How Can We Capture Pro-Competitive Effects from Better Export Opportunities and Import Competition?

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Abstract

Under monopolistic competition, the linear demand à la Melitz and Ottaviano has become the main demand to incorporate pro-competitive effects from trade liberalization. However, is it capable of capturing pro-competitive channels from policies that promote exports or import competition in isolation? In this paper, we investigate this matter and show that the approach chosen to model firm heterogeneity is crucial in this respect. Specifically, a setup with firm heterogeneity à la Melitz exclusively captures pro-competitive effects due to better export opportunities, whereas firm heterogeneity à la Chaney those from tougher import competition. However, none of these approaches are able to capture both mechanisms. Given this trade-off, our results inform about the most suitable firm-heterogeneity approach to study a specific unilateral trade policy.

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1 Introduction

In models of International Trade under monopolistic competition and firm heterogeneity, the demand system by Melitz and Ottaviano (2008) (henceforth, MO) has become the main alternative to the CES demand. This consists of a linear demand that accounts for an endogenous number of varieties, thereby exhibiting endogenous markups unlike the CES.

Due to this property, MO's demand has been utilized to incorporate pro-competitive effects into International-Trade models. Its use has been commonly devoted to the study of trade liberalization, which involves a simultaneous shock to import and export trade costs. Nonetheless, researchers and policymakers could also be interested in policies that promote exports or import competition in isolation. Taking this as the goal, is the linear demand by MO appropriate to study pro-competitive stemming from these policies? And given the different ways to model firm heterogeneity, what kind of monopolistic-competition setting is the most suitable to accomplish this?

In this paper, we provide an answer to this question. Throughout the paper, we define *pro-competitive effects* from trade shocks as the creation of a tougher competitive environment at home that disciplines domestic firms. This benefits a country by decreasing its average prices, through both reductions in the domestic firms' markups and the exit of the least-productive domestic firms.

We investigate in particular two pro-competitive mechanisms through which trade liberalization can generate pro-competitive effects: an import-competition channel and an export-opportunities channel. The *import-competition channel* acts through the exposure of domestic firms to tougher import competition, which could take the form of either lower prices or entry by foreign firms. When this channel is active, it entails a reduction in market power of domestic firms that decreases their markups. Additionally, it forces the least-productive domestic firms to exit, thereby allocating production to more efficient firms and hence reducing the average price at home.

As for the *export-opportunities channel*, it is capable of generating pro-competitive effects through the creation of better business opportunities for domestic firms. This increases the domestic firms' expected profits and induces entry, thus strengthening competition at home when the entrants serve home. Ultimately, it leads to a reduction in the domestic incumbents' markups and makes it harder for the least-productive domestic firms to survive.

Our goal is to analyze these channels under MO's linear demand in settings with firm heterogeneity. The International-Trade literature has considered two approaches to model firm heterogeneity under monopolistic competition. The first one is employed in MO and follows Melitz (2003). This variant assumes that firms are ex-ante homogeneous and do not know their productivity. By paying an entry cost, they are able to discover this information and become heterogeneous ex-post. We refer to this type of heterogeneity as **heterogeneity à la Melitz**.

Alternatively, there is a more standard way to introduce firm heterogeneity, where each firm has some given productivity level. MO refers to this approach as a short-run version of their model, since it is isomorphic to their setting once firms pay the entry cost. It can also be considered as part of a long-run setting in which firms are ex-ante heterogeneous and know their productivity. This interpretation is consistent with Kehoe et al. (2016), who rationalize the inherent heterogeneity of firms as differences in entrepreneurial talent. We refer to this type of heterogeneity as **heterogeneity à la Chaney**, since it has been commonly associated with Chaney (2008) in the International-Trade field.

The main conclusion of our paper is that firm heterogeneity à la Melitz captures pro-competitive effects exclusively due to better export opportunities. On the contrary, firm heterogeneity à la Chaney exhibits pro-competitive effects entirely due to tougher import competition. However, none of them are able to capture both pro-competitive mechanisms, making each approach appropriate for the study of different unilateral trade policies (i.e., reductions in export or import trade costs).

Our paper proceeds as follows. In Section 2 we formalize the setup, consisting of a monopolistic-competition model and MO's linear demand. Unlike MO, which only considers a Pareto distribution, we suppose a general productivity distribution that is possibly country-specific. Moreover, we obtain results for both small and large changes in trade costs.

In Section 3 and Section 4 we analyze the results under heterogeneity à la Melitz and à la Chaney, respectively. Our goal is to determine whether each variant captures the import-competition and export-opportunities channels. Identifying whether these channels generate pro-competitive effects requires considering shocks that isolate the specific mechanism acting in each of them. This means for the import-competition channel that the shock has to exclusively entail an exposure of domestic firms to tougher import competition. On the other hand, isolating the export-opportunities channel requires that the shock solely reflects better business opportunities to sell abroad.

To accomplish this, we consider a small country in the sense of Demidova and Rodríguez-Clare (2009; 2013). This entails that the country studied and its firms have a negligible impact on other countries. Thus, any shock in the small country does not simultaneously impact the foreign countries' domestic conditions, which otherwise would additionally affect the country analyzed's export conditions. This allows us to directly isolate the mechanism of each channel through reductions in import and export trade costs.

At a formal level, identifying pro-competitive effects is equivalent to determining whether there is a decrease in the demand's choke price of the country analyzed. This follows because the choke price is a sufficient statistic for a country's competitive environment. Our results show that tougher import competition under heterogeneity à la Melitz does not affect the domestic choke price. Thus, the domestic firms' prices and the domestic survival marginal-cost cutoff do not change; only the mass of domestic firms paying the entry cost is reduced. On the contrary, tougher import competition under heterogeneity à la Chaney reduces the domestic choke price. This means that competition at home is strengthened, thereby lowering the domestic firms' prices and their marginal-cost cutoff.

As for better export opportunities, they increase domestic competition under heterogeneity à la Melitz. Thus, both the domestic firms' prices and their marginal-cost cutoff become lower. Instead, better export opportunities do not impact the domestic choke price under heterogeneity à la Chaney, and so do not generate pro-competitive effects.

Overall, our results show that the import-competition channel is only active under heterogeneity à la Chaney, whereas the export-opportunities channel is only active under heterogeneity à la Melitz. However, none of these approaches of firm heterogeneity simultaneously capture both channels.

The small-country assumption is used as a way to isolate the effects of tougher import competition and better export opportunities. Its incorporation makes it possible to identify pro-competitive *mechanisms* that are operating in equilibrium outcomes. Nevertheless, the results for a small country are also important on their own: they determine the effects of a reduction in import and export trade costs in small economies. More generally, a researcher could be interested in investigating the total impact that these shocks entail for a particular country, and hence taking as given whether the country is small or large. Due to this, we extend our results to consider the effects of reductions in export and import trade costs under two large countries.

Reductions in trade costs in a large country activate another mechanism: changes in the foreign countries' competitive conditions. Thus, for instance, a decrease in import trade costs entails tougher import competition at home as in the case of a small country, but also better export opportunities for foreign countries. The latter change the foreign country's domestic conditions, and hence the export conditions of the country analyzed.

The conclusions of this case resemble those of small countries. Specifically, even accounting for the additional mechanism activated with a large country, the variant with heterogeneity à la Melitz still generates pro-competitive effects following reductions in export trade costs. On the contrary, decreases in import trade costs even predict anti-competitive effects, i.e., competition is softened at home.[1] This is because the effect exclusively due to tougher import competition is absent, while the country studied now additionally faces more stringent export conditions. The latter induce exit of domestic firms, which has the opposite effects of better export opportunities.

As for reductions in import and export trade costs under heterogeneity à la Chaney, they predict the same outcomes irrespective of whether a small or large country is considered. Thus, reductions in import trade costs generate pro-competitive effects, whereas decreases in export trade costs do not impact the domestic competitive environment.

Our paper touches upon a vast literature studying pro-competitive effects from trade liberalization in a setting with monopolistic competition and firm heterogeneity. While the CES demand has been the standard demand employed in these settings, its particular properties determine that markups are constant. Thus, the effects of tougher competition

[1] A similar result has been noticed by Melitz and Ottaviano (2008) for the Pareto case, and obtained by Alfaro (2020) for other productivity distributions and an infinitesimal change in import trade costs.

on active domestic firms' prices cannot be addressed under this demand. The main alternative to incorporate pro-competitive effects has been proposed by MO. It consists of a linear demand that accounts for an endogenous number of varieties and displays love for variety. Basically, it is an extension of the demand system by Ottaviano et al. (2002) to an international-trade setting.

Prominent applications of this demand system include Chen et al. (2009), Spearot (2013; 2014), Nocco et al. (2014), Ludema and Yu (2016), Demidova (2017), Bagwell and Lee (2020a; 2020b), among others. These papers focus mainly on optimal tariffs and the empirical test of the model predictions. Instead, we concentrate on a different topic, i.e., the pro-competitive channels captured by standard approaches to model firm heterogeneity. Additionally, unlike these papers, we dispense with the Pareto productivity distribution and obtain results that hold for any distribution.

Our main contribution is to show that standard approaches to model firm heterogeneity activate different pro-competitive mechanisms. The closest paper to the topic we analyze is Alfaro (2020). This study exclusively refers to a reduction in import and export trade costs under heterogeneity à la Melitz, and assumes a demand depending on a single sufficient statistic.[2] Unlike this paper, our focus is on a comparison of pro-competitive channels across approaches to model firm heterogeneity, thus encompassing results under heterogeneity à la Chaney. This reveals the existence of a trade-off, since no approach of firm heterogeneity can simultaneously capture the import-competition and export-opportunities channel. Due to this, our results are particularly relevant for practitioners investigating policies that promote exports or import competition in isolation: they indicate which approach of firm heterogeneity is more suitable to analyze a specific unilateral trade policy.

2 Model Setup

We consider a world economy with a set $\mathcal{C}$ of countries, and a unitary measure of identical agents in each of them. Labor is the only production factor and agents supply one unit of labor inelastically.

[2]In ongoing work, Alfaro and Lander (2020) also study the features of marginal entrants in the activation of pro-competitive channels.

There are two sectors. One consists of a horizontally differentiated good with a continuum of varieties and is the focus of our analysis. The other comprises a homogeneous good supplied under perfect competition, with a country-specific technology that displays constant returns to scale. The price of this good is taken as a numéraire, and is freely traded and produced in each country in equilibrium. Consequently, this sector pins down wages in each country, which we denote by w_i for country i.

Throughout the paper, we employ the convention that a subscript ij for any variable refers to i as the origin country and j as the destination country. A subscript ii in particular refers to a domestic variable. We also characterize the model through countries i and j such that $i, j \in \mathcal{C}$, unless otherwise explicitly stated. All the proofs of this paper are relegated to Appendix A.

2.1 Demand Side

We denote the set of all the potentially conceivable varieties in the industry by $\overline{\Omega}$. Moreover, $\Omega_{ij} := [0, M_{ij}]$ refers to the set of varieties produced in i and consumed in j, and $\Omega_j := [0, M_j]$ to the set of total varieties consumed in j, where $M_j := \sum_{i \in \mathcal{C}} M_{ij}$.

A representative consumer from country i has the utility function,

$$U_i := q_0 + \alpha \int_{\omega \in \overline{\Omega}} q(\omega)\, \mathrm{d}\omega - \frac{\gamma}{2} \int_{\omega \in \overline{\Omega}} [q(\omega)]^2\, \mathrm{d}\omega - \frac{\eta}{2} \left[\int_{\omega \in \overline{\Omega}} q(\omega)\, \mathrm{d}\omega \right]^2,$$

where q_0 is the consumption of the homogeneous good, $q(\omega)$ is the consumption of variety ω, and $\alpha, \gamma, \eta > 0$.

