{\widetilde{s}_D^{\omega}}{\varepsilon_D^{\omega}} - \sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{\widetilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}}}{1 + \frac{\sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{\widetilde{s}_D^{\omega}}{\varepsilon_D^{\omega}} \frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega} s_D^{\omega}} + \sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{\widetilde{s}_X^{\omega}}{\varepsilon_X^{\omega}} \frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega} s_X^{\omega}}}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{\widetilde{s}_D^{\omega}}{\varepsilon_D^{\omega}} - \sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{\widetilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}}}, \tag{7}$$

which implies that a trade liberalization always reduces the price index.

Given $\frac{\mathrm{d}\ln \mathbb{P}^*}{\mathrm{d}\ln \tau}$, we are able to recover the total effect of τ on Y^* and, hence, on E^*. This is obtained by $\frac{\mathrm{d}\ln Y^*}{\mathrm{d}\ln \tau} = \frac{\partial \ln Y^*}{\partial \ln \tau} + \frac{\partial \ln Y^*}{\partial \ln \mathbb{P}}\frac{\mathrm{d}\ln \mathbb{P}^*}{\mathrm{d}\ln \tau}$, which establishes that

$$\frac{\mathrm{d}\ln Y^*}{\mathrm{d}\ln \tau} = \frac{\mathrm{d}\ln E^*}{\mathrm{d}\ln \tau} = (\sigma-1)\frac{\left(\sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{\widetilde{s}_D^{\omega}}{\varepsilon_D^{\omega}} \frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega} s_D^{\omega}}\right)\left(\frac{\mathrm{d}\ln \mathbb{P}^*}{\mathrm{d}\ln \tau}\right) - \left(\sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{\widetilde{s}_X^{\omega}}{\varepsilon_X^{\omega}} \frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega} s_X^{\omega}}\right)\left(1-\frac{\mathrm{d}\ln \mathbb{P}^*}{\mathrm{d}\ln \tau}\right)}{1-\sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{\widetilde{s}_D^{\omega}}{\varepsilon_D^{\omega}} - \sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{\widetilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}}. \tag{8}$$

The result indicates that a trade liberalization can increase or reduce the country's income, and the following condition determines when this occurs:

$$\operatorname{sgn}\left(\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}\right)=\operatorname{sgn}\left[\left(\sum_{\omega\in\Omega_{D}^{\mathrm{DL}}}\frac{\widetilde{s}_{D}^{\omega}}{\varepsilon_{D}^{\omega}}\frac{\sigma-\sigma s_{D}^{\omega}}{\sigma-\varepsilon_{D}^{\omega}s_{D}^{\omega}}+\sum_{\omega\in\Omega_{X}^{\mathrm{DL}}}\frac{\widetilde{s}_{X}^{\omega}}{\varepsilon_{X}^{\omega}}\frac{\sigma-\sigma s_{X}^{\omega}}{\sigma-\varepsilon_{X}^{\omega}s_{X}^{\omega}}\right)x^{\mathrm{DNL}}-\sum_{\omega\in\Omega_{D}^{\mathrm{DL}}}\frac{\widetilde{s}_{D}^{\omega}}{\varepsilon_{D}^{\omega}}\right]. \tag{9}$$

To provide an interpretation of (9), suppose that DLs make decisions without taking into account their impact on the price index. In this way, we are able to ignore differences between DLs and DNLs by their price elasticity of demand, and isolate the effect due to net positive profits of DLs. It can be shown that, if this is the case,

$$\operatorname{sgn}\left(\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}\right)=\operatorname{sgn}\left(\frac{\frac{R_{X}^{\mathrm{DNL}}}{\sigma}}{\frac{R_{D}^{\mathrm{DNL}}}{\sigma}}\frac{R_{D}^{\mathrm{DL}}}{\sigma}-\frac{R_{X}^{\mathrm{DL}}}{\sigma}\right) \tag{10}$$

where $R_{D}^{\mathrm{DL}}:=\sum_{\omega\in\Omega_{D}^{\mathrm{DL}}}R_{D}^{\omega}$ and $R_{X}^{\mathrm{DL}}:=\sum_{\omega\in\Omega_{X}^{\mathrm{DL}}}R_{X}^{\omega}$.

Each of the terms in the right-hand side of (10), where revenues are divided by σ, indicates the gross aggregate profits for each type of firm in each market. When there is a trade liberalization, income varies exclusively by the impact on the DLs' profits, which is affected directly and indirectly. The direct effect is summarized by the term $R_{X}^{\mathrm{DL}}/\sigma$ and it measures the total increase in the DLs' profits due to new export opportunities. As for the indirect effects, a trade liberalization creates a tougher competitive environment in each market, which is captured by a reduction in the price index. The term $\frac{R_{X}^{\mathrm{DNL}}/\sigma}{R_{D}^{\mathrm{DNL}}/\sigma}$ reflects its magnitude by representing the proportional variation of price index that ensures zero expected profits, i.e., such that the DNLs' gains from new export opportunities are compensated by their losses in each market. Thus, the total indirect effect on the DLs' profits is summarized by a term reflecting the variation in the price index times their domestic gross profits, i.e., $\frac{\frac{R_{X}^{\mathrm{DNL}}}{\sigma}}{\frac{R_{D}^{\mathrm{DNL}}}{\sigma}}R_{D}^{\mathrm{DL}}/\sigma$. Overall, if the losses of DLs due to tougher competition are equal to the gains they obtain by the new export opportunities, the total profits of DLs do not vary and, so, a trade liberalization has a zero net impact on income.

As for the general case, where DLs take into account their influence on the price index, a similar interpretation can be provided. (9) only differs because it adds that the price elasticity and the impact of the price index on their profits depend on each DL's market share.

Finally, the total change in welfare, measured in terms of real income $\mathbb{W}:=\frac{Y}{\mathbb{P}}$, is

$$\frac{\mathrm{d}\ln\mathbb{W}^{*}}{\mathrm{d}\ln\tau}:=\frac{\mathrm{d}\ln Y^{*}}{\mathrm{d}\ln\tau}-\frac{\mathrm{d}\ln\mathbb{P}^{*}}{\mathrm{d}\ln\tau}. \tag{11}$$

Given that the total effect on income is indeterminate, gains of trade can be positive or negative.[16]

5 Trade Liberalization in a Small Economy

In this section, we consider the experiment of a trade liberalization in a small country, which we label H and refers to it as home. This characterization is consistent with the country we study empirically, i.e., Denmark. Furthermore, in order to concentrate on the effects of the DLs' profits on income, we add the existence of an outside sector that pins down home wages.

The combination of these assumptions enables us to relax several of the assumptions maintained in the previous section, while keeping the model tractable. First, it permits the incorporation of asymmetries between countries in the empirical analysis, without having to make an assumption regarding the market share of firms serving a country through exports exclusively. This is relevant since we do not observe the market shares of Danish firms abroad or the market share of each importer serving Denmark.

In addition, the inclusion of an outside sector determines that incomes and expenditures of the differentiated sector need not be equal. This allows for a distinction between the income share of each DL (which is relative to the country's total income) and their market shares (which are relative to the industry expenditures). In terms of the mechanisms captured, it highlights the differential impact of income and price index on welfare: while reductions in the price index affect only the liberalized sector, variations in income affect all industries simultaneously. Thus, a trade liberalization in the differentiated sector also impacts the rest of the sectors.

Next, we proceed by, first, describing the specifics of the setup considered and, then, deriving and analyzing the results.

5.1 Setup

We focus on some country H and consider the existence of two sectors, with one of them being a differentiated industry with expenditure share $\beta_H < 1$. The other sector comprises a homogeneous good produced and sold under perfect-competition conditions. Moreover, in equilibrium, this good is always produced in H and its technology requires one unit of work to be produced. Consequently, this sector pins down wages in H, which we take as the numéraire.

[16] In Appendix E we provide examples where the effect on income is negative and so pronounced that welfare decreases with a trade liberalization.

We consider the experiment of a trade liberalization in H, where H is a small country in the sense of Demidova and Rodríguez-Clare (2009; 2013). This definition establishes that changes in country H do not affect the aggregate conditions of any foreign country, i.e., $\left(\mathbb{P}_j^*, Y_j^*, E_j^*, M_j^{E*}, w_j^*\right)_{j \in \mathcal{C} \setminus \{H\}}$. As a corollary, even though DLs from H are non-negligible for their local industry, they cannot affect any foreign price index. Thus, DLs have market power domestically but not abroad.

Notice that, under the assumption of a small economy, the model still allows for extensive and intensive margin adjustments of foreign firms after a reduction in importing trade barriers in H. This follows because the survival productivity in H of foreign firms and their price decisions are affected by changes in trade costs.

With a CES demand and a single-sector economy, Demidova and Rodríguez-Clare (2013) show that this conception of a small economy can be justified through a framework where H has a population size that is negligible relative to the rest of the world. When wages are determined exogenously, Alfaro (2019) shows that it can be rationalized through a model in which each country takes H as part of a continuum of trading partners. Importantly, this rationalization also applies to our setting with large firms.

The assumption of a small economy establishes that, for the determination of results, we can treat foreign countries as if they were a single entity. Due to this, we directly describe the model taking F as a composite country representing the rest of the world. In addition, we consider a trade liberalization taking the form of infinitesimal proportional decreases in trade costs. As in the case of symmetric countries, all the results are expressed in elasticity terms.

5.2 Results

Incorporating that H is a small economy and that its wages are already pinned down, the equilibrium can be characterized through values $\left(\mathbb{P}_H^*, Y_H^*, E_H^*, M_H^{E*}\right)$ such that (FE), (MS), (INC), and (EXD) for H hold.

Some properties of the equilibrium are worthy of remark. First, exploiting the separability of the equilibrium-conditions system, we are able to identify $(\mathbb{P}_H^*, Y_H^*, E_H^*)$ without needing to use (MS) or determine M_H^{E*}.

Second, with wages determined exogenously, a (small or large) reduction in the importing trade costs of the sector in H has no impact on its competitive conditions. Rather, since it only affects (MS) and the system is separable, this type of shock entails only variations in M_H^{E*}, without affecting $\mathbb{P}_H^*$. Due to this, the competitive conditions and income of H are affected exclusively by export-related channels. Formally, $(\mathbb{P}_H^*, Y_H^*, E_H^*)$ is only impacted by variations in τ_{HF}. On the other hand, changes in τ_{FH} only impact M_H^{E*} without affecting $(\mathbb{P}_H^*, Y_H^*, E_H^*)$ and, consequently, have a null impact on H's welfare at the sector and country level. This implies that, to determine the effects of a trade liberalization on welfare, it is only necessary to describe how new export opportunities affect H.[17]

To characterize the equilibrium, we proceed as in Section 4. We begin by characterizing the partial effects of the implicit solution to (INC) for H, given by $Y_H(\mathbb{P}_H; \tau_{HF})$. Since $E_H = \beta_H Y_H$, and so $\mathrm{d}\ln E_H = \mathrm{d}\ln Y_H$, this also describes the log changes in expenditures.

The total profits of DLs from H are

$$\Pi_H^{\mathrm{DL}} = \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \left(\frac{R_{HH}(\mathbb{P}_H, E_H, \varphi_\omega)}{\varepsilon\left[s_{HH}(\mathbb{P}_H, \varphi_\omega)\right]} - f_{HH} \right) + \sum_{\omega\in\Omega_{HF}^{\mathrm{DL}}} \left(\frac{R_{HF}(\mathbb{P}_F, E_F, \varphi_\omega; \tau_{HF})}{\sigma} - f_{HF} \right),$$

which, using (INC), determines that

$$\frac{\partial \ln Y_H}{\partial \ln \mathbb{P}_H} = \frac{\partial \ln E_H}{\partial \ln \mathbb{P}_H} = (\sigma - 1) \frac{\sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^\omega}{\varepsilon_{HH}^\omega} \frac{\sigma - \sigma s_{HH}^\omega}{\sigma - \varepsilon_{HH}^\omega s_{HH}^\omega}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^\omega}{\varepsilon_{HH}^\omega}},$$

$$\frac{\partial \ln Y_H}{\partial \ln \tau_{HF}} = \frac{\partial \ln E_H}{\partial \ln \tau_{HF}} = (1 - \sigma) \frac{\sum_{\omega\in\Omega_{HF}^{\mathrm{DL}}} \frac{R_{HF}^\omega}{\sigma}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^\omega}{\varepsilon_{HH}^\omega}}.$$

Regarding (FE) for H, let $E_H^* := E_H(\mathbb{P}_H^*; \tau_{HF})$ be the implicit solution to (EXD). By making use of E_H^*, the expected profits of firms from H evaluated at equilibrium values can be expressed as a function $\pi_H^{\mathbb{E},\mathrm{DNL}}(\mathbb{P}_H^*, E_H^*; \tau_{HF})$. Thus, (FE) for H is given by

$$\pi_H^{\mathbb{E},\mathrm{DNL}} := \int_{\varphi_{HH}^*}^{\overline{\varphi}_H} \left[\frac{R_{HH}(\mathbb{P}_H^*, E_H^*, \varphi)}{\sigma} - f_{HH} \right] \mathrm{d}G_H(\varphi) + \int_{\varphi_{HF}^*}^{\overline{\varphi}_H} \left[\frac{R_{HF}(\mathbb{P}_F^*, E_F^*, \varphi; \tau_{HF})}{\sigma} - f_{HF} \right] \mathrm{d}G_H(\varphi) = F_H, \tag{12}$$

where φ_{ij}^* corresponds to the function $\varphi_{ij}^*\left(\mathbb{P}_j^*, E_j^*; \tau_{ij}\right)$.

