The Microeconomics of New Trade Models*

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Abstract

I provide a unified framework to study monopolistic competition models à la Melitz under any productivity distribution and a flexible demand system encompassing standard demand functions. By disentangling the effects of a trade liberalization on the domestic market into new export opportunities and tougher import competition, I show that the former entails pro-competitive effects. Surprisingly, tougher import competition does not impact either domestic firms' prices, quantities, or their survival productivity cutoff; it only affects the mass of domestic incumbents. The results applied to the case of two big economies imply that a unilateral liberalization only entails tougher export conditions for the liberalizing country, thus determining that anti-competitive effects necessarily emerge under this setup. I conclude by showing that in a model à la Chaney, where heterogeneous firms know their productivity, the disciplinary effects from the import-competition channel are reactivated.

Keywords: Melitz model, Chaney model, import competition, export opportunities, Metzler paradox.

JEL codes: F10, F12, D43, L13

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1 Introduction

In recent decades, firm heterogeneity has been introduced into International Trade models in various ways. The standard approach to incorporate this feature in a monopolistic-competition setting follows Melitz (2003), where ex-ante homogeneous firms make entry decisions without knowing their productivity and learn their efficiency ex-post.

In this paper, I provide a unified framework to analyze the canonical model of Melitz under any productivity distribution and a demand system that encompasses standard demand functions.[1] By making use of this methodology, I analyze the model at the microeconomic level. Specifically, I study how an industry in isolation is affected by new export opportunities and exposure to import competition, when each is treated as a separate channel. The main conclusion I derive is that, in this setup, any pro-competitive effect in the domestic market due to a trade liberalization comes from the export-opportunities channel. Surprisingly, the import-competition channel is inactive and only affects the mass of domestic incumbents. Neither the survival productivity cutoff, prices, nor quantities of domestic firms are impacted by this.[2] More broadly, no domestic firm's decision is affected, hence the conclusion extends to variables such as quality or number of products when firms are multiproduct. In addition, I also establish that, when there is a unilateral liberalization between two large countries, there are necessarily anti-competitive effects in the liberalizing country. The reason is that the trade shock only entails worse export conditions for this country.[3]

The results are obtained for a demand system featuring a property shared by common demand functions: it summarizes market conditions through a single sufficient statistic.

[1] Among others, in terms of demands that account for an endogenous number of varieties, my setup covers: 1) demands from an additively separable direct utility as in Krugman (1979), which includes Simonovska's (2015) Stone-Geary, the generalized CES as in Jung et al. (2015) and Arkolakis et al. (2019), and Behrens and Murata's (2007) CARA, 2) Melitz and Ottaviano's (2008) linear demand, 3) Feenstra's (2003) translog demand, 4) demands from an additive separable indirect utility as in Bertoletti et al. (2018), including their version of the addilog demand, 4) demands from discrete choice models as in Luce (1959) and McFadden (1973), including the Logit, 5) Nocke and Schutz's (2018) demands from discrete-continuous choices models, and 6) constant expenditure demand systems as in Vives (2001) and Bjornerstedt and Verboven (2016). I also extend some of the results to the nested Logit and nested CES with groups defined by either country of origin or the firm's product bundle when firms are multiproduct.

[2] This insight was noticed by Melitz (2003) for a CES demand. He points out that the effects of trade are driven by new export opportunities, but that *"the model should also be interpreted with caution as it precludes another potentially important channel for the effects of trade, which operates through increases in import competition."*

[3] This explains why some studies found the outcome under the Melitz and Krugman models, where the latter is a special case of the former. For instance, see Venables (1987), Melitz and Ottaviano (2008), Spearot (2014), Bagwell and Lee (2015), Demidova (2017).

This can be interpreted as a measure of how tough the competitive environment is and includes examples such as a price index, a demand choke price, or even a more complex function which depends on different price moments. The range of demands encompassed by a demand system with this property is quite broad (see Footnote 1), including demands that seem to depend on more than one statistic but collapse into one when written appropriately.

While the use of a demand with this characteristic in Melitz is not new,[4] the innovation of this paper lies in how I exploit its structure. Specifically, this demand's feature allows me to conceive the model as a large aggregative economy, as defined in Acemoglu and Jensen (2010, 2015).[5] I make use of the techniques provided by these authors and extend them to account for an endogenous number of agents to cope with monopolistic competition. By interpreting the Melitz model through the lens of large aggregative economies, it is possible to obtain general results in a straightforward way and explain their mechanisms by decomposing the outcomes in terms of channels. In addition, by employing monotone comparative statics, all the results that I identify in this paper hold under a set of critical sufficient conditions, thus ignoring assumptions that are ancillary. In particular, in relation to this, the generality of the model allows me to show that assumptions like Marshall's Second Law of Demand (i.e., that the price elasticity of demand is increasing with respect to own price) plays a less important role in the determination of the results relative to assuming strategic complementary of prices (i.e, that tougher competition increases the price elasticity).

Following the literature on large aggregative economies and regarding the single sufficient statistic of the demand system, I distinguish between an *aggregator* and an *aggregate*. The former is the function that determines how tough the competitive environment is in a country, while the latter is an aggregator's value. For a given aggregate, it is illustrative to think of the aggregator as indicating the different compositions that are consistent with that value. The composition, in turn, is given by the prices and mass of active firms serving that country, with the latter depending on the mass of incumbents and the survival

[4]For some recent applications, see Neary and Mrazova (2017), Parenti et al. (2017), Arkolakis et al. (2019), and Fally (2019).

[5]Large aggregative *economies* are different from large aggregative *games*. Both refer to models where agents are atomistic and market conditions are captured through an aggregator. Nonetheless, the latter is a subset of the former because it imposes strongly separability of the aggregator. Under that assumption, a large aggregative economy arises as a limit with infinite players of an aggregative game with a finite number of agents. Since this property does not play a role in monopolistic competition, I do not suppose it holds. This broadens considerably the scope of the demand system and, hence, of the results.

productivity cutoff of each country.

In Section 3, I describe the setup and show that the equilibrium conditions can be reduced to two systems of equations. The first one is given by zero expected profits, which pins down the *value* of each aggregate in the world. The other set of equations describes the equilibrium at the market stage and determines the *composition* of each aggregator that is consistent with the equilibrium aggregate. Crucially, *the system is separable, allowing the aggregates to be determined independently of their composition.* This feature becomes relevant given that firms' optimal prices and the survival productivity cutoffs in each country are completely determined by its aggregate and independently of its composition.

In Section 4, I develop some intuition regarding the mechanisms operating under the Melitz model when there is a trade liberalization. Similar to how increases in market size are commonly used as an analogy for trade liberalizations, I use two shocks in autarky whose effects resemble the impact from the export-opportunities and import-competition channels. I refer to them as, respectively, shocks to expected profits and shocks to the aggregator.

As for a shock to the aggregator, it cannot have an impact on the aggregate since the system of equations is separable. Hence, this type of shock is only capable of affecting the aggregator's composition. Nonetheless, in terms of composition, the prices and survival productivity cutoff are completely determined by the aggregate and, so, they do not vary either. This establishes that the mass of incumbents is the only remaining variable free to adjust. Thus, the mass of incumbents plays the role of a residual variable, which adjusts to ensure that the equilibrium aggregate is consistent with its composition.

Regarding a positive shock to expected profits, it has an impact on the aggregate. In particular, it gives rise to pro-competitive effects because competition has to become tougher (i.e., the aggregate has to be greater) in order to offset the increase in profitability and, so, restore zero expected profits. In turn, optimal prices and the survival productivity cutoff are completely determined by the aggregate. Therefore, given tougher market conditions, the productivity cutoff increases and prices decrease.[6] With these values pinned down, only the mass of incumbents is left to be determined, which acts as a residual variable. Hence, depending on the magnitude of the changes in prices and the productivity cutoff, the mass of incumbents either decreases or increases to ensure consistency between

[6]The effect on prices holds under strategic complementarity of prices, which states that a tougher competitive environment makes the demand more price elastic.

the equilibrium aggregate and its composition. In particular, under a stability condition, consistency with the equilibrium aggregate is achieved through entry of incumbents.

Based on these insights, I proceed to analyze how new export opportunities and an exposure to import competition affect the domestic economy and its domestic firms. I conduct the analysis for small and big economies separately. To define small economies, the literature has generally proceeded in an ad hoc way by assuming that some endogenous variables are treated as parameters. In some cases, this has led to different assumptions about what variables should be kept fixed.[7] Instead, I formalize the concepts of small and large economies by making use of measure theory. Intuitively, I characterize small economies as negligible for the world but non-negligible relative to its domestic market.

In Section 5, I study the results for a small economy. In this case, separating the import-competition and export-opportunities channels can be accomplished through the study of unilateral liberalizations. I show that better export opportunities resemble a shock to expected profits: it creates a tougher competitive environment in the home market, thus increasing the survival productivity cutoff and decreasing firms' prices domestically. On the other hand, tougher import competition acts like a shock to the aggregator. Therefore, it has no impact on the market conditions or either domestic firms' prices, quantities and their survival productivity cutoff. Only the mass of domestic incumbents vary in terms of domestic variables. This result in particular is robust to different variations of the model, encompassing multidimensional firm heterogeneity (i.e., by demand and costs) and applying to setups where firms make multiple country-specific choices (e.g., quality and number of products). A corollary from the latter is that no decision made by a domestic firm is affected.

In Section 6, I study a setup with two large countries. Through this, I establish that a Metzler paradox arises, i.e., unilateral liberalizations generate anti-competitive effects. Unlike the case of a small economy, a unilateral liberalization is not capable of directly isolating each channel. The reason is the emergence of an additional channel, which I refer to as the export-conditions channel. To understand why they are confounded, consider two countries, H and F, with the former unilaterally opening its economy to imports from F. Let's consider the effects on H. Since countries are non-negligible, new export opportunities

[7] For instance, in models of monopolistic competition with a CES demand, Flam and Helpman (1987) consider that a small economy can affect the price index of foreign countries, while Demidova and Rodríguez-Clare (2009, 2013) dispense with this assumption.

in F affect the market conditions of its own domestic market which, in turn, affects H's export conditions. This generates feedback effects, making the import-competition and export-conditions channels operate simultaneously in H. I show that, properly accounting for each mechanism, H's domestic economy is only affected by the changes in H's export conditions. On the other hand, the effects stemming from the import-competition channel only affect domestic firms through their mass of incumbents. As a result, for the liberalizing country, *opening an economy to imports is qualitatively equivalent to only having worse export conditions.*

Finally, in Section 7, I provide a framework in which the import-competition channel is operating. To accomplish this, I proceed in two steps. First, I determine which assumptions are responsible for making it inactive in the baseline framework. The analysis reveals that this is due to the conjunction of free entry, homogeneity (ex ante or ex post) of marginal entrants, and the existence of a single sufficient statistic for optimal prices and profits. I establish that, as long as those assumptions are met, the import-competition channel is inactive even under oligopoly or market structures with large firms coexisting with negligible firms.

Second, based on this, I show that, by dispensing with the assumption of ex-ante homogeneity of firms in the Melitz model, the import-competition channel is reactivated and entails pro-competitive effects. Specifically, I consider a free-entry model where firms are heterogeneous at the moment of making their entry decisions and know their productivity. The assumption is in line with the model by Chaney (2008). Intuitively, under this framework, tougher import competition affects the economy since the firms' behavior at the market stage is what actually delineates the conditions of the competitive environment. In other words, it is the composition of the aggregator which determines the aggregate rather than the other way around.

Contributions and Related Literature. This paper makes several contributions. First, it is framed within the international trade literature that aims at providing general results to models of monopolistic competition with heterogeneous firms.[8] In spirit, it is in line with Arkolakis et al. (2012) who show that standard assumptions in New Trade models might impose a lot of structure and be crucial determinants of the results. Moreover, it is based on the insights of Alfaro and Lander (2017) who characterize the mechanisms

[8] See, for instance, Arkolakis et al. (2012), Zhelobodko et al. (2012), Neary and Mrazova (2017), and Arkolakis et al. (2019).

under operation across models of imperfect competition for the linear demand of Melitz and Ottaviano (2008).

My contribution in this respect is providing a methodology to analyze the Melitz model under any productivity distribution and a demand system which comprises the bulk of standard demand functions. The approach is based on the theory of large aggregative economies as in Acemoglu and Jensen (2010, 2015), extended to incorporate an endogenous number of agents. Moreover, by resorting to the tools of monotone comparative statics, all the results that I identify in this paper hold under a minimum set of assumptions on the primitives of the model.

Second, since the majority of the papers in the literature inquire upon welfare effects, they make use of an aggregate demand that can be derived from a representative consumer. On the contrary, since my focus is on the effects triggered by trade, I take the aggregate demand as a primitive, remaining agnostic about its microfoundation.[9] This makes the results robust to the "Anything Goes" Theorems.[10] Furthermore, putting less structure on the demand side allows me to inquire upon which assumptions are essential for the outcomes. One matter in which I exploit this is in relation to the so-called Marshall's Second Law of Demand. This states that the price elasticity of demand is increasing with respect to own price. Krugman (1979) indicates that assuming it "*seems to be necessary if this model is to yield reasonable results, and I make the assumption without apology*". Different papers have attempted to justify the assumption on theoretical and empirical grounds.[11]

The generality of the demand system allows me to separate the effect of a tougher competitive environment on the price elasticity (i.e., whether prices are strategic complements) and those from own price on the price elasticity (i.e., Marshall's Second Law of Demand). Once each assumption is separated, I show that several results are independent of the latter. For instance, the effects of trade on the behavior of domestic firms only depend

[9]This fact determines that the results in this paper are more general than assuming the GAS demand system of Pollak (1972). While that demand summarizes the market conditions through one aggregator, it is restricted to demands that can be obtained from the utility maximization of a representative consumer. In addition, as pointed out by Blackorby et al. (1993), an aggregate demand can summarize market conditions through an aggregator without entailing that each individual demand depends on an aggregator.

[10]These theorems refer to two different results. For exchange economies, the Sonnenschein-Mantel-Debreu theorem states that there are almost no restrictions on a function to be an excess demand. Chiappori and Ekeland (1999), more than thirty years after those papers due to the technical difficulties associated, generalize the results and show that this is also true for *market* demands.

[11]See, in particular, Mayer et al. (2016) and Neary and Mrazova (2017).

on whether prices are strategic complements. Thus, if the price elasticity increases when there are tougher market conditions, active firms would reduce their prices and markups.

This begs the question of why Marshall's Second Law plays a crucial role in other studies. The reason is that standard demands derived from a representative consumer confound the effects of competition on the price elasticity and those from own price on the price elasticity. Specifically, strategic complementarity of prices holds if and only if Marshall's Second Law is satisfied.[12] For these demands where both concepts cannot be distinguished, strategic complementary of prices provides an alternative way to justify why Marshall's Second Law should be a maintained assumption. Arguably, supposing strategic complementarity of prices is easier to interpret and justify than an assumption about the effect of own prices on the price elasticity of demand. It only requires that general decreases in prices and entry of new firms make the demand more elastic.

Finally, this paper touches upon the so-called Metzler paradox. This refers to the existence of anti-competitive effects following a unilateral liberalization. Specifically, the paradox arises when a reduction in inward trade barriers lessens competition, thus increasing the prices set by active domestic firms and lowering their survival productivity cutoff.[13] My contribution on this matter is to show that, under demand systems with a single aggregator, the outcome emerges because the import-competition channel is inactive. Consequently, this type of liberalization becomes qualitatively equivalent to worse export conditions in the liberalizing country.

2 An Illustration

The goal of the following illustration is to shed some light on the mechanisms at work in a Melitz model. To make them as stark as possible, I appeal to a simple scenario where firms face different type of exogenous shocks in a closed economy. Their effects in the domestic economy resemble the ones triggered by other shocks explored in this paper, such as new export opportunities and tougher import competition in a small economy.

[12]Formally, let ε be the price elasticity of demand, p the price, and $\mathbb{P}$ an aggregate. Standard demands satisfy that p and $\mathbb{P}$ enter into the demand through a function $f\left(p/\mathbb{P}\right)$. This implies that $\operatorname{sgn} \frac{\partial \varepsilon(p,\mathbb{P})}{\partial \mathbb{P}} = \operatorname{sgn} \frac{\partial \varepsilon(p,\mathbb{P})}{\partial p}$. For instance, this holds for demands derived from an additively separable direct utility, where $\mathbb{P}$ becomes the marginal utility of income. It also holds under the Melitz and Ottaviano's (2008) linear demand, and Feenstra's (2003) translog demand, among others.

[13]See Footnote 3 for references.

Given its prominence and adequacy for illustration purposes, the example makes use of the linear demand function of Melitz and Ottaviano (2008).

2.1 Setup and Equilibrium

Consider a closed economy and a specific industry in isolation. By this, it is meant that the sector has a negligible impact on wages and aggregate income. The economy comprises a unitary mass of identical agents. There is a pool of firms considering entry which are ex-ante symmetric and do not know their productivity. By paying a sunk cost F, they get a draw of productivity that determines marginal costs c, as well as an assignation of a variety ω. The productivity distribution determines a cumulative distribution G for marginal costs with support $[\underline{c}, \overline{c}]$. Let M^E be the measure of incumbents that decide to pay F, and M the subset serving the market.

Following Melitz and Ottaviano (2008), suppose there are no overhead costs and that the demand for variety ω is

$$q_\omega\left(p_\omega, \mathbb{P}, M\right) := \frac{\delta}{\eta M + \gamma} - \frac{1}{\gamma} p_\omega + \frac{\eta}{\eta M + \gamma} \frac{1}{\gamma} \mathbb{P},$$

where $\delta, \gamma, \eta > 0$ and $\mathbb{P} := \int_0^M p_\omega \, \mathrm{d}\omega$. It can be shown that this demand displays an increasing price elasticity and has a choke price function given by

$$p^{\max}\left(\mathbb{P}, M\right) := \frac{\delta\gamma + \eta\mathbb{P}}{\eta M + \gamma}.$$

Each value $p^{\max}$ belonging to the range of the function $p^{\max}\left(\mathbb{P}, M\right)$ represents a statistic that summarizes the conditions of the market in which the firm is operating. Consistent with a monopolistic competition structure, its value cannot be influenced by any firm unilaterally.

The demand can be reexpressed in terms of the choke price value,

$$q\left(p^{\max}, p_\omega\right) := \frac{p^{\max} - p_\omega}{\gamma}. \tag{1}$$

Written in this way, it implies that, conditional on values $p^{\max}$ and p_ω, the demand schedule is completely determined. As a result, the composition of $p^{\max}$ is irrelevant from the firm's point of view: different combinations of M and $\mathbb{P}$ that result in the same value of $p^{\max}$ are considered equivalent.

For a given value of $p^{\max}$, an active firm with marginal costs c sets the following optimal prices,

$$p\left(p^{\max};c\right):=\frac{p^{\max}+c}{2}, \tag{2}$$

determining that its optimal domestic profits are

$$\pi\left(p^{\max};c\right):=\frac{\left(p^{\max}-c\right)^{2}}{4\gamma}.$$

The survival marginal cost cutoff corresponds to the cost that makes the firm indifferent between serving the market or not,

$$c\left(p^{\max}\right)=p^{\max}. \tag{3}$$

Furthermore, given a mass of domestic incumbents M^{E}, the equilibrium at the market stage requires that all firms are optimizing simultaneously. Since for each firm with marginal cost c, optimal prices (2) and the marginal-cost cutoff to serve the market (3) are determined by the value of $p^{\max}$, the equilibrium can be characterized in the following way. Define a function Γ such that

$$\Gamma\left(p^{\max},M^{E}\right):=\frac{\delta\gamma+\eta\mathbb{P}\left(p^{\max},M^{E}\right)}{\eta M\left(p^{\max},M^{E}\right)+\gamma},$$

where $\mathbb{P}\left(p^{\max};M^{E}\right):=M^{E}\int_{\underline{c}}^{p^{\max}}p\left(p^{\max};c\right)\mathrm{d}G\left(c\right)$ by using that $\frac{M}{G[c(p^{\max})]}=M^{E}$. The function Γ takes $p^{\max}$ as input, and provides a value $\Gamma\left(p^{\max};M^{E}\right)$ as output. This output is the choke price that would be self-generated by the firms if they were making optimal decisions based on a choke price with value $p^{\max}$. Thus, an equilibrium at the market stage is given by a choke price that constitutes a fixed point of Γ, establishing that it is consistent with the firms' mass and average price of optimizing firms.