Assuming that income is high enough such that there is consumption of both goods, the demand in country j for a variety ω produced in i is given by

$$q_{ij}(\omega) := \frac{\alpha}{\gamma + \eta M_j} - \frac{1}{\gamma} p_{ij}(\omega) - \frac{\eta}{\gamma} \frac{\mathbb{P}_j}{\gamma + \eta M_j},$$

where $\mathbb{P}_j := \sum_{i \in \mathcal{C}} \int_{\omega \in \Omega_{ij}} p_{ij}(\omega)\, \mathrm{d}\omega$. This demand defines a choke price for a variety produced in j, which is denoted by $p_j^{\max}$ and given by

$$p_j^{\max}(\mathbb{P}_j, M_j) := \frac{\alpha\gamma + \eta \mathbb{P}_j}{\gamma + \eta M_j}. \tag{1}$$

Making use of it, we can reexpress ω's demand as

$$q_{ij}(\omega) = \frac{p_j^{\max} - p_{ij}(\omega)}{\gamma}. \tag{2}$$

The choke price is indicative of the toughness of competition in j. Specifically, (1) and (2) determine that the choke price is lowered by both increases in the mass of firms serving j and decreases in the price of its active firms; this, in turn, reduces the demand of variety ω.

2.2 Supply Side

In each country i, there is a mass $\overline{M}_i$ of potential single-product firms of zero measure. These firms are ex-ante identical and do not know their productivity. Moreover, they have the possibility of entering the industry by paying a sunk fixed entry cost $F_i^E > 0$. By paying this entry cost, they get assigned a unique variety ω and a productivity draw φ. Unlike MO, we suppose that productivity draws come from a *firm-specific* distribution. As we expand upon below, this allows us to encompass different types of firm heterogeneity in a unified way.

A firm from i that pays F_i^E has to decide whether to serve country j. If it does so, it incurs a country-specific fixed cost $f_{ij} \geq 0$. Moreover, the cost to have one unit arrive is given by $c_{ij}^{\tau}(\varphi) := c_i(\varphi)\tau_{ij}$, where $c_i(\varphi) := \frac{w_i}{\varphi}$ and τ_{ij} are trade costs such that $\tau_{ii} := 1$. Notice that we are allowing for the particular case considered in MO where $f_{ij} = 0$. When this occurs, the model can still incorporate firm selection into exporting since the demand exhibits a finite choke price.

Each firm ω that decides to serve j makes a price decision $p_{ij}(\omega)$, and it sets a price greater than the demand's choke price in case it does not serve it. We suppose that markets are segmented, so that firms can sell at a different price in each country. Furthermore, home constitutes the most profitable market of each potential firm. This ensures that any firm which is active in at least one country necessarily serves its domestic market.

Following MO, we consider that there is a unique interior equilibrium, which exhibits firm selection. Thus, among the firms that pay the entry cost, some do not survive, others only serve home, and the most productive ones additionally export.

2.3 Types of Heterogeneity

As we have previously indicated, our framework differs from MO in that firms obtain productivity draws from firm-specific distributions. This makes it possible to encompass standard approaches to model firm heterogeneity within a unified framework, thereby considerably simplifying the proofs of results.

Each type of firm heterogeneity arises by establishing specific properties for the firm-specific productivity distribution. The first variant we define considers **heterogeneity à la Melitz**. This supposes that the distribution is continuous and identical for each firm from i. It corresponds to the case utilized in MO, where all firms are ex-ante alike before paying the entry cost, but become heterogeneous after paying it.

The second variant defines **heterogeneity à la Chaney**. In this approach, the distribution is different for each firm from i and degenerate (i.e., it assigns probability one to a specific productivity draw). Thus, even though strictly speaking firms do not know their productivity, they can anticipate the productivity draw they would get. Due to this, firms do not actually face any intrinsic uncertainty before paying the entry cost, and hence heterogeneity can be treated as deterministic for all practical matters.

The name of this variant comes from Chaney (2008), where this type of firm heterogeneity is assumed. It can be interpreted as a short-run version of MO where a set of firms have already paid the entry cost. When this is the case, there is an exogenous mass of firms knowing their productivity and deciding whether to serve each country. Alternatively, it can be conceived within a long-run setting through firms that are inherently heterogeneous. This interpretation is in line with Kehoe et al. (2016), who rationalize the heterogeneity of firms as differences in entrepreneurial talent. Next, we define each variant formally.

Definition 2.1. *The properties of the distribution from which firms obtain productivity draws define two variants according to the type of firm heterogeneity.*

- ***Firm heterogeneity à la Melitz:*** *the distribution is continuous and identical for each firm from the same country.*
- ***Firm heterogeneity à la Chaney:*** *the distribution is firm-specific, and gives probability one to a specific draw that is different for each firm from the same country.*

Throughout the paper, we follow MO and exploit that there is a one-to-one relationship between φ and $c_i(\varphi)$. Thus, we characterize the solutions of the model through marginal-cost distributions, rather than productivity distributions.

2.4 Optimal Choices

We express a firm's optimal variables as functions of marginal costs. Formally, a firm from i with marginal costs c that is active in j sets the following optimal prices and quantities:

$$p_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} + c_{ij}^{\tau}}{2}, \tag{3}$$

$$q_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} - c_{ij}^{\tau}}{2\gamma}. \tag{4}$$

Given optimal prices, we can obtain the firm's optimal markup. In particular, we consider linear and relative markups, since both are usually employed in the literature. They are respectively defined by $\mu := p - c$ and $m := \frac{p}{c}$, and given in equilibrium by

$$\mu_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} - c_{ij}^{\tau}}{2}, \tag{5}$$

$$m_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{p_j^{\max} + c_{ij}^{\tau}}{2c_{ij}^{\tau}}. \tag{6}$$

As for the firm's optimal profit in j, this can be expressed as

$$\pi_{ij}\left(p_j^{\max}, c; \tau_{ij}\right) := \frac{\left(p_j^{\max} - c_{ij}^{\tau}\right)^2}{4\gamma} - f_{ij}. \tag{7}$$

While the characterization of optimal decisions is the same independently of the type of firm heterogeneity, the equilibrium conditions differ according to it. Due to this, we state the rest of the equilibrium conditions when we analyze each type of firm heterogeneity.

3 Firm Heterogeneity à la Melitz

Under this type of firm heterogeneity, all firms from country i obtain productivity draws from the same distribution. We denote the marginal-cost cdf that this generates by G_i, and suppose it is continuous, has density g_i, and non-negative support $[\underline{c}_i, \overline{c}_i]$.

3.1 Equilibrium

We refer to the marginal-cost cutoff in i to serve j by c_{ij}^*, which corresponds to the marginal cost that determines zero profit. Given optimal profits, (7), this is obtained through

$$\frac{\left[p_j^{\max} - \left(c_{ij}^* \tau_{ij}\right)\right]^2}{4\gamma} = f_{ij},$$

thus determining that c_{ij}^* is given by the following function:

$$c_{ij}^*\left(p_j^{\max}; \tau_{ij}\right) := \frac{p_j^{\max} - 2\sqrt{\gamma f_{ij}}}{\tau_{ij}}, \tag{ZCP-M}$$

where "ZCP" stands for zero cutoff profit and "M" for firm heterogeneity à la Melitz.

Given that firms from the same country obtain productivity draws from the same distribution, their expected profits are the same. Moreover, the expected profit in j of a firm from i can be expressed through the following function:

$$\pi_{ij}^{\exp}\left(p_j^{\max}; \tau_{ij}\right) := \int_{\underline{c}_i}^{\frac{p_j^{\max} - 2\sqrt{\gamma f_{ij}}}{\tau_{ij}}} \left[\frac{\left(p_j^{\max} - c_{ij}^{\tau}\right)^2}{4\gamma} - f_{ij}\right] \mathrm{d}G_i\left(c\right). \tag{8}$$

Equation (8) incorporates that the marginal-cost cutoff is given by the function (ZCP-M), so that expected profits in j are a function of j's choke price exclusively. This makes it possible to express the free-entry condition in i as exclusively depending on each country's choke price in terms of endogenous variables:

$$\sum_{j \in \mathcal{C}} \pi_{ij}^{\exp}\left(p_j^{\max}; \tau_{ij}\right) = F_i^E. \tag{FE-M}$$

Furthermore, the equilibrium condition at the market stage in j (i.e., for a given set of firms that paid the entry cost) can be expressed by exploiting that a firm's optimal choice is a function of j's choke price. Thus, evaluating $\mathbb{P}_j$ and M_j at the optimal choices, there is equilibrium at the market stage when $p_j^{\max *}$ constitutes a fixed point of (1). As we show in the appendix, this means that the following condition has to be satisfied:

$$\frac{\eta}{2}\left[\sum_{i \in \mathcal{C}} \lambda_{ij}\left(p_j^{\max}, M_i^E; \tau_{ij}\right)\right] + \gamma p_j^{\max} = \alpha\gamma, \tag{MS-M}$$

where $\lambda_{ij} := M_i^E \left[G_i\left(c_{ij}^*\right) p_j^{\max *} - \int_{\underline{c}_i}^{c_{ij}^*} c_{ij}^{\tau} \, \mathrm{d}G_i\left(c\right)\right]$ and c_{ij}^* is given by (ZCP-M) (the label "MS" is mnemonic for "market stage").

With the equilibrium conditions expressed in this way, we can identify the model's equilibrium in two steps. In the first one, we make use of the system comprising equations (FE-M) and (MS-M) for each $i, j \in \mathcal{C}$ to identify $\left(p_i^{\max *}, M_i^{E*}\right)_{i \in \mathcal{C}}$. After this, we can identify any optimal variable. In particular, a country's choke price pins down the optimal prices, quantities, and markups by active domestic firms, and their survival marginal-cost cutoff.

3.2 Pro-Competitive Channels

Our goal is to analyze whether specific trade channels generate pro-competitive effects. Throughout the paper, we refer to pro-competitive effects as the creation of a tougher competitive environment at home. In our framework, this leads to reductions in the domestic firms' average prices, through both decreases in the domestic firms' markups and in their marginal-cost cutoff. The former does it by directly reducing the prices of active domestic firms, while the latter by increasing the average productivity at home due to the exit of the least-productive domestic firms.

Our focus is on two specific pro-competitive channels. The first one is referred to as the **import-competition channel**, and entails an exposure of domestic firms to lower prices and entry by foreign firms. This channel generates pro-competitive effects by decreasing the market power of domestic firms and by reducing the profitability of the least-productive domestic firms; the latter induces exit of domestic firms and thereby increases average productivity.

The second pro-competitive channel is referred to as the **export-opportunities channel**. Better export access can result in pro-competitive effects by increasing profitability at home and inducing entry of firms that serve their domestic market. This strengthens competition at home, which reduces the domestic firms' markups and makes it harder for the least-productive domestic firms to survive.

Given our goal, we consider a scenario that isolates each mechanism. This is done by establishing conditions under which a reduction in import trade costs solely entails an exposure of domestic firms to tougher import competition, while a reduction in export trade costs exclusively represents better export opportunities for domestic firms.

To accomplish this, we concentrate on a country H that is small in the sense of Demidova and Rodríguez-Clare (2009; 2013). Its definition implies that shocks affecting H's market or its firms do not affect the domestic conditions of any foreign country. Formally, it means that these shocks do not affect $\left(p_j^{\max *}, M_j^{E*}\right)_{j\in\mathcal{C}\setminus\{H\}}$.[3] Notice that, even when each M_j^{E*} is taken as given, the model exhibits extensive-margin adjustments of foreign firms, since their marginal-cost cutoffs are still endogenous.

It is worth remarking that the importance of this case goes beyond its implications for small countries: it allows us to isolate the import-competition and export-opportunities channel through reductions in import and export trade costs. Instead, the usual case of two large countries determines that these trade shocks activate another mechanism: changes in the foreign countries' competitive conditions. Thus, for instance, a reduction in H's import trade costs strengthens competition in H, but also provides non-negligible better export opportunities for foreign countries. The latter could change the foreign countries' domestic conditions, which would affect H's export conditions and hence confound the channels. We formally show this in Section 5, where we study the implications of our results for large countries.

3.3 Identification of the Pro-Competitive Channels

Throughout the analysis, we compare the equilibrium outcomes for two vectors of trade costs. We denote them by $\boldsymbol{\tau}^* := \left(\tau_{ij}^*\right)_{i,j\in\mathcal{C}}$ and $\boldsymbol{\tau}^{**} := \left(\tau_{ij}^{**}\right)_{i,j\in\mathcal{C}}$, and make them vary in alternative ways to study the different pro-competitive channels. This also entails that changes in trade costs are arbitrary in size, making our results apply to both small and large trade shocks.

To distinguish between the equilibrium value of any variable under each vector of trade costs, we respectively use a superscript $*$ and $**$. In case we refer to some generic equilibrium, we use the superscript $*$.