Differentiating (12), it is established that $\frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}} = -\left(\frac{\mathrm{d}\pi_H^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \mathbb{P}_H}\right)^{-1} \frac{\mathrm{d}\pi_H^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \tau_{HF}}$. Using that $\frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln \mathbb{P}_H} = \frac{\sigma-1}{\sigma} r_{HH}^{\mathrm{DNL}}$, $\frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln \tau_{HF}} = \frac{1-\sigma}{\sigma} r_{HF}^{\mathrm{DNL}}$ and $\frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln Y_H} = \frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln E_H} \frac{\partial \ln E_H}{\partial \ln Y_H} = \frac{r_{HH}^{\mathrm{DNL}}}{\sigma}$, it is determined that

$$\frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}} = \frac{\frac{R_{HF}^{\mathrm{DNL}}}{R_{HH}^{\mathrm{DNL}}} + \frac{\sum_{\omega\in\Omega_{HF}^{\mathrm{DL}}} \frac{R_{HF}^\omega}{\sigma}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^\omega}{\varepsilon_{HH}^\omega}}}{1 + \frac{\sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^\omega}{\varepsilon_{HH}^\omega} \frac{\sigma - \sigma s_{HH}^\omega}{\sigma - \varepsilon_{HH}^\omega s_{HH}^\omega}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^\omega}{\varepsilon_{HH}^\omega}}}. \tag{13}$$

From this, we conclude that, as in the case of symmetric countries, $\frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}} > 0$. Thus, a trade

[17]This is part of a more general result regarding the Melitz model. With a CES demand, Melitz (2003) indicates that "the model should also be interpreted with caution as it precludes another potentially important channel for the effects of trade, which operates through increases in import competition." Alfaro (2019) shows that, at the microeconomic level (i.e., without considering variations in wages and income) and, for any demand that depends on a single sufficient statistic, the same conclusion applies.

liberalization always decreases the price index.

To take the model to the data subsequently, we reexpress $\frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}}$. To do this, we define the following variables. Let Y^{sec} be the income of the differentiated sector and $\alpha_H := \frac{Y_H^{\text{sec}}}{Y_H}$ its value expressed relative to the country's total income. Also, as in the case of symmetric countries, define $\widetilde{s}_{Hj}^{\omega} := \frac{R_{Hj}^{\omega}}{Y_H^{\text{sec}}}$ as the fraction of the differentiated sector's income in H due to sales in $j \in \mathcal{C}$ of a DL ω from H. Finally, let $d_H^{\text{DNL}} := \frac{R_{HH}^{\text{DNL}}}{R_{HH}^{\text{DNL}}+R_{HF}^{\text{DNL}}}$ and $x_H^{\text{DNL}} := 1 - d_H^{\text{DNL}}$ be the domestic and export intensity of DNLs from H, respectively.

Making use of these definitions and multiplying and dividing by Y_H^{sec}, (13) can be expressed as

$$\frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}} = \frac{\frac{x_H^{\text{DNL}}}{d_H^{\text{DNL}}} + \frac{\sum_{\omega\in\Omega_{HF}^{\text{DL}}} \frac{\widetilde{s}_{HF}^{\omega}}{\sigma}}{(\alpha_H)^{-1} - \sum_{\omega\in\Omega_{HH}^{\text{DL}}} \frac{\widetilde{s}_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}}}{1 + \frac{\sum_{\omega\in\Omega_{HH}^{\text{DL}}} \frac{\widetilde{s}_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}} \frac{\sigma - \sigma s_{HH}^{\omega}}{\sigma - \varepsilon_{HH}^{\omega} s_{HH}^{\omega}}}{(\alpha_H)^{-1} - \sum_{\omega\in\Omega_{HH}^{\text{DL}}} \frac{\widetilde{s}_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}}}. \tag{14}$$

Furthermore, the total effect on income and expenditure is given by $\frac{\partial \ln Y_H^*}{\partial \ln \tau_{HF}} + \frac{\partial \ln Y_H^*}{\partial \ln \mathbb{P}_H} \frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}}$, so that

$$\frac{\mathrm{d}\ln Y_H^*}{\mathrm{d}\ln \tau_{HF}} = \frac{\mathrm{d}\ln E_H^*}{\mathrm{d}\ln \tau_{HF}} = \frac{(\sigma - 1)}{(\alpha_H)^{-1} - \sum_{\omega\in\Omega_{HH}^{DL}} \frac{\widetilde{s}_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}} \left[\frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}} \left(\sum_{\omega\in\Omega_{HH}^{DL}} \frac{\widetilde{s}_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}} \frac{\sigma - \sigma s_{HH}^{\omega}}{\sigma - \varepsilon_{HH}^{\omega} s_{HH}^{\omega}} \right) - \sum_{\omega\in\Omega_{HF}^{\text{DL}}} \frac{\widetilde{s}_{HF}^{\omega}}{\sigma} \right]. \tag{15}$$

To determine whether a trade liberalization increases the country's income, the following condition can be used:

$$\text{sgn}\left(\frac{\mathrm{d}\ln Y_H^*}{\mathrm{d}\ln \tau_{HF}} \right) = \text{sgn}\left(\frac{x_H^{\text{DNL}}}{d_H^{\text{DNL}}} - \frac{\sum_{\omega\in\Omega_{HF}^{\text{DL}}} \frac{\widetilde{s}_{HF}^{\omega}}{\sigma}}{\sum_{\omega\in\Omega_{HH}^{\text{DL}}} \frac{\widetilde{s}_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}} \frac{\sigma - \sigma s_{HH}^{\omega}}{\sigma - \varepsilon_{HH}^{\omega} s_{HH}^{\omega}}} \right). \tag{16}$$

Remarkably, condition (16) can be interpreted in a similar way to the condition stated for symmetric countries. In fact, if DLs make decisions ignoring their influence on the domestic price index, (16) collapses to (10).

Unlike the case of a single sector, total welfare of the country takes into account that variations in income also affect the consumption of goods in the rest of the economy, captured by the outside sector. Thus, the impact on the country's welfare is

$$\begin{aligned} \frac{\mathrm{d}\ln \mathbb{W}_H^*}{\mathrm{d}\ln \tau_{HF}} &= \frac{\mathrm{d}\ln Y_H^*}{\mathrm{d}\ln \tau_{HF}} - \beta_H \frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}} \\ &= (1 - \beta_H) \left(\frac{\mathrm{d}\ln Y_H^*}{\mathrm{d}\ln \tau_{HF}} \right) + \beta_H \left(\frac{\mathrm{d}\ln Y_H^*}{\mathrm{d}\ln \tau_{HF}} - \frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}} \right). \end{aligned} \tag{17}$$

Consequently, while the effects on the price index are bounded by the relative importance of the differentiated sector (i.e., β_H), income impacts all sectors simultaneously.

As in the case of symmetric countries, gains of trade are not guaranteed. Thus, it is an empirical matter to establish whether welfare increases after a trade liberalization.[18]

6 Empirical Analysis

In this section, we perform an empirical analysis based on information for Denmark. Since this country can be considered a small economy, it constitutes a suitable choice for the approach developed in Section 5. The results draw on information for the manufacturing sector in 2005.

We proceed as follows. First, we establish that a trade liberalization increases Danish income, which guarantees the existence of gains of trade. After this, we inquire upon which channel is driving the result. Specifically, we compare whether the total effect on income or price index has a greater importance in the magnitude of welfare gains. The calculations reveal that they are almost completely due to the latter.

Second, we investigate how gains of trade compare with a monopolistic-competition model à la Melitz. The main conclusion is that, for Denmark, our setting predicts gains of trade that are considerably lower. By disentangling the effects, it is revealed that this is because each model implies different calibrations of average export intensity, which affects the magnitude of the price-index decrease.

6.1 Data Description

As indicated in Section 2, the information at our disposal is expressed at the year-firm-product level. Moreover, in terms of goods, it is disaggregated at the 8-digit level according to the Combined Nomenclature. The two datasets collected by Statistics Denmark that we utilize have information regarding production value of manufacturing firms, and exports and imports by both manufacturing and non-manufacturing firms.

To obtain results that are representative for a typical manufacturing industry, first, we aggregate goods at a 4-digit NACE industry level. After this, we average variables and outcomes across them using industry-revenue weights. For the different definitions, we follow standard practices utilized in studies of European countries based on similar datasets. See, for instance, Amiti et al. (2018) and Gaubert and Itskhoki (2018).

We consider a firm as domestic if it is included in the Danish Prodcom dataset. Consequently, the classification is according to whether they have production activities in Denmark. Furthermore, each of the variables is defined as follows. First, regarding market shares, they

[18] In Appendix E, we provide an example where welfare decreases for any value of α_H such that $\alpha_H = \beta_H$.

are expressed relative to industry expenditures, which are the sum of domestic sales and imports. In turn, to obtain a measure of these variables at the industry level, we compute each firm's domestic sales as the difference between a firm's total turnover and its exports value.[19] Concerning imports, we encompass those belonging to the industry that are acquired by domestic firms which are inactive in the industry. Thus, they cover imports by both non-manufacturing firms and domestic firms not producing in that industry. This allows us to obtain an accurate measure of import penetration in each industry, such that a DL's market share can be interpreted as a reflection of its market power.

Based on the market shares computed, we classify firms into DLs and DNLs by industry. Specifically, a domestic firm is considered as a DL in an industry if it has a market share greater than 3%, while any domestic firm with a lower market share is taken as a DNL. All the results that we obtain are almost identical if we use 5% as threshold.

Regarding revenue, we take total turnover as its measure and, for each industry, we split it into domestic and export sales. With this information, we construct domestic and export intensities of DNLs, which are calculated, respectively, as domestic and export sales by DNLs relative to the total income generated by them. In addition, we use it to calculate the income shares of each DL, whose values are expressed relative to the industry income.

Finally, for σ, we make use of the estimations by Soderbery (2015), who employs the methodology by Broda and Weinstein (2006) and improves upon it by accounting for small-sample biases. Averaging across industries with revenue weights, we obtain a value of $\sigma := 3.53$, which we use throughout the paper.

6.2 Computation

In order to define a representative Danish manufacturing industry, we base our calculations on a set of industries that are consistent with our theoretical framework. This implies that we only keep industries where DNLs coexist with DLs. In particular, this requires that in each industry there are several domestic firms with negligible market shares serving the market. This is accomplished by following the procedure in Section 2, as outlined in Footnote 11.

After this, for each of these industries, we take the top four Danish firms by domestic

[19] Total turnover is defined by economic ownership of the goods sold and produced by Danish firms. Thus, its definition is not related to the physical territory of the production. Specifically, turnover includes sales of own goods (either produced, processed or assembled by the firm), goods produced by a subcontractor established abroad (if the firm owns the inputs of the subcontracted firm), and resales of goods bought from other domestic firms and sold with any processing. However, it excludes sales of goods imported that are produced by foreign firms not owned by the Danish firm. Due to this, in Appendix C, we define domestic sales of a firm as the sum of a firm's total turnover and its imports. We show that, by proceeding in this way, the results for a representative industry are virtually identical.

market share, and consider them as DLs. The rest of the firms are taken as DNLs.[20] Relying on this classification, we compute the market and revenue shares of DLs, as well as the domestic and export intensities of DNLs. Using industry revenues as weights, the quantitative description of a Danish representative industry is as follows.[21]

Table 1. *Features of the Representative Industry*

(a) Features of DLs (in %)

Firm	Domestic Market Share	Domestic Revenues as % of sector income	Export Revenues as % of sector income
Top 1	17.83	18.90	11.11
Top 2	7.27	8.20	2.00
Top 3	4.65	5.14	3.41
Top 4	3.36	3.75	2.75

(b) Revenue Intensities (in %)

	Domestic Intensity	Export Intensity
DNLs	68.62	31.38
All Firms	58.12	41.88

Note: Calculations based on industries with coexistence of DNLs and DLs, and using industry revenues as weights. Domestic market shares are calculated relative to expenditures, which includes both domestic sales and imports. Domestic and export intensity calculated, respectively, as domestic and export sales of the group considered (i.e., DNLs and all firms) relative to the total sales of the group.

For the analysis, we consider an infinitesimal proportional decrease in trade costs. More specifically, we consider a 1% reduction in trade costs and express all the results in percentage terms. The effects are obtained by computing (7), (8), and (11) for the case of two symmetric countries, and (14), (15), and (17) for the small-economy framework.

One advantage of considering a small reduction in trade costs is that these effects can be computed without making a specific distributional assumption for the productivity of DNLs. Thus, we are able to estimate the model by using exclusively the information provided in Table 1.

6.3 Welfare Results

We start the analysis by checking whether a trade liberalization increases or decreases Danish income. For each variant of the model, this can be done by (9) and (16), respectively. The results indicate that a trade liberalization increases profits and, hence, the country's income. As a corollary, since the price index decreases for any value of parameters, there are gains of trade. In particular, for the small-economy scenario, this result holds irrespective of the values assigned to α_H and β_H, where recall that α_H is defined as the proportion of income of the differentiated sector relative to the income of the country, while β_H has a similar definition but regarding expenditures.