Adding the possibility that Γ is subject to an exogenous shock α that decreases the function, the equilibrium condition is

$$\Gamma\left(p^{\max},M^{E};\alpha\right)-p^{\max}=0. \tag{4}$$

Finally, making use of (3), the free-entry condition is

$$\widetilde{\pi}\left(p^{\max}\right)+\epsilon=F, \tag{5}$$

where $\widetilde{\pi}\left(p^{\max *}\right):=\int_{\underline{c}}^{p^{\max *}}\pi\left(p^{\max *};c\right)\mathrm{d}G\left(c\right)$ and ϵ is a shock to expected profits.

The equilibrium is defined by a vector $\left(p^{\max *},M^{E*}\right)$ such that equations (4) and (5) hold, $c^* := c(p^{\max *})$ satisfies (3), $M^* := M^{E*}G(c^*)$, and $(p^*(c), q^*(c))_{c \leq c^*}$ where $p^*(c) := p(p^{\max *}; c)$ and $q^*(c) := q(p^{\max *}, p^*(c))$.

2.2 Analysis of the Equilibrium

By the specific way in which I have set the equilibrium conditions, we can observe that, once $(p^{\max *}, M^{E*})$ is identified through conditions (4) and (5), the rest of the equilibrium values are also pinned down. In particular, regarding the decisions of firms, equations (1), (2), and (3) reveal that $p^{\max *}$ is a sufficient statistic for the equilibrium prices, quantities, and marginal cost cutoff. Therefore, conditional on $p^{\max *}$, their choices are independent of M^{E*}.

Delving into how (4) and (5) identify $(p^{\max *}, M^{E*})$, notice that the system is separable: *the free-entry condition* (5) *does not depend on* M^{E*} *and, so, it pins down* $p^{\max *}$. Taking $p^{\max}$ as a measure of competition, this fact provides some interpretations of what each condition (4) and (5) is accomplishing.

First, the free-entry condition (5) identifies how tough the competitive environment has to be in order to ensure that firms have zero expected profits. Thus, (5) determines the *level* of competition $p^{\max *}$.

Second, condition (4) establishes the *composition* that validates it. However, once that $p^{\max *}$ has been pinned down, the only variable free to vary in terms of the composition is M^{E*}. This is because prices of active firms and the proportion of active firms out of the mass of incumbents are completely determined by p^{max*}. Hence, M^{E*} plays the role of a *residual variable* which adjusts to validate the value of $p^{\max *}$ that ensures zero expected profits.

2.3 Effects of the Exogenous Shocks

Consider the shock α. Since α is a shock to (4), but not to (5), $p^{\max *}$ does not vary. The key to understanding this is that (5) on its own identifies $p^{\max *}$. As a corollary, given that the choke price is a sufficient statistic for firms' decisions, the equilibrium prices, quantities, and marginal cost cutoff do not vary either. Any variation in α translates exclusively into a change in M^{E*}.

An increase in α resembles the case of tougher import competition in a small economy, which might take the form of either a reduction of import prices or an increase in the mass of active foreign firms. More generally, when accounted for properly, it is akin to the impact from the import-competition channel.

Unlike the shock α, a change in ϵ affects condition (5) directly. A positive shock to expected profits creates pro-competitive effects, triggering a decrease in $p^{\max *}$. This follows because, if expected profits increase, tougher conditions in the market are required to restore zero expected profits. In turn, the tougher competitive environment decreases both prices and the marginal cost cutoff. The effects of a shock to expected profits are similar to those triggered by better export opportunities in a small economy and, more broadly, to those of the export-opportunities channel.

3 The Model

In this section, I proceed in several steps. First, I establish a framework in line with trade models of monopolistic competition and firm heterogeneity as in Melitz (2003). I extend the setup to account for the existence of small and big economies. I then add some structure to the demand in order to encompass functions that summarize market conditions through a single sufficient statistic. Finally, I solve for the equilibrium and highlight some of its properties. All the proofs are relegated to the appendix.

3.1 Structure of the Model

There is a world economy with a bounded set of countries $\mathcal{C} \subset \mathbb{R}_{++}$. Throughout the paper, any subscript ij refers to i as the origin country and j as the destination country. In the setup, I consider the possibility that countries can be small or big. I incorporate the distinction in the following way. Denote by λ the Lebesgue measure and # the counting measure. I suppose that $\mathcal{C}$ can be partitioned into sets $\mathcal{C}^S$ and $\mathcal{C}^B$, such that $\mathcal{C}^S \subset \mathbb{R}_{++}$ and $\mathcal{C}^B \subset \mathbb{N}_+$, and is endowed with a mixed measure $\mu(\cdot) := \lambda\left(\cdot \cap \mathcal{C}^S\right) + \#\left[\cdot \cap \mathcal{C}^B\right]$. The measure μ captures that, depending on the size that a country attains, it should be considered either small or big relative to the world.[14]

In addition, there is a country-specific measure μ_i over $\mathcal{C}$ which indicates the size of i's trading partners (including i). This measure is incorporated to account for the fact that, even if i is a small economy, it should be considered non-negligible relative to its own

[14] The idea of treating small economies as part of a continuum has been also used by Lucas (1988) and Matsuyama (1996).

country. Formally, this is captured by defining $\mu_i(\cdot) := \lambda\left(\cdot \cap \mathcal{C}^S\right) + \#\left[\cdot \cap \left(\mathcal{C}^B \cup \{i\}\right)\right]$.[15] To avoid uninteresting cases, I suppose that there is always at least one trading partner $j \neq i$ that is big for each $i \in \mathcal{C}$. Throughout the text, any integral that appears should be understood as a Lebesgue integral. In this way, sums and integrals are captured within a unified notation.[16]

I study a specific industry in isolation, which allows me to focus on the effects at the microeconomic level. By this, I mean that the industry does not affect aggregate conditions such as the wages or income of a country.[17] The sector consists of a differentiated good with a set $\overline{\Omega}_i := \left[0, \overline{M}_i\right]$ of horizontally differentiated varieties (endowed with the Lebesgue measure), where $\overline{M}_i$ is an arbitrary large mass of all conceivable varieties in i. This mass is taken as given throughout the analysis. I denote by $\Omega_{ji} := [0, M_{ji}]$ the subset of varieties from j which are sold in i, and $M_i := \int_{j \in \mathcal{C}} M_{ji}\, \mathrm{d}\mu_i(j)$ the total mass of varieties consumed in i. I denote by $q_{ji}(\omega)$ the demand of a variety ω in i produced by a firm from j. The demand of each variety is taken as a primitive of the model and I relegate its characterization to the next subsection.

The differentiated sector in i has a supply side as in Melitz (2003). There is a mass $\overline{M}_i$ of prospective entrants which are ex-ante identical. To enter, they consider paying a fixed (sunk) entry cost $F_i > 0$ that enables them to receive a productivity draw φ and an assignation of a unique variety ω. The random variable representing productivity has non-negative support $\left[\underline{\varphi}_i, \overline{\varphi}_i\right]$, with $\underline{\varphi}_i \in \mathbb{R}_+$, $\overline{\varphi}_i \in \mathbb{R}_{++} \cup \{\infty\}$, and a continuous strictly increasing cdf G_i with density g_i.

Once firms know their productivity, they make decisions regarding whether to serve each country. They can choose not to sell in country j or do so by incurring in an overhead fixed cost $f_{ij} \geq 0$ (with strict inequality if the demand's choke price is infinite). Producing in i to serve j entails constant marginal costs $c_i(\varphi, \tau_{ij})$, where τ_{ij} is a trade cost that a

[15] To keep matters simple, I have assumed that a country that is small for the world is also small for each country, except for itself. However, a small economy could constitute a non-negligible trading partner for some specific country. For instance, while Sweden could be considered a small economy in the world, it is not for Denmark. A case like this can be easily captured by partitioning $\mathcal{C}$ in subsets that are country specific. Formally, $\mathcal{C}_i^S$ and $\mathcal{C}_i^B$ for each $i \in \mathcal{C}$, where μ_i is defined accordingly.

[16] Thus, for instance, the total measure of countries $\mathcal{C}$ is $\mu(\mathcal{C}) = \int_{j \in \mathcal{C}} \mathrm{d}\mu(j)$, which is a compact way to express $\int_{j \in \mathcal{C}^S} \mathrm{d}j + \sum_{j \in \mathcal{C}^B} 1$.

[17] With demands derived from a representative consumer, this could be rationalized by the standard assumption of a quasilinear utility function and the existence of a homogeneous good that pins down wages and absorbs any income effect. More generally, following Neary (2016), the industry under consideration could be thought as part of a continuum, so that it has a negligible impact on the aggregate conditions of the country.

firm in i has to incur to sell in j. I adopt the convention that there are no trade costs within the domestic market and assume c_i is smooth, decreasing in φ and increasing in τ_{ij}. Notice I allow for marginal costs that differ by country, reflecting the possibility that the cost of inputs is different across them.

Each firm from i makes a decision in j over prices $p_{ij}(\omega) \in P_j := \left[\underline{p}_j, \overline{p}_j\right]$, with $\underline{p}_j \in \mathbb{R}_+$ and $\overline{p}_j \in \mathbb{R}_{++} \cup \{\infty\}$, where $\overline{p}_j$ is equal to or greater than the demand's choke price. Markets are segmented with resale forbidden and firms can charge different prices in each country. Also, I suppose that, in equilibrium, every firm that exports also sells domestically. I denote by $\mathbf{p}_{ij} := (p_{ij}(\omega))_{\omega \in \overline{\Omega}_i}$ the vector of prices of all varieties from i in j and endow it with the pointwise order relation.[18] Entry and exit of firms is incorporated by assuming that any unavailable variety in j has a price $\overline{p}_j$.

The mass of incumbents in country i is denoted by M_i^E, with a mass of active firms in i selling to j given by $M_{ij} := [1 - G_i(\varphi_{ij})] M_i^E$, where φ_{ij} is the productivity cutoff of a firm from i to break even in country j.

Definition 1. *In a country, **the market structure is à la Melitz** when it is given by the setup described above.*

Notice that the Melitz model with a degenerate productivity distribution and $f_{ij} = 0$ for all $i, j \in \mathcal{C}$ leads to a **market structure à la Krugman**. Thus, the results of this paper are also valid for that framework.

3.2 Demand System

I add some structure to the demand side in order to get a demand system that summarizes market conditions through a single sufficient statistic. The assumption is satisfied for a large group of standard demand functions used in the literature.[19] I start with some definitions.

Definition 2. *A **price aggregator** for country i is a function $\mathcal{P}_i : \left(\times_{\overline{\Omega}_j} P_i\right)_{j \in \mathcal{C}} \to \mathbb{R}_+$ with $(\mathbf{p}_{ji})_{j \in \mathcal{C}} \mapsto \mathcal{P}_i\left[(\mathbf{p}_{ji})_{j \in \mathcal{C}}\right]$ such that any measure-zero set of firms cannot influence its*

[18] I have chosen this order relation for simplicity. The first-order dominance relation could be assumed.

[19] See Footnote 1 for a list. Also, several examples of demands can be found in Appendix C, along with the way in which they can be rewritten to reflect the assumption.

value.[20] *A* ***price aggregate*** *for country i is a value $\mathbb{P}_i \in \text{range}\,\mathcal{P}_i$ where range $\mathcal{P}_i$ is a convex set.*

A price aggregator represents a statistic that summarizes information related to country's prices. It could be, for instance, the average, variance, or a more complex function which depends on different price statistics. Some properties of a price aggregator are worth elaborating on. First, the general form in which I have defined it allows for asymmetries. Thus, different subsets of firms' prices might impact the aggregator differently. Furthermore, $\mathcal{P}_i$ takes as input the vector of prices and provides a real number as an output. The intuition is that in large economies, under some appropriate law of large numbers, the price statistic vanishes any individual uncertainty and, so, there is no uncertainty in the aggregate.[21]

Usually price aggregators take a more specific form, which I refer to as standard price aggregators. I make use of them throughout the paper, although the conclusions of the paper could also be obtained by using a characterization as in Definition 2.

Definition 3. *A* ***standard price aggregator*** *for country i is a function $\mathcal{P}_i\left[(\mathbf{p}_{ji})_{j\in\mathcal{C}}\right] := \int_{j\in\mathcal{C}}\left[\int_{\omega\in\overline{\Omega}_j} h_{ji}\left(p_{ji}\left(\omega\right)\right)\mathrm{d}\omega\right]\mathrm{d}\mu_i\left(j\right)$ where the absolute value of h_{ji} and all its derivatives are dominated by integrable positive functions.*[22]

Basically, this type of price aggregator ensures that, even if optimal prices are discontinuous as happens when we consider entry and exit, it has a compact convex range due to Lyapunov's Convexity Theorem.[23] It also allows us to apply the technique of differentiation under the integral sign whenever it simplifies the calculations. Notice that the mass of varieties is, in fact, a standard price aggregator since $M_i := \int_{j\in\mathcal{C}}\left[\int_{\omega\in\overline{\Omega}_j} \mathbb{1}\left(p_{ji}\left(\omega\right) < \overline{p}_i\right)\mathrm{d}\omega\right]\mathrm{d}\mu_i\left(j\right).$ Next, I define the concepts of an aggregator and an aggregate.

Definition 4. *Let $\boldsymbol{\mathcal{P}}_i := \left(\mathcal{P}_i^k\right)_{k=1}^K$ and $\mathbf{P}_i := \left(\mathbb{P}_i^k\right)_{k=1}^K$ with $K < \infty$ and where each $\mathcal{P}_i^k$ is a price aggregator and $\mathbb{P}_i^k$ a price aggregate as in Definition 3 with functions $\left(h_{ji}^k\right)_{j\in\mathcal{C}}$. An* ***aggregator*** *for country i is a smooth real-valued function $\mathcal{A}_i$ with $\mathbf{P}_i \mapsto \mathcal{A}_i\left(\mathbf{P}_i\right)$ which is*

[20] Formally, let $\left(\mathbf{p}'_{ji}\right)_{j\in\mathcal{C}}$ and $\left(\mathbf{p}''_{ji}\right)_{j\in\mathcal{C}}$ be two vectors such that the set of prices with $p'_{ji}\left(\omega\right) \neq p''_{ji}\left(\omega\right)$ is of measure zero, then $\mathbb{P}'_i = \mathbb{P}''_i$ where $\mathbb{P}'_i := \mathcal{P}_i\left[\left(\mathbf{p}'_{ji}\right)_{j\in\mathcal{C}}\right]$ and $\mathbb{P}''_i := \mathcal{P}_i\left[\left(\mathbf{p}''_{ji}\right)_{j\in\mathcal{C}}\right]$.

[21] Under some assumptions, this real number corresponds to a degenerate random variable that takes a value $\mathbb{P}_i$ with probability 1. For further details, see Uhlig (1996) and Acemoglu and Jensen (2010).

[22] Consistent with Definition 2, the standard price aggregator could be defined through functions h_ω such that asymmetries between varieties, and not only among countries, are allowed.

[23] See, for instance, Aliprantis and Border (2006).

decreasing in $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ *when it is defined through* $\boldsymbol{\mathcal{P}}_i$*. An* ***aggregate*** *for country* i *is a value* $\mathbb{A}_i \in \text{range } \mathcal{A}_i$.[24]

The aggregator plays the role of a single sufficient statistic that sums up the conditions of the sector in country i.[25] Since reductions of prices increase the aggregate (which includes entry of firms by setting a lower price than the choke price), it can be interpreted as a measure of the competitive environment's toughness. Making use of these definitions, the demand is specified in the following way.

Assumption DEM. *The* ***demand of a variety*** ω *in* j *produced by a firm from* i *is given by*

$$q_{ij}(\omega) := \max\{0, q_j[\mathbb{A}_j, p_{ij}(\omega)]\},$$

where $\mathbb{A}_j$ *is as in Definition 4 and* q_j *is a smooth function such that* q_j *is decreasing in* $\mathbb{A}_j$ *and* $p_{ij}(\omega)$*, and it has a choke price belonging to* $\mathbb{R}_{++} \cup \{\infty\}$.

Mathematically, Assumption DEM states that the demand satisfies weak separability of $\left(\mathbb{P}_j^k\right)_{k=1}^K$ from $p_{ij}(\omega)$.[26] Hence, from the firm's point of view, the only relevant piece of information regarding the environment in j is $\mathbb{A}_j$, while its composition is irrelevant.

3.3 Equilibrium Conditions

The first-order condition gives an implicit characterization of the optimal prices set by a φ-type firm from i which is active in j,[27]

$$p_{ij} = m_j(\mathbb{A}_j, p_{ij})\, c_i(\varphi, \tau_{ij}), \tag{6}$$

where $m_j(\mathbb{A}_j, p_{ij}) := \frac{\varepsilon_j(\mathbb{A}_j, p_{ij})}{\varepsilon_j(\mathbb{A}_j, p_{ij})-1}$ is the firm's markup, with $\varepsilon_j(\mathbb{A}_j, p_{ij}) := -\left.\frac{\partial \ln q_j(\mathbb{A}_j, p)}{\partial \ln p}\right|_{p=p_{ij}}$. Let $p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ be the implicit p_{ij} that satisfies (6), and $m_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ the markups

[24] It can be shown that, by the assumptions made, range $\mathcal{A}_i$ is convex. This is important for having well-defined equilibrium conditions. See Appendix E for further details.

[25] The aggregator is not uniquely defined. Any monotone transformation defines a new aggregator. This determines that the aggregator defines, in fact, a class of functions.

[26] Exploiting this feature and based on the differential characterization of weak separability due to Leontief (1947) and Sono (1961), in Appendix D I provide conditions to check whether a demand satisfies Assumption DEM.

[27] As is standard in the monotone comparative statics literature, in order to not confound the assumptions necessary for comparative statics with those ensuring a well-defined solution, throughout the text I assume that any optimal solution or equilibrium condition exists, and is unique and interior. For details about existence and uniqueness, see Appendix E.

evaluated at this price.[28] Taking into account that firms also decide whether to be active in each market, optimal prices for each $\varphi \in \left[\underline{\varphi}_i, \overline{\varphi}_i\right]$ are then

$$p^*_{ij}\left(\mathbb{A}_j, \varphi_{ij}, \varphi; \tau_{ij}\right) := \begin{cases} p_{ij}\left(\mathbb{A}_j, \varphi; \tau_{ij}\right) & \text{if } \varphi \geq \varphi_{ij} \\ \overline{p}_j & \text{otherwise} \end{cases} \quad \begin{array}{l} \text{for } \mu\text{-almost all } i \in \mathcal{C} \\ \mu_i\text{-almost all } j \in \mathcal{C}, \end{array} \tag{PRICE}$$

where φ_{ij} is a productivity cutoff of a firm from i in j. This, in turn, determines the optimal quantities for each $\varphi \in \left[\underline{\varphi}_i, \overline{\varphi}_i\right]$:

$$q^*_{ij}\left(\mathbb{A}_j, \varphi_{ij}, \varphi; \tau_{ij}\right) := \begin{cases} q_{ij}\left[\mathbb{A}_j, p_{ij}\left(\mathbb{A}_j, \varphi; \tau_{ij}\right)\right] & \text{if } \varphi \geq \varphi_{ij} \\ 0 & \text{otherwise} \end{cases} \quad \begin{array}{l} \text{for } \mu\text{-almost all } i \in \mathcal{C} \\ \mu_i\text{-almost all } j \in \mathcal{C}. \end{array} \tag{QTY}$$

Conditional on entry, the optimal gross profits of an active φ-type firm from i in market j are

$$\pi_{ij}\left(\mathbb{A}_j, \varphi; \tau_{ij}\right) := q_{ij}\left[\mathbb{A}_j, p_{ij}\left(\mathbb{A}_j, \varphi; \tau_{ij}\right)\right]\left[p_{ij}\left(\mathbb{A}_j, \varphi; \tau_{ij}\right) - c_i\left(\varphi, \tau_{ij}\right)\right].$$

Thus, optimal profits are

$$\pi^*_{ij}\left(\mathbb{A}_j, \varphi_{ij}, \varphi; \tau_{ij}, f_{ij}\right) := \mathbb{1}_{(\varphi \geq \varphi_{ij})}\left[\pi_{ij}\left(\mathbb{A}_j, \varphi; \tau_{ij}\right) - f_{ij}\right].$$

Given a value of $\mathbb{A}_i$, the zero-profit condition of a firm from i selling in j determines the productivity cutoff φ_{ij}. This is given by the infimum productivity such that profits in j are zero. Given the continuum of firms, the system of zero-profits conditions around the world is

$$\pi_{ij}\left(\mathbb{A}_j, \varphi_{ij}; \tau_{ij}\right) = f_{ij} \text{ for } \mu\text{-almost all } i \in \mathcal{C} \text{ and } \mu_i\text{-almost all } j \in \mathcal{C}. \tag{ZCP}$$

I denote the φ_{ij} that satisfies equation (ZCP) by $\varphi^*_{ij}\left(\mathbb{A}_j; \tau_{ij}, f_{ij}\right)$.