Identifying the existence of pro-competitive effects is equivalent to investigating whether a trade shock reduces H's choke price. This follows because H's choke price is a sufficient statistic for the variables that define the existence of pro-competitive effects (i.e., the domestic markups and marginal-cost cutoff of firms from H). Intuitively, the choke price acts

[3] The implications of this small-country definition can be formalized by assuming that H is part of a continuum of trading partners in each foreign country. See Alfaro (2020).

as a measure of competition toughness in a country.

We also streamline notation for the equilibrium variables. Specifically, consider a firm from i that is active in j and has marginal cost c. Given j's equilibrium choke price, we denote its equilibrium prices and quantities given by (3) and (4) through $p^*_{ij}(c)$ and $q^*_{ij}(c)$, and the equilibrium markups (5) and (6) through $\mu^*_{ij}(c)$ and $m^*_{ij}(c)$.

Finally, we derive results by leaving the type of heterogeneity for foreign firms unspecified, since the results hold independently of this feature when the country analyzed is small.[4]

3.4 The Import-Competition Channel

Next, we study the effect of a reduction in H's import trade costs. Incorporating that H is a small economy, this trade shock exclusively captures the exposure of H's domestic firms to tougher import competition.

We first present the results and then proceed to explain them. In addition to the effect on markups and marginal-cost cutoff of domestic firms, we include results for other domestic variables to provide a complete picture of how the model adjusts.

Proposition 3.1. *Suppose that H is a small country. Let trade costs $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ be such that $\tau^{**}_{jH} \leq \tau^*_{jH}$ for each $j \neq H$, with strict inequality for at least one country. If firms from H exhibit heterogeneity à la Melitz, then, relative to the equilibrium with $\boldsymbol{\tau}^*$:*

- *The competitive environment in H does not change. Formally, $p^{\max **}_H = p^{\max *}_H$.*
- *The survival marginal-cost cutoff of domestic firms does not change. Formally, $c^{**}_{HH} = c^*_{HH}$.*
- *Prices, markups, and quantities of H's domestic firms do not change. Formally, $p^{**}_{HH}(c)$, $q^{**}_{HH}(c)$, $m^{**}_{HH}(c)$, and $\mu^{**}_{HH}(c)$ do not vary.*
- *The mass of incumbents from H becomes lower. Formally, $M^{E**}_H < M^{E*}_H$.*

The proposition establishes that the import-competition channel is inactive, in the sense that it does not affect competition at home. Thus, the exposure of domestic firms to tougher foreign competition does not result in reductions of average prices set by domestic firms. More generally, H's domestic firms do not vary any of their decisions. This includes their

[4] In the proofs of propositions, we show that a reduction in H's import trade costs always triggers more aggressive pricing and entry by foreign firms. This is all we need to analyze how trade shocks affect outcomes.

prices and quantities, and the decision of whether to serve H since the domestic marginal-cost cutoff does not change.

Instead, the more aggressive behavior by foreign firms triggers a reduction in the mass of firms from H that pay the entry cost. The magnitude of this reduction is such that it completely offsets the increase in competition. Thus, tougher import competition solely replaces domestic firms, without any consequence on H's choke price and therefore on H's competitive environment.

Mathematically, this outcome can be noticed through inspection of the equilibrium conditions. The existence of pro-competitive effects depends on how a trade shock affects the domestic choke price, $p_H^{\max}$. In turn, we have set the equilibrium conditions to completely identify the equilibrium choke prices by solving the system (FE-M) for each $i \in \mathcal{C}$. Incorporating that H is a small country, the system (FE-M) collapses to simply one equation, given by the free-entry condition in H:

$$\pi_{HH}^{\exp}\left(p_H^{\max *}\right) + \sum_{j \neq H} \pi_{Hj}^{\exp}\left(p_j^{\max *}; \tau_{Hj}^{*}\right) = F_H^E, \tag{9}$$

where $\left(p_j^{\max *}\right)_{j \in \mathcal{C} \setminus \{H\}}$ are given and not affected by shocks in the small country H. Examination of (9) determines that this equation is independent of import trade costs. Consequently, there is only one choke price in H consistent with zero expected profits, and this is not affected in equilibrium by tougher import competition. On the contrary, import trade costs directly affect the equation (MS-M) for H, explaining the reduction in the mass of domestic incumbents from H.

3.5 Export-Opportunities Channel

Since H is a small economy, the study of the export-opportunities channel can be done through a reduction in export trade costs. In other words, the effects of this trade shock exclusively capture new business opportunities for firms from H. The results are summarized in the following proposition.

Proposition 3.2. *Suppose that H is a small country. Let trade costs $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ be such that $\tau_{HF}^{**} < \tau_{HF}^{*}$ for some country $F \neq H$. If firms from H exhibit heterogeneity à la Melitz, then, relative to the equilibrium with $\boldsymbol{\tau}^*$:*

- *Competition in H becomes tougher. Formally, $p_H^{\max **} < p_H^{\max *}$.*
- *It is harder for a domestic firm from H to survive. Formally, $c_{HH}^{**} < c_{HH}^{*}$.*
- *Prices, markups, and quantities of H's domestic firms become lower. Formally, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, and $\mu_{HH}^{**}(c)$ decrease.*
- *The mass of incumbents from H becomes greater. Formally, $M_H^{E**} > M_H^{E*}$.*

The proposition establishes that the export-opportunities channel is active, i.e., it generates pro-competitive effects. The mechanism is as follows. The better business opportunities for firms from H increase their expected profits, thereby inducing more firms from H to pay the entry cost. Eventually, not all the firms survive, but those that do it serve the domestic market and hence make competition at home tougher. This is reflected through a decrease in H's choke price.

The fact that competition is strengthened in H determines that the domestic firms' markups decrease, which reduces prices in H. Additionally, the survival marginal-cost cutoff of domestic firms decreases. This implies that the least-productive domestic firms exit, which increases the average productivity in H and reduces the average domestic prices.

Mathematically, the results can be appreciated through H's free-entry condition, (9). This completely identifies H's choke price, determining that a reduction in export trade costs increases expected profits. Thus, a lower domestic choke price is required to restore zero expected profits.

From this, we also infer that the key for the activation of the export-opportunities channel is that firms expect to export with some positive probability. In this way, a reduction in export trade costs can affect the expected profits of inactive firms, and hence influence their entry decisions. As we show below, the existence of such a mechanism establishes a clear contrast with what occurs under firm heterogeneity à la Chaney.

4 Firm Heterogeneity à la Chaney

Under this type of firm heterogeneity, each firm that pays the entry cost obtains a specific productivity draw with probability one. As a result, firms act as if they knew their productivity for all practical matters.

This has some implications for the analysis. First, any firm that pays the entry cost necessarily obtains non-negative profits; otherwise, it would prefer to stay inactive. Second, recall that we are focusing on an equilibrium with firm selection at the market: some firms exit, others are only active at home, and more productive ones additionally export. This implies that the least-productive firms that are active in the industry serve home exclusively. Consequently, they enter the industry if they are able to garner enough domestic profits to cover both the entry cost and the fixed domestic cost. Formally, given optimal profits (7), this is reflected in the free-entry condition in i, which becomes

$$\frac{\left(p_i^{\max} - c_{ii}^*\right)^2}{4\gamma} - f_{ii} = F_i^E. \tag{10}$$

From (10), we can identify the domestic survival marginal-cost cutoff. Additionally, the decision to export to j can be determined as in the variant with firm heterogeneity à la Melitz, so that the marginal-cost cutoff in $j \neq i$ is given by (ZCP-M). All this establishes that the marginal-cost cutoff to serve each country is

$$c_{ij}^*\left(p_j^{\max};\tau_{ij}\right) := \begin{cases} p_i^{\max} - 2\sqrt{\gamma f_{ii} + F_i^E} & \text{if } i = j \\ \frac{p_j^{\max} - 2\sqrt{\gamma f_{ij}}}{\tau_{ij}} & \text{otherwise.} \end{cases} \tag{ZCP-C}$$

As for the equilibrium at the market stage in j, it requires that $p_j^{\max *}$ constitutes a fixed point of (1). As we show in the appendix, this defines the following condition

$$\frac{\eta}{2}\left[\sum_{i\in\mathcal{C}} \phi_{ij}\left(p_j^{\max};\tau_{ij}\right)\right] + \gamma p_j^{\max} = \alpha\gamma, \tag{MS-C}$$

with $\phi_{ij} := \overline{M}_i \left[G_i\left(c_{ij}^*\right) p_j^{\max *} - \int_{\underline{c}_i}^{c_{ij}^*} c_{ij}^{\tau}\, \mathrm{d}G_i\left(c\right)\right]$ and where c_{ij}^* is given by (ZCP-C).

Overall, the equilibrium of the model can be obtained by identifying the vector $(p_i^{\max *})_{i\in\mathcal{C}}$ that solves the system (MS-C) for each $j \in \mathcal{C}$. After this, we can proceed as in the setup with firm heterogeneity à la Melitz. Specifically, given the equilibrium choke price in each country, we can identify the equilibrium variables of a firm from i with marginal cost c. These variables include $p_{ij}^*\left(c\right)$, $q_{ij}^*\left(c\right)$, $\mu_{ij}^*\left(c\right)$ and $m_{ij}^*\left(c\right)$, which are still respectively given by (3), (4), (5), and (6).

4.1 The Import-Competition Channel

To determine whether a pro-competitive channel is active, we proceed analogously to the case with heterogeneity à la Melitz. Specifically, we focus on some country H, and compare outcomes when trade costs are $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$.

Determining whether a trade shock generates pro-competitive effects requires identifying the effect on H's choke price. Inspection of the equilibrium conditions establishes that this can be done through the equation (MS-C) for H:

$$\frac{\eta}{2}\left[\phi_{HH}\left(p_H^{\max *}\right)+\sum_{j\neq H}\phi_{jH}\left(p_H^{\max *};\tau_{jH}\right)\right]+\gamma p_H^{\max *}=\alpha\gamma. \tag{11}$$

Condition (11) identifies H's choke price, irrespective of whether H is a small or large country. The fact that the country's size does not affect the results is because markets are segmented and there are no feedback effects due to exports. Thus, the equilibrium at the market stage in H is determined irrespective of what occurs in other countries.

Taking this into account, a reduction in import trade costs exclusively represents an exposure of domestic firms to tougher competition, irrespective of H's size. We present the results for this trade shock and then proceed to explain them.

Proposition 4.1. *Let trade costs $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ be such that $\tau_{jH}^{**}\leq\tau_{jH}^{*}$ for each $j\neq H$ with strict inequality for at least one country. If firms from H exhibit heterogeneity à la Chaney, then, relative to the equilibrium with $\boldsymbol{\tau}^*$:*

- *Competition in H becomes tougher. Formally, $p_H^{\max **}<p_H^{\max *}$.*
- *It is harder for a domestic firm from H to survive. Formally, $c_{HH}^{**}<c_{HH}^{*}$.*
- *Prices, markups, and quantities of H's domestic firms become lower. Formally, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, and $\mu_{HH}^{**}(c)$ decrease.*
- *The mass of domestic firms in H decreases. Formally, $M_{HH}^{**}<M_{HH}^{*}$.*

The proposition establishes that the import-competition channel is active, so that exposure to tougher import competition generates pro-competitive effects. This is captured through a lower choke price in H, which decreases the average price set by domestic firms. It occurs through reductions in both the domestic firms' markups and their marginal-cost cutoff, where the latter acts by inducing the exit of the least-productive domestic firms.

The outcome differs from the case with heterogeneity à la Melitz. In that variant, firms are ex-ante homogeneous, and so all make their entry decisions sharing the same expected profits. Therefore, all firms pay the entry cost if H's choke price guarantees non-negative expected profits. This implies that there is only one H's choke price that satisfies zero expected profits, and this is independent of import trade costs. Thus, tougher import competition cannot affect H's equilibrium choke price; it can only have an impact on the mass of firms from H paying the entry cost. In contrast, firms in the variant à la Chaney are ex-ante heterogeneous, determining that free entry does not impose such a type of restriction on H's choke price. In other words, exit of firms is able to occur without free entry imposing that the same choke price in H has to hold.

4.2 The Export-Opportunities Channel

Recall that the export-opportunities channel can generate pro-competitive effects if it induces entry of firms that serve the domestic market. Under heterogeneity à la Chaney, this requires that better business opportunities translate into increases in profits of the least-productive firms. However, this mechanism is not operative.