To quantify the relative importance of each term in welfare, we proceed by calculating the total impact of a trade liberalization on the country's price index, income, and welfare.

[20] In Appendix B we obtain results under an alternative where we classify firms according to their total revenue, rather than domestic market shares. We show that the conclusions are the same.

[21] Considering other DLs in addition to the top four does not affect the results. This is because the domestic shares of these firms become quite small and, so, their aggregate effects are insignificant.

All the results are expressed in elasticity terms, relative to a decrease of 1% in trade costs.

Regarding the symmetric-countries scenario, income has a modest increase of 0.007% while the price index decreases by 0.316%, determining an increase in total welfare of 0.324%. Thus, gains of trade stem almost exclusively from the variation in the price index.

Similar conclusions regarding welfare are obtained under the small-country setup. To show the results under this framework, however, in addition to the information included in Table 1, it is also necessary to calibrate α_H and β_H. Rather than doing this, we opt to consider them as unobservables. In particular, exploiting that differences in expenditure and income in Denmark are small, we take as a baseline case that $\alpha_H = \beta_H$, and present results in Figure 3 for all the possible range of values, i.e., $(0, 1)$.

Figure 3. *Impact of a Trade Liberalization in the Small-Country Framework*

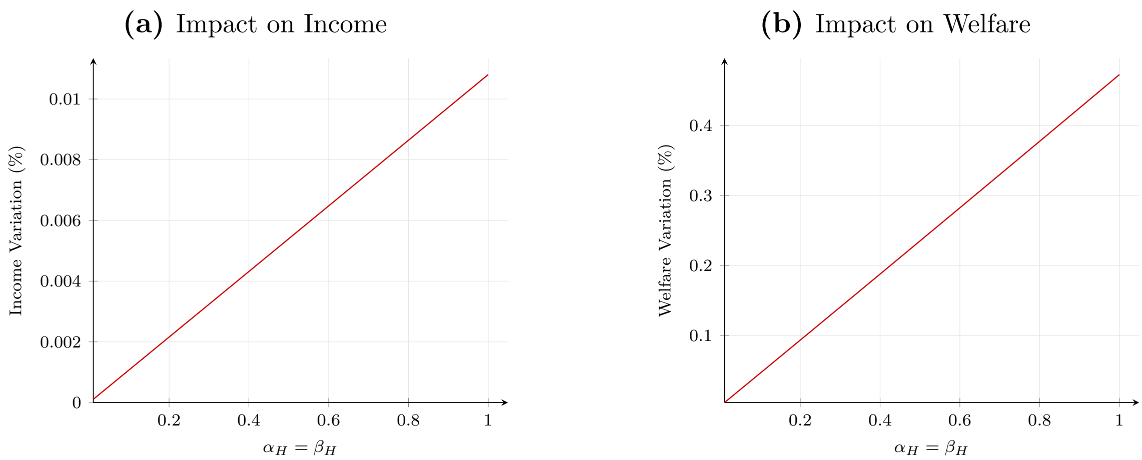


Figure 3a indicates that the maximum possible effect of a trade liberalization on income is slightly above 0.011%. Hence, the gains of trade are primarily driven by the reductions in price index. Furthermore, this determines increases in welfare as high as 0.472%. The result is demonstrated in Figure 3b.

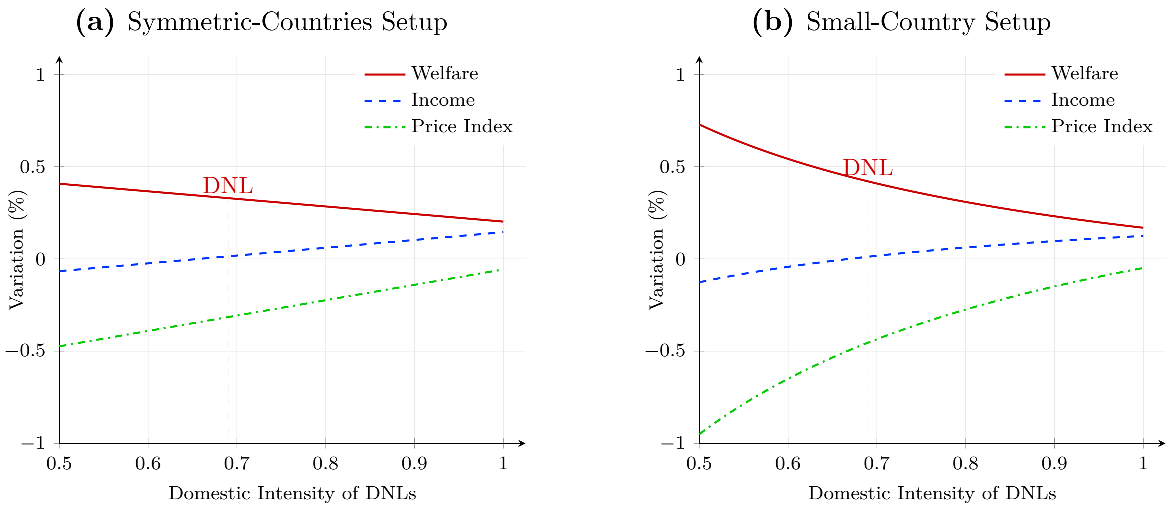
The fact that the variation in income is almost null responds to how profits of each DL are affected by a trade liberalization. This experiment creates two opposing effects. On the one hand, for the Danish economy, all DLs are exporters and, hence, better export access increases their profits. On the other hand, lower trade costs create tougher competitive conditions, which reduces their profits. Overall, although the effect of better export conditions dominates, this is almost entirely offset by the tougher competitive conditions.

To provide some illustrations regarding these offsetting effects, next we resort to some visual aids. Through them, we keep the benefits of DLs from new export opportunities fixed, but vary the losses they have from tougher competitive conditions. Specifically, we do this by showing how income and the price index are affected by the export intensity of DNLs, which is a key determinant of how tough competitive conditions become after a trade liberalization.

To explain why this variable plays such a role, consider the setting where Denmark is treated as a small economy, so that the price index in the foreign country is taken as given. The export intensity of DNLs provides information about the magnitude in which export opportunities impact a DNL's expected profits. Specifically, a greater export intensity (or, equivalently, a lower domestic intensity) is associated with a higher impact on expected profits. In turn, a more pronounced increase in expected exports profits requires a more marked reduction in the domestic price index to restore zero expected profits, determining that the competitive environment becomes even tougher. Intuitively, the mechanism is such that a lower export intensity makes a trade liberalization induce less entry and, hence, the domestic competitive conditions are less affected.

In Figure 4, we consider a reduction in trade costs of 1% and show the impact of this on welfare, the price index, and income. Consistent with the empirical regularity of home bias, we indicate values of domestic intensity greater than 50%. Also, we indicate through the label "DNL" the domestic intensity that is observed for Danish DNLs. For the small-economy setup, the graph is drawn supposing that $\alpha_H = \beta_H = 0.9$, with the same conclusions holding for any other value.

Figure 4. *Impact of a Trade Liberalization on Welfare, Price Index, and Income*



The graphs capture the mechanism described above. Thus, when the domestic intensity approaches one, we can observe that the price index barely decreases. Consequently, DLs are mainly benefited from the better export access without being so affected by the tougher competition. On the other hand, if the domestic intensity is low, so that DNLs export more intensively, a trade liberalization induces more entry and, therefore, it determines a more pronounced decrease in the price index. This, in turn, impacts a DL's profits negatively. Thus, if the export intensity of DNLs is high enough, the graphs reveal that income could even potentially decrease following a trade liberalization.

Regarding the Danish economy, Table 1b indicates that the domestic intensity of DNLs is 68%. For that level of domestic intensity, Figure 4 points out that the variation in income is close to zero, revealing that each mechanism affecting the DLs' profits is similar in magnitude.

6.4 Comparison with Monopolistic Competition

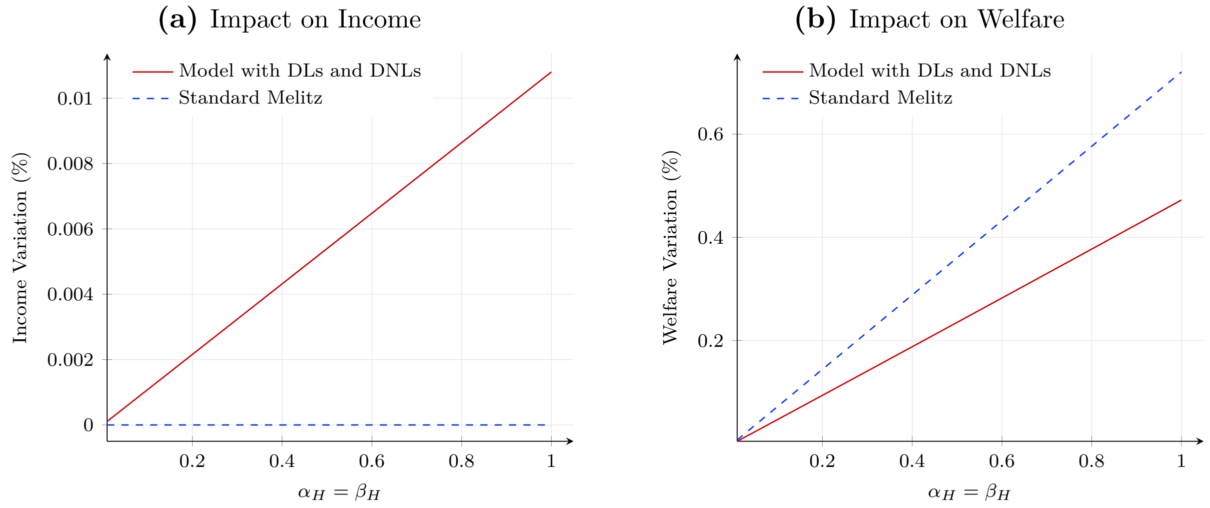
Next, we proceed to compare the welfare calculations of our model relative to an alternative where all firms are considered negligible. In particular, we do this relative to the standard Melitz model. This requires us to estimate the gains of trade in the symmetric-countries and small-country scenarios under this setup, which we do as follows.

First, we exploit the fact that our setting collapses to the Melitz model when the set of DLs is empty. In other terms, Melitz constitutes a special case of our framework in which all firms are regarded as DNLs. Thus, by construction, aggregate profits are not impacted by a trade liberalization, and welfare changes are completely determined by variations in the price index. Due to this, it can be shown that the welfare variations in the symmetric-countries and small-economy scenarios become x_H^{DNL} and $\frac{x_H^{\mathrm{DNL}}}{d_H^{\mathrm{DNL}}}$, respectively.

Second, in order to compute welfare, it is necessary to recalibrate the domestic and export intensities of DNLs. This follows because the characterization of firms in Melitz entails that any DNL makes entry decisions considering the whole productivity distribution for the calculation of its expected profits, including the range corresponding to DLs. Thus, by utilizing the information in Table 1b, it is established that the domestic and export intensities are 58.12% and 41.88%, respectively. Notice that, compared to the DNLs in our model, the average domestic intensity in Melitz is higher and, hence, the export intensity lower.

Using this calibration, welfare in Melitz is 0.419% in the symmetric-countries scenario, which determines that gains of trade in our setting are 22.6% lower. Furthermore, in Figure 5, we contrast the results for the small-country scenario. For values of α_H and β_H such that $\alpha_H = \beta_H$, the comparison of outcomes predicts an almost constant percentage relation, with gains of trade in our model that are around 35% lower.

Figure 5. *Impact of a Trade Liberalization in the Small-Country Framework*



The question that arises in this context is what explains these differences in welfare gains. To provide an answer, recall that, once that the optimal survival productivity cutoffs are substituted in, the price index can be obtained by the free-entry condition of each model. This, in turn, depends on the features of DNLs exclusively. Consequently, DLs only affect welfare indirectly through its impact on income. From this, we conclude that, primarily, there are two potential sources of differences between our setting and Melitz's:

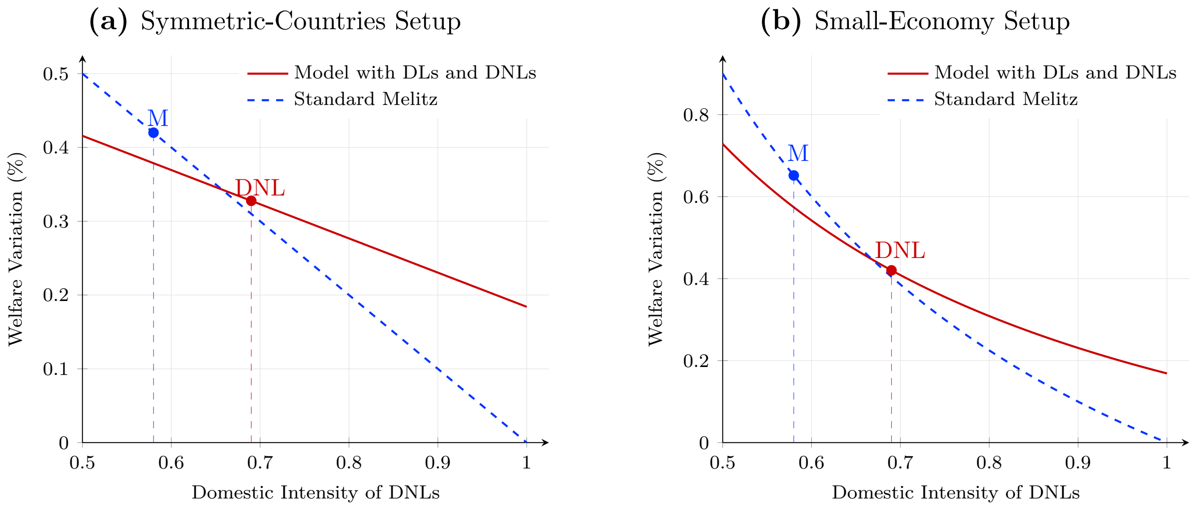
[i] *Changes in income*: In Melitz, a trade liberalization has no impact on profits and, hence, income. On the contrary, in our setting, income affects welfare directly and, also, indirectly through its impact on the price index.