As for the market-clearing conditions for i, given a mass of incumbents $\mathbf{M}^E := \left(M^E_j\right)_{j\in\mathcal{C}}$, an equilibrium for the market stage requires that, up to a set of measure zero, all firms make pricing decisions optimally. This condition can be characterized in a straightforward way by exploiting the structure of the demand system.

First, notice that all firms' decisions are determined exclusively by $\mathbb{A}_i$, which is a value belonging to the range of the aggregator $\mathcal{A}_i\left(\mathbf{P}_i\right)$. Likewise, each $\mathbb{P}^k$ is a value determined

[28] There is some abuse of notation by denoting optimal prices of active firms by $p_{ij}\left(\mathbb{A}_j, \varphi; \tau_{ij}\right)$. Since the demand function with positive quantities is a function $q_j\left(\cdot\right)$ and, conditional on τ_{ij}, the demand does not depend on i, strictly speaking the function is $p_j\left(\mathbb{A}_j, \varphi; \tau_{ij}\right)$. I do this to make it clear what the country of origin is. A similar remark applies to the rest of the variables.

by $\mathcal{P}_i^k$ which requires firms' optimal prices from $j \in \mathcal{C}$ as inputs of the function. Optimal prices are completely characterized by the price decision $p_{ji}^*(\mathbb{A}_i, \varphi_{ji}, \varphi; \tau_{ji})$, the survival productivity cutoff $\varphi_{ji}^*(\mathbb{A}_i; \tau_{ji}, f_{ji})$, and the density of firms' mass for each productivity level, which for country j is $M_j^E g_j(\varphi)$. Thus, in equilibrium, the coordinate k of $\boldsymbol{\mathcal{P}}_i^*$ can be described by a function $\mathcal{P}_i^{k*}\left(\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j\in\mathcal{C}}\right)$.[29] This implies that the aggregator can be expressed by a function $\mathcal{A}_i^*(\cdot) := \mathcal{A}_i[\boldsymbol{\mathcal{P}}_i^*(\cdot)]$ where $(\cdot) := \left(\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j\in\mathcal{C}}\right)$. In each country i, the market stage is in equilibrium when there exists an $\mathbb{A}_i$ such that each firm's optimal decision self-generates that $\mathbb{A}_i$. Formally, this means that, for a given $\mathbf{M}^E$, the aggregate $\mathbb{A}_i$ has to be a fixed point of $\mathcal{A}_i^*$:

$$\mathbb{A}_i = \mathcal{A}_i^*\left(\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j\in\mathcal{C}}\right) \text{ for } \mu\text{-almost all } i \in \mathcal{C}, \tag{MS}$$

The label "MS" is applied to (MS) because, for a given mass of incumbents, it constitutes the condition for equilibrium at the market stage.

Finally, plugging in $\varphi_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij})$, the free-entry conditions in the world are

$$\int_{j\in\mathcal{C}} \widetilde{\pi}_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij})\, \mathrm{d}\mu_i(j) = F_i \text{ for } \mu\text{-almost all } i \in \mathcal{C}, \tag{FE}$$

where $\widetilde{\pi}_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij}) := \int_{\varphi_{ij}(\mathbb{A}_j;\tau_{ij},f_{ij})}^{\overline{\varphi}_i} [\pi_{ij}(\mathbb{A}_j, \varphi; \tau_{ij}) - f_{ij}]\, \mathrm{d}G_i(\varphi)$.[30]

With all the conditions established, the equilibrium can be defined in the following way.

Definition 5. *An* ***equilibrium*** *is defined as values* $\mathbb{A}_i^*$, M_i^{E*}, M_{ij}^*, M_i^*, φ_{ij}^*, $p_{ij}^*(\varphi)$ *and* $q_{ij}^*(\varphi)$ *for* $\varphi \in \left[\underline{\varphi}_i, \overline{\varphi}_i\right]$ *and* $i, j \in \mathcal{C}$ *such that (PRICE), (QTY), (ZCP), (MS) and (FE) hold, and where* $M_i^* := \int_{j\in\mathcal{C}} M_{ji}^*\, \mathrm{d}\mu_i(j)$, $M_{ji}^* := \left[1 - G_j\left(\varphi_{ji}^*\right)\right] M_j^{E*}$, $p_{ij}^*(\varphi) := p_{ij}^*\left(\mathbb{A}_j^*, \varphi_{ij}^*, \varphi; \tau_{ij}\right)$ *and* $q_{ij}^*(\varphi) := q_{ij}^*\left(\mathbb{A}_j^*, \varphi_{ij}^*, \varphi; \tau_{ij}\right)$.

3.4 Equilibrium Properties

Next, I delve into how the equilibrium values are identified. Given how I have set the equilibrium conditions, the following lemma follows by simple observation. I state it and then proceed to explain its implications.

[29] To see this more clearly, under the usual assumption that price aggregators are defined in terms of the prices set by active firms, the coordinate k of $\boldsymbol{\mathcal{P}}_i^*$ would be given by $\int_{j\in\mathcal{C}} M_j^E \left[\int_{\varphi_{ji}^*(\mathbb{A}_i;\tau_{ji},f_{ji})}^{\overline{\varphi}_j} h_{ji}[p_{ji}(\mathbb{A}_i, \varphi; \tau_{ji})]\, \mathrm{d}G_j(\varphi)\right] \mathrm{d}\mu_i(j)$.

[30] Notice that the left-hand side of (FE) is equivalent to $\int_{j\in\mathcal{C}^S} \widetilde{\pi}_{ij}(\cdot)\, \mathrm{d}\mu_i(j) + \sum_{j\in\mathcal{C}^B\setminus\{i\}} \widetilde{\pi}_{ij}(\cdot) + \widetilde{\pi}_{ii}(\cdot)$. This definition applies irrespective of whether i is a small or big country. In the former case, the integral over $\mathcal{C}^S\setminus\{i\}$ or $\mathcal{C}^S$ gives the same result because i is negligible and, also, $\mathcal{C}^B\setminus\{i\} = \mathcal{C}^B$ because i does not belong to $\mathcal{C}^B$.

Lemma 1. *Suppose the market structure in each country $i \in \mathcal{C}$ is à la Melitz with demands for each variety as in Assumption DEM. Then:*

- *all the equilibrium values can be obtained by knowledge of $\left(\mathbb{A}_i^*, M_i^{E*}\right)_{i\in\mathcal{C}}$ which is determined by conditions (FE) and (MS), and*
- *the system of equations formed by (FE) and (MS) is separable such that $(\mathbb{A}_i^*)_{i\in\mathcal{C}}$ is pinned down exclusively by (FE) and independently of $\mathbf{M}^E$.*

One important consequence of Lemma 1 is that the equilibrium prices, quantities, and survival productivity cutoffs in country i are completely determined in equilibrium by $\mathbb{A}_i^*$. Thus, only the system of conditions (FE) is relevant for those variables, while the system (MS) is not.

In addition, an interpretation of what conditions (FE) and (MS) accomplish can be provided. First, as I mentioned above, we can interpret the aggregate $\mathbb{A}_i^*$ as a measure of competition in country i. As such, (FE) pins down the **level** of competition in each country that is consistent with zero expected profits. Second, a value for the aggregate belongs to the range of the aggregator $\mathcal{A}_i$. By (FE), regardless of how $\mathcal{A}_i^*$ is composed of in country i, its value has to be $\mathbb{A}_i^*$. Thus, (MS) determines the **composition** of the aggregator that is consistent with the $\mathbb{A}_i^*$ identified in (FE).

To understand how the composition is determined, notice that the aggregator depends on the mass of incumbents and the pricing of firms, where the latter includes the decision to become inactive and, hence, the survival productivity cutoff. However, the prices and productivity cutoffs are completely determined by $\mathbb{A}_i^*$, so that $\mathbf{M}^E$ is the only variable free to adjust. Therefore, $\mathbf{M}^E$ acts as a residual vector that adjusts to make the composition of the aggregator validate $\mathbb{A}_i^*$, thereby ensuring it is consistent with an equilibrium at the market stage.

3.5 Assumptions for Comparative Statics

I state three assumptions to obtain definite comparative statics when neutrality of effects do not arise.

I begin by adding an assumption which only affects (MS). Thus, it is only necessary for comparative statics results for the mass of incumbents.

Assumption STB. *For any $\left(\mathbb{A}_i, \mathbf{M}^E\right)$, $\frac{\partial \mathcal{A}_i^*\left(\mathbb{A}_i, \mathbf{M}^E\right)}{\partial \mathbb{A}_i} < 1$.*

Assumption STB has some specific interpretations in terms of the market stage. First, in case the condition holds only when evaluated at the equilibrium value $\mathbb{A}_i^*$ rather than globally, it becomes the usual Poincaré-Hopf index condition for the model with a given mass of incumbents. This constitutes an “almost” if and only if condition for uniqueness of the equilibrium at the market stage.[31] In addition, when it holds for any $\mathbb{A}_i$, it establishes that $\mathcal{A}_i$ is a contraction along any optimal path. Hence, the assumption can be justified as a global stability condition for the market stage in line with Samuelson's Correspondence Principle (this explains the label “STB”). To get unambiguous global comparative-statics results, stability of some sort is usually required.[32]

Finally, I state two additional assumptions concerning how the price elasticity of demand is affected by the aggregate and own prices.

Assumption SC. *For any* $(p, \mathbb{A}_i)$, $\frac{\partial \varepsilon_i(\mathbb{A}_i,p)}{\partial \mathbb{A}_i} > 0$.

Assumption MSLD. *For any* $(p, \mathbb{A}_i)$, $\frac{\partial \varepsilon_i(\mathbb{A}_i,p)}{\partial p} > 0$.

Assumption MSLD has become standard since at least Krugman (1979), who supposed it “without apologies”, and it is usually referred to as Marshall's Second Law of Demand. As for Assumption SC, it indicates that a tougher competitive environment increases the price elasticity of demand. It is equivalent to assuming that prices are strategic complements. These two facts justify the labels “MSLD” and “SC”.

One advantage of taking a demand as a primitive is that it allows me to distinguish between these two assumptions. This is not always possible. For instance, when demands derive from additively separable utility functions or for a linear demand, $\operatorname{sgn} \frac{\partial \varepsilon(\mathbb{A},p)}{\partial \mathbb{A}} = \operatorname{sgn} \frac{\partial \varepsilon(\mathbb{A},p)}{\partial p}$ is satisfied. As I show below, the behavior of domestic firms depend on Assumption SC rather than Assumption MSLD. Arguably, assuming that tougher competition makes demand more price elastic is more intuitive than supposing that the Marshall's Second Law of Demand holds.

[31]Formally, the local condition is $\left.\frac{\partial \mathcal{A}_i(\mathbb{A}_i;\mathbf{M}^E)}{\partial \mathbb{A}_i}\right|_{\mathbb{A}_i=\mathbb{A}_i^*} < 1$. Under compactness and smoothness conditions, the Poincaré-Hopf condition is an "almost" if and only if, in the sense that if the term is strictly greater than 1 there are necessarily multiple equilibria. Therefore, if we can rule out the possibility that $\neq 1$, the condition is also necessary.

[32]This is true even in well-behaved models with complementarities as in Milgrom and Shannon (1994). Under supermodularity assumptions, only a characterization for extremal equilibria is obtained and this is silent about the behavior of the remaining equilibria. Echenique (2002) shows that, if our goal is to obtain definite comparative statics for *any* equilibrium, stability conditions are required.

4 Shocks to the Aggregator and to Expected Profits

In this section, I present results similar to those obtained in Section 2. I consider a closed economy that faces two types of shock: α that hits the aggregator and ϵ that affects the expected profits. Similar to the usual analogy of a trade liberalization to an increase in the market size, the effects of these shocks resemble the impact in a small domestic economy from tougher import competition and better export opportunities, respectively. More broadly, when large countries are incorporated, they correspond to the effects from the import-competition and export-opportunities channels.

Incorporating the shocks, the equilibrium conditions (MS) and (FE) for a closed economy evaluated at the optimal values are:

$$\widetilde{\pi}\left(\mathbb{A}^*\right)+\epsilon=F, \tag{FE-CL}$$

$$\mathcal{A}^*\left(\mathbb{A}^*, M^{E*}; \alpha\right)=\mathbb{A}^*, \tag{MS-CL}$$

where $\widetilde{\pi}\left(\mathbb{A}\right) := \int_{\varphi^*(\mathbb{A})}^{\overline{\varphi}} \left[\pi\left(\mathbb{A},\varphi\right)-f\right] \mathrm{d}G\left(\varphi\right)$. Without loss of generality, I assume that $\mathcal{A}^*$ is increasing in α.

I analyze variations in α and ϵ separately. The following proposition formalizes the effects of the former.

Proposition 1: ***Shock to the Aggregator***

Suppose a closed economy with a market structure à la Melitz and demands for varieties as in Assumption DEM. If there is a positive shock to the aggregator (i.e., an increase in α) then:

- *the aggregate remains the same (i.e., the level of competition does not vary),*
- *the prices, markups, and quantities of active firms remain the same,*
- *the survival productivity cutoff remains the same, and*
- *the mass of incumbents and the mass of active firms decrease.*

As I show below, the effects of a positive shock to the aggregator resemble those triggered by tougher import competition in a small economy and, more broadly, the import-competition channel. The intuition behind the result can be seen clearly by the application of Lemma 1 to a closed economy. This implies that all the equilibrium values can be determined by $\left(\mathbb{A}^*, M^{E*}\right)$. Furthermore, by inspection of (FE-CL) and (MS-CL), we can see clearly that the system is separable and, thereby, $\mathbb{A}^*$ is pinned down by (FE-CL) and independently of M^{E*}. This establishes that only shocks to (FE-CL) can affect $\mathbb{A}^*$. In addition, since the aggregate is a sufficient statistic for the survival productivity cutoff and the prices and quantities of active domestic firms, these variables are not affected by a shock to (MS-CL).

The next proposition summarizes the effects of the shock ϵ.

Proposition 2: ***Shock to the Expected Profits***

Suppose a closed economy with a market structure à la Melitz and demands for varieties as in Assumption DEM. If there is a positive shock to expected profits (i.e., an increase in ϵ) then:

- *the aggregate increases (i.e., competition becomes tougher),*
- *the survival productivity cutoff increases,*
- *if Assumption SC holds, the prices and markups of active domestic firms decrease, and*
- *if Assumption STB holds, the mass of incumbents increases.*

As I show below, the effects of a shock to expected profits are akin to those stemming from new export opportunities in a small economy and, more generally, the export-opportunities channel. Intuitively, Proposition 2 follows because an exogenous increase in expected profits makes the left-hand side of (FE-CL) greater and, so, it needs tougher competitive conditions to restore zero expected profits. The tougher competitive environment increases the minimum productivity to survive in the market and, when tougher competition makes the demand more price elastic, prices and markups fall.

Depending on the size of the price decreases, an increase or decrease in the mass of incumbents might be required to validate the increase in the aggregate. Assumption STB ensures that the decreases in prices are not so pronounced that there has to be a reduction in the mass of incumbents to validate the new value of aggregate.

Figure 1 summarizes the results for a closed economy. Notice that none of them require assuming that Marshall's Second Law of Demand holds.

Figure 1. *Shocks in a Closed Economy: Summary of the Results*

Shock to the Aggregator

$$= \mathbb{A}^* \Rightarrow \begin{cases} = \varphi^* \\ = p^* \\ = m^* \\ = q^* \\ \downarrow M^{E*} \; (\downarrow M^*) \end{cases}$$

Shock to the Expected Profits

$$\uparrow \mathbb{A}^* \Rightarrow \begin{cases} \uparrow \varphi^* \\ \downarrow p^* \text{ if SC} \\ \downarrow m^* \text{ if SC} \\ \uparrow M^{E*} \text{ if STB} \end{cases}$$

Note: SC refers to Assumption SC and STB to Assumption STB. In Section 5, I show that these results resemble the impact on the domestic economy from a trade liberalization. Specifically, the effects of a shock to the aggregator are akin to those triggered by tougher import competition in a small economy and, more generally, the import-competition channel. The impact of a shock to the expected profits is similar to that from better export opportunities in a small economy and, more broadly, to the export-opportunities channel.

5 Small-Economy Case

In this section, I consider the effects of a trade liberalization in a small country i accounting for the equilibrium in the world economy. Formally, I consider a country $H \in \mathcal{C}^S$ and distinguish between the effects on the domestic market stemming from the import-competition and export-opportunities channels. To isolate each of them, since H is a small economy, it is enough to consider a unilateral liberalization. As I show in Section 6, when there is trade between big economies, unilateral liberalizations cannot disentangle the channels directly.

I begin by considering the effects of tougher import competition.

Proposition 3: ***Import Competition in a Small Economy***

Suppose that for any country $k \in \mathcal{C}$ the market structure is à la Melitz and the demands for varieties are as in Assumption DEM. Consider a country $H \in \mathcal{C}^S$, so that it is small, and suppose there is a reduction of τ_{jH} or f_{jH} for each $j \in \mathcal{C} \backslash \{H\}$. Then,

- *$(\mathbb{A}_k^*)_{k \in \mathcal{C}}$ remains the same (i.e., the level of competition in each country does not vary),*
- *Regarding firms from H,*
  - *φ_{Hk}^*, $p_{Hk}^*(\varphi)$ and $q_{Hk}^*(\varphi)$ for each $\varphi \geq \varphi_{Hk}^*$ and $k \in \mathcal{C}$ do not vary, and*
  - *M_H^{E*} and M_{Hk}^* decrease for each $k \in \mathcal{C}$.*
- *Regarding firms from each country $j \in \mathcal{C} \backslash \{H\}$:*
  - *φ_{jH}^* decreases and M_{jH}^* increases,*
  - *if τ_{jH} decreases:*
    for each $\varphi \geq \varphi_{jH}^*$, $p_{jH}^*(\varphi)$ decreases and, if Assumption MSLD holds for H, $m_{jH}^*(\varphi)$ increases, and
  - *if f_{jH} decreases:*
    $p_{jH}^*(\varphi)$ and $m_{jH}^*(\varphi)$ for each $\varphi \geq \varphi_{jH}^*$ remain the same.

Proposition 3 establishes that tougher import competition does not entail pro-competitive effects in the domestic market. This follows because the more aggressive behavior of foreign firms can be interpreted as a shock to the aggregator. Since this does not affect the expected profits in H, the aggregate in H remains the same, and only the mass of domestic incumbents varies to restore the equilibrium.

In the appendix, I extend the scope of Proposition 3 in different ways. First, in Appendix B.1, I prove that the results hold under multidimensional firm heterogeneity. Specifically, I consider a framework where, by paying the entry cost, a firm gets draws of productivity and variety appeal (i.e., quality or taste). Second, in Appendix B.2, I show that the result applies to any country-specific decision of a domestic firm. This encompasses pervasive cases considered in the literature, such as quality and number of products when firms are multiproduct. Thus, no domestic decision at the market stage is impacted.

Finally, in Appendix B.3, I show that the results are robust to the nested versions of CES and Logit when varieties are partitioned by country of origin (i.e., domestic or foreign) or by varieties produced by the same multiproduct firm.

Regarding the effects of better export opportunities, the results are the following.