To explain why this is so, it is crucial that there is firm selection into exporting. This property is consistent with the mounting evidence collected since Bernard et al. (1995), which documents that only the most productive firms export.[5] As a corollary, the least-efficient active firms serve the domestic market exclusively.

Joint with this, the least-productive firms know their productivity under heterogeneity à la Chaney, and hence they pay the entry cost knowing that they will not export. This entails that better export opportunities do not benefit them; only affect the most profitable firms from H, including some firms that were only serving home but now start exporting. Due to this, better export opportunities do not induce entry of previously inactive firms, and so there are no additional firms serving the domestic market. As a result, competition at home does not become tougher.

This contrasts with what occurs under heterogeneity à la Melitz. In that case, firms are ex-ante alike and do not know their productivity. Thus, when they pay the entry cost, they consider it possible to obtain high-productivity draws that make them export. Given this, better export opportunities increase expected profits and additional firms pay the entry cost. Eventually, not all of them survive, but those that do so end up serving home and hence increasing competition at home.

[5]For up-to-day evidence regarding this, see Bernard et al. (2012) for the USA and Mayer and Ottaviano (2008) for several European countries.

For completeness, we summarize the results in the following proposition.

Proposition 4.2. *Let trade costs $\boldsymbol{\tau}^{*}$ and $\boldsymbol{\tau}^{**}$ be such that $\tau_{HF}^{**} < \tau_{HF}^{*}$ for some country $F \neq H$. If firms from H exhibit heterogeneity à la Chaney, then, relative to the equilibrium with $\boldsymbol{\tau}^{*}$:*

- *The competitive environment in H does not change. Formally, $p_{H}^{\max **} = p_{H}^{\max *}$.*
- *The survival marginal-cost cutoff of domestic firms does not change. Formally, $c_{HH}^{**} = c_{HH}^{*}$.*
- *Prices, markups, and quantities of H's domestic firms do not vary. Formally, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, and $\mu_{HH}^{**}(c)$ do not change.*
- *The mass of domestic firms in H does not change. Formally, $M_{HH}^{**} = M_{HH}^{*}$.*

5 Large Countries

So far, we have analyzed the emergence of pro-competitive channels according to the type of firm heterogeneity assumed. We have investigated in particular whether the import-competition and export-opportunities channels are active. With this goal, we have considered settings and shocks that exclusively capture effects due to tougher import competition and better export opportunities.

Nonetheless, these results have been derived by stating the effects of reductions in import and export trade costs in a small country. Following this interpretation, a researcher could also be interested in the effects of these trade shocks taking as given the size of the country. Taking this into account, next we provide results for the case of large countries.

The outcomes when countries are large and exhibit heterogeneity à la Chaney are the same as under a small country. This follows because Propositions 4.1 and 4.2 are valid irrespective of the size of the country under analysis. Due to this, next we concentrate on the variant with heterogeneity à la Melitz,

5.1 Heterogeneity à la Melitz

We consider a world economy with a set of large countries $\mathcal{C} := \{H, F\}$, where the focus is on a country H that exhibits heterogeneity à la Melitz. When a small country is studied, it is possible to derive results without the need to assume the type of heterogeneity in foreign countries. On the contrary, with two large countries it is necessary to state what occurs in this regard for both H and F. We assume in particular that F also exhibits heterogeneity à la Melitz.

To investigate the existence of pro-competitive effects, we need to identify H's choke price. In the variant with heterogeneity à la Melitz, this can be done by solving the system (FE-M), which for two large countries becomes

$$\begin{aligned}\pi_{HH}^{\exp}\left(p_H^{\max *}\right)+\pi_{HF}^{\exp}\left(p_F^{\max *};\tau_{HF}\right)&=F_H^E,\\ \pi_{FF}^{\exp}\left(p_F^{\max *}\right)+\pi_{FH}^{\exp}\left(p_H^{\max *};\tau_{FH}\right)&=F_F^E.\end{aligned}\tag{12}$$

The analysis of trade shocks in a large country differs from the small-country case due to the existence of feedback effects that need to be taken into account. Specifically, if we want to study what occurs in H following a trade shock in H, we need to incorporate that F's market conditions are affected in a non-negligible way, which in turn affects H's own conditions to export. The system (12) reflects this by incorporating F's zero-expected-profits condition. Thus, variations in H's choke price affect the expected profits of firms from F, and vice versa.

This fact also explains why capturing the effects of tougher import competition or export opportunities in isolation requires assuming that H is a small country. Otherwise, it would also entail a simultaneous change in H's export conditions and confound the channels.

To perform comparative statics when there are two large countries, we need to add a stability condition for the system (12). The condition we add is always satisfied under two symmetric countries, and is needed in our framework since wages and the productivity distribution could differ by country.

Specifically, we suppose that changes in a country's market conditions have a greater impact on the domestic firms' expected profits than on the exporters' expected profits. Formally, given

$$\Delta\pi_{ij}^{\exp}:=\pi_{ij}^{\exp}\left(p_j^{\max **};\tau_{ij}\right)-\pi_{ij}^{\exp}\left(p_j^{\max *};\tau_{ij}\right),$$

where $p_i^{\max **}>p_i^{\max *}$, we assume that $\Delta\pi_{ii}^{\exp}>\Delta\pi_{ji}^{\exp}$ for $i\in\mathcal{C}$ and $j\neq i$. The assumption is a relatively mild, and the existence of trade costs ensures that is always satisfied if countries are symmetric. Moreover, it determines a positive Jacobian of (12), which is the usual stability condition under infinitesimal changes.

We begin by analyzing the effects of (small or large) reductions in export trade costs.

Proposition 5.1. *Suppose a world economy with a set of large countries* $\mathcal{C} := \{H, F\}$, *and that* $\Delta\pi_{ii}^{exp} > \Delta\pi_{ji}^{exp}$ *for* $i \in \mathcal{C}$ *and* $j \neq i$. *Let trade costs* $\boldsymbol{\tau}^{*}$ *and* $\boldsymbol{\tau}^{**}$ *be such that* $\tau_{HF}^{**} < \tau_{HF}^{*}$. *If firms from* H *and* F *exhibit heterogeneity à la Melitz, then, relative to the equilibrium with* $\boldsymbol{\tau}^{*}$*:*

- *Competition in* H *becomes tougher. Formally,* $p_H^{\max **} < p_H^{\max *}$.
- *It is harder for a domestic firm from* H *to survive. Formally,* $c_{HH}^{**} < c_{HH}^{*}$.
- *Prices, markups, and quantities of* H*'s domestic firms become lower. Formally,* $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, *and* $\mu_{HH}^{**}(c)$ *decrease.*

The main conclusion we derive is that the existence of feedback effects does not qualitatively affect the results: a reduction in export trade costs still generates pro-competitive effects. Intuitively, this occurs because the feedback effects reinforce the effects arising in a small country.

Specifically, as in a small country, better export opportunities in H increase H's expected profits and induce entry of firms, thereby making competition in H tougher. This additionally entails that firms from F serving H face worse conditions to export. Unlike the case of a small country, this has now a non-negligible impact on F's expected profits, with effects akin to worse export opportunities. Therefore, fewer firms from F pay the entry cost and competition in F is reduced. Due to this, it becomes even easier for firms from H to export, which reinforces the original effect of better export opportunities.

Matters are different when we consider reductions in import trade costs: they generate anti-competitive effects. In other terms, unilateral liberalization dampens competition in H, which is similar to the result obtained by MO for the Pareto case.[6] We first present the results, which are valid for small and large changes in import trade costs, and then provide an intuition for them.

Proposition 5.2. *Suppose a world economy with a set of large countries* $\mathcal{C} := \{H, F\}$, *and that* $\Delta\pi_{ii}^{exp} > \Delta\pi_{ji}^{exp}$ *for* $i \in \mathcal{C}$ *and* $j \neq i$. *Let trade costs* $\boldsymbol{\tau}^{*}$ *and* $\boldsymbol{\tau}^{**}$ *be such that* $\tau_{FH}^{**} < \tau_{FH}^{*}$. *If firms from* H *and* F *exhibit heterogeneity à la Melitz, then, relative to the equilibrium with* $\boldsymbol{\tau}^{*}$*:*

[6] This is also obtained in Alfaro (2020) for an infinitesimal change in import trade costs.

- *Competition in H is softened. Formally,* $p_H^{\max **} > p_H^{\max *}$*.*
- *It is easier for a domestic firm from H to survive. Formally,* $c_{HH}^{**} > c_{HH}^{*}$*.*
- *Prices, markups, and quantities of H's domestic firms become greater. Formally,* $p_{HH}^{**}(c)$*,* $q_{HH}^{**}(c)$*,* $m_{HH}^{**}(c)$*, and* $\mu_{HH}^{**}(c)$ *increase.*

To explain the proposition, recall that the import-competition channel is inactive when heterogeneity is à la Melitz. Considering trade between two large countries does not modify the conclusion, since this is a result about the operating mechanisms of a model. In other terms, if we disentangle all the channels in equilibrium, the effect on the domestic competitive environment exclusively due to tougher import competition would be null. As a corollary, the existence of anti-competitive effects in equilibrium must necessarily come from another channel.

Delving into the channels that are operating, notice that a reduction in H's import trade costs entails tougher import competition, but also a reduction in export trade costs for F. Moreover, unlike the case of a small economy, better export opportunities in F have a non-negligible impact on F's expected profits. This induces entry in F and strengthens competition in that country. Consequently, firms from H face worse export conditions, and so fewer firms are willing to pay the entry cost. Ultimately, this determines less entry in H and hence a decrease of competition in that country.

The result can alternatively be explained by considering a small reduction in H's import trade costs, $\mathrm{d}\tau_{FH} < 0$. We formally prove in Appendix A.1 that this determines

$$\operatorname{sgn}\left\{\mathrm{d}p_H^{\max *}\right\} = \operatorname{sgn}\left\{\frac{\partial p_H^{\max *}}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}}{\partial \tau_{FH}} \mathrm{d}\tau_{FH}\right\}, \tag{13}$$

where $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} > 0$ and $\frac{\partial p_H^{\max *}}{\partial p_F^{\max}} < 0$. (13) indicates that $p_H^{\max *}$ is exclusively impacted by how its export conditions (i.e., $p_F^{\max}$) change. Specifically, the variation in import trade costs affect F's domestic market in a magnitude $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}}\mathrm{d}\tau_{FH}$; this, in turn, affects H's choke price in a magnitude captured by $\frac{\partial p_H^{\max *}}{\partial p_F^{\max}}$.

In words, (13) shows that H's choke price is exclusively affected by the changes in export conditions, without any direct impact from tougher import competition (i.e., the import-competition channel).

6 Conclusion

Better export opportunities and import competition can benefit a country by increasing domestic competition, and hence disciplining domestic firms. This results in pro-competitive effects, which in our paper are reflected through decreases in the domestic firms' average prices. They occur through both reductions in domestic firms' markups and the exit of the least-productive domestic firms.

The CES demand, which has been the standard system in the field since at least Krugman (1980), predicts constant markups in a monopolistic-competition setting. Thus, it cannot reflect how the pricing of a domestic firm is affected by trade. Due to this, the linear demand à la Melitz and Ottaviano has emerged as the main alternative to study pro-competitive effects in a monopolistic-competition setting.

Our paper has investigated the pro-competitive outcomes under such a demand. The goal was twofold. First, to analyze whether specific pro-competitive mechanisms due to international trade emerge. In particular, our focus was on what we called the import-competition and the export-opportunities channels. The former acts through the exposure of domestic firms to more aggressive behavior by foreign firms, whereas the latter is triggered by an increase in profits that induce entry of domestic firms.

The second goal, tightly related to the first one, was to identify the total effects emerging when import competition or better export opportunities are promoted in isolation. In this respect, we characterized the outcomes for reductions in import and export trade costs for both small and large countries. Our results were derived without assuming a particular productivity distribution and allowing for large changes in trade costs.

The main conclusion of our paper was that the approach to model firm heterogeneity is crucial for the mechanism that is captured, and hence for the unilateral trade policy studied (i.e., reductions in export or import trade costs). Specifically, firm heterogeneity à la Melitz exclusively captures pro-competitive effects due to better export opportunities, while firm heterogeneity à la Chaney those from tougher import competition. However, none of them are capable of capturing both pro-competitive mechanisms.