[ii] *Changes in the average export intensity of firms*: In our model, DNLs discard from consideration the productivity draws of DLs. This entails that they make entry decisions conceiving more likely that, eventually, they either serve the domestic market exclusively or export at a lower scale. In other words, they make entry choices by expecting a lower export intensity relative to what happens in Melitz. This factor is crucial for the determination of the price index: the lower the export intensity of DNLs, the lower the impact that a trade liberalization has on DNLs' expected profits and, so, the less the domestic price index has to decrease to restore zero expected profits.

In Figure 6 we present the welfare outcomes of each model within the same graph. The equilibrium in our setting is indicated by a red dot, while a blue dot references the equilibrium in Melitz. Also, to facilitate the comparison, we indicate through the label "DNL" the domestic intensity that is observed for Danish DNLs, while "M" refers to the domestic intensity calculated considering all Danish firms. Next, we make use of these graphs to investigate the role that *i)* and *ii)* play in the differences in gains between models.

Suppose that both DNLs and firms in Melitz had the same domestic intensity, so that *ii)* plays no role and the models differ only by the effects operating through income that they entail. In particular, assume that the domestic intensity in both models is that of DNLs (i.e., 68%). If this were the case, we can appreciate that the differences between the red dot and the blue line at a domestic intensity of 68% are quite modest and, in fact, predict that gains in our model would be higher. Thus, the direct and indirect impact of income on welfare is not explaining the outcome.

Figure 6. *Welfare Comparison Across Models*



As for *ii)*, once we let the domestic intensity vary, the decrease in the price index is greater in Melitz. This is because, in the Danish economy, the calibration of domestic intensity for all firms is lower than the one of DNLs in our setting. Therefore, we conclude that *ii)* causes that gains of trade in our model are lower.

To explain why the price index decreases more in Melitz, recall that the calibration of domestic intensity provides information about the magnitude in which a better export access impacts a DNL's expected profits. Therefore, the lower domestic intensity in Melitz implies a greater export intensity and, hence, a greater impact on expected profits following a trade liberalization. Due to this, the price index decreases more markedly to restore zero expected profits in Melitz.

6.5 Additional Results

In the appendices, we provide some additional results. First, in Appendix B, we recalculate all the empirical results for a calibration where DLs and DNLs are classified according to total revenue, rather than domestic market shares. This broadens the scope of large firms to include big exporters. Under this alternative, we show that the same qualitative conclusions are obtained, with quantitative results that are quite similar. This is consistent with the empirical fact stated in Section 2, which indicates that being a DL is an almost sufficient condition for being among the firms with highest revenues in each industry.

Furthermore, in our empirical application, we have obtained results based on the calibration for the Danish economy. In turn, the features of the country determine that the effect of a trade liberalization on income is quite small. This could wrongly convey the idea that, in our model, the impact on income is always insignificant or, at least, always lower than the variations in the price index. Due to this, we show that, for different shares distributions of firms, the model can accommodate various outcomes. In particular, we demonstrate the following.

- In Appendix D, we study the magnitude and determinants of mechanisms operating in the model when the distributions of DLs' shares vary. We demonstrate that, unlike what happens in the Danish case, the impact on income can be quite significant. A corollary of this is that gains of trade in our model might be greater than in the Melitz model. In addition, the examples underscore the trade-off in profits following a trade liberalization: depending on whether DLs have incomes coming mainly from its domestic or foreign market, a trade liberalization is able to create a negative or positive effect on income.
- In Appendix E, we provide examples where a trade liberalization decreases the country's income in such a magnitude that it ends up reducing welfare.

7 Conclusion

In this paper, we studied the effects of trade liberalization accounting for the presence of large firms. We started the analysis by showing some empirical facts regarding Danish manufacturing. More precisely, we showed that the bulk of revenue comes from industries where there is a coexistence of a few firms with great domestic market shares and numerous firms with insignificant domestic market shares. Furthermore, in each of these industries, we established that the leaders generate a sizable portion of the total revenues.

After this, we proposed a model to structurally estimate gains of trade accounting for this fact. This framework features firm-level heterogeneity, extensive-margin adjustments, and large firms garnering positive profits that are passed back to consumers. Basically, it consists of embedding a set of large firms into an otherwise standard Melitz model.

We considered two different variants of our model. In the first one, we supposed a world economy with two symmetric countries. Obtaining outcomes for this scenario facilitates the understanding of how our model differs relative to standard frameworks. In the second variant, we proposed a framework in line with our empirical application, where the country under analysis was treated as a small economy. This made it possible to, subsequently, obtain estimations accounting for asymmetries between countries without requiring further information relative to the symmetric-countries scenario.

Empirically, we estimated the model for Danish manufacturing and assessed quantitatively the effects of a trade liberalization. The conclusions were twofold. First, a trade liberalization has a positive impact on income, although this is negligible. The reason is that the better export opportunities created are almost entirely offset by tougher competitive conditions.

Second, we compared welfare outcomes relative to the standard monopolistic competition framework, where all firms are considered negligible in their industries. In particular, in comparison to the Melitz model, the results indicated that the gains of trade predicted in our model are lower. We showed that the reason for this does not lie in the variations in income stemming from non-zero aggregate profits, since the direct and indirect impact of income on welfare is quite small. Rather, it responds to the calibration of export intensity implied by each model, which affects the magnitude in which the price index decreases. This can be understood by the fact that Melitz is a particular case of our model where all firms are considered DNLs. In addition, in both our model and Melitz, the price index is identified by the zero-expected-profits condition once optimal variables are substituted in. Thus, given that in Melitz the export intensity of firms is greater than that of DNLs in our model, firms make entry decisions expecting, on average, to export more intensively. This implies that export opportunities have a greater impact on expected profits in Melitz, thereby inducing more entry to the industry and creating a more pronounced decrease in the price index to restore zero expected profits.

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Online Appendix - not for publication

In these appendices, we include the following. In Appendix A, we provide derivations for the results in the main part of the paper. In Appendix B, we estimate the model for an alternative where we identify top Danish firms by total industry revenues. This establishes that, qualitatively, the same conclusions as in the main part of paper hold. In Appendix C, we consider alternative approaches to compute domestic market shares, which point out that they are almost identical to those included in the main part of the paper. In Appendix D, we illustrate the operating mechanisms of the model by considering different shares distributions. The goal is to show how outcomes differ according to the features of DLs, thus remarking on other results that might arise for other parameters calibrations. Finally, in Appendix E, we provide examples demonstrating that gains of trade are not guaranteed in our model. In particular, for both the symmetric-countries and small-country scenarios, the impact of a trade liberalization on income can be substantially negative to determine losses from trade.

A Derivations

In Appendix A.1, we provide some intermediate results that help us derive results more easily subsequently. In Appendix A.2, we do it for the case of symmetric countries while, in Appendix A.3, we do it for the small-country scenario.

To keep notation simple, we only include those variables that are relevant for the derivations as arguments of the functions.

A.1 Intermediate Results

Next, we derive some intermediate results for prices and market shares (Appendix A.1.1) and total profits of DLs (Appendix A.1.2).

A.1.1 Prices and Market Shares

Given optimal prices, we proceed to determine the partial effect of the price index on the market share and price elasticity of a DL ω from $i \in \mathcal{C}$ in $j \in \mathcal{C}$. Regarding the effect of market share on prices, by (4), we can express domestic prices by $\ln p_{ij}^{\omega} = \ln m_{ij}^{\omega} + \ln c_{ij}^{\omega}$. Therefore,

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{\partial \ln m_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{\partial \ln m_{ij}^{\omega}}{\partial \ln \varepsilon_{ij}^{\omega}} \frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}}.$$

In turn, $\ln m_{ij}^{\omega} = \ln \varepsilon_{ij}^{\omega} - \ln \left(\varepsilon_{ij}^{\omega} - 1\right)$ and $\varepsilon_{ij}^{\omega} = \sigma + s_{ij}^{\omega}(1-\sigma)$. Consequently, $\frac{\partial \ln m_{ij}^{\omega}}{\partial \ln \varepsilon_{ij}^{\omega}} = 1 - \frac{\varepsilon_{ij}^{\omega}}{\varepsilon_{ij}^{\omega}-1} = 1 - m_{ij}^{\omega}$, and $\frac{\partial \varepsilon_{ij}^{\omega}}{\partial s_{ij}^{\omega}} = 1 - \sigma$ so that $\frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{s_{ij}^{\omega}(1-\sigma)}{\varepsilon_{ij}^{\omega}}$. Thus,

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \left(1 - m_{ij}^{\omega}\right) \frac{s_{ij}^{\omega}(1-\sigma)}{\varepsilon_{ij}^{\omega}},$$

which, by using that $1 - m_{ij}^{\omega} = \frac{-1}{\varepsilon_{ij}^{\omega} - 1}$ and $\varepsilon_{ij}^{\omega} - 1 = (\sigma - 1)\left(1 - s_{ij}^{\omega}\right)$, becomes

$$\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}} = \frac{s_{ij}^{\omega}}{\left(1 - s_{ij}^{\omega}\right)\varepsilon_{ij}^{\omega}}. \tag{18}$$

Substituting (4) into (2), market shares satisfy $s_{ij}^{\omega} = \left(\frac{p_{ij}^{\omega}\left(s_{ij}^{\omega}\right)}{\mathbb{P}_j}\right)^{1-\sigma}$, which determines an implicit function $s_{ij}^{\omega}\left(\mathbb{P}_i\right)$. Differentiating it, we obtain $\mathrm{d}\ln s_{ij}^{\omega}\left[1 - (1-\sigma)\frac{\partial \ln p_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}}\right] = (1-\sigma)\,\mathrm{d}\ln \mathbb{P}_j$. Working out the expression, and using (18) and that $(\sigma - 1)\, s_{ij}^{\omega} = \sigma - \varepsilon_{ij}^{\omega}$, we obtain the following

$$\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} = (\sigma - 1)\frac{\varepsilon_{ij}^{\omega} - \varepsilon_{ij}^{\omega} s_{ij}^{\omega}}{\sigma - \varepsilon_{ij}^{\omega} s_{ij}^{\omega}}. \tag{19}$$

Moreover, using (19), we establish that

$$\frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} = \frac{\partial \ln \varepsilon_{ij}^{\omega}}{\partial \ln s_{ij}^{\omega}}\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} = \frac{s_{ij}^{\omega}(1-\sigma)}{\varepsilon_{ij}^{\omega}}\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j}. \tag{20}$$

As for a DNL ω, by using (4), the partial effects of price index and trade costs on its market share in j is

$$\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} = -\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \tau_{ij}} = \sigma - 1. \tag{21}$$

A.1.2 Total Profits of DLs

Next, we determine the partial effects of expenditure, price index, and trade costs on total profits of DLs. The sum of profits of DLs from $i \in \mathcal{C}$ can be expressed as

$$\Pi_i^{\mathrm{DL}} = \sum_{\omega \in \Omega_{ii}^{\mathrm{DL}}}\left(\frac{E_i s_{ii}^{\omega}\left(\mathbb{P}_i\right)}{\varepsilon_{ii}^{\omega}\left(\mathbb{P}_i\right)} - w_i f_{ii}\right) + \sum_{j \in \mathcal{C}\setminus\{i\}}\sum_{\omega \in \Omega_{ij}^{\mathrm{DL}}}\left(\frac{E_j s_{ij}^{\omega}\left(\mathbb{P}_j; \tau_{ij}\right)}{\varepsilon_{ij}^{\omega}\left(\mathbb{P}_j\right)} - w_i f_{ij}\right), \tag{22}$$

which determines that if $\mathrm{d}\ln E_k \neq 0$, $\mathrm{d}\ln \mathbb{P}_i \neq 0$, $\mathrm{d}\ln \mathbb{P}_j \neq 0$, and $\mathrm{d}\ln \tau_{ij} \neq 0$, with $j \neq i$ and $k \in \mathcal{C}$, then

$$\mathrm{d}\Pi_i^{\mathrm{DL}} = \left(\sum_{\omega \in \Omega_{ik}^{\mathrm{DL}}}\frac{R_{ik}^{\omega}}{\varepsilon_{ik}^{\omega}}\right)\mathrm{d}\ln E_k + \left[\sum_{\omega \in \Omega_{ii}^{\mathrm{DL}}}\frac{R_{ii}^{\omega}}{\varepsilon_{ii}^{\omega}}\left(\frac{\partial \ln s_{ii}^{\omega}}{\partial \ln \mathbb{P}_i} - \frac{\partial \ln \varepsilon_{ii}^{\omega}\left(\mathbb{P}_i\right)}{\partial \ln \mathbb{P}_i}\right)\right]\mathrm{d}\ln \mathbb{P}_i + \left[\sum_{\omega \in \Omega_{ij}^{\mathrm{DL}}}\frac{R_{ij}^{\omega}}{\varepsilon_{ij}^{\omega}}\left(\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \mathbb{P}_j} - \frac{\partial \ln \varepsilon_{ij}^{\omega}\left(\mathbb{P}_j\right)}{\partial \ln \mathbb{P}_j}\right)\right]\mathrm{d}\ln \mathbb{P}_j + \left[\sum_{\omega \in \Omega_{ij}^{\mathrm{DL}}}\frac{R_{ij}^{\omega}}{\varepsilon_{ij}^{\omega}}\frac{\partial \ln s_{ij}^{\omega}}{\partial \ln \tau_{ij}}\right]\mathrm{d}\ln \tau_{ij}.$$