Proposition 4: ***Export Opportunities in a Small Economy***

Suppose that for any country $k \in \mathcal{C}$ the market structure is à la Melitz and the demands for varieties are as in Assumption DEM. Consider a country $H \in \mathcal{C}^S$, so that it is small, and suppose there is a reduction of τ_{HF} or f_{HF} for some $F \in \mathcal{C} \backslash \{H\}$. Then:

- *$\mathbb{A}_H^*$ increases (i.e., competition becomes tougher) and each $\mathbb{A}_j^*$ with $j \in \mathcal{C} \backslash \{H\}$ remains the same (i.e., j's level of competition does not vary),*
- *Regarding domestic firms in H:*
  - *φ_{HH}^* increases,*
  - *if Assumption SC holds for H, then $p_{HH}^*(\varphi)$ and $m_{HH}^*(\varphi)$ decrease for each $\varphi \geq \varphi_{HH}^*$, and*
  - *if Assumption STB holds, M_H^{E*} increases.*
- *Regarding exporters from each $j \in \mathcal{C} \backslash \{H\}$ and destination H:*
  - *φ_{jH}^* increases and M_{jH}^* decreases, and*
  - *if Assumption SC holds for H, then $p_{jH}^*(\varphi)$ and $m_{jH}^*(\varphi)$ decrease for each $\varphi \geq \varphi_{jH}^*$.*
- *Regarding exporters from H:*
  - *φ_{HF}^* decreases and M_{Hj}^* increases for each $j \in \mathcal{C} \backslash \{H\}$,*
  - *when τ_{HF} decreases:*
    for each $\varphi \geq \varphi_{HF}^*$, $p_{HF}^*(\varphi)$ decreases and, if Assumption MSLD holds for F, then $m_{HF}^*(\varphi)$ increases, and
  - *when f_{HF} varies:*
    $p_{HF}^*(\varphi)$ and $m_{HF}^*(\varphi)$ for each $\varphi \geq \varphi_{HF}^*$ do not vary.

Unlike the case of variations in inward trade barriers, better export opportunities create pro-competitive effects in the domestic market. They are akin to a positive shock to the expected profits. As such, the aggregate increases to restore the zero expected profits, thus affecting the prices, markups, and survival productivity cutoff of domestic firms.

6 Trade Between Two Big Countries

In this section, I consider trade in a world economy with a set of countries $\mathcal{C} := \{H, F\}$ where H and F are two big countries. I begin by presenting the total effects of a unilateral liberalization in H and show that a Metzler paradox arises: anti-competitive effects emerge in the country that opens its economy to imports. This is reflected in a decrease of the aggregate which, in turn, determines that active domestic firms increase their prices and face a lower survival productivity cutoff.

Proposition 5: ***Unilateral Liberalization with Two Big Countries***

Consider a world economy with set of countries $\mathcal{C} := \{H, F\}$ where $H, F \in \mathcal{C}^B$, so that they are big, and where each country has a market structure à la Melitz with demands for varieties as in Assumption DEM. Suppose that $|J_{FE}| > 0$ and $|J_{MS}| > 0$, where J_{FE} and J_{MS} refer to the Jacobians of the free-entry and market-stage conditions, respectively. If there is a decrease of τ_{FH} or f_{FH}, with no changes in τ_{HF} or f_{HF}, then, taking $k \in \{H, F\}$:

- *In country H:*
  - *$\mathbb{A}_H^*$ decreases (i.e., competition becomes less tough),*
  - *each φ_{kH}^* decreases,*
  - *if Assumption SC holds for H, then $p_{HH}^*(\varphi)$ and $m_{HH}^*(\varphi)$ increase for each $\varphi \geq \varphi_{HH}^*$,*
  - *when τ_{FH} decreases:*
    if Assumptions SC and MSLD hold for H, then $m_{FH}^*(\varphi)$ increases for each $\varphi \geq \varphi_{FH}^*$,[33]
  - *when f_{FH} decreases:*
    if Assumption SC holds for H, then $m_{FH}^*(\varphi)$ increases for each $\varphi \geq \varphi_{FH}^*$, and,
  - *M_H^{E*} decreases.*
- *In country F:*
  - *$\mathbb{A}_F^*$ increases (i.e., competition becomes tougher),*
  - *each φ_{kF}^* increases,*
  - *if Assumption SC holds for F, $p_{kF}^*(\varphi)$ and $m_{kF}^*(\varphi)$ decrease for each $\varphi \geq \varphi_{kF}^*$, and*
  - *M_F^{E*} increases.*

To provide an explanation of Proposition 5, I decompose the effects in each country in terms of channels. If we were dealing with a small economy, unilateral liberalizations would allow us to account for the effects from the export-opportunities and import-competition channels directly. Nonetheless, when countries are non-negligible, isolating the effects of each mechanism is a more complex task due to the presence of feedback effects. This determines that one more channel is under operation, which I refer to as the export-conditions channel.

To understand this, consider a reduction of trade costs to serve H from F. First, country H is subject to tougher import competition from F. Second, unlike the case of small economies, country F also constitutes H's export market. Thus, variations in trade costs trigger changes in F's domestic sector, determining different export conditions for H. Overall, H now faces more import competition *and* tougher access conditions to F, thus

[33] Without further assumptions, the effect on prices set by firms from F in H are indeterminate because the reduction of trade costs gives incentives to decrease prices while less competition does the opposite.

confounding the export-conditions and import-competition channels. Likewise, country F has new export opportunities and potentially different access conditions to H.

To separate each effect, consider the free-entry conditions in H and F:

$$\int_{\varphi_{HH}(\mathbb{A}_H)}^{\overline{\varphi}_H} \left[\pi_{HH}\left(\mathbb{A}_H,\varphi\right)-f_{HH}\right]\mathrm{d}G_H\left(\varphi\right)+\int_{\varphi_{HF}(\mathbb{A}_F)}^{\overline{\varphi}_H} \left[\pi_{HF}\left(\mathbb{A}_F,\varphi\right)-f_{HF}\right]\mathrm{d}G_H\left(\varphi\right)=F_H, \tag{FE-H}$$

$$\int_{\varphi_{FF}(\mathbb{A}_F)}^{\overline{\varphi}_F} \left[\pi_{FF}\left(\mathbb{A}_F,\varphi\right)-f_{FF}\right]\mathrm{d}G_F\left(\varphi\right)+\int_{\varphi_{FH}(\mathbb{A}_H,\tau_{FH})}^{\overline{\varphi}_F} \left[\pi_{FH}\left(\mathbb{A}_H,\varphi;\tau_{FH}\right)-f_{FH}\right]\mathrm{d}G_F\left(\varphi\right)=F_F. \tag{FE-F}$$

From (FE-H) and (FE-F), implicit solutions $\mathbb{A}_H\left(\mathbb{A}_F;\tau_{FH}\right)$ and $\mathbb{A}_F\left(\mathbb{A}_H;\tau_{FH}\right)$ can be obtained. Let $(\mathbb{A}_H^*,\mathbb{A}_F^*)$ be a pair of values such that both (FE-H) and (FE-F) hold. Then,

$$\frac{\mathrm{d}\mathbb{A}_H^*}{\mathrm{d}\tau_{FH}}=\underbrace{\frac{\partial\mathbb{A}_H\left(\mathbb{A}_F^*;\tau_{FH}\right)}{\partial\tau_{FH}}\kappa}_{\text{import-competition channel }(=0)}+\underbrace{\frac{\partial\mathbb{A}_H\left(\mathbb{A}_F^*;\tau_{FH}\right)}{\partial\mathbb{A}_F}\frac{\partial\mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial\tau_{FH}}\kappa}_{\text{export-conditions channel }(>0)}, \tag{AGG-H}$$

$$\frac{\mathrm{d}\mathbb{A}_F^*}{\mathrm{d}\tau_{FH}}=\underbrace{\frac{\partial\mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial\tau_{FH}}\kappa}_{\text{export-opportunities channel }(<0)}+\underbrace{\frac{\partial\mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial\mathbb{A}_H}\frac{\partial\mathbb{A}_H\left(\mathbb{A}_F^*;\tau_{FH}\right)}{\partial\tau_{FH}}\kappa}_{\text{export-conditions channel }(=0)}, \tag{AGG-F}$$

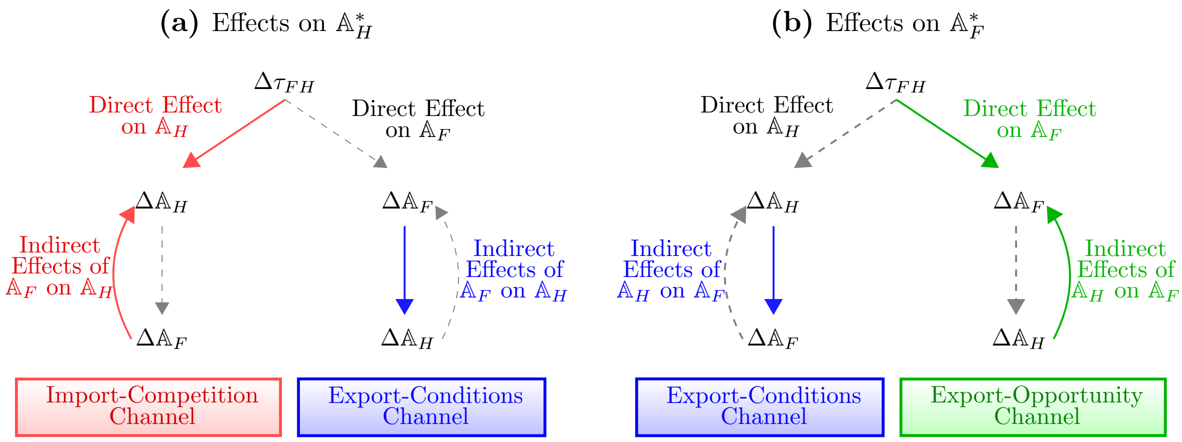
where $\kappa:=\left(1-\frac{\partial\mathbb{A}_H\left(\mathbb{A}_F^*;\tau_{FH}\right)}{\partial\mathbb{A}_F}\frac{\partial\mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial\mathbb{A}_H}\right)^{-1}>1$ is a multiplier of the direct effects capturing those that are indirect.[34]

Consider country H. The total variation of H's aggregate is given by equation (AGG-H) and described visually in Figure 2a. The total effect is split into two channels. I define the *import-competition channel for H* as the total effect on $\mathbb{A}_H^*$ (direct and indirect) triggered by the direct impact of τ_{FH} on its own aggregate $\mathbb{A}_H$. Likewise, the *export-conditions channel for H* is defined as the total effect on $\mathbb{A}_H^*$ (direct and indirect) triggered by the direct impact of τ_{FH} on the trading partner's aggregate $\mathbb{A}_F$.

The import-competition channel is the one operating in a small economy under a unilateral liberalization. It has a null impact on H since it represents a shock to its aggregator. Unlike the case of a small economy, now a decrease in τ_{FH} also creates new export opportunities in F which affect F's domestic conditions. For F, the reduction of τ_{FH} represents a shock to its expected profits, generating a tougher competitive environment in its own country. From H's point of view, this channel entails harder conditions to export and, therefore, a negative shock to its expected profits. For this reason, competition in H has to be more lenient to counterbalance this effect and restore the zero expected profit condition. The result provides a *rationalization of the Metzler paradox* in the Melitz model:

[34]The fact that $\kappa>1$ follows from $|J_{FE}|>0$.

Figure 2. *A Unilateral Trade Liberalization in Country H*



a unilateral liberalization in H is equivalent to simply tougher export conditions.

Regarding F's aggregate, the effect is given by equation (AGG-F) and described visually in Figure 2b. Its total variation operates though two channels. The *export-opportunities channel for F* comprises the total effects (direct and indirect) on $\mathbb{A}_F^*$ triggered by the direct impact of τ_{FH} on $\mathbb{A}_F$. Likewise, the *export-conditions channel for F* accounts for the total effects (direct and indirect) on $\mathbb{A}_F^*$ triggered by the direct impact of τ_{FH} on the trading partner's aggregate $\mathbb{A}_H$. The export-conditions channel for F has a null effect since it captures the effect of a shock to the aggregator in H. Thus, the total effects in F's aggregate resembles a shock to its expected profits. However, unlike the case of a small economy where $\kappa = 1$, the effect is amplified through a multiplier $\kappa > 1$ that takes into account how variations in $\mathbb{A}_F$ affects $\mathbb{A}_H$ and makes the competitive environment in H less tough.

7 Reactivating the Import-Competition Channel

To propose an alternative setup in which the import-competition channel is active, it is necessary to first understand why it is inactive. Therefore, I start by considering different market structures where the insensitivity of the domestic market to import competition is also present. To make the intuition clear, I keep that part simple by considering a closed economy which faces a shock to the aggregator.

The cases I analyze indicate that three assumptions are key: free entry, homogeneity of the marginal entrants, and the existence of a non-stochastic sufficient statistic to determine optimal firms' choices and profits. I show that if those conditions are satisfied, , even under market structures with large firms, the import-competition channel is inactive.

Then, I consider a framework where heterogeneous firms are aware of their productivity. I show that, when that is the case, the import-competition channel is reactivated. Supposing that firms know their productivity turns the model isomorphic to a version of Melitz where firms have already paid the entry cost and, so, the mass of incumbents is given. This setup corresponds to that in Chaney (2008) and it is usually interpreted as a short-run version of the Melitz model.[35]

7.1 Oligopoly and Alternative Market Structures: a Negative Result

The fact that the import-competition channel has no impact on domestic firms' decisions might be considered specific to the existence of zero-measure firms. The goal of this section is to consider some alternative market structures with non-negligible firms and show that the intuition is misdirected.

Consider a closed economy where the set $\overline{\Omega}$ of total varieties can be partitioned into sets $\overline{\Omega}^{\mathcal{E}}$ and $\overline{\Omega}^{\mathcal{I}}$. The subscripts are mnemonics for "entrants" and "insiders", consistent with some of the assumptions I establish below. $\overline{\Omega}^{\mathcal{I}}$ is a discrete set (endowed with the counting measure) of non-negligible firms. Each of these firms has a unique variety assigned and knows its productivity, which is potentially heterogeneous across firms. As for $\overline{\Omega}^{\mathcal{E}}$, I suppose that the minimum productivity of firms in $\overline{\Omega}^{\mathcal{I}}$ is greater than the maximum productivity of any firm in $\overline{\Omega}^{\mathcal{E}}$ and that firms enter sequentially in each market according to a productivity order. Therefore, when both types of firms are active, any extensive-margin adjustment occurs within firms belonging to $\overline{\Omega}^{\mathcal{E}}$.[36] This establishes that the features of the marginal entrants correspond to those of $\overline{\Omega}^{\mathcal{E}}$.

[35] The framework can also be considered as a long-run version of Melitz in which firms do not know their productivity but each obtains a draw from a degenerate firm-specific productivity distribution. In other words, firms are heterogeneous even ex ante and they know exactly how efficient they are. For an interpretation along these lines, see Alfaro and Lander (2017). Also, Kehoe et al. (2016) provide an interpretation of Melitz model à la Chaney in terms of the span of control model by Lucas (1978). In that setup, firms are heterogeneous because entrepreneurs have different levels of talent.

[36] Entry according to a productivity order is standard in oligopoly models and it has the aim of ruling out equilibria where inefficient firms drive more productive ones out of the market. In the International Trade literature, it is supposed in, for example, Atkeson and Burstein (2008), Eaton et al. (2012), Edmond et al. (2015), and Gaubert and Itskhoki (2016). Also, in case there is a subset of firms which have the same productivity, I suppose there is some arbitrary fixed order among them.

Two cases are considered distinguished by the nature of firms in $\overline{\Omega}^{\mathcal{E}}$. In the first case, which I label as "*oligopoly with a fringe à la Melitz*", firms in $\overline{\Omega}^{\mathcal{E}}$ are characterized as in the Melitz model. In the other, which I refer to as *"oligopoly with symmetric entrants"*, firms in $\overline{\Omega}^{\mathcal{E}}$ are of nonzero measure and have the same level of productivity.[37] Notice that, in both cases, insiders are non-negligible and possibly heterogeneous. To keep notation simple, I endow the set $\overline{\Omega}^{\mathcal{E}}$ with a general measure ρ that would correspond to the Lebesgue measure or the counting measure, depending on the case considered.

What do these cases have in common? That entry and exit to the industry takes place between firms that are homogeneous. In both cases, the profits of marginal entrants put enough structure such that, ignoring the integer constraint, there is only one value of aggregate that is consistent with the zero expected profits.[38]

I suppose that firms decide on prices so that the competition is à la Bertrand. The same results hold if there is Cournot competition. The only difference is that in Cournot the setup has to be specified in terms of an inverse demand with a quantity aggregator.

Given that non-negligible firms can influence the sector's conditions, more stringent conditions over the aggregator are needed to get an aggregate that is a sufficient statistic to determine optimal choices and profits. I accomplish this by assuming that the aggregator can be expressed as a monotone transformation of a standard price aggregator. Then, the demand would be as in Assumption DEM with an aggregator that is additively separable. This turns the model into a fully aggregative game in the sense of Cornes and Hartley (2012). Demand functions with an additively separable aggregator encompass an important subset of the demands included in Assumption DEM.[39] Next, I present the definition of an aggregator displaying this feature. For completeness, I do it for the case of an open economy.

Definition 6. *A* ***standard price aggregator accounting for large firms*** *for country i is a function* $\mathcal{P}_i\left[(\mathbf{p}_{ji})_{j\in\mathcal{C}}\right] := \int_{j\in\mathcal{C}}\left[\sum_{\omega\in\overline{\Omega}_j^{\mathcal{I}}} h_{ji}^{\mathcal{I}}(p_{ji}(\omega)) + \int_{\omega\in\overline{\Omega}_j^{\mathcal{E}}} h_{ji}^{\mathcal{E}}(p_{ji}(\omega))\,\mathrm{d}\rho(\omega)\right]\mathrm{d}\mu_i(j)$ *where the absolute value of $h_{ji}^{\mathcal{I}}$ and $h_{ji}^{\mathcal{E}}$ and all their derivatives are dominated by integrable*

[37] Anderson et al. (2016) use this particular market structure to study the behavior of insiders.

[38] The case of oligopoly with symmetric entrants could be interpreted it as a scenario where firms do not know their productivity but, if each pays the entry cost, it would obtain a draw from a degenerate distribution. Thus, there is only one aggregate that is consistent with zero expected profits which, given a degenerate distribution, coincide with the realized profits.

[39] For instance, the assumption is satisfied for direct demands derived from either a discrete or discrete-continuous choice model, constant expenditure demand systems, and demands derived from an additively separable indirect utility. Without accounting for love of variety, it also includes the translog and linear demand. As for Cournot competition, the scope is even broader.

positive functions. The price aggregate for i is a value $\mathbb{P}_i \in \text{range}\,\mathcal{P}_i$. An ***aggregator accounting for large firms*** *for country i is a smooth bijective real-valued function $\mathcal{A}_i$ with $\mathbb{P}_i \mapsto \mathcal{A}_i(\mathbb{P}_i)$ which is decreasing in $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ when is it defined through and $\mathcal{P}_i$. An* ***aggregate*** *for i is a value $\mathbb{A}_i \in \text{range}\,\mathcal{A}_i$.*

Under this setup, it can be shown that the model behaves as in the baseline case regarding the import-competition channel. I reflect this by considering a shock to the aggregator in a closed economy.

Proposition 6: ***Import-Competition Channel with Non-Negligible Firms***

Consider a closed economy with a market structure given by either an oligopoly with a fringe à la Melitz or an oligopoly with symmetric entrants. Suppose that the demands for varieties are as in Assumption DEM with aggregators as in Definition 6. Moreover, suppose that firms enter sequentially by a productivity order and that a subset of firms from $\overline{\Omega}^{\mathcal{E}}$ are active in each equilibrium. Then, ignoring the integer constraint, if there is a shock to the aggregator,

- *the aggregate remains the same (i.e., the level of competition does not vary),*
- *the prices, markups, and quantities of active firms remain the same,*
- *the survival productivity cutoff remains the same, and*
- *only the mass/number of entrants varies.*

In Appendix B.4, I generalize the results of Proposition 6 to a setup where firms make choices over a vector of country-specific variables. Thus, under these market structures too, the import-competition channel is inactive and has no effect on any decision made by a domestic active firm.

7.2 Ex-Ante Heterogeneous Firms

Some lessons can be derived from Section 7.1 with the goal of modifying the baseline Melitz framework and reactivate the import-competition channel. First, the existence of large firms is not sufficient for this to happen. Second, the features ascribed to marginal entrants under free entry are crucial. The reason is that ex-ante homogeneity of entrants joint with zero profits (expected or deterministic, depending on the case considered) put enough structure in the model to pin down the equilibrium aggregate and make it independent of any shock to the aggregator.

Taking these conclusions into consideration, I modify the Melitz model by dispensing with the assumption of ex-ante identical firms. Instead, I suppose that in each country $i \in \mathcal{C}$, there is a continuum of prospective entrants, with mass $\overline{M}_i$, each having a unique variety ω and a productivity φ assigned. These firms are indexed from the most productive to the least, with a distribution characterized by a strictly increasing and continuous cdf G_i.