A corollary of this finding is that each approach to model firm heterogeneity is suitable for the study of different unilateral trade policies. In this sense, our results inform setting choice when researchers are analyzing policies that promote better export conditions or import competition in isolation.

References

Alfaro, M. (2020). The Microeconomics of New Trade Models. *Mimeo*.

Alfaro, M. and D. Lander (2020). *Marginal Entrants and Trade-Liberalization Effects Across Models of Imperfect Competition.*

Bagwell, K. and S. H. Lee (2020a). Entry and welfare in general equilibrium with heterogeneous firms and endogenous markups. In *manuscript*.

Bagwell, K. and S. H. Lee (2020b). Trade policy under monopolistic competition with firm selection. *Journal of International Economics 127*, 103379.

Bernard, A., J. Jensen, S. Redding, and P. Schott (2012). The Empirics of Firm Heterogeneity and International Trade. *Annual Review of Economics 4*(1), 283–313.

Bernard, A. B., J. B. Jensen, and R. Z. Lawrence (1995). Exporters, jobs, and wages in us manufacturing: 1976-1987. *Brookings papers on economic activity. Microeconomics 1995*, 67–119.

Chaney, T. (2008). Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review 98*(4), 1707–1721.

Chen, N., J. Imbs, and A. Scott (2009). The dynamics of trade and competition. *Journal of International Economics 77*(1), 50–62.

Demidova, S. (2017). Trade policies, firm heterogeneity, and variable markups. *Journal of International Economics 108*, 260 – 273.

Demidova, S. and A. Rodríguez-Clare (2009). Trade policy under firm-level heterogeneity in a small economy. *Journal of International Economics 78*(1), 100 – 112.

Demidova, S. and A. Rodríguez-Clare (2013). The Simple Analytics of the Melitz Model in a Small Economy. *Journal of International Economics 90*(2), 266–272.

Kehoe, T. J., P. S. Pujolàs, and K. J. Ruhl (2016). The opportunity costs of entrepreneurs in international trade. *Economics Letters 146*, 1–3.

Krugman, P. (1980). Scale economies, product differentiation, and the pattern of trade. *The American Economic Review 70*(5), 950–959.

Ludema, R. D. and Z. Yu (2016). Tariff pass-through, firm heterogeneity and product quality. *Journal of International Economics 103*, 234–249.

Mayer, T. and G. Ottaviano (2008). The Happy Few: The Internationalisation of European Firms. *Intereconomics- Review of European Economic Policy 43*(3), 135–148.

Melitz, M. J. (2003). The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity. *Econometrica 71*(6), 1695–1725.

Melitz, M. J. and G. I. P. Ottaviano (2008). Market Size, Trade, and Productivity. *The Review of Economic Studies 75*(1), 295–316.

Nocco, A., G. I. Ottaviano, and M. Salto (2014). Monopolistic competition and optimum product selection. *American Economic Review 104*(5), 304–09.

Ottaviano, G., T. Tabuchi, and J.-F. Thisse (2002). Agglomeration and trade revisited. *International Economic Review 43*(2), 409–435.

Spearot, A. (2014). Tariffs, Competition, and the Long of Firm Heterogeneity Models. *Mimeo*.

Spearot, A. C. (2013). Variable demand elasticities and tariff liberalization. *Journal of International Economics 89*(1), 26–41.

Online Appendix - not for publication

A Derivations and Proofs

We keep respectively indicating the equilibrium value of any variable under trade costs $\boldsymbol{\tau}^*$ or $\boldsymbol{\tau}^{**}$ by a superscript $*$ and $**$, with generic equilibrium $*$.

To unify the proofs for each type of firm heterogeneity, we make some definitions. First, the marginal-cost cdf of all potential firms $\overline{M}_i$ is denoted by G_i in both variants, with density g_i and non-negative support $[\underline{c}_i, \overline{c}_i]$. This coincides with the marginal-cost distribution from which each firms obtain a marginal-cost draw when heterogeneity is à la Melitz.

We also streamline notation by defining some variables that apply to both variants. Let $G_{ij}^* := G_i\left(c_{ij}^*\right)$ and $g_{ij}^* := g_i\left(c_{ij}^*\right)$ be the cdf and density of firms from i evaluated at the marginal-cost cutoff, and $\mathbb{C}_{ij}^* := \int_{\underline{c}_i}^{c_{ij}^*} c_{ij}^\tau \, \mathrm{d}G_i(c)$ the sum of unit costs inclusive of trade costs. Moreover, we unify the notation for marginal-cost cutoffs by defining

$$c_{ij}^*\left(p_j^{\max};\tau_{ij}\right) := \frac{p_j^{\max} - \xi_{ij}}{\tau_{ij}}, \tag{ZCP}$$

where each type of heterogeneity defines ξ_{ij} differently. More precisely, $\xi_{ij} := 2\sqrt{\gamma f_{ij}}$ for any $i,j \in \mathcal{C}$ in the variant with heterogeneity à la Melitz. Instead, $\xi_{ij} := 2\sqrt{\gamma f_{ij}}$ for $i \neq j$ and $\xi_{ii} := 2\sqrt{\gamma\left(f_{ii} + F_i^E\right)}$ in the variant with heterogeneity à la Chaney.

Lemma 1. *$p_{ij}\left(p_j^{\max};c_{ij}^\tau\right)$, $q_{ij}\left(p_j^{\max};c_{ij}^\tau\right)$, $c_{ij}^*\left(p_j^{\max};\tau_{ij}\right)$, $m_{ij}\left(p_j^{\max};c_{ij}^\tau\right)$, and $\mu_{ij}\left(p_j^{\max};c_{ij}^\tau\right)$ are increasing in $p_j^{\max}$. Moreover, $c_{ij}^*\left(p_j^{\max};\tau_{ij}\right)$ is decreasing in τ_{ij}.*

Proof of Lemma 1. Taking derivatives of each function, we obtain that $\frac{\partial p_{ij}(\cdot)}{\partial p_j^{\max}} = \frac{1}{2}$, $\frac{\partial q_{ij}(\cdot)}{\partial p_j^{\max}} = \frac{1}{2\gamma}$, $\frac{\partial c_{ij}^*(\cdot)}{\partial p_j^{\max}} = \frac{1}{\tau_{ij}}$, $\frac{\partial m_{ij}(\cdot)}{\partial p_j^{\max}} = \frac{1}{2c_{ij}^\tau}$, and $\frac{\partial \mu_{ij}(\cdot)}{\partial p_j^{\max}} = \frac{1}{2}$. Besides, $\frac{\partial c_{ij}^*}{\partial \tau_{ij}} = -\frac{p_j^{\max}-\xi_{ij}}{(\tau_{ij})^2}$. ■

The density of active firms is expressed differently in each variant of firm heterogeneity. A corollary of this is that the equilibrium at the market stage is expressed differently. Due to this, we begin by establishing the following lemma.

Lemma 2. *The mass of active firms from i with marginal costs c has density $M_i^E g_i(c)$ in the variant with heterogeneity à la Melitz, and density $\overline{M}_i g_i(c)$ in the variant with heterogeneity à la Chaney.*

Proof of Lemma 2. By definition, the mass of firms from i active in j that have marginal costs c is $M_{ij}\frac{g_i(c)}{G_{ij}^*}$. Regarding the variant with heterogeneity à la Melitz, we can reexpress this by using that $M_{ij} = M_i^E G_{ij}^*$. As for the case with heterogeneity à la Chaney, the distribution of marginal costs for the mass $\overline{M}_i$ of firms is given by G_i, and only a mass $M_{ij} = \overline{M}_i G_{ij}^*$ is active in j. Substituting in by M_{ij}, the result follows. ■

Next, we characterize the equilibrium condition at the market stage, i.e., for a given set of firms that paid the entry cost. Continuing with the approach of unifying notation, we replace the use of ϕ under heterogeneity à la Chaney for λ. This notation is utilized for all the proofs.

Lemma 3. *In either the setup with heterogeneity à la Melitz or Chaney, the equilibrium at the market stage in $j \in \mathcal{C}$ is given by*

$$\frac{\eta}{2}\sum_{i\in\mathcal{C}}\lambda_{ij}+\gamma p_j^{\max *}=\gamma\alpha, \tag{MS}$$

where $\lambda_{ij} := M_i^E\left(G_{ij}^* p_j^{\max *}-\mathbb{C}_{ij}^*\right)$ for the setup with heterogeneity à la Melitz, and $\lambda_{ij} := \overline{M}_i\left(G_{ij}^* p_j^{\max *}-\mathbb{C}_{ij}^*\right)$ for the setup with heterogeneity à la Chaney.

Proof of Lemma 3. In any of the variants, the equilibrium at the market stage in j requires that $p_j^{\max *}$ is a fixed point of (1). Next, we focus on this condition for the case with heterogeneity à la Melitz, since the derivation for a setup with heterogeneity à la Chaney is similar.

The choke price in j is given by a function $p_j^{\max}(\mathbb{P}_j, M_j) := \frac{\alpha\gamma+\eta\mathbb{P}_j}{\gamma+\eta M_j}$, where $\mathbb{P}_j := \sum_{i\in\mathcal{C}}\int_{\omega\in\Omega_{ij}} p_{ij}(\omega)\,\mathrm{d}\omega$ and $M_j := \sum_{i\in\mathcal{C}} M_{ij}$. Decomposing the terms of $\mathbb{P}_j$, we can reexpress $\mathbb{P}_j := \sum_{i\in\mathcal{C}}\mathbb{P}_{ij}$ where $\mathbb{P}_{ij} := \int_{\omega\in\Omega_{ij}} p_{ij}(\omega)\,\mathrm{d}\omega$. Thus, using the optimal prices and marginal-cost cutoffs, we obtain that $\mathbb{P}_{ij}$ and M_{ij} are given by the following functions in equilibrium

$$\mathbb{P}_{ij}\left(p_j^{\max}, M_i^E; \tau_{ij}\right) := M_i^E \int_{\underline{c}_i}^{\frac{p_j^{\max}-\xi_{ij}}{\tau_{ij}}} p_{ij}\left(p_j^{\max}; c, \tau_{ij}\right)\mathrm{d}G_i(c), \tag{A1}$$

$$M_{ij}\left(p_j^{\max}, M_i^E; \tau_{ij}\right) := M_i^E G_{ij}^*. \tag{A2}$$

Thus, the equilibrium condition in j is given by a value $p_j^{\max *}$ such that

$$p_j^{\max *} = \frac{\alpha\gamma+\eta\sum_{i\in\mathcal{C}}\mathbb{P}_{ij}\left(p_j^{\max}, M_i^E; \tau_{ij}\right)}{\gamma+\eta\sum_{i\in\mathcal{C}} M_{ij}\left(p_j^{\max}, M_i^E; \tau_{ij}\right)}. \tag{A3}$$

To reexpress (A3) in the form (MS), we reexpress it as

$$2\left(p_j^{\max *}\sum_{i\in\mathcal{C}} M_{ij}-\sum_{i\in\mathcal{C}}\mathbb{P}_{ij}\right)+\frac{2\gamma}{\eta}p_j^{\max *}=\frac{2\gamma\alpha}{\eta}. \tag{A4}$$

Define

$$\lambda_{ij} := 2\left(p_j^{\max *} M_{ij}-\mathbb{P}_{ij}\right). \tag{A5}$$

Using (A1) and (A2), and that $p_{ij}\left(p_j^{\max}; c, \tau_{ij}\right)$ is given by (3), we can reexpress (A5) by

$$\begin{aligned}
\lambda_{ij} &= 2\left[p_j^{\max *} M_i^E G_{ij}^* - M_i^E \int_{\underline{c}_i}^{c_{ij}^*} \frac{p_j^{\max *}+c_{ij}^{\tau}}{2}\mathrm{d}G_i(c)\right], \\
&= 2\left[p_j^{\max *} M_i^E G_{ij}^* - \frac{p_j^{\max *} M_i^E G_{ij}^*}{2} - M_i^E \int_{\underline{c}_i}^{c_{ij}^*} \frac{c_{ij}^{\tau}}{2}\mathrm{d}G_i(c)\right],
\end{aligned}$$

which determines that $\lambda_{ij} := M_i^E\left(G_{ij}^* p_j^{\max *}-\mathbb{C}_{ij}^*\right)$. Substituting this in (A4), then (MS) is obtained.