To obtain each of the partial derivatives of this expression, by (19), (20) and working out the expression, it is determined that, for $k \in \mathcal{C}$,

$$\frac{\partial \ln s_{ik}^{\omega}\left(\mathbb{P}_k\right)}{\partial \ln \mathbb{P}_k} - \frac{\partial \ln \varepsilon_{ik}\left[s_{ik}^{\omega}\left(\mathbb{P}_k\right)\right]}{\partial \ln \mathbb{P}_k} = (\sigma - 1)\frac{\varepsilon_{ik}^{\omega} - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}{\sigma - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}\left[1 - \frac{s_{ik}^{\omega}(1-\sigma)}{\varepsilon_{ik}^{\omega}}\right],$$

which, using that $s_{ik}^{\omega}(1-\sigma) = \varepsilon_{ik}^{\omega} - \sigma$, becomes

$$\frac{\partial \ln s_{ik}^{\omega}\left(\mathbb{P}_k\right)}{\partial \ln \mathbb{P}_k} - \frac{\partial \ln \varepsilon_{ik}\left[s_{ik}^{\omega}\left(\mathbb{P}_k\right)\right]}{\partial \ln \mathbb{P}_k} = \frac{\sigma(\sigma - 1)}{\varepsilon_{ik}^{\omega}}\frac{\varepsilon_{ik}^{\omega} - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}{\sigma - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}} = (\sigma - 1)\frac{\sigma - \sigma s_{ik}^{\omega}}{\sigma - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}.$$

In addition, by using (21) and for $k \in \mathcal{C}$ and $j \in \mathcal{C} \setminus \{i\}$, it is determined that

$$\frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln E_k} = \sum_{\omega \in \Omega_{ik}^{\mathrm{DL}}} \frac{R_{ik}^{\omega}}{\varepsilon_{ik}^{\omega}}, \tag{23}$$

$$\frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln \mathbb{P}_k} = (\sigma - 1) \sum_{\omega \in \Omega_{ik}^{\mathrm{DL}}} \frac{R_{ik}^{\omega}}{\varepsilon_{ik}^{\omega}} \frac{\sigma - \sigma s_{ik}^{\omega}}{\sigma - \varepsilon_{ik}^{\omega} s_{ik}^{\omega}}, \tag{24}$$

$$\frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln \tau_{ij}} = -(\sigma - 1) \sum_{\omega \in \Omega_{ij}^{\mathrm{DL}}} \frac{R_{ij}^{\omega}}{\varepsilon_{ij}^{\omega}}. \tag{25}$$

A.2 Section 4

Next, we provide derivations for the results included in Section 4, where it is considered a trade liberalization between two symmetric countries. We perform the calculations to obtain the partial effects of the trade shock on income/expenditure (Appendix A.2.1), the total effects on price index and income/expenditure (Appendix A.2.2), and the condition for the sign of the income effect (Appendix A.2.3).

A.2.1 Partial Effects on Income/Expenditure

We start by characterizing the partial effects of price index and trade costs on income and expenditure. Since $E_i = Y_i$ for each $i \in \mathcal{C}$, by describing the impact on income we also determine the effect on expenditures.

Given the symmetry of countries and wages chosen as the numéraire, the sum of profits generated by DLs from $i \in \mathcal{C}$ can be expressed as

$$\Pi_i^{\mathrm{DL}} = \left[\sum_{\omega \in \Omega_D^{\mathrm{DL}}} \left(\frac{E(\mathbb{P};\tau)\, s_D^{\omega}(\mathbb{P})}{\varepsilon\left[s_D^{\omega}(\mathbb{P})\right]} - f_D \right) + \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \left(\frac{E(\mathbb{P};\tau)\, s_X^{\omega}(\mathbb{P};\tau)}{\varepsilon\left[s_X^{\omega}(\mathbb{P};\tau)\right]} - f_X \right) \right]. \tag{26}$$

By (23), (24), and (25), we establish that, given $i, j \in \mathcal{C} := \{H, F\}$ with $i \neq j$, the partial effects on total profits are

$$\frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln E} = \frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln E_i} + \frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln E_j} = \sum_{\omega \in \Omega_D^{\mathrm{DL}}} \frac{R_D^{\omega}}{\varepsilon_D^{\omega}} + \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \frac{R_X^{\omega}}{\varepsilon_X^{\omega}},$$

$$\frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln \mathbb{P}} = \frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln \mathbb{P}_i} + \frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln \mathbb{P}_j} = (\sigma - 1) \left[\sum_{\omega \in \Omega_D^{\mathrm{DL}}} \frac{R_D^{\omega}}{\varepsilon_D^{\omega}} \frac{\sigma - \sigma s_D^{\omega}}{\sigma - \varepsilon_D^{\omega} s_D^{\omega}} + \sum_{\omega \in \Omega_X^{\mathrm{DL}}} \frac{R_X^{\omega}}{\varepsilon_X^{\omega}} \frac{\sigma - \sigma s_X^{\omega}}{\sigma - \varepsilon_X^{\omega} s_X^{\omega}} \right],$$

$$\frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln \tau} = \frac{\partial \Pi_i^{\mathrm{DL}}}{\partial \ln \tau_{ij}} = \sum_{\omega \in \Omega_X^{\mathrm{DL}}} (1 - \sigma) \frac{R_X^{\omega}}{\varepsilon_X^{\omega}}.$$

By the fact that $Y = E$ and differentiating (26), the partial effects on income are

$$\mathrm{d}\ln Y\left[Y-\frac{\partial\Pi_i^{\mathrm{DL}}}{\partial\ln Y}\right]=\frac{\partial\Pi_i^{\mathrm{DL}}}{\partial\ln\mathbb{P}}\mathrm{d}\ln\mathbb{P},$$
$$\mathrm{d}\ln Y\left[Y-\frac{\partial\Pi_i^{\mathrm{DL}}}{\partial\ln Y}\right]=\frac{\partial\Pi_i^{\mathrm{DL}}}{\partial\ln\tau}\mathrm{d}\ln\tau,$$

and, so,

$$\frac{\partial\ln Y}{\partial\ln\mathbb{P}}=\frac{\partial\ln E}{\partial\ln\mathbb{P}}=(\sigma-1)\frac{\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega}+\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}},\tag{27}$$
$$\frac{\partial\ln Y}{\partial\ln\tau}=\frac{\partial\ln E}{\partial\ln\tau}=(1-\sigma)\frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}.\tag{28}$$

A.2.2 Total Impact on Price Index and Income/Expenditure

Now, we proceed to determine the total effects of a trade liberalization on the price index and income/expenditure. We begin with the total effect of τ on the equilibrium price index. Differentiating (6), this is given by

$$\frac{\mathrm{d}\ln\mathbb{P}^*}{\mathrm{d}\ln\tau}=-\left(\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln\tau}\right)\left(\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln\mathbb{P}}\right)^{-1},$$

where

$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln\mathbb{P}}=\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln\mathbb{P}}+\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln E}\frac{\partial\ln E^*}{\partial\ln Y}\frac{\partial\ln Y^*}{\partial\ln\mathbb{P}},$$
$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln\tau}=\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln\tau}+\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln E}\frac{\partial\ln E^*}{\partial\ln Y}\frac{\partial\ln Y^*}{\partial\ln\tau}.$$

To get an expression for each term, let $r_{ij}^{\mathrm{DNL}}:=\frac{R_{ij}^{\mathrm{DNL}}}{M_i^E}$. Following the same procedure that we used for the profits of DLs in Appendix A.1.2, it can be shown that $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln\mathbb{P}}=\frac{\sigma-1}{\sigma}\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)$, $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln\tau}=\frac{1-\sigma}{\sigma}r_X^{\mathrm{DNL}}$, and $\frac{\partial\pi_i^{\mathbb{E},\mathrm{DNL}}}{\partial\ln Y}=\frac{r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}}{\sigma}$. Thus,

$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln\mathbb{P}}=\frac{\sigma-1}{\sigma}\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)\left[1+\frac{\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega}+\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}\right],$$
$$\frac{\mathrm{d}\pi_i^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln\tau}=\frac{1-\sigma}{\sigma}\left[r_X^{\mathrm{DNL}}+\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)\frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}\right].$$

Therefore,

$$\frac{\mathrm{d}\ln\mathbb{P}^*}{\mathrm{d}\ln\tau}=\frac{r_X^{\mathrm{DNL}}+\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)\frac{\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}}{\left(r_D^{\mathrm{DNL}}+r_X^{\mathrm{DNL}}\right)\left[1+\frac{\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}\frac{\sigma-\sigma s_D^\omega}{\sigma-\varepsilon_D^\omega s_D^\omega}+\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}\frac{\sigma-\sigma s_X^\omega}{\sigma-\varepsilon_X^\omega s_X^\omega}}{Y-\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{R_D^\omega}{\varepsilon_D^\omega}-\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{R_X^\omega}{\varepsilon_X^\omega}}\right]}.$$

To obtain the expression (7) included in the main part of the paper, we have to take the following steps. First, we divide numerator and denominator by $r_D^{\text{DNL}} + r_X^{\text{DNL}}$ and Y:

$$\frac{\mathrm{d}\ln\mathbb{P}^*}{\mathrm{d}\ln\tau} = \frac{\frac{r_X^{\text{DNL}}}{r_D^{\text{DNL}}+r_X^{\text{DNL}}} + \frac{\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{\tilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}}{1-\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{\tilde{s}_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{\tilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}}}{1+\frac{\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{\tilde{s}_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}+\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{\tilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}}{1-\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{\tilde{s}_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{\tilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}}}.$$

After this, for the term $\frac{r_X^{\text{DNL}}}{r_D^{\text{DNL}}+r_X^{\text{DNL}}}$, we divide numerator and denominator by the revenue generated by DNLs divided by M^{E*}. Then, (7) is obtained.

Regarding the total effect of τ on the equilibrium income, given by (8), this is determined by

$$\frac{\mathrm{d}\ln Y^*}{\mathrm{d}\ln\tau} = \frac{\partial\ln Y^*}{\partial\ln\tau} + \frac{\partial\ln Y^*}{\partial\ln\mathbb{P}}\frac{\mathrm{d}\ln\mathbb{P}^*}{\mathrm{d}\ln\tau}.$$

By using (27) and (28), we obtain

$$\frac{\mathrm{d}\ln Y^*}{\mathrm{d}\ln\tau} = (1-\sigma)\frac{\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}}{Y-\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}} + (\sigma-1)\frac{\left(\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}+\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_X^{\omega}s_X^{\omega}}\right)\frac{\mathrm{d}\ln\mathbb{P}^*}{\mathrm{d}\ln\tau}}{Y-\sum_{\omega\in\Omega_D^{\text{DL}}}\frac{R_D^{\omega}}{\varepsilon_D^{\omega}}-\sum_{\omega\in\Omega_X^{\text{DL}}}\frac{R_X^{\omega}}{\varepsilon_X^{\omega}}},$$

and, dividing and multiplying numerator and denominator by Y and gathering terms, we end up with (8).

A.2.3 Sign of Income Effect

In the main part of the paper, we have claimed that it is possible to establish the sign of $\frac{\mathrm{d}\ln Y^*}{\mathrm{d}\ln\tau}$ through the condition stated in (9). To show this, we first define some variables to streamline notation. Let $\chi_1 := \sum_{\omega\in\Omega_X^{\text{DL}}}\frac{\tilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}$, $\chi_2 := \sum_{\omega\in\Omega_X^{\text{DL}}}\frac{\tilde{s}_X^{\omega}}{\varepsilon_X^{\omega}}\frac{\sigma-\sigma s_X^{\omega}}{\sigma-\varepsilon_D^{\omega}s_X^{\omega}}$, $\delta_1 := \sum_{\omega\in\Omega_D^{\text{DL}}}\frac{\tilde{s}_D^{\omega}}{\varepsilon_D^{\omega}}$, and $\delta_2 := \sum_{\omega\in\Omega_D^{\text{DL}}}\frac{\tilde{s}_D^{\omega}}{\varepsilon_D^{\omega}}\frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega}s_D^{\omega}}$.