With this modification, the model is mathematically isomorphic to the baseline Melitz model, but where $\overline{M}_i = M_i^E$ in each country i. Thus, the equilibrium conditions are the same as in Section 3 without the zero expected profits condition (FE). Given this property, I refer to this version of monopolistic competition as **à la Chaney**.

Proposition 7: ***Unilateral Trade Liberalization in a Model à la Chaney***

Suppose that for any country $k \in \mathcal{C}$ the market structure is à la Chaney and that $H \in \mathcal{C}^S$, so that it is small. Suppose that the demands for varieties are as in Assumption DEM and that Assumption STB holds for H.

If there is a reduction of τ_{jH} or f_{jH} for each $j \in \mathcal{C} \backslash \{H\}$, then in country H:

- *$\mathbb{A}_H^*$ increases (i.e., competition becomes tougher in H),*
- *φ_{HH}^* increases and M_{HH}^* decreases, and*
- *if Assumption SC holds for H, then $p_{jH}^*(\varphi)$ and $m_{HH}^*(\varphi)$ decrease for each $\varphi \geq \varphi_{HH}^*$ and $j \in \mathcal{C} \backslash \{H\}$.*

The key of the result is that, under the assumptions established, the equilibrium aggregate is determined by condition (MS) for H. Thus, unlike the Melitz model, it is the composition of the aggregator which determines the value of the aggregate. On the other hand, in the Melitz model, the equilibrium aggregate is determined and the composition of the aggregator only varies to validate that value.

8 Conclusions

In this paper, I developed a methodology to analyze the Melitz model under any productivity distribution and a demand system that comprises standard demand functions. The approach is based on the theory of large aggregative economies, which I extended to account for an endogenous number of agents. In addition, I made use of monotone comparative statics and did not assume that the demand system comes from a representative consumer. This allowed me show that Marshall's Second Law of Demand (i.e., that the price elasticity of demand is increasing with respect to own price) plays a less determinant role in the results relative to assuming strategic complementary of prices (i.e., that tougher competition increases the price elasticity).

Using this framework, I analyzed the channels through which trade liberalizations affect economies. First, I showed that the export-opportunities channel is active and gives rise to pro-competitive effects. Thus, it decreases the prices of active firms and increases the survival productivity cutoff. In addition, I established that the import-competition channel is inactive. It only affects the mass of domestic firms but it does not alter any domestic firms' decisions or their productivity cutoff to serve the domestic market.

To address the second result, I proposed a model where the import-competition channel is reactivated. This was accomplished by identifying what assumptions play a crucial role in the deactivation of this channel. In particular, I remarked on the fact that the same outcome can arise under oligopolistic competition and, hence, frameworks with large firms do not necessarily activate the channel. Then, I showed that in a monopolistic competition model where firms know their productivity and are heterogeneous, the import-competition channel is reactivated.

One of the main implications of this paper is in relation to applied work and, in particular, for structural estimation: the choice of a specific monopolistic competition model has a bearing on the channels that are incorporated. Specifically, in the standard version of the Melitz model with demands summarizing market conditions through a single sufficient static, only the impact stemming from export-related channels is captured.

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Online Appendix - not for publication

A Proofs and Derivations

Conventions: I use the notation $\widehat{\cdot}$ to define the natural logarithm of any function or variable $\cdot$. To avoid cumbersome notation, occasionally, if a parameter remains fixed along the analysis, I omit it from the arguments of the functions.

Throughout the proofs, I make use of several standard results from the monotone comparative statics literature. I state the main ones I use in Lemma 2. The reader is referred to Topkis (1998) for further details.[40]

Lemma 2. *Let $f : X \times \Theta_1 \times \Theta_2 \to \mathbb{R}$ with $X := [\underline{x}, \overline{x}]$ and $\Theta_n := \left[\underline{\theta}_n, \overline{\theta}_n\right]$ for $n = 1, 2$. Then:*

- *if f is quasisupermodular on $X \times \Theta_1 \times \Theta_2$ then $\arg\max_{x \in X} f(x, \theta_1, \theta_2)$ is increasing in θ_1 and θ_2,*
- *if f is log-supermodular on $X \times \Theta_1 \times \Theta_2$ then f is quasisupermodular on $X \times \Theta_1 \times \Theta_2$, and*
- *f is log-supermodular on $X \times \Theta_1 \times \Theta_2$ iff f is pairwise log-supermodular.*

Next, I add some lemmas which will be used in subsequent proofs.

Lemma 3. *$\pi_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ is decreasing in $\mathbb{A}_j$ and τ_{ij}, and increasing in φ. Moreover, $\varphi^*_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij})$ is increasing in all its arguments.*

Lemma 4. *$p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ is decreasing in φ and increasing in τ_{ij}. Moreover, if Assumption MSLD holds for j, then markups $m_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ are increasing in φ, and decreasing in τ_{ij}.*

Lemma 5. *If Assumption SC holds for j, then prices $p_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ and markups $m_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ are decreasing in $\mathbb{A}_j$.*

Lemma 6. *$\mathcal{A}^*_i\left(\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j \in \mathcal{C}}\right)$ is increasing in M^E_j for any $j \in \mathcal{C}^B \cup \{i\}$.*

Lemma 7. *$\mathcal{A}^*_i\left(\mathbb{A}_i, \mathbf{M}^E; (\tau_{ji}, f_{ji})_{j \in \mathcal{C}}\right)$ is decreasing in τ_{ji} and f_{ji}, for any $j \in \mathcal{C}^B$.*

[40] The results are special cases of several results from Topkis (1978) and Milgrom and Shannon (1994) applied to real-valued functions defined over a compact Euclidean domain.

Proof of Lemma 3. Gross profits are $\pi_{ij}(\mathbb{A}_j, p_{ij}, \varphi; \tau_{ij}) := q_j(\mathbb{A}_j, p_{ij})[p_{ij} - c_i(\varphi, \tau_{ij})]$. The domain of prices does not depend on any of the parameters, q_j is decreasing in $\mathbb{A}_j$, and $c_i(\varphi, \tau_{ij})$ is decreasing in φ and increasing in τ_{ij}. Thus, by a revealed-preference argument (or, alternatively, the envelope theorem), the optimal gross profits for active firms $\pi_{ij}(\mathbb{A}_j, \varphi; \tau_{ij})$ are decreasing in $\mathbb{A}_j$ and τ_{ij}, and increasing in φ.

Regarding $\varphi^*_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij})$, by definition, it is the value φ^*_{ij} such that $\pi_{ij}\left(\mathbb{A}_j, \varphi^*_{ij}; \tau_{ji}\right) = f_{ij}$. Let $\beta \in \{\mathbb{A}_j, \tau_{ij}\}$, so that $-\frac{\partial \pi_{ij}(\cdot)/\partial \beta}{\partial \pi_{ij}(\cdot)/\partial \varphi} = \frac{\partial \varphi^*_{ij}}{\partial \beta}$. Then, using that $\pi_{ij}(\mathbb{A}_j, \varphi; \tau_{ji})$ is decreasing in $\mathbb{A}_j$ and τ_{ij}, and increasing in φ, we determine the sign of $-\frac{\partial \pi_{ij}(\cdot)/\partial \beta}{\partial \pi_{ij}(\cdot)/\partial \varphi}$ which gives that $\varphi^*_{ij}(\mathbb{A}_j; \tau_{ij}, f_{ij})$ is increasing in $\mathbb{A}_j$ and τ_{ij}. Moreover, $\frac{1}{\partial \pi_{ij}(\cdot)/\partial \varphi} = \frac{\partial \varphi^*_{ij}}{\partial f_{ij}}$ and, so, φ^*_{ij} is increasing in f_{ij}. ■

Proof of Lemma 4. Let $\beta \in \{-\varphi, \tau_{ij}\}$. Profits are $\pi_{ij}(\mathbb{A}_j, p_{ij}, \varphi; \beta) = q_j(\mathbb{A}_j, p_{ij})[p_{ij} - c_i(\beta)] - f_{ij}$. For any $p > c$, we can apply logs and get $\frac{\partial^2 \hat{\pi}_{ij}(\mathbb{A}_j, p_{ij}, \varphi; \beta)}{\partial \beta \partial p_{ij}} = \frac{\partial c_i(\beta)}{\partial \beta} \frac{1}{(p_{ij} - c_i(\beta))^2} > 0$ and so, by Lemma 2, $\pi_{ij}(\mathbb{A}_j, p_{ij}, \varphi; \beta)$ is log-supermodular in (p_{ij}, β). This establishes that prices are decreasing in φ and increasing in τ_{ij}. Regarding markups, let $m_{ij} := \frac{p_{ij}}{c_i(\beta)}$ and reexpress the gross profits as a function of markups: $\pi_{ij}(\mathbb{A}_j, m_{ij}; \beta) = q_j[\mathbb{A}_j, m_{ij} c_i(\beta)](m_{ij} - 1) c_i(\beta)$. Then, $\frac{\partial^2 \hat{\pi}_{ij}(\mathbb{A}_j, m_{ij}; \beta)}{\partial m_{ij} \partial \beta} = \left[\frac{\partial^2 \hat{q}_j(\mathbb{A}_j, p_{ij})}{\partial p^2_{ij}} m_{ij} c_i(\beta) + \frac{\partial \hat{q}_j(\mathbb{A}_j, p_{ij})}{\partial p_{ij}}\right] \frac{\partial c_i(\beta)}{\partial \beta}$, and it can be shown that the term in brackets is equal to $-\frac{\partial \varepsilon_j(\cdot, p)}{\partial p}$. Thus, given Assumption MSLD, gross profits are log-submodular in (m_{ij}, β), or equivalently, log-supermodular in $(m_{ij}, -\beta)$ which, by Lemma 2, proves that markups are increasing in φ and decreasing in τ_{ij}. ■

Proof of Lemma 5. By Lemma 2, if we show that $\pi_{ij}(\mathbb{A}_j, p_{ij}, \varphi)$ is log-supermodular in $(p_{ij}, -\mathbb{A}_j)$ or, equivalently, log-submodular in $(p_{ij}, \mathbb{A}_j)$, then optimal prices are decreasing in $\mathbb{A}_j$. Since $\frac{\partial^2 \hat{\pi}_{ij}(\mathbb{A}_j, p_{ij}, \varphi)}{\partial \mathbb{A}_j \partial p_{ij}} = \frac{\partial^2 \hat{q}_j(\mathbb{A}_j, p_{ij})}{\partial \mathbb{A}_j \partial p_{ij}}$ is negative if and only if $\frac{\partial \varepsilon_j(\mathbb{A}_j, \cdot)}{\partial \mathbb{A}_j} > 0$, and the latter holds by Assumption SC for j, the result follows. Also, reexpressing the gross profits as function of markups, then $\frac{\partial^2 \hat{\pi}_{ij}(\mathbb{A}_j, m_{ij})}{\partial m_{ij} \partial \mathbb{A}_j} = \frac{\partial^2 \hat{q}_j(\mathbb{A}_j, p_{ij})}{\partial \mathbb{A}_j \partial p_{ij}} c_i(\varphi, \tau_{ij})$ and so markups are decreasing in $\mathbb{A}_j$ by the same argument. ■

Proof of Lemma 6. To show that $\mathcal{A}^*_i$ is increasing in M^E_j, we need to show that $\boldsymbol{\mathcal{P}}^*_i\left(\mathbb{A}_i, \mathbf{M}^E\right)$ is decreasing in M^E_j. Suppose that M^E_j increases for a given $\mathbb{A}_i$. Each of these additional firms will get a productivity draw. We also know that $\mathcal{A}_i$ is decreasing in $(\mathbf{p}_{ji})_{j \in \mathcal{C}}$ through $\boldsymbol{\mathcal{P}}_i$, where $\mathbf{p}_{ji}$ comprises the prices of all firms, including unavailable varieties. Depending on the productivity draw that the firm gets, it either keeps setting $\overline{p}_i$, so that it continues not serving the market, or it becomes active and sets $p_{ji}(\mathbb{A}^*_i, \varphi) < \overline{p}_i$.

Thus, since $\boldsymbol{\mathcal{P}}_i^*$ is decreasing when a nonzero set of firms increase their prices and j is non-negligible for i (and assuming the non-trivial case where there is no φ that makes a firm from j serve i), the increase of M_j^E decreases $\boldsymbol{\mathcal{P}}_i^*$. ■

Proof of Lemma 7. Let $\beta \in \{\tau_{ji}, f_{ji}\}$. Since $\mathcal{A}_i$ is decreasing in $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ through $\boldsymbol{\mathcal{P}}_i$, the result follows if $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ is increasing in β when evaluated at the optimal values. Suppose that β increases. Prices are affected by two different channels. First, since, by Lemma 3, $\varphi_{ji}^*(\mathbb{A}_i;\beta_{ji})$ increases when β increases, some of the firms that were setting $p_{ji}(\mathbb{A}_i,\varphi;\tau_{ji})$ now become inactive and set $\overline{p}_i > p_{ji}(\mathbb{A}_i,\varphi;\tau_{ji})$. Second, firms that remain active before and after the change in β set a price $p_{ji}(\mathbb{A}_i,\varphi;\tau_{ji})$, which is increasing in τ_{ji} by Lemma 4 and remains constant if $\beta = f_{ji}$. Thus, $(\mathbf{p}_{ji})_{j\in\mathcal{C}}$ is increasing in β when evaluated at the optimal values and the result follows. ■

A.1 Section 4

Proof of Proposition 1. Applying Lemma 1, $\mathbb{A}^*$ is determined completely by condition (MS-CL) and independently of α and M^{E*}. Thus, $\mathbb{A}^*$ does not vary. Also, $p(\mathbb{A}^*,\varphi)$, $m^*(\mathbb{A}^*,\varphi)$, and $\varphi^*(\mathbb{A}^*)$ are independent of α and M^{E*}, and $\mathbb{A}^*$ does not vary. Thus, they do not vary either. Since prices do not vary, $q[\mathbb{A},p^*(\mathbb{A}^*,\varphi)]$ does not vary either. By Lemma 1, M^{E*} is determined by condition (FE-CL) and, by Lemma 6, $\mathcal{A}^*\left(\mathbb{A}^*,M^E;\alpha\right)$ is increasing in M^E. Besides, $\mathcal{A}^*\left(\mathbb{A}^*,M^E;\alpha\right)$ is increasing in α by assumption. Hence, positive variations in α determine that M^{E*} decreases to restore the equilibrium. Since φ^* does not vary and M^{E*} is lower, then M^* decreases.

Proof of Proposition 2. Assume an increase in ϵ. Applying Lemma 1, $\mathbb{A}^*$ is determined completely by condition (MS-CL) and independently of M^{E*}. By Lemma 3, $\pi(\mathbb{A},\varphi)$ is decreasing in $\mathbb{A}$, and $\varphi^*(\mathbb{A})$ increasing in $\mathbb{A}$. Hence, $\widetilde{\pi}(\mathbb{A})$ is decreasing in $\mathbb{A}$. Thus, if ϵ increases, then $\mathbb{A}^*$ has to increase so that $\widetilde{\pi}(\mathbb{A})$ decreases and makes condition (MS-CL) hold again. Given a greater aggregate, by Lemma 5 and $\frac{\partial\varepsilon(\mathbb{A},\cdot)}{\partial\mathbb{A}} > 0$, then $p(\mathbb{A}^*,\varphi)$ and $m^*(\mathbb{A}^*,\varphi)$ for each $\varphi \geq \varphi^*$ decrease. Moreover, by Lemma 3, $\varphi^*(\mathbb{A})$ increases.

Regarding the mass of incumbents, M^{E*} is determined through condition (FE-CL). By Assumption STB, $\mathbb{A} - \mathcal{A}^*\left(\mathbb{A},M^E;\alpha\right)$ is increasing in $\mathbb{A}$. Also, by Lemma 6, $\mathcal{A}^*\left(\mathbb{A},M^E\right)$ is increasing in M^E. Hence, if $\mathbb{A}^*$ increases then M^{E*} has to increase to restore the equality of equation (FE-CL). ■

A.2 Section 5

Proof of Proposition 3. Consider a reduction of $\beta_{jH} \in \{\tau_{jH}, f_{jH}\}$ for each $j \in \mathcal{C}\backslash\{H\}$. By Lemma 1, condition (FE) pins down $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$ and it is independent of $\mathbf{M}^E$. I show that the system of equations (FE) is not affected by the shock. First, $(\beta_{jH})_{j\in\mathcal{C}\backslash\{H\}}$ does not directly affect condition (FE) for country H. Besides, since H is a small economy, the profits garnered by any firm from $j \neq H$ in H are negligible. Thus, variations in $(\beta_{jH})_{j\in\mathcal{C}\backslash\{H\}}$ have a negligible impact on any of the equations (FE) for any $j \neq H$. Hence, $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$ does not vary.

Consider firms from H serving any country $k \in \mathcal{C}$. Since $(\mathbb{A}_k^*, \beta_{Hk})_{k\in\mathcal{C}}$ does not vary, neither $p_{Hk}^*(\varphi)$ for $\varphi \geq \varphi_{Hk}^*$ nor $\varphi_{Hk}^*(\mathbb{A}_k^*; \beta_{Hk})$ vary. Moreover, since neither $p_{Hk}^*(\varphi)$ nor $\mathbb{A}_k^*$ vary, $q_{Hk}^*(\varphi)$ does not vary.

Regarding firms from $j \in \mathcal{C}\backslash\{H\}$, $\mathbb{A}_H^*$ does not vary but there is a reduction of β_{jH} for each j. Thus, $\varphi_{jH}^*(\mathbb{A}_H^*; \beta_{jH})$ decreases for each $j \in \mathcal{C}\backslash\{H\}$ by Lemma 3. Moreover, firms with productivity $\varphi \geq \varphi_{jH}^*$ that are active before and after the shock set $p_{jH}^*(\varphi)$, which decreases by Lemma 4 if $\beta_{jH} = \tau_{jH}$, and remains the same if $\beta_{jH} = f_{jH}$. Moreover, by Assumption MSLD $\frac{\partial \varepsilon_H(p_{jH},\cdot)}{\partial p_{jH}} > 0$, and, so for each $\varphi \geq \varphi_{jH}^*$, $m_{jH}^*(\varphi)$ increases by Lemma 4 if $\beta_{jH} = \tau_{jH}$, and remains the same if $\beta_{jH} = f_{jH}$.

Regarding the measure of firms, consider the system of equations given by condition (MS) for each country $j \in \mathcal{C}\backslash\{H\}$. Since H is a small economy, changes in M_H^E have a negligible impact on any of those conditions. Moreover, $\left(\mathbb{A}_j^*, (\beta_{kj})_{k\in\mathcal{C}}\right)_{j\in\mathcal{C}\backslash\{H\}}$ for any $k \in \mathcal{C}$ does not vary, which determines that M_j^{E*} does not vary for any $j \neq H$. This implies that, for condition (MS) in H to hold, M_H^{E*} has to adjust. In equilibrium, condition (MS) in H can be expressed as $\mathbb{A}_H^* = \mathcal{A}_H^*\left(\mathbb{A}_H^*, M_H^{E*}, (\beta_{jH})_{j\in\mathcal{C}\backslash\{H\}}\right)$. By Lemma 7, $\mathcal{A}_H^*$ is decreasing in $(\beta_{jH})_{j\in\mathcal{C}^B}$ and $\mathbb{A}_H^*$ is the same before and after the shock. Thus, condition (MS) in H can only hold as an equality if M_H^{E*} decreases. For $k \in \mathcal{C}$, since φ_{Hk}^* has not changed but M_H^{E*} decreases, then each M_{Hk}^* decreases. Regarding any country $j \neq H$ serving H, since φ_{jH}^* decreases and M_j^{E*} remains the same, then M_{jH}^* increases. ■

Proof of Proposition 4. Consider a reduction of $\beta_{HF} \in \{\tau_{HF}, f_{HF}\}$ for some $F \neq H$. By Lemma 1, the system of conditions (FE) pins down $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$. The parameter β_{HF} does not affect directly the condition (FE) of any country $j \in \mathcal{C}\backslash\{H\}$. Moreover, since H is a small economy, the profits garnered by any firm from $j \in \mathcal{C}\backslash\{H\}$ in H are negligible.