Consider now the setup with heterogeneity à la Chaney. Then, we can reexpress the choke price in j in the same way, but replacing (A1) and (A2) by the following functions

$$\begin{aligned}\mathbb{P}_{ij}\left(p_j^{\max};\tau_{ij}\right) :=& \overline{M}_i \int_{\underline{c}_i}^{\frac{p_j^{\max}-\xi_{ij}}{\tau_{ij}}} p_{ij}\left(p_j^{\max};c,\tau_{ij}\right) \mathrm{d}G_i\left(c\right), \\ M_{ij}\left(p_j^{\max};\tau_{ij}\right) :=& \overline{M}_i G_{ij}^*.\end{aligned}$$

The equilibrium at the market stage also requires that (A4) holds. Moreover, using the definition of λ_{ij} in (A5), this variant entails that

$$\lambda_{ij} = 2\left[p_j^{\max *}\overline{M}_i G_{ij}^* - \overline{M}_i \int_{\underline{c}_i}^{c_{ij}^*} \frac{p_j^{\max *} + c_{ij}^{\tau}}{2} \mathrm{d}G_i\left(c\right)\right],$$

which can be reexpressed as $\lambda_{ij} := \overline{M}_i\left(G_{ij}^* p_j^{\max *} - \mathbb{C}_{ij}^*\right)$. Substituting this in (A4), then we obtain (MS). ■

Lemma 4. *In either the variant with heterogeneity à la Melitz or Chaney, λ_{ij}^* is increasing in $p_j^{\max *}$ and decreasing in τ_{ij}.*

Proof of Lemma 4. By Lemma 3, we know that $\lambda_{ij} := M_i^E\left(G_{ij}^* p_j^{\max *} - \mathbb{C}_{ij}^*\right)$ in the variant with heterogeneity à la Melitz, and $\lambda_{ij} := \overline{M}_i\left(G_{ij}^* p_j^{\max *} - \mathbb{C}_{ij}^*\right)$ with heterogeneity à la Chaney. Without loss of generality, we make use of infinitesimal variations.

In both variants, $\mathbb{C}_{ij}^*$ determines $\frac{\partial \mathbb{C}_{ij}^*}{\partial p_j^{\max *}} = \left(c_{ij}^*\tau_{ij}\right) g_{ij}^* \frac{\partial c_{ij}^*}{\partial p_j^{\max *}}$ and $\frac{\partial \mathbb{C}_{ij}^*}{\partial \tau_{ij}} = \left(c_{ij}^*\tau_{ij}\right) g_{ij}^* \frac{\partial c_{ij}^*}{\partial \tau_{ij}} + \frac{\mathbb{C}_{ij}^*}{\tau_{ij}}$. Moreover, $\frac{\partial c_{ij}^*(\cdot)}{\partial p_j^{\max}} = \frac{1}{\tau_{ij}}$ and $\frac{\partial c_{ij}^*}{\partial \tau_{ij}} = -\frac{p_j^{\max *}-\xi_{ij}}{(\tau_{ij})^2}$.

Consider the setup with heterogeneity à la Melitz. Then, $\frac{\partial \lambda_{ij}^*}{\partial p_j^{\max *}} = M_i^E\left(G_{ij}^* + p_j^{\max *} g_{ij}^* \frac{\partial c_{ij}^*}{\partial p_j^{\max *}} - \frac{\partial \mathbb{C}_{ij}^*}{\partial p_j^{\max *}}\right)$, and we can reexpress it by

$$\begin{aligned}\frac{\partial \lambda_{ij}^*}{\partial p_j^{\max *}} &= M_i^E\left[G_{ij}^* + g_{ij}^* \frac{\partial c_{ij}^*}{\partial p_j^{\max *}}\left(p_j^{\max *} - c_{ij}^*\tau_{ij}\right)\right], \\ &= M_i^E\left[G_{ij}^* + \frac{g_{ij}^*}{\tau_{ij}}\left(p_j^{\max *} - c_{ij}^*\tau_{ij}\right)\right] > 0.\end{aligned}$$

Also, $\frac{\partial \lambda_{ij}^*}{\partial \tau_{ij}} = M_i^E\left(g_{ij}^* p_j^{\max *} \frac{\partial c_{ij}^*}{\partial \tau_{ij}} - \frac{\partial \mathbb{C}_{ij}^*}{\partial \tau_{ij}}\right)$, which becomes

$$\begin{aligned}\frac{\partial \lambda_{ij}^*}{\partial \tau_{ij}} &= M_i^E\left[g_{ij}^* \frac{\partial c_{ij}^*}{\partial \tau_{ij}}\left(p_j^{\max *} - c_{ij}^*\tau_{ij}\right) - \frac{\mathbb{C}_{ij}^*}{\tau_{ij}}\right], \\ &= -\frac{M_i^E}{\tau_{ij}}\left[g_{ij}^* \frac{p_j^{\max *} - \xi_{ij}}{\tau_{ij}}\left(p_j^{\max *} - c_{ij}^*\tau_{ij}\right) + \mathbb{C}_{ij}^*\right] < 0.\end{aligned}$$

As for the variant with heterogeneity à la Chaney, the results follow verbatim by replacing M_i^E by $\overline{M}_i$. ■

Lemma 5. *π_{ij}^{exp} is increasing in $p_j^{\max}$ and decreasing in τ_{ij}, where $\pi_{ij}^{exp}\left(p_j^{\max};\tau_{ij}\right)$ defined as in* (8).

Proof of Lemma 5. $\pi_{ij}^{\exp}$ can be expressed as

$$\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}\right) := \int_{\underline{c}_i}^{c_{ij}^*\left(p_j^{\max};\tau_{ij}\right)} \left[\frac{\left(p_j^{\max} - c_{ij}^{\tau}\right)^2}{4\gamma} - f_{ij}\right] \mathrm{d}G_i\left(c\right),$$

where $c_{ij}^{*}\left(p_j^{\max};\tau_{ij}\right)$ is given by (ZCP). Suppose that $p_j^{\max}$ does not vary, but $\tau_{ij}^{*}<\tau_{ij}^{**}$. Then,

$$\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}^{**}\right)-\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}^{*}\right)=$$
$$\int_{\underline{c}_i}^{c_{ij}^{*}\left(p_j^{\max};\tau_{ij}^{**}\right)}\left[\frac{\left(p_j^{\max}-c_{ij}^{\tau**}\right)^2}{4\gamma}-f_{ij}\right]\mathrm{d}G_i\left(c\right)-\int_{\underline{c}_i}^{c_{ij}^{*}\left(p_j^{\max};\tau_{ij}^{*}\right)}\left[\frac{\left(p_j^{\max}-c_{ij}^{\tau*}\right)^2}{4\gamma}-f_{ij}\right]\mathrm{d}G_i\left(c\right).$$

By Lemma 1, we know that c_{ij}^{*} is decreasing in τ_{ij}, so that $c_{ij}^{*}\left(p_j^{\max};\tau_{ij}^{**}\right)<c_{ij}^{*}\left(p_j^{\max};\tau_{ij}^{*}\right)$. Therefore,

$$\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}^{**}\right)-\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}^{*}\right)=$$
$$\underbrace{\int_{\underline{c}_i}^{c_{ij}^{*}\left(p_j^{\max};\tau_{ij}^{**}\right)}\left[\frac{\left(p_j^{\max}-c_{ij}^{\tau**}\right)^2-\left(p_j^{\max}-c_{ij}^{\tau*}\right)^2}{4\gamma}\right]\mathrm{d}G_i\left(c\right)}_{=:\kappa_1}-\underbrace{\int_{c_{ij}^{*}\left(p_j^{\max};\tau_{ij}^{**}\right)}^{c_{ij}^{*}\left(p_j^{\max};\tau_{ij}^{*}\right)}\left[\frac{\left(p_j^{\max}-c_{ij}^{\tau*}\right)^2}{4\gamma}-f_{ij}\right]\mathrm{d}G_i\left(c\right)}_{=:\kappa_2}.$$

Since $c_{ij}^{\tau**}>c_{ij}^{\tau*}$ and $p_j^{\max}>c_{ij}^{\tau}$, then $\kappa_1<0$. Moreover, $\kappa_2>0$ given that its term in brackets represents the profit of a firm with unit cost $c_{ij}^{\tau*}$. Thus, $\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}^{**}\right)<\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}^{*}\right)$, and so $\pi_{ij}^{\exp}$ is decreasing in τ_{ij}.

Consider now that τ_{ij} does not vary, but $p_j^{\max **}>p_j^{\max *}$. Then,

$$\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}^{**}\right)-\pi_{ij}^{\exp}\left(p_j^{\max};\tau_{ij}^{*}\right)=$$
$$\int_{\underline{c}_i}^{c_{ij}^{*}\left(p_j^{\max **};\tau_{ij}\right)}\left[\frac{\left(p_j^{\max **}-c_{ij}^{\tau}\right)^2}{4\gamma}-f_{ij}\right]\mathrm{d}G_i\left(c\right)-\int_{\underline{c}_i}^{c_{ij}^{*}\left(p_j^{\max *};\tau_{ij}\right)}\left[\frac{\left(p_j^{\max *}-c_{ij}^{\tau}\right)^2}{4\gamma}-f_{ij}\right]\mathrm{d}G_i\left(c\right).$$

By Lemma 1, c_{ij}^{*} is increasing in $p_j^{\max}$, and so $c_{ij}^{*}\left(p_j^{\max **};\tau_{ij}\right)>c_{ij}^{*}\left(p_j^{\max *};\tau_{ij}\right)$. Therefore, proceeding in the same fashion as for changes in τ_{ij},

$$\pi_{ij}^{\exp}\left(p_j^{\max **};\tau_{ij}\right)-\pi_{ij}^{\exp}\left(p_j^{\max *};\tau_{ij}\right)=$$
$$\underbrace{\int_{\underline{c}_i}^{c_{ij}^{*}\left(p_j^{\max *};\tau_{ij}\right)}\left[\frac{\left(p_j^{\max **}-c_{ij}^{\tau}\right)^2-\left(p_j^{\max *}-c_{ij}^{\tau}\right)^2}{4\gamma}\right]\mathrm{d}G_i\left(c\right)}_{=:\kappa_1}+\underbrace{\int_{c_{ij}^{*}\left(p_j^{\max *};\tau_{ij}\right)}^{c_{ij}^{*}\left(p_j^{\max **};\tau_{ij}\right)}\left[\frac{\left(p_j^{\max **}-c_{ij}^{\tau}\right)^2}{4\gamma}-f_{ij}\right]\mathrm{d}G_i\left(c\right)}_{=:\kappa_2}.$$

Since $p_j^{\max **}>p_j^{\max *}$, then $\kappa_1>0$. Furthermore, $\kappa_2>0$ because the term in brackets is the profit of a firm with unit cost c_{ij}^{τ}. Therefore, $\pi_{ij}^{\exp}\left(p_j^{\max **};\tau_{ij}\right)>\pi_{ij}^{\exp}\left(p_j^{\max *};\tau_{ij}\right)$, which determines that $\pi_{ij}^{\exp}$ is increasing in $p_j^{\max}$. ■

Proof of Proposition 3.1. The fact that H is a small economy implies that the choke price in any country $j\neq H$ is not affected by trade shocks in H. Formally, this means that $p_j^{\max *}=p_j^{\max **}$ for any $j\neq H$. Furthermore, we have established the equilibrium conditions such that $p_H^{\max}$ can be identified by using (FE-M), which implies that

$$\sum_{j\in\mathcal{C}}\pi_{Hj}^{\exp}\left(p_j^{\max *};\tau_{Hj}^{*}\right)=\sum_{j\in\mathcal{C}}\pi_{Hj}^{\exp}\left(p_j^{\max **};\tau_{Hj}^{**}\right)=F_H^{E}. \tag{A6}$$

Since export trade costs do not vary, then $\tau_{Hj}^{*}=\tau_{Hj}^{**}$ for any $j\in\mathcal{C}$. Moreover, taking into account that $p_j^{\max *}=p_j^{\max **}$ for any $j\neq H$, then (A6) can only hold if $p_H^{\max *}=p_H^{\max **}$. Thus, H's choke price does not vary. Due to this, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$, and $\mu_{HH}^{**}(c)$ are equal as in the equilibrium with $\boldsymbol{\tau}^*$ for any active domestic firm. Moreover, $c_{HH}^* = c_{HH}^{**}$, so that the marginal-cost cutoff at home is also the same.