We begin by reexpressing the equilibrium effect on price index by using these terms:

$$\frac{\mathrm{d}\ln\mathbb{P}^*}{\mathrm{d}\ln\tau} = \frac{\left(1-d^{\text{DNL}}\right)+\frac{\chi_1}{1-\delta_1-\chi_1}}{1+\frac{\delta_2+\chi_2}{1-\delta_1-\chi_1}} = \frac{(1-\delta_1)-d^{\text{DNL}}(1-\delta_1-\chi_1)}{1-\delta_1-\chi_1+\delta_2+\chi_2}, \tag{29}$$

where the second equality follows by multiplying numerator and denominator by $1-\delta_1-\chi_1$, and gathering some of the terms. Reexpressed in terms of δ_1, δ_2, χ_1, and χ_2, the effect on the equilibrium income is

$$\frac{\mathrm{d}\ln Y^*}{\mathrm{d}\ln\tau} = \frac{(\sigma-1)}{1-\delta_1-\chi_1}\left[(\delta_2+\chi_2)\frac{\mathrm{d}\ln\mathbb{P}^*}{\mathrm{d}\ln\tau}-\chi_1\right],$$

which establishes that

$$\frac{\mathrm{d}\ln Y^*}{\mathrm{d}\ln\tau} > 0 \text{ iff } (\delta_2+\chi_2)\frac{\mathrm{d}\ln\mathbb{P}^*}{\mathrm{d}\ln\tau} > \chi_1.$$

Substituting in by (29), the condition is

$$\delta_2\left(\frac{(1-\delta_1)-d^{\text{DNL}}(1-\delta_1-\chi_1)}{1-\delta_1-\chi_1+\delta_2+\chi_2}\right)-\chi_1 > 0,$$

and, by working it out, it is established that the condition is reduced to $(\delta_2+\chi_2)\,x^{\mathrm{DNL}}>\chi_1$, which is (9).

In addition, we have stated that, when DLs have no market power, (9) becomes

$$\operatorname{sgn}\left(\frac{\mathrm{d}\ln Y^*}{\mathrm{d}\ln\tau}\right)=\operatorname{sgn}\left(\frac{\frac{R_X^{\mathrm{DNL}}}{\sigma}}{\frac{R_D^{\mathrm{DNL}}}{\sigma}}\frac{R_D^{\mathrm{DL}}}{\sigma}-\frac{R_X^{\mathrm{DL}}}{\sigma}\right).$$

$$=\operatorname{sgn}\left(\frac{\frac{R_X^{\mathrm{DNL}}}{\sigma}}{\frac{R_D^{\mathrm{DNL}}}{\sigma}}-\frac{\frac{R_X^{\mathrm{DL}}}{\sigma}}{\frac{R_D^{\mathrm{DL}}}{\sigma}}\right).$$

To prove this, notice that if $s_X^\omega\to0$ then $\chi_2=\chi_1=:\chi$ where $\chi:=\sum_{\omega\in\Omega_X^{\mathrm{DL}}}\frac{\widetilde{s}_X^\omega}{\sigma}$. Therefore, the sign is positive when $(\delta_2+\chi_2)\,x^{\mathrm{DNL}}>\chi_1$ or, equivalently, $\delta_2 x^{\mathrm{DNL}}>\chi d^{\mathrm{DNL}}$ where we have used that $1-d^{\mathrm{DNL}}=x^{\mathrm{DNL}}$. From this, it is established that

$$\frac{x^{\mathrm{DNL}}}{d^{\mathrm{DNL}}}-\frac{\chi}{\delta_2}>0\Leftrightarrow\frac{\frac{R_X^{\mathrm{DNL}}}{\sigma}}{\frac{R_D^{\mathrm{DNL}}}{\sigma}}-\frac{\frac{R_X^{\mathrm{DL}}}{\sigma}}{\frac{R_D^{\mathrm{DL}}}{\sigma}}>0,$$

where we have used the fact that $\delta_2:=\sum_{\omega\in\Omega_D^{\mathrm{DL}}}\frac{\widetilde{s}_D^\omega}{\sigma}$ when $s_D^\omega\to0$, and that for any firm ω it is true by definition that $\frac{\widetilde{s}_D^\omega}{\widetilde{s}_X^\omega}=\frac{R_D^\omega}{R_X^\omega}$.

A.3 Section 5

Next, we provide derivations for the results included in Section 5, where the country under analysis is small. We show the necessary calculations to obtain the partial effects of the trade shock on income/expenditure (Appendix A.3.1), the total effects on price index and income/expenditure (Appendix A.3.2), and the condition for the sign of the income effect (Appendix A.3.3).

A.3.1 Income and Expenditures

We focus on the partial effects of the price index and trade costs on income and expenditure. In order to do this, we exploit the fact that, since $E_H=\beta_H Y_H$, then $\frac{\partial\ln E_H}{\partial\ln Y_H}=1$ regardless of the value of β_H. Thus, by describing the impact on income, we also determine the effect on expenditures.

To perform the calculations, it is convenient to express the total profits of DLs in the following way:

$$\Pi_H^{\mathrm{DL}}=\sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}}\left(\frac{E_H s_{HH}^\omega(\mathbb{P}_H)}{\varepsilon_{HH}^\omega\left[s_{HH}^\omega(\mathbb{P}_H)\right]}-f_{HH}\right)+\sum_{\omega\in\Omega_{HF}^{\mathrm{DL}}}\left(\frac{E_F s_{HF}^\omega(\mathbb{P}_F;\tau_{HF})}{\sigma}-f_{HF}\right).\qquad(30)$$

Given (INC) and $E_H = \beta_H Y_H$, we determine the following partial effects on total profits

$$\frac{\partial \Pi_H^{\mathrm{DL}}}{\partial \ln Y_H} = \sum_{\omega \in \Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}},$$
$$\frac{\partial \Pi_H^{\mathrm{DL}}}{\partial \ln \mathbb{P}_H} = (\sigma - 1) \sum_{\omega \in \Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}} \frac{\sigma - \sigma s_{HH}^{\omega}}{\sigma - \varepsilon_{HH}^{\omega} s_{HH}^{\omega}},$$
$$\frac{\partial \Pi_H^{\mathrm{DL}}}{\partial \ln \tau_{HF}} = \sum_{\omega \in \Omega_{HF}^{\mathrm{DL}}} (1 - \sigma) \frac{R_{HF}^{\omega}}{\sigma},$$

Differentiating (INC), we obtain the partial effects regarding income:

$$\mathrm{d} \ln Y_H \left[Y_H - \frac{\partial \Pi_H^{\mathrm{DL}}}{\partial \ln Y_H} \right] = \frac{\partial \Pi_H^{\mathrm{DL}}}{\partial \ln \mathbb{P}_H} \mathrm{d} \ln \mathbb{P}_H,$$
$$\mathrm{d} \ln Y_H \left[Y_H - \frac{\partial \Pi_H^{\mathrm{DL}}}{\partial \ln Y_H} \right] = \frac{\partial \Pi_H^{\mathrm{DL}}}{\partial \ln \tau_{HF}} \mathrm{d} \ln \tau_{HF},$$

which establishes that

$$\frac{\partial \ln Y_H}{\partial \ln \mathbb{P}_H} = \frac{\partial \ln E_H}{\partial \ln \mathbb{P}_H} = (\sigma - 1) \frac{\sum_{\omega \in \Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}} \frac{\sigma - \sigma s_{HH}^{\omega}}{\sigma - \varepsilon_{HH}^{\omega} s_{HH}^{\omega}}}{Y_H - \sum_{\omega \in \Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}}, \tag{31}$$
$$\frac{\partial \ln Y_H}{\partial \ln \tau_{HF}} = \frac{\partial \ln E_H}{\partial \ln \tau_{HF}} = (1 - \sigma) \frac{\sum_{\omega \in \Omega_{HF}^{\mathrm{DL}}} \frac{R_{HF}^{\omega}}{\sigma}}{Y_H - \sum_{\omega \in \Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}}. \tag{32}$$

A.3.2 Total Impact on Price Index, Income, and Expenditure

Now, we proceed to determine the total effects of a trade liberalization on the price index, income, and expenditure. We begin by deriving the total effect on price index. Differentiating (12), it is established that $\frac{\mathrm{d} \ln \mathbb{P}_H^*}{\mathrm{d} \ln \tau_{HF}} = -\left(\frac{\mathrm{d}\pi_H^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d} \ln \mathbb{P}_H} \right)^{-1} \frac{\mathrm{d}\pi_H^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d} \ln \tau_{HF}}$. Regarding each term,

$$\frac{\mathrm{d}\pi_H^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d} \ln \mathbb{P}_H} = \frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln \mathbb{P}_H} + \frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln Y_H} \frac{\partial \ln Y_H}{\partial \ln \mathbb{P}_H},$$
$$\frac{\mathrm{d}\pi_H^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d} \ln \tau_{HF}} = \frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln \tau_{HF}} + \frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln Y_H} \frac{\partial \ln Y_H}{\partial \ln \tau_{HF}}.$$

For the calculations, we use the fact that infinitesimal variations of the survival productivity cutoff in any country have a second-order impact on expected profits. Let $r_{ij}^{\mathrm{DNL}} := \frac{R_{ij}^{\mathrm{DNL}}}{M_i^E}$. Therefore, we obtain that $\frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln \mathbb{P}_H} = \frac{\sigma - 1}{\sigma} r_{HH}^{\mathrm{DNL}}$, $\frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln \tau_{HF}} = \frac{1-\sigma}{\sigma} r_{HF}^{\mathrm{DNL}}$ and $\frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln Y_H} = \frac{\partial \pi_H^{\mathbb{E},\mathrm{DNL}}}{\partial \ln E_H} \frac{\partial \ln E_H}{\partial \ln Y_H} = \frac{r_{HH}^{\mathrm{DNL}}}{\sigma}$. Thus, by (31) and (32),

$$\frac{\mathrm{d}\pi_H^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \mathbb{P}_H} = \frac{\sigma-1}{\sigma} r_{HH}^{\mathrm{DNL}} \left[1 + \frac{\sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}} \frac{\sigma-\sigma s_{HH}^{\omega}}{\sigma-\varepsilon_{HH}^{\omega} s_{HH}^{\omega}}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}} \right],$$

$$\frac{\mathrm{d}\pi_H^{\mathbb{E},\mathrm{DNL}}}{\mathrm{d}\ln \tau_{HF}} = \frac{1-\sigma}{\sigma} \left[r_{HF}^{\mathrm{DNL}} + r_{HH}^{\mathrm{DNL}} \frac{\sum_{\omega\in\Omega_{HF}^{\mathrm{DL}}} \frac{R_{HF}^{\omega}}{\sigma}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}} \right].$$

By making use of these results, we obtain that

$$\frac{\mathrm{d}\ln \mathbb{P}_H^{*}}{\mathrm{d}\ln \tau_{HF}} = \frac{r_{HF}^{\mathrm{DNL}} + r_{HH}^{\mathrm{DNL}} \frac{\sum_{\omega\in\Omega_{HF}^{\mathrm{DL}}} \frac{R_{HF}^{\omega}}{\sigma}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}}}{r_{HH}^{\mathrm{DNL}} \left[1 + \frac{\sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}} \frac{\sigma-\sigma s_{HH}^{\omega}}{\sigma-\varepsilon_{HH}^{\omega} s_{HH}^{\omega}}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}} \right]}.$$

Dividing numerator and denominator by r_{HH}^{DNL}

$$\frac{\mathrm{d}\ln \mathbb{P}_H^{*}}{\mathrm{d}\ln \tau_{HF}} = \frac{\frac{r_{HF}^{\mathrm{DNL}}}{r_{HH}^{\mathrm{DNL}}} + \frac{\sum_{\omega\in\Omega_{HF}^{\mathrm{DL}}} \frac{R_{HF}^{\omega}}{\sigma}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}}}{1 + \frac{\sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}} \frac{\sigma-\sigma s_{HH}^{\omega}}{\sigma-\varepsilon_{HH}^{\omega} s_{HH}^{\omega}}}{Y_H - \sum_{\omega\in\Omega_{HH}^{\mathrm{DL}}} \frac{R_{HH}^{\omega}}{\varepsilon_{HH}^{\omega}}}}.$$

By multiplying numerator and denominator of $\frac{r_{HF}^{\mathrm{DNL}}}{r_{HH}^{\mathrm{DNL}}}$ by M_H^{E*}, we get $\frac{R_{HF}^{\mathrm{DNL}}}{R_{HH}^{\mathrm{DNL}}}$. Then, multiplying and dividing by the sector income Y_H^{sec}, we obtain (14).

With these results, we can obtain the total effect on income, which also determines the total effect on expenditure since $E_H = \beta_H Y_H$ and, so, $\frac{\partial \ln E_H}{\partial \ln Y_H} = 1$. This is given by,

$$\frac{\mathrm{d}\ln Y_H^{*}}{\mathrm{d}\ln \tau_{HF}} = \frac{\partial \ln Y_H^{*}}{\partial \ln \tau_{HF}} + \frac{\partial \ln Y_H^{*}}{\partial \ln \mathbb{P}_H} \frac{\mathrm{d}\ln \mathbb{P}_H^{*}}{\mathrm{d}\ln \tau_{HF}}.$$

By substituting in (31) and (32), we obtain (15).