Hence, the equation (FE) for any $j \in \mathcal{C}\setminus\{H\}$ does not change through this channel either, determining that $\left(\mathbb{A}_j^*\right)_{j\in\mathcal{C}\setminus\{H\}}$ does not vary. Besides, since H is a small economy, it has no impact on each equation of (MS) for country $j \in \mathcal{C}\setminus\{H\}$, determining that $\left(M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$ does not vary.

Regarding country H, the reduction of β_{HF} represents a positive shock to the expected profits in H. Proceed in a similar fashion as in the proof of Proposition 2, this determines that there is an increase in $\mathbb{A}_H^*$. Besides, for any $k \in \mathcal{C}$, since $\varphi_{kH}^*\left(\mathbb{A}_H, \beta_{kH}\right)$ is increasing in $\mathbb{A}_H$, and β_{kH} does not vary, then $\varphi_{kH}^*(\cdot)$ increases by Lemma 3. Also, for any $j \in \mathcal{C}\setminus\{H\}$, M_{jH}^* decreases because M_j^{E*} does not vary and $\varphi_{jH}^*(\cdot)$ increases. Regarding firms from $k \in \mathcal{C}$ with $\varphi \geq \varphi_{kH}^*$, since by Assumption SC $\frac{\partial \varepsilon_H(\cdot,\mathbb{A}_H)}{\partial \mathbb{A}_H} > 0$, then $p_{kH}^*(\varphi)$ and $m_{kH}^*(\varphi)$ decrease by Lemma 5.

Regarding firms from H in F, $\mathbb{A}_F^*$ does not vary and $\varphi_{HF}^*(\cdot)$ is increasing in β_{HF}, so that $\varphi_{HF}^*\left(\mathbb{A}_F^*; \beta_{HF}\right)$ decreases by Lemma 3. In addition, if $\beta_{HF} = \tau_{HF}$, since $\mathbb{A}_F^*$ does not vary and τ_{HF} decreases, then by Lemma 4, $p_{HF}^*(\varphi)$ decreases for each $\varphi \geq \varphi_{HF}^*$. Given that, additionally by Assumption MSLD $\frac{\partial \varepsilon_F(p_{HF},\cdot)}{\partial p_{HF}} > 0$, $m_{HF}^*(\varphi)$ decreases too. If $\beta_{HF} = f_{HF}$, $p_{HF}^*(\varphi)$ and $m_{HF}^*(\varphi)$ for $\varphi \geq \varphi_{HF}^*$ do not change.

Concerning M_H^{E*}, since M_j^{E*} is the same for each $j \neq H$ and β_{kH} does not vary for any $k \in \mathcal{C}$, condition (MS) in H can be expressed as $\mathbb{A}_H^* = \mathcal{A}_H^*\left(\mathbb{A}_H^*, M_H^{E*}\right)$ When Assumption STB holds, $\mathbb{A}_H^* - \mathcal{A}_H^*\left(\mathbb{A}_H^*, M_H^{E*}\right)$ is increasing in $\mathbb{A}_H^*$ and, using Lemma 6 and the fact $\mathbb{A}_H^*$ is greater, M_H^{E*} has to increase to restore the equality of (MS) in H. This also determines that M_{Hj}^* increases for any $j \in \mathcal{C}\setminus\{H\}$. ■

A.3 Section 6

Proof of Proposition 5. Let $\beta_{HF} \in \{\tau_{FH}, f_{FH}\}$. I first show that $\mathbb{A}_H^*$ is increasing in β_{FH}, and $\mathbb{A}_F^*$ decreasing in β_{FH}. Differentiating conditions (FE-H) and (FE-F),

$$\begin{pmatrix} \frac{\partial \tilde{\pi}_{HH}\left(\mathbb{A}_H^*\right)}{\partial \mathbb{A}_H} & \frac{\partial \tilde{\pi}_{HF}\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F} \\ \frac{\partial \tilde{\pi}_{FH}\left(\mathbb{A}_H^*;\beta_{FH}\right)}{\partial \mathbb{A}_H} & \frac{\partial \tilde{\pi}_{FF}\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F} \end{pmatrix} \begin{pmatrix} \frac{\partial \mathbb{A}_H^*}{\partial \beta_{FH}} \\ \frac{\partial \mathbb{A}_F^*}{\partial \beta_{FH}} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\partial \tilde{\pi}_{FH}\left(\mathbb{A}_H^*;\beta_{FH}\right)}{\partial \beta_{FH}} \end{pmatrix}.$$

From this, we get $\frac{\partial \mathbb{A}_H^*}{\partial \beta_{FH}} = \frac{\frac{\partial \tilde{\pi}_{FH}\left(\mathbb{A}_H^*;\beta_{FH}\right)}{\partial \beta_{FH}} \frac{\partial \tilde{\pi}_{HF}\left(\mathbb{A}_F^*\right)}{\partial \mathbb{A}_F}}{|J_{FE}|}$ and $\frac{\partial \mathbb{A}_F^*}{\partial \beta_{FH}} = -\frac{\frac{\partial \tilde{\pi}_{FH}\left(\mathbb{A}_H^*;\beta_{FH}\right)}{\partial \beta_{FH}} \frac{\partial \tilde{\pi}_{HH}\left(\mathbb{A}_H^*\right)}{\partial \mathbb{A}_H}}{|J_{FE}|}$. By Lemma 3, $\tilde{\pi}_{FH}\left(\mathbb{A}_H, \beta_{FH}\right)$ is decreasing in β_{FH} and $\tilde{\pi}_{kl}\left(\mathbb{A}_l, \cdot\right)$ is decreasing in $\mathbb{A}_l$ for $k, l \in \{H, F\}$. Hence, since $|J_{FE}| > 0$, then $\frac{\partial \mathbb{A}_H^*}{\partial \tau_{FH}} > 0$ and $\frac{\partial \mathbb{A}_F^*}{\partial \tau_{FH}} < 0$.

Regarding domestic firms in H, by the decrease in $\mathbb{A}_H^*$ and Lemma 3, φ_{HH}^* decreases. Furthermore, by the decrease in $\mathbb{A}_H^*$, Lemma 5, and Assumption SC in H, then $p_{HH}^*(\varphi)$

and $m^*_{HH}(\varphi)$ increase for each $\varphi \geq \varphi^*_{HH}$.

Regarding exporters from F selling in H, they are affected by both the decrease of $\mathbb{A}^*_H$ and the decrease of β_{FH}. By Lemmas 4 and 5, given that Assumptions SC and MSLD hold, then $m_{FH}(\mathbb{A}^*_H)$ increases when $\beta_{FH} = \tau_{FH}$. If Assumption SC holds, the same result holds in the case $\beta_{FH} = f_{FH}$. Moreover, by Lemma 3, φ^*_{FH} decreases.

For given values $\mathbb{A}^*_H$ and $\mathbb{A}^*_F$, we have that M^{E*}_H and M^{E*}_F are obtained through the following system of equations:

$$\mathbb{A}^*_H = \mathcal{A}^*_H\left(M^{E*}_H, M^{E*}_F; \mathbb{A}^*_H, \beta_{FH}\right),$$
$$\mathbb{A}^*_F = \mathcal{A}^*_F\left(M^{E*}_H, M^{E*}_F; \mathbb{A}^*_F\right).$$

Taking into account the variation of $\mathbb{A}^*_H$ and $\mathbb{A}^*_F$ due to the change in β_{FH}, differentiating the system we obtain

$$\begin{pmatrix} \frac{\partial \mathcal{A}^*_H(\cdot)}{\partial M^E_H} & \frac{\partial \mathcal{A}^*_H(\cdot)}{\partial M^E_F} \\ \frac{\partial \mathcal{A}^*_F(\cdot)}{\partial M^E_H} & \frac{\partial \mathcal{A}^*_F(\cdot)}{\partial M^E_F} \end{pmatrix} \begin{pmatrix} \frac{\partial M^{E*}_H}{\partial \tau_{FH}} \\ \frac{\partial M^{E*}_F}{\partial \tau_{FH}} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbb{A}^*_H}{\partial \beta_{FH}}\left(1 - \frac{\partial \mathcal{A}^*_H(\cdot)}{\partial \mathbb{A}^*_H}\right) - \frac{\partial \mathcal{A}^*_H(\cdot)}{\partial \beta_{FH}} \\ \frac{\partial \mathbb{A}^*_F}{\partial \beta_{FH}}\left(1 - \frac{\partial \mathcal{A}^*_F(\cdot)}{\partial \mathbb{A}^*_F}\right) \end{pmatrix}.$$

By assumption, $|J_{MS}| > 0$. Let $\Delta_1 := \frac{\partial \mathbb{A}^*_H}{\partial \beta_{FH}}\left(1 - \frac{\partial \mathcal{A}^*_H(\cdot)}{\partial \mathbb{A}^*_H}\right) - \frac{\partial \mathcal{A}^*_H(\cdot)}{\partial \beta_{FH}}$ and $\Delta_2 := \frac{\partial \mathbb{A}^*_F}{\partial \beta_{FH}}\left(1 - \frac{\partial \mathcal{A}^*_F(\cdot)}{\partial \mathbb{A}^*_F}\right)$. Given that Assumption STB holds, then $\Delta_1 > 0$ and $\Delta_2 < 0$. Moreover all the entries in J_{MS} are positive. Hence, $\frac{\partial M^{E*}_H}{\partial \tau_{FH}} = \frac{\Delta_1 \frac{\partial \mathcal{A}^*_F(\cdot)}{\partial M^E_F} - \Delta_2 \frac{\partial \mathcal{A}^*_H(\cdot)}{\partial M^E_F}}{|J_{MS}|} > 0$, and $\frac{\partial M^{E*}_F}{\partial \tau_{FH}} = \frac{\Delta_2 \frac{\partial \mathcal{A}^*_H(\cdot)}{\partial M^E_H} - \Delta_1 \frac{\partial \mathcal{A}^*_F(\cdot)}{\partial M^E_H}}{|J_{MS}|} < 0$.
■

Derivation of channels in Section 6. From conditions (FE-H) and (FE-F), we get $\mathbb{A}_H(\mathbb{A}_F; \tau_{FH})$ and $\mathbb{A}_F(\mathbb{A}_H; \tau_{FH})$, respectively. An equilibrium is a pair $(\mathbb{A}^*_H, \mathbb{A}^*_F)$ such that $\mathbb{A}^*_H = \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})$ and $\mathbb{A}^*_F = \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})$. Differentiating $\mathbb{A}_H(\mathbb{A}_F; \tau_{FH})$ and $\mathbb{A}_F(\mathbb{A}_H; \tau_{FH})$, and evaluating the expressions at the equilibrium, we get

$$\frac{\mathrm{d}\mathbb{A}^*_H(\mathbb{A}^*_F; \tau_{FH})}{\mathrm{d}\tau_{FH}} = \frac{\partial \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})}{\partial \tau_{FH}} + \frac{\partial \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})}{\partial \mathbb{A}_F} \frac{\mathrm{d}\mathbb{A}^*_F(\mathbb{A}^*_H; \tau_{FH})}{\mathrm{d}\tau_{FH}},$$
$$\frac{\mathrm{d}\mathbb{A}^*_F(\mathbb{A}^*_H; \tau_{FH})}{\mathrm{d}\tau_{FH}} = \frac{\partial \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})}{\partial \tau_{FH}} + \frac{\partial \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})}{\partial \mathbb{A}_H} \frac{\mathrm{d}\mathbb{A}^*_H(\mathbb{A}^*_F; \tau_{FH})}{\mathrm{d}\tau_{FH}}.$$

Working out the expressions, we get the following equations,

$$\frac{\mathrm{d}\mathbb{A}^*_H(\mathbb{A}^*_F; \tau_{FH})}{\mathrm{d}\tau_{FH}} = \frac{\partial \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})}{\partial \tau_{FH}}\kappa + \frac{\partial \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})}{\partial \mathbb{A}_F} \frac{\partial \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})}{\partial \tau_{FH}}\kappa,$$
$$\frac{\mathrm{d}\mathbb{A}^*_F(\mathbb{A}^*_H; \tau_{FH})}{\mathrm{d}\tau_{FH}} = \frac{\partial \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})}{\partial \tau_{FH}}\kappa + \frac{\partial \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})}{\partial \mathbb{A}_H} \frac{\partial \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})}{\partial \tau_{FH}}\kappa,$$

with $\kappa := \left(1 - \frac{\partial \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})}{\partial \mathbb{A}_F} \frac{\partial \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})}{\partial \mathbb{A}_H}\right)^{-1}$. By Lemma 3, $\widetilde{\pi}_{kl}(\mathbb{A}_l, \cdot)$ is decreasing in $\mathbb{A}_l$ for $k, l \in \{H, F\}$. Hence, $\frac{\partial \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})}{\partial \mathbb{A}_F} < 0$ and $\frac{\partial \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})}{\partial \mathbb{A}_H} < 0$. Since $|J_{FE}| > 0$, then $\frac{\partial \mathbb{A}_H(\mathbb{A}^*_F; \tau_{FH})}{\partial \mathbb{A}_F} \frac{\partial \mathbb{A}_F(\mathbb{A}^*_H; \tau_{FH})}{\partial \mathbb{A}_H} < 1$. Furthermore, since neither $\widetilde{\pi}_{HH}$ nor $\widetilde{\pi}_{HF}$ depend on τ_{FH}

directly, then $\frac{\partial \mathbb{A}_H\left(\mathbb{A}_F^*;\tau_{FH}\right)}{\partial \tau_{FH}} = 0$. Finally, by Lemma 3, $\widetilde{\pi}_{FH}\left(\mathbb{A}_H, \tau_{FH}\right)$ is decreasing in τ_{FH} and $\widetilde{\pi}_{kl}\left(\mathbb{A}_l, \cdot\right)$ is decreasing in $\mathbb{A}_l$ for $k, l \in \{H, F\}$. Both determine that $\frac{\partial \mathbb{A}_F\left(\mathbb{A}_H^*;\tau_{FH}\right)}{\partial \tau_{FH}} < 0$.

A.4 Section 7

Proof of Proposition 6. For the proof, I proceed in several steps. First, I begin by showing that the aggregate is a sufficient statistic for optimal prices and quantities of any firm belonging to $\overline{\Omega}^{\mathcal{I}}$. Then, I show that the same is true for firms in $\overline{\Omega}^{\mathcal{E}}$, including their profits.

Regarding the first step, the first-order condition of an active firm $\omega \in \overline{\Omega}^{\mathcal{I}}$ with productivity φ determines the following equation,

$$p = m\left(\mathbb{A}, p\right) c\left(\varphi\right),$$

where $m\left(\mathbb{A}, p\right) := \frac{\varepsilon(\mathbb{A},p)}{\varepsilon(\mathbb{A},p)-1}$ is the firm's markup. The price elasticity of demand $\varepsilon\left(\mathbb{A}, p\right)$ incorporates that large firms can affect market conditions. Specifically, $\varepsilon\left(\mathbb{A}, p\right) := -\left[\frac{\partial \ln q(\mathbb{A},p)}{\partial \ln p(\omega)} + \frac{\partial \ln q(\mathbb{A},p)}{\partial \ln \mathbb{A}}\frac{\partial \mathcal{A}(\cdot)}{\partial p(\omega)}\right]$. Next, I show that ε is a function of $(\mathbb{A}, p)$. First $\frac{\partial \mathcal{A}(\cdot)}{\partial p(\omega)} = \frac{\partial \mathcal{A}(\cdot)}{\partial \mathbb{P}}\frac{\partial \mathcal{P}}{\partial p(\omega)}$ and, since $\mathcal{A}$ is bijective, if $\mathcal{A}\left(\mathbb{P}\right) = \mathbb{A}$ then $\mathbb{P} = \mathcal{A}^{-1}\left(\mathbb{A}\right)$. This implies that $\frac{\partial \mathcal{A}(\cdot)}{\partial \mathbb{P}}$ can be expressed as a function of $\mathbb{A}$. Besides, since the price aggregator is additively separable, $\frac{\partial \mathcal{P}}{\partial p(\omega)}$ is a function of $p\left(\omega\right)$. Both facts prove the claim. Therefore, optimal prices can be expressed as a function $p_{\mathcal{I}}\left(\mathbb{A}, \varphi\right)$. In turn, this determines that optimal quantities are a function $q_{\mathcal{I}}\left(\mathbb{A}, \varphi\right)$. Also, by assumption, firms in $\overline{\Omega}^{\mathcal{I}}$ are always active and, so, there is no need to determine if any firm belonging to $\overline{\Omega}^{\mathcal{I}}$ serves the market or not.

Regarding firms in $\overline{\Omega}^{\mathcal{E}}$, we have two possibilities. Either, they have a characterization as in Melitz or they comprise symmetric non-negligible firms. In the former case, the equations describing the equilibrium are the same as in the baseline model. The only difference is that condition (MS) is defined in terms of an aggregator as in Definition 6. Thus, the proof would follow the same steps as the proof of Proposition 3.

Consider now the case of $\overline{\Omega}^{\mathcal{E}}$ composed of symmetric large firms. Let $\varphi_{\mathcal{E}}$ be the productivity shared by these firms. The optimal price of an active firm would be the same as that obtained for a firm in $\overline{\Omega}^{\mathcal{I}}$ but for productivity $\varphi_{\mathcal{E}}$. Moreover, since optimal prices and quantities are a function of the aggregates, their profits are a function $\pi_{\mathcal{E}}\left(\mathbb{A}, \varphi_{\mathcal{E}}\right)$. Since we are assuming that a subset of $\overline{\Omega}^{\mathcal{E}}$ is active in each equilibrium and that there is free entry, then, ignoring the integer constraint, the profits of these firms have to satisfy

$$\pi_{\mathcal{E}}\left(\mathbb{A}, \varphi_{\mathcal{E}}\right) - f = F,$$

which pins down $\mathbb{A}^*$. Therefore, prices, quantities, and markups of active firms do not vary. Also, the survival productivity cutoff does not vary since $\mathbb{A}^*$ does not change. Finally, in this setup, condition (MS) constitutes the Nash equilibrium condition of the model. Thus, all the adjustment due to the shock to the aggregator takes place through the number of active firms among the entrants. ■

Proof of Proposition 7. First, since H is a small economy, it does not affect the domestic conditions of any country $j \in \mathcal{C} \setminus \{H\}$. Moreover, since the model is isomorphic to the baseline Melitz model with $M_i^E := \overline{M}_i$ for each $i \in \mathcal{C}$, the equation (MS) for H constitutes the equilibrium condition and determines $\mathbb{A}_H^*$. Let $\beta_{jH} \in \{\tau_{jH}, f_{jH}\}$ and consider variations $\mathrm{d}\beta_{jH} < 0$ for each $j \in \mathcal{C} \setminus \{H\}$. Regarding condition (MS) for H, by Assumption STB, we know $\mathbb{A}_H^* - \mathcal{A}_H^* \left(\mathbb{A}_H^*, M_H^{E*}\right)$ is increasing in $\mathbb{A}_H^*$. Moreover, by Lemma 7, $\mathcal{A}_H^* (\cdot)$ is decreasing in β_{jH} for each $j \in \mathcal{C}^B$and we are considering the case $\mathrm{d}\beta_{jH} < 0$. Hence, $\mathbb{A}_H^*$ increases. Moreover, by Lemma 3, φ_{HH}^* is increasing in $\mathbb{A}_H^*$, so that φ_{HH}^* increases too. Both facts imply that M_{HH}^* decreases. Finally, since Assumption 5 holds, prices and markups of active firms are decreasing in $\mathbb{A}_H^*$, so that $p_{HH}^* (\varphi)$ and $m_{HH}^* (\varphi)$ for each $\varphi \geq \varphi_{HH}^*$ decrease. Finally, by Lemmas 5 and STB, since both $\mathbb{A}_H^*$ and τ_{jH} decrease, then $p_{jH}^* (\varphi)$ for each $\varphi \geq \varphi_{jH}^*$ decreases if $\beta_{jH} = \tau_{jH}$. For the case $\beta_{jH} = f_{jH}$, the result follows just by the fact that $\mathbb{A}_H^*$ decreases. ■

B Extensions with an Inactive Import-Competition Channel

In this section, I modify the baseline model and establish some alternative setups in which the import-competition channel is inactive. In particular, I consider three extensions.