Now, we show that the variation in import trade costs is exclusively reflected in a reduction in the mass of incumbents from H. This can be done through condition (MS-M) in H for each equilibrium, which we reexpress as

$$M_H^{E*}\left(G_{HH}^* p_H^{\max *} - \mathbb{C}_{HH}^*\right) + \lambda_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^*\right) = \frac{2\gamma}{\eta}\left(\alpha - p_H^{\max *}\right), \tag{A7}$$

$$M_H^{E**}\left(G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}\right) + \lambda_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right) = \frac{2\gamma}{\eta}\left(\alpha - p_H^{\max **}\right), \tag{A8}$$

where $\boldsymbol{\tau}_{\cdot H} := \left(\tau_{jH}\right)_{j \in \mathcal{C}\setminus\{H\}}$ are H's import trade costs and

$$\lambda_{-H}\left(p_H^{\max}; \boldsymbol{\tau}_{\cdot H}\right) := \sum_{j \in \mathcal{C}\setminus\{H\}} \lambda_{jH}\left(p_H^{\max}; \tau_{jH}\right). \tag{A9}$$

Notice that, since H is a small economy, $\left(M_j^E\right)_{j \in \mathcal{C}\setminus\{H\}}$ is taken as given since it is not affected by shocks in H. Thus, the results make use of that λ_{-H} can be defined for any type of firm heterogeneity in $j \in \mathcal{C}\setminus\{H\}$.

Given that $p_H^{\max *} = p_H^{\max **}$ and $c_{HH}^* = c_{HH}^{**}$, then $\mathbb{C}_{HH}^* = \mathbb{C}_{HH}^{**}$ and $G_{HH}^* = G_{HH}^{**}$. This implies that $\lambda_{HH}^* = \lambda_{HH}^{**}$ or, what is same, $G_{HH}^* p_H^{\max *} - \mathbb{C}_{HH}^* = G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}$. Therefore, by taking the difference of (A7) and (A8),

$$M_H^{E**} - M_H^{E*} = \frac{\lambda_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^*\right) - \lambda_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right)}{G_{HH}^* p_H^{\max *} - \mathbb{C}_{HH}^*}.$$

Since $p_H^{\max *} = p_H^{\max **}$ and $\tau_{jH}^* > \tau_{jH}^{**}$ for any $j \in \mathcal{C}\setminus\{H\}$, then $\lambda_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^*\right) < \lambda_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right)$ by using Lemma 4. Thus, the numerator of the RHS is negative, and so $M_H^{E**} < M_H^{E*}$. ■

Proof of Proposition 3.2. Given how we have set the equilibrium conditions in a setup with heterogeneity à la Melitz, the system (FE-M) for each $i \in \mathcal{C}$ pins down the choke prices of all countries. Besides, since H is a small economy, $p_j^{\max *} = p_j^{\max **}$ for any $j \in \mathcal{C}\setminus\{H\}$, and (FE-M) for H completely identifies H's domestic choke price. This condition for trade costs $\boldsymbol{\tau}^*$ and $\boldsymbol{\tau}^{**}$ is respectively given by

$$\pi_{HH}^{\exp}\left(p_H^{\max *}\right) + \sum_{j \neq H} \pi_{Hj}^{\exp}\left(p_j^{\max *}; \tau_{Hj}^*\right) = F_H^E,$$

$$\pi_{HH}^{\exp}\left(p_H^{\max **}\right) + \sum_{j \neq H} \pi_{Hj}^{\exp}\left(p_j^{\max **}; \tau_{Hj}^{**}\right) = F_H^E.$$

Since $\tau_{Hj}^* = \tau_{Hj}^{**}$ for any $j \in \mathcal{C}\setminus\{F\}$, we can take the difference of these equations and obtain

$$\underbrace{\left[\pi_{HH}^{\exp}\left(p_H^{\max **}\right) - \pi_{HH}^{\exp}\left(p_H^{\max *}\right)\right]}_{=:\kappa_1} + \underbrace{\left[\pi_{HF}^{\exp}\left(p_F^{\max **}; \tau_{HF}^{**}\right) - \pi_{HF}^{\exp}\left(p_F^{\max *}; \tau_{HF}^*\right)\right]}_{=:\kappa_2} = 0. \tag{A10}$$

We know that $p_F^{\max *} = p_F^{\max **}$ since H is a small economy, and that $\pi_{HF}^{\exp}$ is decreasing in τ_{HF} by Lemma 5. Therefore, $\kappa_2 > 0$ since $\tau_{HF}^* > \tau_{HF}^{**}$. This implies that $\kappa_1 < 0$ necessarily, so that $\pi_{HH}^{\exp}\left(p_H^{\max **}\right) < \pi_{HH}^{\exp}\left(p_H^{\max *}\right)$. By Lemma 5, we know that $\pi_{HH}^{\exp}$ is increasing in $p_H^{\max}$, and therefore $\pi_{HH}^{\exp}\left(p_H^{\max **}\right) < \pi_{HH}^{\exp}\left(p_H^{\max *}\right)$ requires that $p_H^{\max **} < p_H^{\max *}$.

The fact that $p_H^{\max **} < p_H^{\max *}$ implies by Lemma 1 that c_{HH}^{**}, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ are lower for each active firm with marginal cost c, relative to the equilibrium with τ_{HF}^{*}.

As for the mass of incumbents, we make use of that (MS-M) in H can be expressed as

$$M_H^{E*}\left(G_{HH}^{*} p_H^{\max *} - \mathbb{C}_{HH}^{*}\right) + \lambda_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^{*}\right) = \frac{2\gamma}{\eta}\left(\alpha - p_H^{\max *}\right), \tag{A11}$$

$$M_H^{E**}\left(G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}\right) + \lambda_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right) = \frac{2\gamma}{\eta}\left(\alpha - p_H^{\max **}\right), \tag{A12}$$

where λ_{-H} is defined as in (A9) and so we are allowing for heterogeneity à la Melitz or Chaney in each foreign country.

Taking the difference of (A11) and (A12),

$$M_H^{E*}\left(G_{HH}^{*} p_H^{\max *} - \mathbb{C}_{HH}^{*}\right) - M_H^{E**}\left(G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}\right) = \tag{A13}$$

$$\underbrace{\frac{2\gamma}{\eta}\left(p_H^{\max **} - p_H^{\max *}\right)}_{=:\kappa_1} + \underbrace{\lambda_{-H}\left(p_H^{\max **}; \tau_{\cdot H}^{**}\right) - \lambda_{-H}\left(p_H^{\max *}; \tau_{\cdot H}^{*}\right)}_{=:\kappa_2}. \tag{A14}$$

We have already proven that $p_H^{\max **} < p_H^{\max *}$, which implies that $\kappa_1 < 0$. Moreover, λ_{-H} is increasing in $p_H^{\max}$ by Lemma 4, and H's import trade cost do not vary, so that $\tau_{jH}^{*} = \tau_{jH}^{**}$ for any $j \in \mathcal{C} \backslash \{H\}$. The combination of both facts determine that $\kappa_2 < 0$.

Taking into account that $\kappa_1 < 0$ and $\kappa_2 < 0$, the RHS of (A13) necessarily has to be negative, so that

$$M_H^{E*}\left(G_{HH}^{*} p_H^{\max *} - \mathbb{C}_{HH}^{*}\right) < M_H^{E**}\left(G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}\right),$$

or, what is same,

$$\frac{M_H^{E**}}{M_H^{E*}} > \frac{G_{HH}^{*} p_H^{\max *} - \mathbb{C}_{HH}^{*}}{G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}}, \tag{A15}$$

where we have used that $G_{HH} p_H^{\max} - \mathbb{C}_{HH} > 0$. Noticing that $\lambda_{HH} = G_{HH} p_H^{\max} - \mathbb{C}_{HH}$, it follows that $G_{HH}^{*} p_H^{\max *} - \mathbb{C}_{HH}^{*} > G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}$., since λ_{HH} is increasing in $p_H^{\max}$ by Lemma 4. Therefore,

$$\frac{G_{HH}^{*} p_H^{\max *} - \mathbb{C}_{HH}^{*}}{G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}} > 1,$$

which using (A15) implies that $M_H^{E**} > M_H^{E*}$. ■

Proof of Proposition 4.1. The equilibrium conditions under heterogeneity à la Chaney have been set in a way that (MS-C) for H pins down H's choke prices. This condition under $\boldsymbol{\tau}^{*}$ and $\boldsymbol{\tau}^{**}$ is respectively given by

$$\overline{M}_H\left(G_{HH}^{*} p_H^{\max *} - \mathbb{C}_{HH}^{*}\right) + \lambda_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^{*}\right) = \frac{2\gamma}{\eta}\left(\alpha - p_H^{\max *}\right),$$

$$\overline{M}_H\left(G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}\right) + \lambda_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right) = \frac{2\gamma}{\eta}\left(\alpha - p_H^{\max **}\right),$$

where λ_{-H} is given by (A9) and so we are allowing for heterogeneity à la Melitz or Chaney in each foreign country. Taking the difference of these equations,

$$\begin{aligned} \tfrac{2\gamma}{\eta}\left(p_H^{\max **} - p_H^{\max *}\right) + \overline{M}_H\left[\left(G_{HH}^{**} p_H^{\max **} - \mathbb{C}_{HH}^{**}\right) - \left(G_{HH}^{*} p_H^{\max *} - \mathbb{C}_{HH}^{*}\right)\right] &= \\ \lambda_{-H}\left(p_H^{\max *}; \boldsymbol{\tau}_{\cdot H}^{*}\right) - \lambda_{-H}\left(p_H^{\max **}; \boldsymbol{\tau}_{\cdot H}^{**}\right). \end{aligned} \tag{A16}$$

Towards a contradiction, assume that $p_H^{\max **} \geq p_H^{\max *}$. Then, the LHS in (A16) is non-negative by Lemma 4, whereas the RHS is negative Lemma 4. This is a contradiction, and hence $p_H^{\max **} < p_H^{\max *}$.

Taking into account that $p_H^{\max **} < p_H^{\max *}$, then $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ are lower relative to the equilibrium with $\boldsymbol{\tau}^*$ by Lemma 1. By the same lemma, it follows that $c_{HH}^{**} < c_{HH}^*$, and this implies that $M_{HH}^{**} < M_{HH}^*$. ■

Proof of Proposition 4.2. The condition (MS-C) for H identifies H's choke price and this is not affected by τ_{HF}. Therefore, $p_H^{\max *} = p_H^{\max **}$, which entails that $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ are the same as in the equilibrium with $\boldsymbol{\tau}^*$. Besides, $c_{HH}^* = c_{HH}^{**}$ since (ZCP-C) for firms from H is not affected, and so the mass of domestic firms serving H do not change either. ■

A.1 Large Countries

We use the notation $\Delta\pi_{ij}^{\exp}(\tau_{ij}) := \pi_{ij}^{\exp}\left(p_j^{\max **};\tau_{ij}\right) - \pi_{ij}^{\exp}\left(p_j^{\max *};\tau_{ij}\right)$, and occasionally omit the term τ_{ij} as argument of it in case it does not vary.

Proofs of Proposition 5.1 and Proposition 5.2. We can cover both proofs at the same time by noticing that if $\tau_{HF}^{**} < \tau_{HF}^*$ then H faces a reduction in its import trade costs and F in its export trade costs. Thus, the effects in H cover those in Proposition 5.1, while the effects in F those in Proposition 5.2.

The system of equations (FE-M) comprising countries H and F for trade costs $\boldsymbol{\tau}^*$ is

$$\pi_{HH}^{\exp}(p_H^{\max *}) + \pi_{HF}^{\exp}(p_F^{\max *}) = F_H^E, \tag{A17a}$$

$$\pi_{FF}^{\exp}(p_F^{\max *}) + \pi_{FH}^{\exp}(p_H^{\max *};\tau_{FH}^*) = F_F^E, \tag{A17b}$$

whereas for trade costs $\boldsymbol{\tau}^{**}$ is

$$\pi_{HH}^{\exp}(p_H^{\max **}) + \pi_{HF}^{\exp}(p_F^{\max **}) = F_H^E, \tag{A18a}$$

$$\pi_{FF}^{\exp}(p_F^{\max **}) + \pi_{FH}^{\exp}(p_H^{\max **};\tau_{FH}^{**}) = F_F^E. \tag{A18b}$$

Taking the difference of (A18a) and (A17a), and of (A18b) and (A17b),

$$\Delta\pi_{HH}^{\exp} = -\Delta\pi_{HF}^{\exp},$$

$$\Delta\pi_{FF}^{\exp} + \Delta\pi_{FH}^{\exp}(\tau_{FH}^{**}) = \pi_{FH}^{\exp}(p_H^{\max *};\tau_{FH}^*) - \pi_{FH}^{\exp}(p_H^{\max *};\tau_{FH}^{**}),$$

where the expression in the second line comes from adding and subtracting $\pi_{FH}^{\exp}(p_H^{\max *};\tau_{FH}^{**})$.