A.3.3 Sign of Income Effect

In the main part of the paper, we have claimed that the sign of $\frac{\mathrm{d}\ln Y_H^{*}}{\mathrm{d}\ln \tau_{HF}}$ is determined by condition (16). Next, we show this. In order to simplify the derivations, we make a change of variables similar to the case of two symmetric countries. Specifically, let $\chi := \sum_{\omega\in\Omega_X^{\mathrm{DL}}} \frac{\widetilde{s}_X^{\omega}}{\sigma}$, $\delta_1 := \sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{\widetilde{s}_D^{\omega}}{\varepsilon_D^{\omega}}$, and $\delta_2 := \sum_{\omega\in\Omega_D^{\mathrm{DL}}} \frac{\widetilde{s}_D^{\omega}}{\varepsilon_D^{\omega}} \frac{\sigma-\sigma s_D^{\omega}}{\sigma-\varepsilon_D^{\omega} s_D^{\omega}}$. In addition, let $\rho := (\alpha_H)^{-1}$.

In terms of these variables, the total effects on each variable are

$$\frac{\mathrm{d}\ln Y_H^{*}}{\mathrm{d}\ln \tau_{HF}} = \frac{(\sigma-1)}{\rho-\delta_1} \left[\frac{\mathrm{d}\ln \mathbb{P}_H^{*}}{\mathrm{d}\ln \tau_{HF}} \delta_2 - \chi \right],$$

$$\frac{\mathrm{d}\ln \mathbb{P}_H^{*}}{\mathrm{d}\ln \tau_{HF}} = \frac{\frac{1-d_H^{\mathrm{DNL}}}{d_H^{\mathrm{DNL}}} (\rho-\delta_1) + \chi}{\rho-\delta_1+\delta_2}.$$

From this, we infer that

$$\frac{\mathrm{d}\ln Y_H^*}{\mathrm{d}\ln \tau_{HF}} > 0 \text{ iff } \frac{\mathrm{d}\ln \mathbb{P}_H^*}{\mathrm{d}\ln \tau_{HF}}\delta_2 > \chi,$$

and, thus, the condition is

$$\frac{\frac{1-d_H^{\mathrm{DNL}}}{d_H^{\mathrm{DNL}}}(\rho-\delta_1)+\chi}{\rho-\delta_1+\delta_2}\delta_2 > \chi,$$

or, just

$$\delta_2\frac{(\rho-\delta_1)-d_H^{\mathrm{DNL}}(\rho-\delta_1-\chi)}{d_H^{\mathrm{DNL}}(\rho-\delta_1+\delta_2)}-\chi > 0.$$

Working out the expression, we end up with the condition $\delta_2\left(1-d_H^{\mathrm{DNL}}\right) > d_H^{\mathrm{DNL}}\chi$ and, using that $1-d_H^{\mathrm{DNL}} = x_H^{\mathrm{DNL}}$, the result follows.

B Results for Large Firms Defined By Total Revenue

In the main part of the paper, we obtained results within a setting based on empirical regularities for Danish manufacturing. This determined that, in first place, the market structure displayed a coexistence of DLs and DNLs, thus allowing for some firms having market power. Second, the process in which firms enter and exit the market was supposed to be different in nature. More specifically, some firms did not know their profitability with certainty and, ex post, there were zero average profits. In addition, there was a subset of firms that were well established in the industry and garnered positive profits.

Regarding this second point, there is some latitude relative to what constitutes that a firm is well established in the market. In our baseline case, we took the model to the data by classifying firms according to its domestic market share. Implicitly, we were based on the empirical fact that domestic leadership is persistent over time, where we even provided evidence that this acts as an almost sufficient condition for being among those firms with greatest revenues in their industry.

Alternatively, we could conceive that well-established firms are those that generate the greatest revenue in their industry, irrespective if this is due to the sales in the domestic or a foreign market. Due to this, in this appendix, we proceed to recalculate outcomes using total revenues to classify firms into groups. To distinguish between this case and the baseline scenario with DNLs and DLs, we refer to each type of firm as high-revenue firms (HRFs) and low-revenue firms (LRFs). Thus, LRFs are characterized as in Melitz while HRFs comprise large firms that have positive profits.

In order to construct a representative industry, we follow similar steps to those in Section 6. First, we only consider those industries with coexistence of DLs and DNLs, since the characterization of market structure is still the same as in the baseline case. For these industries, we define the domestic market and income shares for each HRFs, and the domestic and export intensities of LRFs. After this, we define a representative industry by taking the top four HRFs and using industry revenues as weights. Notice that, still, $\sigma := 3.53$ since this calculation does not depend on how we classify firms. Performing the corresponding calculations, the description of the representative industry is as follows.

Table 2. *Features of a Representative Industry*

(a) Features of DLs (in %)

	Domestic Market Share	Domestic Revenues as % of sector income	Export Revenues as % of sector income
Top 1	16.31	17.46	15.23
Top 2	7.28	8.11	4.48
Top 3	4.89	5.38	2.56
Top 4	3.38	3.68	2.03

(b) Revenue Intensities (in %)

	Domestic Intensity	Export Intensity
LRFs	75.82	24.18
DNLs	68.62	31.38
All Firms	58.12	41.88

Note: Calculations based on industries with coexistence of DNLs and DLs, and using industry revenues as weights. Domestic market shares are calculated relative to expenditures, which includes both domestic sales and imports. Domestic and export intensity calculated, respectively, as domestic and export sales of the group considered (i.e., LRFs, DNLs, and all firms) relative to the total sales of the group.

Comparing the description in Table 2 with the features of a representative industry in our baseline scenario in Table 1, we obtain two conclusions. First, domestic market shares and domestic income shares of HRFs are quite similar to those corresponding to the top four DLs. Second, the top HRF has a considerable higher contribution to the income of the sector through its export sales.

As for the information in Table 2b, we have included the domestic intensity of DNLs to compare it with that of LRFs. The numbers indicate that LRFs as a group have a greater home bias.

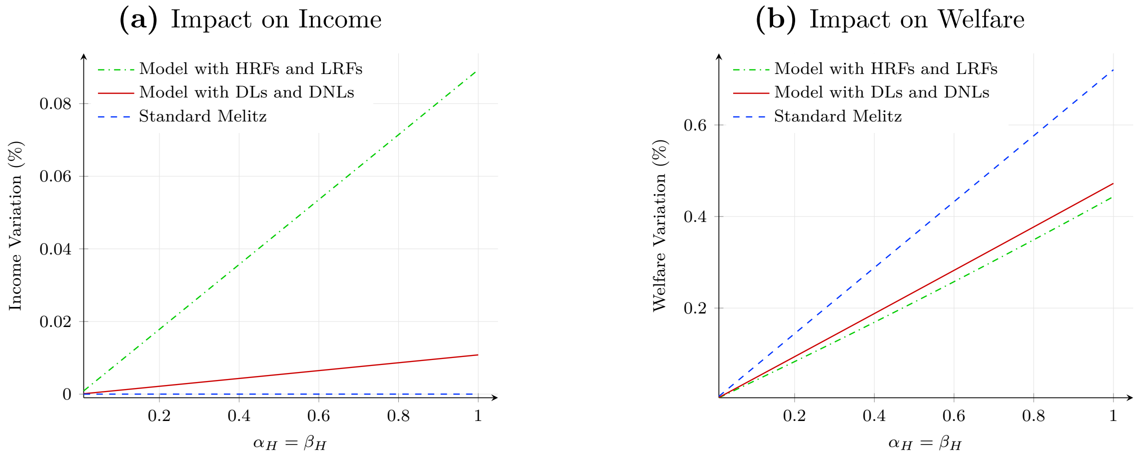
In the following, we study the impact on the economy of an infinitesimal variation of trade costs. Similar to what happens with the baseline case, the first conclusion we obtain is that there are gains of trade: by using (9) and (16), it is established that a trade liberalization increases the country's income in each variant of the model. Thus, since additionally the price index always decreases, welfare is greater. In particular, for the small-economy scenario, this result holds irrespective of the values assigned to α_H and β_H. Recall that they are, respectively, the proportion of income and expenditure of the differentiated sector relative to the country's income.

Next, we proceed to quantify the relative importance of income and price index in welfare. We express the results in elasticity terms, relative to a decrease in 1% of trade costs.

Regarding the symmetric-countries scenario, income increases by 0.068% and the price index decreases by 0.268%. Relative to the baseline case, this establishes that the change in income is greater but the reduction in price index lower. Overall, welfare increases by 0.336%, which is slightly greater than what we got for the baseline case. Also, in line with the baseline scenario, the welfare increase is lower than that obtained in Melitz.

Concerning the small-economy scenario, next we present results for different values of α_H with $\alpha_H = \beta_H$.

Figure 7. *Impact of a Trade Liberalization in the Small-Country Framework*



From Figure 7, we can establish the following conclusions. First, by comparing the results with the Melitz model, and in line with the baseline case, gains of trade are between 38% and 43% lower. Second, relative to the baseline case, gains of trade are also lower, in a range between 6% and 13%. This holds in spite of the setting with LRFs and HRFs predicting a greater income increase: it is as big as 0.089% in this setting, while in the baseline case this number is 0.011%.

These facts can be explained by the calibration of domestic intensity for LRFs which, by Table 2b, is greater than the calibrations for both Melitz and the baseline case. Due to this, in the next figures we present results for $\alpha_H = 0.9$ where we vary the domestic intensity of LRFs and DNLs. The graphs are drawn in a same scale, so that we are able to compare the vertical differences in each figure.

Figure 8. *Symmetric-Countries Framework*

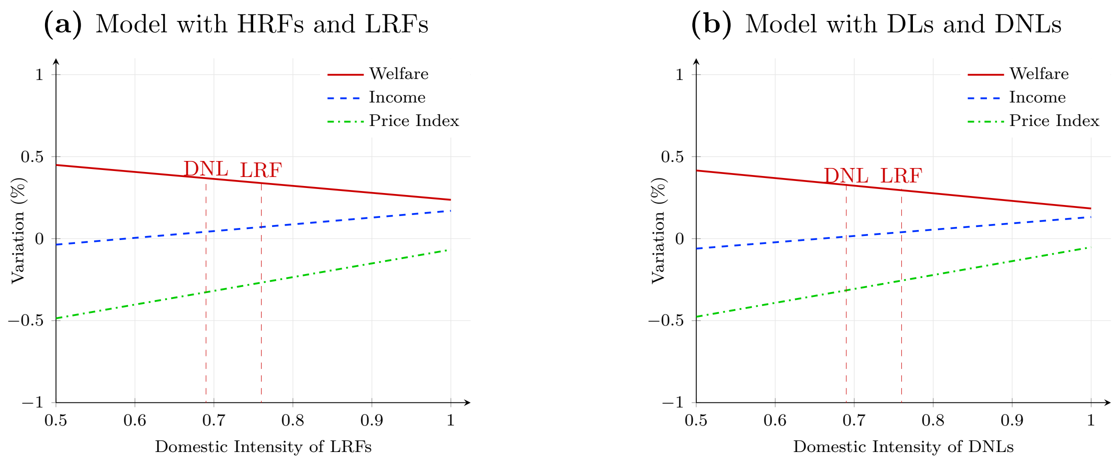
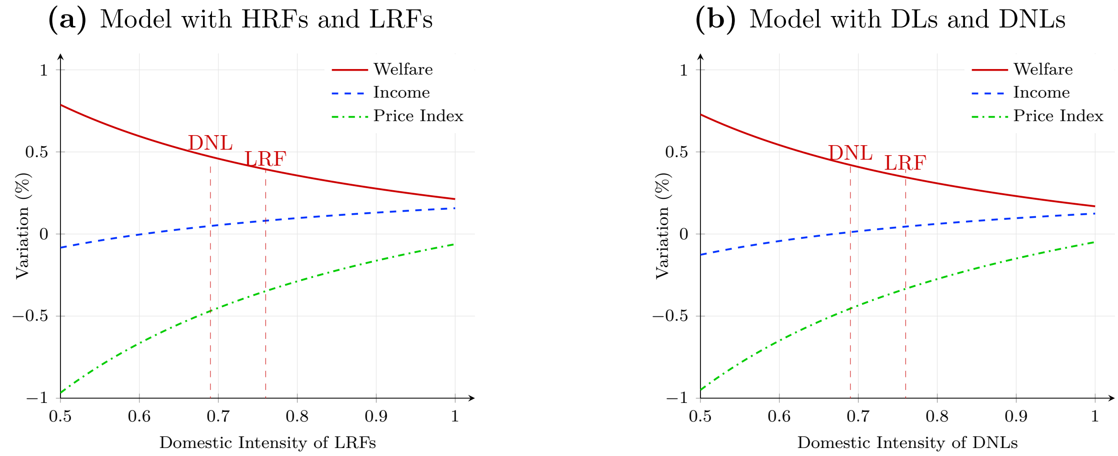


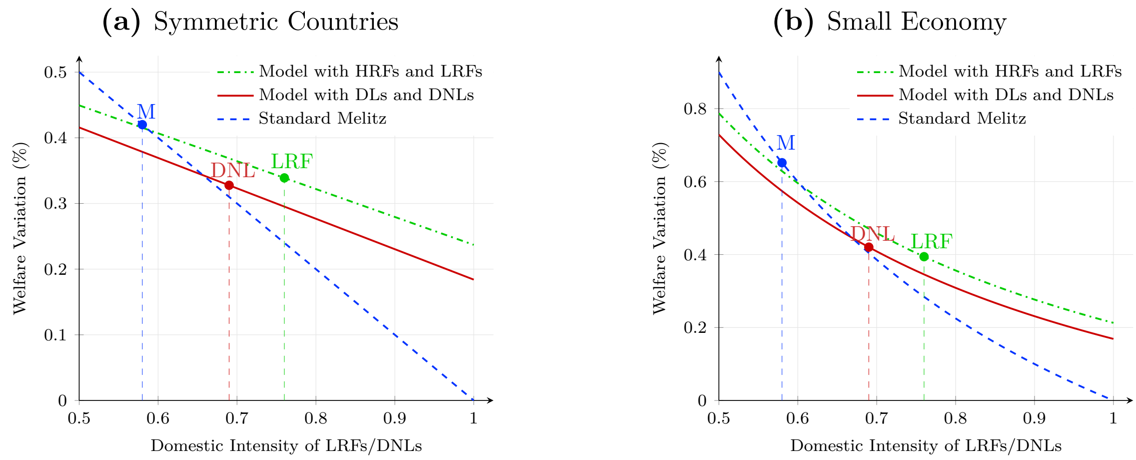
Figure 9. *Small-Country Framework*



From Figure 8 and Figure 9, we can appreciate that, if DNLs and LRFs had a similar domestic intensity, the results would not differ much. Thus, income variations do not play a major role in the differences between both settings.