First, in Appendix B.1, I demonstrate that the result holds for the case of multidimensional firm heterogeneity. Specifically, I consider a model where firms receive draws of efficiency and variety appeal (quality or taste). In this way, firms are heterogeneous in terms of cost and demand. Then, I extend the setup to account for a vector of firm's country-specific decisions. This encompasses choice variables considered in the literature, such as quality and number of goods when firms are multiproduct. The results are presented for the Melitz model (Appendix B.2) and the alternative frameworks of Section 7.1 (Appendix B.4). Finally, in Appendix B.3, I show that the result also holds under standard demands with nested structures (i.e., the nested versions of the CES and Logit) when two types of varieties' partitions: by country of origin and by varieties belonging to the same multiproduct firm.

B.1 Multidimensional Heterogeneity

In the baseline model, firms are heterogeneous exclusively in terms of their productivity. Regarding the demand side, a demand system as in Definition DEM allows for only country-specific heterogeneity. Next, I show that, in fact, the neutrality to import competition also arises when there is firm heterogeneity in terms of demand and productivity.

To illustrate this, consider the case of a closed economy. The setup is the same as in the baseline case but with the following modifications. By paying the entry cost, a firm gets the assignation of a unique variety ω with feature σ_ω and a productivity draw φ_ω. I denote the type of a firm by $\boldsymbol{\theta} := (\sigma, \varphi)$ and assume $\boldsymbol{\theta}$ has support Θ and is distributed according to a joint cdf G.

The demand that a firm ω faces is given by

$$q_\omega := \max\left\{0, q\left(\mathbb{A}, p_\omega, \sigma_\omega\right)\right\}.$$

With this demand function for some (σ, φ), profits of ω are

$$\pi\left(\mathbb{A}, p; \boldsymbol{\theta}\right) := q_\omega\left(\mathbb{A}, p, \sigma\right)\left[p - c\left(\varphi\right)\right],$$

which, conditional on (σ, φ), are completely determined by $\mathbb{A}$. Consequently, optimal prices and quantities of active firms are functions $p\left(\mathbb{A}, \sigma, \varphi\right)$ and $q\left(\mathbb{A}, \sigma, \varphi\right)$, and they determine that an active firm $\boldsymbol{\theta}$ has optimal profits given by

$$\pi\left(\mathbb{A}, \boldsymbol{\theta}\right) := q_\omega\left(\mathbb{A}, \sigma, \varphi\right)\left[p\left(\mathbb{A}, \sigma, \varphi\right) - c\left(\varphi\right)\right].$$

Unlike the baseline case, now the entry decision rule cannot be described by some cutoff of productivity. Nonetheless, it still depends on $\mathbb{A}$ such that, for a given $\mathbb{A}$, different combinations of σ and φ determine whether a firm serves the market. Formally, defining $\mathcal{E}\left(\mathbb{A}\right) := \left\{\boldsymbol{\theta} : \pi\left(\mathbb{A}, \boldsymbol{\theta}\right) \geq f\right\}$, a firm $\boldsymbol{\theta}$ enters if $\boldsymbol{\theta} \in \mathcal{E}\left(\mathbb{A}\right)$. Thus, expected profits can be written as

$$\widetilde{\pi}\left(\mathbb{A}\right) := \int_{\theta \in \Theta} \mathbb{1}_{\left(\boldsymbol{\theta} \in \mathcal{E}\left(\mathbb{A}\right)\right)}\left[\pi\left(\mathbb{A}, \boldsymbol{\theta}\right) - f\right] \mathrm{d}G\left(\boldsymbol{\theta}\right).$$

By expressing expected profits in this way, it can be seen that they are completely determined by $\mathbb{A}$. As a result, $\mathbb{A}$ is still a sufficient statistic for optimal firm's decisions (both prices and entry) and profits. From this, it is concluded that any shock to the aggregator does not affect the equilibrium value of $\mathbb{A}$.

B.2 Vector of Country-Specific Decisions in Melitz

In the baseline model, the only decision made by firms at the market stage is regarding prices. Next, I consider the possibility that each decides on a vector of country-specific variables. To show the result as starkly as possible, I consider a closed economy which faces a shock to the aggregator.

The framework is as in the baseline model but incorporating that each firm ω makes a decision on $\mathbf{x}_\omega \in X := \times_{n=1}^{N} [\underline{x}_n, \overline{x}_n] \cup \{\mathbf{x}_0\}$ with $N < \infty$, where $\mathbf{x}_0$ represents inaction (exit of the market). Given a vector of choices, the aggregator is defined in the following way.

Definition 7. *Let* $\boldsymbol{\mathcal{X}} := \left(\mathcal{X}^k\right)_{k=1}^{K}$ *with* $K < \infty$ *such that* $\mathcal{X}^k \left[(\mathbf{x}_\omega)_{\omega\in\Omega}\right] := \int_{\omega\in\overline{\Omega}} u_\omega^k (\mathbf{x}_\omega)\, \mathrm{d}\omega$, $\mathbb{X}^k \in \operatorname{range} \mathcal{X}^k$, *and* $\mathbb{X} := \left(\mathbb{X}^k\right)_{k=1}^{K}$, *where the absolute value of* u_ω^k *and all its derivatives are dominated by integrable positive functions. An* ***aggregator*** *is a smooth real-valued function* $\mathcal{A}$ *with* $\mathbb{X} \mapsto \mathcal{A}(\mathbb{X})$. *An* ***aggregate*** *is a value* $\mathbb{A} \in \operatorname{range} \mathcal{A}$.

Making use of this definition, I specify the demand function.

Assumption DEM". *The* ***demand of a variety*** ω *is given by,*

$$q(\omega) := \max\{0, q[\mathbb{A}, \mathbf{x}(\omega)]\},$$

where $\mathbb{A}$ *is as in Definition 7 and* q *is a smooth function such that* $q(\mathbb{A}, \mathbf{x}_0) = 0$.

Notice that I have not imposed any restriction on how the choice vector is incorporated in either the demand or the aggregator. This provides a wide scope in terms of the possible functional forms that can be considered. Regarding costs, I assume a general function that might depend on $\mathbf{x}(\omega)$. Formally, $C[q(\omega), \mathbf{x}(\omega)] := q(\omega)\, c(\varphi, \mathbf{x}(\omega)) + f^{\mathbf{x}}[\mathbf{x}(\omega)]$, allowing for the possibility that some of the choices entail fixed costs, affect marginal costs, or both.

The optimization problem of a firm with productivity φ is

$$\max_{\mathbf{x}_\omega} \pi(\mathbf{x}_\omega, \mathbb{A}, \varphi).$$

The system of first-order conditions which characterize the optimal decisions of a φ-type active firm is

$$\frac{\partial \pi(\mathbf{x}_\omega, \mathbb{A}, \varphi)}{\partial \mathbf{x}_\omega} = 0. \tag{7}$$

Let $\mathbf{x}(\mathbb{A},\varphi)$ be the implicit solution to the system (FE-H). Incorporating that firms also decide whether to serve the market, the optimal vector of decisions for each $\varphi \in [\underline{\varphi},\overline{\varphi}]$ is

$$\mathbf{x}^*(\mathbb{A},\varphi^*,\varphi) := \begin{cases} \mathbf{x}(\mathbb{A},\varphi) & \text{if } \varphi \geq \varphi^* \\ \mathbf{x}_0 & \text{otherwise,} \end{cases} \tag{$\mathbf{x}$-vec}$$

where φ^* represents the survival productivity cutoff of the firm.

If a φ-type firm serves the market, its optimal gross profits are $\pi(\mathbb{A},\varphi) := \pi[\mathbb{A},\mathbf{x}(\mathbb{A},\varphi),\varphi]$. Therefore, the survival productivity cutoff $\varphi^*(\mathbb{A})$ is the implicit solution to the following equation

$$\pi(\mathbb{A},\varphi^*) = f.$$

All this determines that the aggregate is a sufficient statistic for firm's decisions and its profits. Thus, the equilibrium is determined just like in the baseline case with the zero expected profits condition determining the equilibrium aggregate $\mathbb{A}^*$. Formally,

$$\widetilde{\pi}(\mathbb{A}^*) = F, \tag{FE-vec}$$

where $\widetilde{\pi}(\mathbb{A}) := \int_{\varphi^*(\mathbb{A})}^{\overline{\varphi}} [\pi(\mathbb{A},\varphi) - f]\,\mathrm{d}G(\varphi)$. Therefore, since (FE-vec) pins down $\mathbb{A}^*$, a shock to the aggregator determines that $\varphi^*(\mathbb{A}^*)$ and the vector of optimal decisions $\mathbf{x}^*(\varphi) := \mathbf{x}^*[\mathbb{A}^*,\varphi^*(\mathbb{A}^*),\varphi]$ do not vary.

As in the baseline case too, M^{E*} is determined by the equilibrium condition of the market stage. Evaluating $\mathcal{X}$ at $\mathbf{x}^*(\varphi)$, we obtain a vector $\boldsymbol{\mathcal{X}}^*\left(\mathbb{A}^*,M^E\right)$ and, so, the aggregator becomes a function $\widetilde{\mathcal{A}}^*\left(\mathbb{A}^*,M^E\right) := \mathcal{A}\left[\mathcal{X}^*\left(\mathbb{A}^*,M^E\right)\right]$. Then, for a given $\mathbb{A}^*$, M^{E*} has to satisfy that

$$\mathbb{A}^* = \widetilde{\mathcal{A}}^*\left(\mathbb{A}^*,M^{E*};\alpha\right), \tag{MS-vec}$$

where α is a shock to the aggregator.

In the following proposition I summarize the findings. With some slight abuse of terminology, I refer to Assumption STB as if it were specified in terms of $\widetilde{\mathcal{A}}^*$ rather than $\mathcal{A}^*$.

Proposition 8: ***Shock to the Aggregator with a Vector of Decisions***

Suppose a closed economy with a market structure à la Melitz, extended to account for a vector of decision variables and with demands for varieties as in Assumption DEM". If there is a positive shock to the aggregator, then:

- *the aggregate remains the same (i.e., the level of competition does not vary),*
- *all active firms' choices remain the same,*

- *the survival productivity cutoff remains the same, and*
- *if Assumption STB holds, the mass of incumbents decreases.*

B.3 Country-Specific Aggregators and Nested Demand Structures

I proceed to show conditions under which the import-competition channel is inactive when demand systems have nested structures. In particular, I consider scenarios where varieties are partitioned by country of origin or by varieties produced by the same multiproduct firm. In the latter case, the result holds without any additional assumption. In the case of nests defined by country of origin, it requires to put some structure to the demand. The structure added, nonetheless, covers the nested versions of the CES and Logit as special cases.

Before studying these cases, I define generally demands with nested structures. Suppose that the set of total varieties are partitioned into L nests with nest l defining a subset of varieties Ω^l. To deal with a continuum and a discrete number of nests in a unified framework, I endow the subsets of nests with a measure ρ that is either the Lebesgue or the counting measure. The demand of a firm producing a variety ω and belonging to nest l is defined by

$$q^l(\omega) := q^l\left(\mathbb{P}, \mathbb{P}^l, p_\omega\right), \tag{DEM-NEST}$$

with $\mathcal{P}\left[\left(\mathbb{P}^l\right)_0^L\right] := \int_0^L U^l\left(\mathbb{P}^l\right)\rho(\mathrm{d}l)$, $\mathcal{P}^l\left(\left(p_{\omega'}\right)_{\omega'\in\Omega^l}\right) := \int_{\omega'\in\Omega^l} u^l\left(p\left(\omega'\right)\right)\mathrm{d}\omega'$, where U^l and u^l are monotone functions, $\mathbb{P}\in \operatorname{range}\mathcal{P}$, and $\mathbb{P}^l \in \operatorname{range}\mathcal{P}^l$.

Next, I divide the analysis for each type of nested structure considered.

B.3.1 Nested Demands with Nests Defined by Own-Firm Varieties

Suppose the case of multiproduct firms with nest l defined by all the varieties produced by firm l. Incorporating this feature to the setup, the framework is, in fact, a particular case of the model presented in Appendix B.2, where the decisions are the prices of varieties belonging to the firm. Formally, in terms of a demand as in Assumption DEM", a firm l chooses $\mathbf{x}^l := (p(\omega))_{\omega\in\Omega^l}$, with $\mathcal{P}^l$ and $\mathcal{P}$ replacing $\mathcal{X}^k$ and $\mathcal{A}$. Given this, the import-competition channel is inactive by Proposition 8.

B.3.2 Nested Demands with Nests Defined by Country of Origin

To focus on the import-competition channel, I consider the case of a country $H \in \mathcal{C}$ that is small. In addition, I suppose that the demand in country $j \in \mathcal{C}\backslash\{H\}$ of a variety ω produced in $k \in \mathcal{C}$ is as in Assumption DEM. In this way, only the demand in H has a nested structure. In particular, suppose that the demands in H are as in (DEM-NEST) with two nests that partition the varieties according to its origin, i.e., domestic or foreign. Then, the demands in H for, respectively, a variety produced in H and for a variety produced in $j \in \mathcal{F}$ with $\mathcal{F} := \mathcal{C}\backslash\{H\}$ can be expressed by

$$q_{HH}(\omega) := q_H[\mathbb{P}_H, \mathbb{P}_{HH}, p_{HH}(\omega)],$$
$$q_{jH}(\omega) := q_j[\mathbb{P}_H, \mathbb{P}_{FH}, p_{jH}(\omega)],$$

with $\mathcal{P}_H[\mathbb{P}_{FH}, \mathbb{P}_{HH}] := U^F(\mathbb{P}_{FH}) + U^H(\mathbb{P}_{HH})$ where $\mathbb{P}_{kH} \in \operatorname{range}\mathcal{P}_{kH}$ for $k \in \{H, F\}$, $\mathcal{P}_{FH}\left[(\mathbf{p}_{jH})_{j\in\mathcal{F}}\right] := \int_{j\in\mathcal{F}}\left[\int_{\omega\in\Omega_{jH}} u_{jH}[p_{jH}(\omega)]\,\mathrm{d}\omega\right]\mathrm{d}\mu_H(j)$, and $\mathcal{P}_{HH}(\mathbf{p}_{HH}) := \int_{\omega\in\Omega_{HH}} u_{HH}[p_{HH}(\omega)]\,\mathrm{d}\omega$.

Next, I add some structure to the demand in H. Specifically, let $i \in \mathcal{C}$ and $k \in \{H, F\}$, and suppose that each function q_i is weakly separable in $(\mathbb{P}_H, \mathbb{P}_{kH})$ from $p_{iH}(\omega)$. Two demands consistent with this property are the nested CES and the nested Logit (see Appendix C). Incorporating this, the demand of a variety ω produced domestically and produced in $j \in \mathcal{F}$ can then be expressed, respectively, by

$$q_{HH}(\omega) := q_H[\mathbb{A}_{HH}, p_{HH}(\omega)], \qquad \text{(DEM-}H\text{)}$$
$$q_{jH}(\omega) := q_j[\mathbb{A}_{FH}, p_{jH}(\omega)], \qquad \text{(DEM-}F\text{)}$$

where $\mathcal{A}_{kH}$ is a smooth real-valued function $(\mathbb{P}_H, \mathbb{P}_{kH}) \mapsto \mathcal{A}_{kH}(\mathbb{P}_H, \mathbb{P}_{kH})$, and $\mathbb{A}_{kH} \in \operatorname{range}\mathcal{A}_{kH}$ with $k \in \{H, F\}$ is a country-specific aggregate. The demand function determines that, for any domestic firm, $\mathbb{A}_{HH}$ is a sufficient statistic for its profits and decisions in H. Specifically, the aggregate determines domestic optimal prices, quantities, markups and survival productivity cutoff.

Consider a variation in $(\tau_{jH}, f_{jH})_{j\in\mathcal{F}}$. If we can show that $\mathbb{A}^*_{HH}$ is determined by a system of equations that is independent of these parameters, then changes in import competition do not affect any of the variables mentioned. To show this, take $j \in \mathcal{F}$. The free-entry conditions are

$$\widetilde{\pi}_{HH}\left(\mathbb{A}_{HH}^{*}\right)+\int_{k\in\mathcal{F}}\widetilde{\pi}_{Hk}\left(\mathbb{A}_{k}^{*};\tau_{Hk}\right)\mathrm{d}\mu_{H}\left(k\right)=F_{H}, \tag{8}$$

$$\int_{k\in\mathcal{F}}\widetilde{\pi}_{jk}\left(\mathbb{A}_{k}^{*};\tau_{jk},f_{jk}\right)\mathrm{d}\mu_{j}\left(k\right)=F_{j}, \tag{9}$$

for μ-almost all $j\in\mathcal{F}$. Equation (9) incorporates the fact that H is a small economy, so that $\widetilde{\pi}_{jH}$ has a negligible impact on j's expected profits. This establishes that (9) for any $j\in\mathcal{F}$ is not affected by either τ_{jH} or f_{jH}, and so each $\mathbb{A}_{j}^{*}$ is not affected. Since each $\mathbb{A}_{j}^{*}$ summarizes the export conditions of H and none of them vary, the shocks under consideration do not affect (8). Therefore, since $\mathbb{A}_{HH}^{*}$ is pinned down by (8), which is not affected directly by τ_{jH} or f_{jH}, $\mathbb{A}_{HH}^{*}$ does not vary and the result follows.

B.4 Vector of Country-Specific Decisions with Large Firms

I consider the model of Section 7.1, but with the following modifications. Firms make decisions on a vector of variables belonging to $X:=\mathbb{R}_{+}^{N}\cup\left\{\mathbf{x}_{0}\right\}$, with $N<\infty$ and where $\mathbf{x}_{0}$ represents inaction (exit of the market). I suppose that price is a choice variable, so that there is Bertrand competition and the model is still defined in terms of direct demands. Same results hold by assuming Cournot competition and establishing the setup in terms of inverse demands.

Just like in the baseline model with large firms, the aggregator needs to satisfy additive separability. However, now, it has to be in terms of the vector of choices. I keep using the measure ρ to cover within a unified notation the cases of oligopoly with a fringe à la Melitz and with symmetric entrants.

Definition 8. *Let* $\mathcal{X}\left[\left(\mathbf{x}_{\omega}\right)_{\omega\in\overline{\Omega}}\right]:=\sum_{\omega\in\overline{\Omega}^{\mathcal{I}}}u^{\mathcal{I}}\left(\mathbf{x}_{\omega}\right)+\int_{\omega\in\overline{\Omega}^{\mathcal{E}}}u^{\mathcal{E}}\left(\mathbf{x}_{\omega}\right)\mathrm{d}\rho\left(\omega\right)$ *and* $\mathbb{X}\in\operatorname{range}\mathcal{X}$, *where the absolute value of* $u^{\mathcal{I}}$ *and* $u^{\mathcal{E}}$ *and all its derivatives are dominated by some integrable positive functions. An* ***aggregator*** *is a smooth bijective real-valued function* $\mathcal{A}$ *with* $\mathbb{X}\mapsto\mathcal{A}\left(\mathbb{X}\right)$. *An* ***aggregate*** *is a value* $\mathbb{A}\in\operatorname{range}\mathcal{A}$.

As far as costs go, I assume a general function $C\left[q\left(\omega\right),\mathbf{x}\left(\omega\right)\right]:=q\left(\omega\right)c\left(\varphi,\mathbf{x}\left(\omega\right)\right)+f^{\mathbf{x}}\left[\mathbf{x}\left(\omega\right)\right]$. The next proposition establishes that the model behaves as in the baseline case regarding shocks to aggregator.

Proposition 9: ***Import-Competition Channel with Non-Negligible Firms***

Consider a closed economy with a market structure given by either an oligopoly with a fringe *à la Melitz or an oligopoly with symmetric entrants. Suppose that the demand for each variety is as in Assumption DEM" with an aggregator as in Definition 8, that firms enter sequentially by a productivity order, and also that a subset of firms from* $\overline{\Omega}^{\mathcal{E}}$ *are active. Then, ignoring the integer constraint, if there is a shock to the aggregator,*

- *the aggregate remains the same (i.e., the level of competition does not vary),*
- *all active firms' choices remain the same,*
- *the survival productivity cutoff in the oligopoly with fringe à la Melitz remains the same, and*
- *only the mass/number of entrants varies.*

Proof of Proposition 9. Next, I consider the optimal choices of a large firm. Then, I consider the case of a negligible firm whose decisions are characterized in a similar way but assuming that it cannot affect the aggregate conditions of the market.