Given that the cases differ by the calibration of domestic intensity, next we present welfare outcomes according to the domestic intensities of DNLs and LRFs. We also include the case of Melitz.

Figure 10. *Welfare Comparison Across Models*



Remarkably, the graphs indicate that, if the baseline case had predicted the same calibration of domestic intensity, the welfare gains with HRFs and LRFs would have been greater under any variant of the model. This result primarily holds because the export share of the top HRF is greater and, hence, this firm benefits more from a trade liberalization. This, in turn, increases the country's income. Nonetheless, once that we account for differences in the domestic intensity of LRFs and DNLs, this scenario predicts greater gains in the symmetric variant but lower gains under the small-economy scenario.

C Market Shares Based on Alternative Definitions of Sales

In the main part of the paper, we measured domestic market shares by taking a firm's domestic turnover as a firm's home sales. However, as we explain below, this leaves a portion of sales out of consideration. In this appendix, we show that domestic market shares are not sensitive to calculation procedures based on alternative definitions of sales.

To understand the motivation of the analysis we present, notice that classifying a firm as DL is based on its domestic market share which, in turn, is defined by its domestic sales relative to the industry's expenditure.

Ideally, total domestic sales of a firm should include those goods supplied to the market, irrespective if they are produced by itself or other firms. As a baseline case, we have used total turnover as the firm's domestic supply, which already includes some imports. Specifically, in our data, total turnover includes sales of goods produced by the firm itself, goods produced by a subcontractor established abroad when the firm owns the inputs of the subcontracted firm, and resales of goods bought from other domestic firms if they are sold with any processing. Nonetheless, there is a portion of total supply that is not covered by turnover: goods bought to firms established abroad that the firm does not own.

The reason to use total turnover as a baseline scenario is that, even though we have information about firms' imports, we do not know whether they are inputs or final goods for the firm. Thus, by defining sales as total turnover we are taken a conservative position: we are implicitly assuming that any import is either an input or a final good that has been reprocessed by a firm, which in our data is already included in total turnover.

Next, we show that, if we use measures of DLs sales that account for imported goods, domestic market shares are almost identical to those in the baseline case. Since we do not have information regarding whether each import constitutes an input, we consider two alternatives.

- **Alternative A**: domestic sales of a firm are defined as the sum of its turnover and imports minus its exports of any 8-digit product that belongs to the firm's industry and is also produced by the firm.
- **Alternative B**: domestic sales of a firm are defined as the sum of its turnover and imports minus its exports of any 8-digit product that belongs to the firm's industry.

Alternative A is based on the assumption that, if a firm produces and sells domestically a good, an import of this good constitutes a product to be resold without further processing, rather than an input. Regarding Alternative B, it supposes that any good imported that belongs to the industry is resold.

Using these definitions, the domestic market shares of the top four DLs are reported in Table 3. The results indicate that the domestic shares of DLs are virtually identical.

Table 3. *Domestic Market Share of DLs - Total Sales*

	Baseline Case (Turnover)	Alternative A	Alternative B
Top 1	17.83	17.96	17.84
Top 2	7.27	7.26	7.24
Top 3	4.65	4.89	4.92
Top 4	3.36	3.55	3.73

D DLs' Shares Distributions and their Impact on Income

In the main part of the paper, we considered empirical results for the Danish economy. This entailed a specific distribution of shares for DLs and DNLs that were representative for that country. As a corollary, for the different empirical exercises, we obtained conclusions conditional on those shares and, hence, for a specific impact of a trade liberalization on income.

In this appendix, we show that different shares distributions of DLs are capable of capturing other plausible mechanisms and effects. In particular, the different examples we present highlight that the impact of trade liberalizations on income can have a substantial effect on welfare gains.

To accomplish this, we consider several cases. In all of them, we keep assuming that $\sigma := 3.53$ and, for the small-economy scenario, $\alpha_H = \beta_H$ with $\alpha_H = 0.9$. By assuming a high value for α_H, we aim at capturing that the representative differentiated industry is relevant for the conditions of the country. We study cases where there is only one DL and this has the following shares.

Table 4. *Features of DLs - One DL*

	Domestic Share	Domestic Revenues as % of sector income	Export Revenues as % of sector income
Case 1	50	50	0
Case 2	0	0	50
Case 3	50	50	50
Case 4	0	0	0

In words, the cases proposed have the goal of reflecting the following situations. Case 1 supposes that the DL does not export and can be considered as "local superstar": a Danish firm that only has success in its home market. Alternatively, it could correspond to a multinational firm that is established only with the purpose of serving the local market.

Case 2 represents a situation where we isolate the effect of the DL's export revenues on outcomes. We accomplish this by considering an extreme situation where the DL has no domestic revenues.

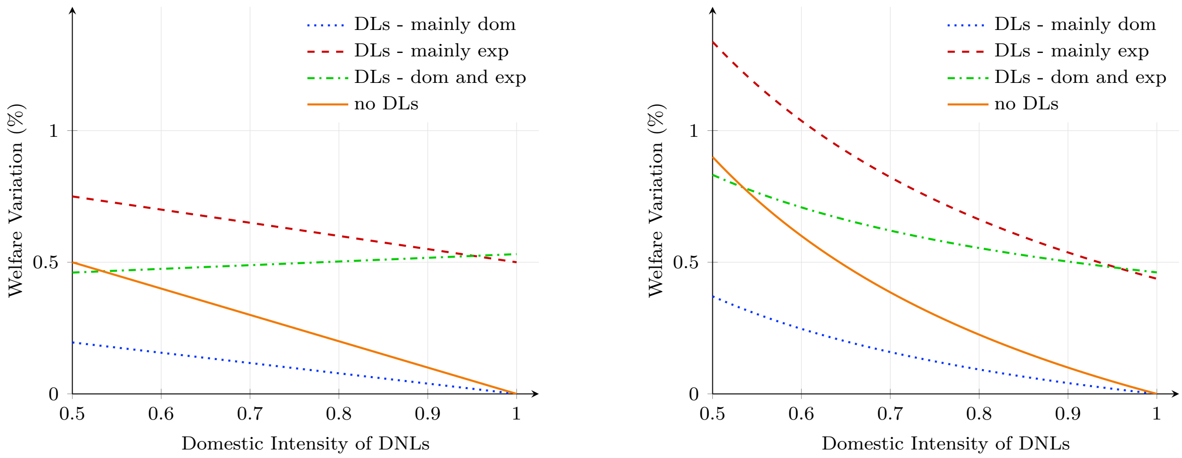
Case 3 captures a scenario where the DL has well diversified sales across markets. Thus, it has a big presence in the domestic market but, also, generates a substantial portion of the country's exports.

Finally, Case 4 acts as a benchmark where we suppose that there are no DLs, as in Melitz.

In the next figure, we present results for the scenarios with symmetric countries and a small country. We focus on the effects for different values of DNLs' domestic intensity, which reflects the impact of new export opportunities on the creation of a tougher competitive environment.

Figure 11. *Welfare Comparison Across Models*

(a) Symmetric-Countries Scenario **(b)** Small-Country Scenario



Note: Case 1 corresponds to "DLs - Mainly Dom", Case 2 is "DLs - Mainly Exp", Case 3 is "DLs - dom and exp", and Case 4 is "no DLs".

Case 1 and Case 2 represent polar cases whose effects constitute bounds for Case 4, where there are no DLs. In particular, Case 1 represents situations where DLs do not benefit from new export opportunities and are negatively impacted by tougher competitive conditions in the domestic market. Due to this, a trade liberalization decreases income and, hence, increases in welfare are lower relative to Case 4.

On the other hand, Case 2 captures scenarios where DLs are not impacted by tougher domestic competition and benefit from better export opportunities. Thus, a trade liberalization increases the country's income and, therefore, gains of trade are greater relative to Case 4.

As for Case 3, it represents intermediate situations where DLs have a great presence in both the domestic and foreign market. Thus, these firms are simultaneously affected by tougher competition and better export opportunities. Consistent with this interpretation, the gains of trade predicted by this case are bounded by Case 1 and Case 2.

E Welfare Losses

In the main part of the paper, we showed that it is possible that a trade liberalization reduces a country's income. Nonetheless, if the reduction in the price index always dominates any income variation, welfare gains would be always positive. Next, we provide a counterexample which indicates that this is not the case. Thus, a trade liberalization may actually lead to welfare losses.

A counterexample for the case of a small scenario is easy to provide if we let α_H and β_H vary independently. For instance, if β_H is low enough, reductions in the price index have a small impact on welfare. Thus, if income decreases enough, there would be losses from trade. Next, we show that, in fact, a counterexample can be provided even if $\alpha_H = \beta_H$. Also, we present a counterexample for the scenario with symmetric countries.

The counterexamples require that a trade liberalization determines a negative impact on income and that this effect is pronounced enough to surmount any reduction in the price index. Based on Appendix D, this can be achieved if DLs do not export (so that they do not benefit from new export opportunities) and have a great presence in the domestic market (so that tougher domestic conditions substantially impact the home profits of DLs).

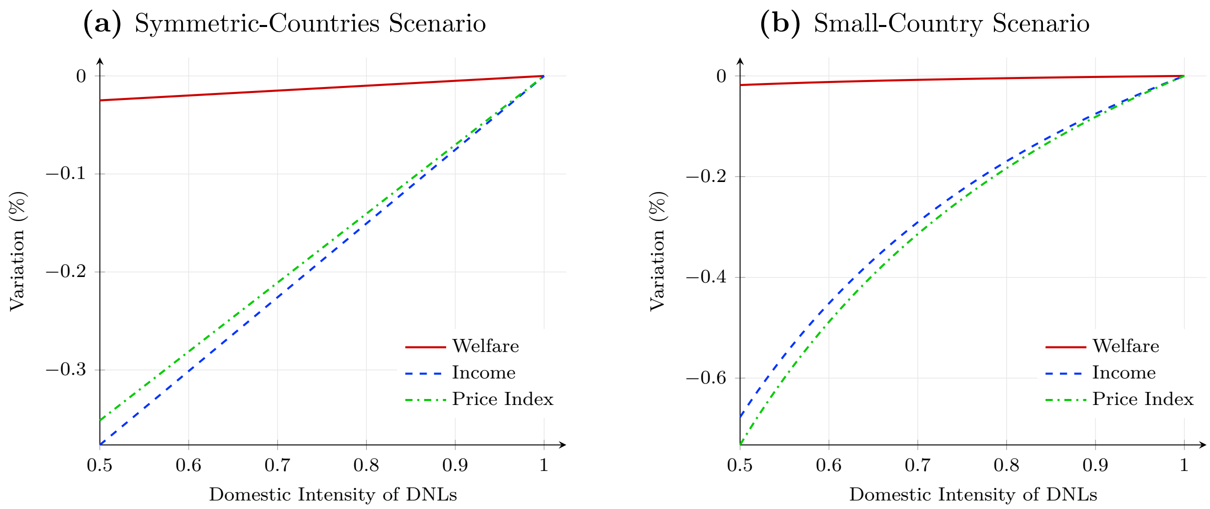
In the following, we suppose that $\sigma := 3.53$, as in the baseline case. Moreover, for the small-country setting, it is assumed that $\alpha_H = \beta_H$ with $\alpha_H = 0.9$. The counterexamples for the different scenarios are based on the following shares distribution of DLs.

Table 5. *Features of DLs*

	Domestic Market Share	Domestic Revenues as % of sector income	Export Revenues as % of sector income
Top 1	30	30	0
Top 2	30	30	0
Top 3	30	30	0

Based on these shares distributions, we present graphs that depict outcomes for different levels of domestic intensity of DNLs.

Figure 12. *Welfare Comparison Across Models*



These graphs establish that, irrespective of the DNLs' domestic intensity, there are always welfare losses. Intuitively, this occurs because the losses in the DLs' domestic profits are more significant for the country's welfare than the reductions in the price index.