The optimization of a large firm (irrespective if it is an insider or an entrant) producing variety ω with productivity φ is

$$\max_{\mathbf{x}_\omega} \pi\left(\mathbf{x}_\omega, \mathbb{A}; \varphi\right).$$

Large firms make their choices taking into account that they can affect $\mathcal{X}\left[\left(\mathbf{x}_{\omega'}\right)_{\omega' \in \overline{\Omega}}\right]$. Therefore, the optimal vector of decisions for an active firm can be characterized by the following system of first-order conditions:

$$\frac{\partial \pi\left(\mathbf{x}_\omega, \mathcal{A}\left(\mathbb{X}\right); \varphi\right)}{\partial \mathbf{x}_\omega} + \frac{\partial \pi\left(\mathbf{x}_\omega, \mathcal{A}\left(\mathbb{X}\right); \varphi\right)}{\partial \mathbb{A}} \frac{\partial \mathcal{A}\left(\mathbb{X}\right)}{\partial \mathbb{X}} \frac{\partial \mathcal{X}\left(\cdot\right)}{\partial \mathbf{x}_\omega} = 0. \tag{10}$$

Consider the case of an active firm with $\omega \in \overline{\Omega}^{\mathcal{I}}$. I show that the implicit solution to equation (10) can be expressed as a function $\mathbf{x}_{\mathcal{I}}\left(\mathbb{A}, \varphi\right)$. Since $\mathcal{X}$ is strongly separable, $\frac{\partial \mathcal{X}(\cdot)}{\partial \mathbf{x}_\omega}$ is a function of $\mathbf{x}_\omega$ exclusively. Moreover, since $\mathcal{A}$ is bijective, if $\mathcal{A}\left(\mathbb{X}\right) = \mathbb{A}$ then $\mathbb{X} = \mathcal{A}^{-1}\left(\mathbb{A}\right)$. Hence, $\frac{\partial \mathcal{A}(\mathbb{X})}{\partial \mathbb{X}}$ can be expressed as a function of $\mathbb{A}$. Then, the result follows.

By the same token, for an active firm $\omega \in \overline{\Omega}^{\mathcal{E}}$ that is large, $\mathbf{x}_{\mathcal{E}}\left(\mathbb{A}, \varphi\right)$ is the implicit solution to equation (10). Regarding a firm $\omega \in \overline{\Omega}^{\mathcal{E}}$ that is active and has a characterization as in Melitz, its vector of decisions is characterized by (10) but with $\frac{\partial \mathcal{X}(\cdot)}{\partial \mathbf{x}_\omega} = 0$. With some abuse of notation, let's denote the optimal vector of decisions of an entrant by $\mathbf{x}^*_{\mathcal{E}}$ for both types of entrants. What matters here is that $\mathbf{x}^*_{\mathcal{E}}$ takes $\mathbb{A}$ as a sufficient statistic, irrespective of the specific functional form considered.

Regarding the optimal decisions, we are supposing that any firm $\omega \in \overline{\Omega}^{\mathcal{I}}$ is active in any equilibrium. Thus, $\mathbf{x}_{\mathcal{I}}\left(\mathbb{A}, \varphi\right)$ characterizes the choices of any insider firm. As for a firm $\omega \in \overline{\Omega}^{\mathcal{E}}$, $\mathbf{x}^*_{\mathcal{E}}$ is given for each $\varphi \in \left[\underline{\varphi}, \overline{\varphi}\right]$ by

$$\mathbf{x}^*_{\mathcal{E}}\left(\mathbb{A}, \varphi^*, \varphi\right) := \begin{cases} \mathbf{x}_{\mathcal{E}}\left(\mathbb{A}, \varphi\right) & \text{if } \varphi \geq \varphi^* \\ \mathbf{x}_0 & \text{otherwise,} \end{cases} \tag{$\mathbf{x}^*_{\mathcal{E}}$-vec}$$

where φ^* represents the survival productivity cutoff of the firm. Denote the optimal gross profits of an active entrant with productivity φ by

$$\pi_{\mathcal{E}}\left(\mathbb{A},\varphi\right) := \pi\left[\mathbb{A},\mathbf{x}_{\mathcal{E}}\left(\mathbb{A},\varphi\right),\varphi\right].$$

Now, let's consider each case separately and show that the import-competition channel is inactive.

Fringe à la Melitz. The proof follows verbatim the proof of Proposition 8.

Symmetric Entrants Case. Let $\varphi_{\mathcal{E}}$ be the productivity shared by the entrants. Since it is assumed that in each equilibrium there is some subset of firms from $\overline{\Omega}^{\mathcal{E}}$ which are active then, ignoring the integer constraint, the free-entry condition is given by

$$\pi\left(\mathbb{A},\varphi_{\mathcal{E}}\right) = f. \tag{11}$$

Since $\varphi_{\mathcal{E}}$ is given, equation (11) pins down $\mathbb{A}^*$. In addition, as in the baseline case, the number of active entrants is determined by the equilibrium condition of the market stage. Denote $N_{\mathcal{E}}$ the number of these firms. Given the optimal decisions $\mathbf{x}_{\mathcal{E}}^*$ and $\mathbf{x}_{\mathcal{I}}^*$, the function $\mathcal{X}$ evaluated at these vectors can be described by a function $\mathcal{X}^*\left(\mathbb{A},N_{\mathcal{E}}\right)$. Thus, the aggregator becomes a function $\mathcal{A}^*\left(\mathbb{A},N_{\mathcal{E}}\right) := \mathcal{A}\left[\mathcal{X}^*\left(\mathbb{A},N_{\mathcal{E}}\right)\right]$. Given the value $\mathbb{A}^*$ determined by condition (11), $N_{\mathcal{E}}^*$ ensures that $\mathbb{A}^*$ is a fixed point of $\mathcal{A}^*$. Thus,

$$\mathbb{A}^* = \mathcal{A}^*\left(\mathbb{A}^*,N_{\mathcal{E}}^*;\alpha\right), \tag{12}$$

where α is a shock to the aggregator.

Let $\mathbf{x}_{\mathcal{I}}^*\left(\varphi\right)$ and $\mathbf{x}_{\mathcal{E}}^*\left(\varphi\right)$ be the vector of decisions evaluated at $\mathbb{A}^*$. Since $\mathbb{A}^*$ is determined by condition (11), α does not affect its value. Hence, neither $\mathbf{x}_{\mathcal{I}}^*\left(\varphi\right)$ and $\mathbf{x}_{\mathcal{E}}^*\left(\varphi\right)$ are affected by α, and all the adjustment takes place through variations in $N_{\mathcal{E}}^*$. ■

C Demand Systems: Examples

I provide some examples of demands consistent with Assumption DEM. Some of these demands have particular cases which overlap with other families considered. However, it is illustrative to show separately how they can be explicitly rewritten in terms of an aggregate.

I consider the demand q_ω of a variety ω, given total measure of varieties sold M and price p_ω. Also, E is an exogenous demand shifter (e.g., income). Any Greek letter refers to a positive parameter.

- **Demands from an additively separable direct utility** as in Krugman (1979). Given utility $U\left[(q_{\omega'})_{\omega'\in\Omega}\right] := \int_{\omega'\in\Omega} u\left(q_{\omega'}\right)\mathrm{d}\omega'$ with u monotone, let $g := (u')^{-1}$. Then, $q_\omega := g\left(\frac{p_\omega}{\mathbb{A}}\right)$ where $\mathbb{A}$ is the inverse of the marginal utility of income. This demand includes as special cases:
  - **Demands derived from an exponential utility** as in Behrens and Murata (2007): $q_\omega := \mathbb{A} - \frac{1}{\alpha}\ln p_\omega$ where $\mathbb{A} := \frac{E}{\mathbb{P}_1} + \frac{\ln\mathbb{P}_1}{\alpha} + \frac{\mathbb{P}_2}{a}$ with $\mathbb{P}_1 := \int_{\omega'\in\Omega} p_{\omega'}\mathrm{d}\omega'$ and $\mathbb{P}_2 := \int_{\omega'\in\Omega}\ln\left(\frac{p_{\omega'}}{\mathbb{P}_1}\right)\frac{p_{\omega'}}{\mathbb{P}_1}\mathrm{d}\omega'$.
  - **Generalized CES** as in Jung et al. (2015) and Arkolakis et al. (2019)**:** $q_\omega := \mathbb{A}p_\omega^{-\sigma} - \alpha$ with $\mathbb{A} := \frac{E+\alpha\mathbb{P}_1}{\mathbb{P}_2}$, $\mathbb{P}_1 := \int_{\omega'\in\Omega} p_{\omega'}\mathrm{d}\omega'$ and $\mathbb{P}_2 := \int_{\omega'\in\Omega}\left(p_{\omega'}\right)^{1-\sigma}\mathrm{d}\omega'$
  - **Stone-Geary** (Generalized CES with $\sigma \to 1$) as in Simonovska (2015): $q_\omega := \frac{\mathbb{A}}{p_\omega} - \alpha$ with $\mathbb{A} := \frac{E+\alpha\mathbb{P}}{M}$ and $\mathbb{P} := \int_{\omega'\in\Omega} p_{\omega'}\mathrm{d}\omega'$.
- **Melitz and Ottaviano's (2008) linear demand** $q_\omega := \mathbb{A} - \frac{p_\omega}{\gamma}$ with $\mathbb{A} := \frac{\alpha+\eta\mathbb{P}}{\eta M+\gamma}$ and $\mathbb{P} := \frac{1}{\gamma}\int_{\omega'\in\Omega} p_{\omega'}\mathrm{d}\omega'$.
- **Feenstra's (2003) translog demand** $q_\omega := \frac{E}{p_\omega}\left[\mathbb{A} - \ln\left(p_\omega\right)\right]$ where $\mathbb{A} := \frac{1+\gamma\mathbb{P}}{M}$ and $\mathbb{P} := \int_{\omega'\in\Omega}\ln p_{\omega'}\mathrm{d}\omega'$.
- **Demands from a discrete choice model** (Luce, 1959; McFadden, 1973)**:** $q_\omega := \frac{h_\omega(p_\omega)}{\mathbb{A}}$ with $\mathbb{A} := H\left(\int_{\omega'\in\Omega} h_{\omega'}\left(p_{\omega'}\right)\mathrm{d}\omega'\right)$. It includes as special case:
  - Multinomial Logit demand: $h_\omega\left(p_\omega\right) := \exp\left(\alpha - \beta p_\omega\right)$
  - Multiplicative Competitive Interaction demand: $h_\omega\left(p_\omega\right) := \alpha\left(p_\omega\right)^{-\beta}$.
- **Demands from discrete-continuous choices model** as in Nocke and Schutz (2018): $q_\omega := \frac{\partial h_\omega(p_\omega)/\partial p_\omega}{\mathbb{A}}$ with $\mathbb{A} := H\left(\int_{\omega'\in\Omega} h_{\omega'}\left(p_{\omega'}\right)\mathrm{d}\omega'\right)$. It includes the Logit and the CES without income effects as special cases.
- **Constant expenditure demands** (Vives, 2001): they are isomorphic to demands from a discrete choice model but accounting for income effects. Formally, $q_\omega := \frac{E}{p_\omega}\frac{h_\omega(p_\omega)}{\mathbb{A}}$ with $\mathbb{A} := H\left(\int_{\omega'\in\Omega} h_{\omega'}\left(p_{\omega'}\right)\mathrm{d}\omega'\right)$. It includes:
  - CES: $h\left(p_\omega\right) := \alpha\left(p_\omega\right)^{-\beta}$.
  - Exponential demand: $h\left(p_\omega\right) := \exp\left(\alpha - \beta p_\omega\right)$.
- **Demands from an additively separable indirect utility** as in Bertoletti and Etro (2015): given an indirect utility $V\left[(p_{\omega'})_{\omega'\in\Omega}, E\right] := \int_{\omega'\in\Omega} v_{\omega'}\left(\frac{p_{\omega'}}{E}\right)\mathrm{d}\omega'$, demands are $q_\omega := \frac{v'_{\omega'}\left(\frac{p_\omega}{E}\right)}{\mathbb{A}}$ with $\mathbb{A} := \int_{\omega'\in\Omega} v'_{\omega'}\left(\frac{p_{\omega'}}{E}\right)\frac{p_{\omega'}}{E}\mathrm{d}\omega'$.

In Appendix B, I consider an extension to nested demands with groups of varieties defined by their origin (domestic or foreign). There, I show that the import-competition channel is inactive when the demand satisfies weak separability with respect to the aggregators. Following the definition of nested demands used there, I prove that the cases of Nested CES and Nested Logit satisfy the assumption.

- **Nested CES**

$$q^l(\omega) := \alpha \mathbb{P}^\sigma \left(\mathbb{P}^l\right)^{\varepsilon-\sigma} p_\omega^{-\varepsilon}$$

where α is a demand shifter, $\left(\mathbb{P}^l\right)^{1-\varepsilon} := \int_{\omega' \in \Omega^l} (p_{\omega'})^{1-\varepsilon} \mathrm{d}\omega'$ and $\mathbb{P}^{1-\sigma} := \int_0^L \left(\mathbb{P}^l\right)^{1-\sigma} \mathrm{d}l$. Thus, $\mathbb{P}$ and $\mathbb{P}^l$ are weakly separable from p_ω.

- **Nested Logit**

$$q^l(\omega) := \alpha \exp\left(-\frac{p_\omega}{\lambda^l}\right) \frac{\left(\mathbb{P}^l\right)^{\lambda^l - 1}}{\mathbb{P}}$$

with α a demand shifter, $\mathbb{P}^l := \int_{\omega' \in \Omega^l} \exp\left(-\frac{p_{\omega'}}{\lambda^l}\right) \mathrm{d}\omega'$, and $\mathbb{P} := \int_0^L \left(\mathbb{P}^l\right)^{\lambda^l} \mathrm{d}l$. Thus, $\mathbb{P}$ and $\mathbb{P}^l$ are weakly separable from p_ω.

D Differential Characterization of Demand Systems

Checking whether a demand system satisfies Assumption DEM requires verifying that there exists an aggregate $\mathbb{A}$ such that the demand can be expressed as $q_\omega := \max\{0, q(\mathbb{A}, p_\omega)\}$. Mathematically, this is equivalent to weak separability of $\left(\mathbb{P}^k\right)_{k=1}^K$ from p_ω in q_ω. Using some results from the separability literature, there is a differential characterization of weak separability that allows us to identify if a demand is consistent with Assumption DEM. I start by stating the differential characterization of weak separability generically.

Lemma 8. *(Leontief, 1947; Sono, 1961). Let $f : X \to \mathbb{R}$ with $X \subseteq \mathbb{R}_+^N$ with $N < \infty$. Consider a partition of the N variables into R groups $\left\{I^1, I^2, ..., I^R\right\}$ so that $X := \times_{r=1}^R X^r$. Denote generic elements by $\mathbf{x} \in X$ and $\mathbf{x}^r \in X^r$. We say that each group $r = 1, ...R$ is **weakly separable** from all other variables in f if there exist real-valued functions H and $(h^r)_{r=1}^R$ such that $f(\mathbf{x}) = H\left[h^1(\mathbf{x}^1), ..., h^R\left(\mathbf{x}^R\right)\right]$.*

*If $f \in \mathbf{C}^1$, group r is **weakly separable** from the rest of the variables in f if and only if the marginal rate of substitution between any two variables belonging to the group r are independent of any variable which does not belong to r. Formally,*
$$\frac{\partial\left(\frac{\partial f(\mathbf{x})/\partial x_{i'}}{\partial f(\mathbf{x})/\partial x_{i''}}\right)}{\partial x_j} = 0 \text{ for } i', i'' \in r \text{ and } j \notin r.$$

Making use of this Lemma, the following corollary can be applied to identify whether a particular demand satisfies Assumption DEM.

Corollary 1. *p_ω is weakly separable from $\left(\mathbb{P}^k\right)_{k=1}^K$ in q_ω if and only $\frac{\partial\left(\frac{\partial q_\omega/\partial \mathbb{P}^{k'}}{\partial q_\omega/\partial \mathbb{P}^{k''}}\right)}{\partial p_\omega}=0$ for all $k',k''=1,...,K$.*

E Well-Defined Equilibrium and Uniqueness

In the main text, I have not delve into conditions for a unique and well defined equilibrium. While some of the assumptions needed for this are standard, one feature of the equilibrium deserves some comments: the free-entry conditions pin down the aggregates which are not variables themselves. Rather, each aggregate is a value belonging to the range of a function that, in turn, depends on endogenous variables. I show that the equilibrium is well defined since range $\mathcal{A}$ is compact and convex under standard assumptions. To illustrate this as simply as possible, I focus on the case of a closed economy.

First of all, the price domain is $P := \left[\underline{p},\overline{p}\right]$, with $\underline{p}\in\mathbb{R}_+$ and $\overline{p}\in\mathbb{R}_{++}\cup\{\infty\}$, and so compact. Thus, optimal prices, characterized by (PRICE), exist and are unique under standard Inada conditions and strict quasiconcavity of profits in own prices.

In addition, we know that $\mathcal{P}^k(\mathbf{p}) := \int_{\omega\in\overline{\Omega}} h^k(p(\omega))\,\mathrm{d}\omega$ where each h^k is an integrable function. Also, $\boldsymbol{\mathcal{P}} := \left(\mathcal{P}^k\right)_{k=1}^K$ with $\boldsymbol{\mathbb{P}} := \left(\mathbb{P}^k\right)_{k=1}^K$ and $K<\infty$. In the literature, it is usually assumed that price aggregators are expressed in terms of varieties actually sold in the market. This is formalized by defining $\mathcal{P}^k(\mathbf{p}) := \int_{\omega\in\overline{\Omega}} \mathbb{1}_{(p(\omega)<\overline{p})} h^k(p(\omega))\,\mathrm{d}\omega$ or, simply, $\mathcal{P}^k(\mathbf{p}) := \int_{\omega\in[0,M]} h^k(p(\omega))\,\mathrm{d}\omega$. Thus, given optimal prices, any price aggregator takes the form $\mathcal{P}^k(\mathbf{p}) := M^E \int_{\varphi^*(\mathbb{A})}^{\overline{\varphi}} h^k\left[p(\mathbb{A},\varphi)\right] g(\varphi)\,\mathrm{d}\varphi$. With this characterization, we can apply Lyapunov's Convexity Theorem and conclude that range $\mathcal{P}^k$ is compact and convex.[41] Moreover, since $\mathcal{A}$ is continuous on $\times_{k=1}^K \text{range}\,\mathcal{P}^k$, then range $\mathcal{A}$ is compact and convex too. Therefore, range $\mathcal{A}\in\left[\underline{\mathbb{A}},\overline{\mathbb{A}}\right]$ for some $\underline{\mathbb{A}},\overline{\mathbb{A}}\in\mathbb{R}_+$.

Applying Berge's maximum theorem, if the profits function is continuously differentiable $n+1$ times, then the value function $\pi(\mathbb{A};\varphi)$ is continuously differentiable n times. Joint with the smoothness of the costs functions and Lemma 3, it determines that $\varphi^*(\mathbb{A})$ is a single-valued correspondence which is continuously differentiable. If there exists an integrable function g such that $\left|\frac{\partial\pi}{\partial\mathbb{A}}\right|\leq g$, then we can apply Leibniz rule and, so, $\widetilde{\pi}(\mathbb{A})$ is

[41] See, for instance, Aliprantis and Border (2006).

continuously differentiable too. Thus, since $\mathbb{A} \in \left[\underline{\mathbb{A}}, \overline{\mathbb{A}}\right]$ and supposing that $\widetilde{\pi}\left(\underline{\mathbb{A}}\right) > F$ and $\widetilde{\pi}\left(\overline{\mathbb{A}}\right) < F$, an equilibrium $\mathbb{A}^*$ exists. By Lemma 3, $\pi\left(\mathbb{A}, \varphi\right)$ is decreasing in $\mathbb{A}$ and, also, $\varphi^*\left(\mathbb{A}\right)$ is increasing in $\mathbb{A}$. Thus, $\widetilde{\pi}\left(\mathbb{A}\right)$ is decreasing in $\mathbb{A}$, implying that the equilibrium is unique.

Finally, suppose $\mathbb{A} > \mathcal{A}^*\left(\mathbb{A}, 0\right)$ and $\mathbb{A} < \mathcal{A}^*\left(\mathbb{A}, \overline{M}\right)$ for any $\mathbb{A} > \underline{\mathbb{A}}$. Then, by continuity, a solution set $M^E\left(\mathbb{A}^*\right)$ exists. To show that it is unique and so a function, assume that $\frac{\partial \mathcal{A}^*\left(\mathbb{A}, M^E\right)}{\partial \mathbb{A}} \neq 1$ (or simply Assumption STB) for any M^E and $\mathbb{A}$, and then the result follows.