By Lemma 5, $\pi_{FH}^{\exp}(p_H^{\max *};\tau_{FH}^{**}) > \pi_{FH}^{\exp}(p_H^{\max *};\tau_{FH}^*)$, and so

$$\Delta\pi_{HH}^{\exp} = -\Delta\pi_{HF}^{\exp}, \tag{A19a}$$

$$\Delta\pi_{FF}^{\exp} < -\Delta\pi_{FH}^{\exp}(\tau_{FH}^{**}). \tag{A19b}$$

We also know by Lemma 5 that $\pi_{ij}^{\exp}$ is increasing in $p_j^{\max}$. Therefore, (A19a) implies that necessarily either i) $p_H^{\max **} > p_H^{\max *}$ and $p_F^{\max **} < p_F^{\max *}$, or ii) $p_H^{\max **} < p_H^{\max *}$ and $p_F^{\max **} > p_F^{\max *}$.

Towards a contradiction, suppose ii). Then, $\Delta\pi_{HH}^{\exp} < 0$, $\Delta\pi_{HF}^{\exp} > 0$, $\Delta\pi_{FF}^{\exp} > 0$, $\Delta\pi_{FH}^{\exp}(\tau_{FH}^{**}) <$

0. But, starting from the inequality in (A19b), multiplying both sides by $\Delta\pi_{HF}^{\exp}$, and using (A19a),

$$-\Delta\pi_{FF}^{\exp}\Delta\pi_{HH}^{\exp} < -\Delta\pi_{FH}^{\exp}\left(\tau_{FH}^{**}\right)\Delta\pi_{HF}^{\exp},$$

which implies that

$$\frac{\Delta\pi_{FF}^{\exp}\Delta\pi_{HH}^{\exp}}{\Delta\pi_{HF}^{\exp}\Delta\pi_{FH}^{\exp}\left(\tau_{FH}^{**}\right)} < 1. \tag{A20}$$

Furthermore, we are assuming that $\Delta\pi_{ii}^{\exp} > \Delta\pi_{ji}^{\exp}$ for $j \neq i$ if $p_i^{\max **} > p_i^{\max *}$. Therefore, $\Delta\pi_{HH}^{\exp} < \Delta\pi_{FH}^{\exp}\left(\tau_{FH}^{**}\right)$ given $p_H^{\max **} < p_H^{\max *}$, and $\Delta\pi_{FF}^{\exp} > \Delta\pi_{HF}^{\exp}$ given $p_F^{\max **} > p_F^{\max *}$. The former in particular implies that $\frac{\Delta\pi_{HH}^{\exp}}{\Delta\pi_{FH}^{\exp}\left(\tau_{FH}^{**}\right)} > 1$ since $\Delta\pi_{HH}^{\exp} < 0$ and $\Delta\pi_{FH}^{\exp} < 0$. Consequently,

$$\frac{\Delta\pi_{FF}^{\exp}}{\Delta\pi_{HF}^{\exp}}\frac{\Delta\pi_{HH}^{\exp}}{\Delta\pi_{FH}^{\exp}} > 1,$$

which contradicts (A20). Hence, i) has to hold, and so $p_H^{\max **} > p_H^{\max *}$ and $p_F^{\max **} < p_F^{\max *}$.

Finally, applying Lemma 1, we obtain that c_{HH}^{**}, $p_{HH}^{**}(c)$, $q_{HH}^{**}(c)$, $m_{HH}^{**}(c)$ and $\mu_{HH}^{**}(c)$ are greater relative to the equilibrium with $\boldsymbol{\tau}^*$ since $p_H^{\max **} > p_H^{\max *}$. By the same token, c_{FF}^{**}, $p_{FF}^{**}(c)$, $q_{FF}^{**}(c)$, $m_{FF}^{**}(c)$ and $\mu_{FF}^{**}(c)$ are greater relative to the equilibrium with $\boldsymbol{\tau}^*$ since $p_F^{\max **} < p_F^{\max *}$. ■

Next, we formalize the result expressed in (13), which indicates that

$$\operatorname{sgn}\left\{\mathrm{d}p_H^{\max *}\right\} = \operatorname{sgn}\left\{\frac{\partial p_H^{\max *}}{\partial p_F^{\max}}\frac{\partial p_F^{\max *}}{\partial \tau_{FH}}\mathrm{d}\tau_{FH}\right\},$$

with $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} > 0$ and $\frac{\partial p_H^{\max *}}{\partial p_F^{\max}} < 0$.

The system of equilibrium conditions for the choke prices are

$$\pi_{HH}^{\exp}\left(p_H^{\max *}\right) + \pi_{HF}^{\exp}\left(p_F^{\max *}; \tau_{HF}\right) = F_H^E, \tag{A21}$$
$$\pi_{FF}^{\exp}\left(p_F^{\max *}\right) + \pi_{FH}^{\exp}\left(p_H^{\max *}; \tau_{FH}\right) = F_F^E. \tag{A22}$$

Equation (A21) determines an implicit solution $p_H^{\max *} = p_H^{\max}\left(p_F^{\max *}; \tau_{FH}\right)$, whereas (A22) an implicit solution $p_F^{\max *} = p_F^{\max}\left(p_H^{\max *}; \tau_{FH}\right)$. Differentiating both expressions,

$$\begin{aligned}\frac{\mathrm{d}p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\mathrm{d}\tau_{FH}} &= \frac{\partial p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\partial \tau_{FH}} + \frac{\partial p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\partial p_F^{\max}}\frac{\mathrm{d}p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\mathrm{d}\tau_{FH}},\\ \frac{\mathrm{d}p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\mathrm{d}\tau_{FH}} &= \frac{\partial p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\partial \tau_{FH}} + \frac{\partial p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\partial p_H^{\max}}\frac{\mathrm{d}p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\mathrm{d}\tau_{FH}}.\end{aligned} \tag{A23}$$

Solving the system (A23), we obtain

$$\frac{\mathrm{d}p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\mathrm{d}\tau_{FH}} = \frac{\frac{\partial p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\partial \tau_{FH}}}{1 - \frac{\partial p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\partial p_F^{\max}}\frac{\partial p_F^{\max *}\left(p_H^{\max *}, \tau_{FH}\right)}{\partial p_H^{\max}}} + \frac{\frac{\partial p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\partial p_F^{* \max}}\frac{\partial p_F^{\max *}\left(p_H^{\max *}; \tau_{FH}\right)}{\partial \tau_{FH}}}{1 - \frac{\partial p_H^{\max *}\left(p_F^{\max *}; \tau_{FH}\right)}{\partial p_F^{\max}}\frac{\partial p_F^{\max *}\left(p_H^{\max *}, \tau_{FH}\right)}{\partial p_H^{\max}}}. \tag{A24}$$

To show that (A24) implies (13), we provide some lemmas.

Lemma 6. $\frac{\partial p_i^{\max *}}{\partial p_j^{\max}} < 0$ *given* $i \in \mathcal{C}$ *and* $j \neq i$, *whereas* $\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} > 0$ *and* $\frac{\partial p_H^{\max *}}{\partial \tau_{FH}} = 0$.

Proof of Lemma 6. Differentiating (A21) and (A22) under $\mathrm{d}\tau_{HF} = 0$,

$$\frac{\partial \pi_{HH}^{\exp}}{\partial p_H^{\max}} \mathrm{d}p_H^{\max *} + \frac{\partial \pi_{HF}^{\exp}}{\partial p_F^{\max}} \mathrm{d}p_F^{\max *} = 0, \tag{A25}$$

$$\frac{\partial \pi_{FH}^{\exp}}{\partial p_H^{\max}} \mathrm{d}p_H^{\max *} + \frac{\partial \pi_{FF}^{\exp}}{\partial p_F^{\max}} \mathrm{d}p_F^{\max *} + \frac{\partial \pi_{FH}^{\exp}}{\partial \tau_{HF}} \mathrm{d}\tau_{FH} = 0. \tag{A26}$$

From (A25) and (A26) we determine that, for $i \in \mathcal{C}$ and $j \neq i$,

$$\frac{\partial p_i^{\max *}}{\partial p_j^{\max}} = -\frac{\partial \pi_{ij}^{\exp}}{\partial p_j^{\max}} \left(\frac{\partial \pi_{ii}^{\exp}}{\partial p_i^{\max}} \right)^{-1} < 0, \tag{A27}$$

and from (A26),

$$\frac{\partial p_F^{\max *}}{\partial \tau_{FH}} = -\frac{\partial \pi_{FH}^{\exp}}{\partial \tau_{FH}} \left(\frac{\partial \pi_{FF}^{\exp}}{\partial p_F^{\max}} \right)^{-1} > 0,$$

where the signs follow from Lemma 5. Notice also that (A21) trivially determines that $\frac{\partial p_H^{\max *}}{\partial \tau_{FH}} = 0$. ■

Lemma 7. *Suppose that* $\Delta\pi_{ii}^{exp} > \Delta\pi_{ji}^{exp}$ *for* $i \in \mathcal{C}$ *and* $j \neq i$ *when* $p_i^{\max **} > p_i^{\max *}$. *Then,* $\det J^{FE} > 0$ *where*

$$J^{FE} := \begin{pmatrix} \frac{\partial \pi_{HH}^{exp}}{\partial p_H^{\max}} & \frac{\partial \pi_{HF}^{exp}}{\partial p_F^{\max}} \\ \frac{\partial \pi_{FH}^{exp}}{\partial p_H^{\max}} & \frac{\partial \pi_{FF}^{exp}}{\partial p_F^{\max}} \end{pmatrix}.$$

Moreover, $\det J^{FE} > 0$ *iff*

$$\frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}(p_H^{\max *}, \tau_{FH})}{\partial p_H^{\max}} < 1.$$

Proof of Lemma 7. By assumption, given $p_H^{\max **} > p_H^{\max *}$ and $p_F^{\max **} > p_F^{\max *}$, then $\frac{\Delta\pi_{FF}^{\exp}}{\Delta\pi_{HF}^{\exp}} > 1$ and $\frac{\Delta\pi_{HH}^{\exp}}{\Delta\pi_{FH}^{\exp}} > 1$. This implies $\frac{\Delta\pi_{FF}^{\exp}}{\Delta\pi_{HF}^{\exp}} \frac{\Delta\pi_{HH}^{\exp}}{\Delta\pi_{FH}^{\exp}} > 1$ or, what is same,

$$\frac{\Delta\pi_{FF}^{\exp} \Delta\pi_{HH}^{\exp}}{\Delta p_H^{\max} \Delta p_F^{\max}} > \frac{\Delta\pi_{HF}^{\exp} \Delta\pi_{FH}^{\exp}}{\Delta p_H^{\max} \Delta p_F^{\max}}.$$

where $\Delta p_i^{\max} := p_i^{\max **} - p_i^{\max *}$, and we have used that $\Delta\pi_{ij}^{\exp} > 0$ for $i, j \in \mathcal{C}$ due to Lemma 5. Taking limits for $\Delta p_i^{\max} \to 0$ with $i \in \mathcal{C}$, then

$$\frac{\partial \pi_{HH}^{\exp}}{\partial p_H^{\max}} \frac{\partial \pi_{FF}^{\exp}}{\partial p_F^{\max}} > \frac{\partial \pi_{HF}^{\exp}}{\partial p_F^{\max}} \frac{\partial \pi_{FH}^{\exp}}{\partial p_H^{\max}}, \tag{A28}$$

which indicates that $\det J^{FE} > 0$.

Furthermore, using (A27) for $\frac{\partial p_i^{\max *}}{\partial p_j^{\max}}$, we obtain that

$$\frac{\partial p_H^{\max *}}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}}{\partial p_H^{\max}} = \frac{\frac{\partial \pi_{FH}^{\exp}}{\partial p_H^{\max}} \frac{\partial \pi_{HF}^{\exp}}{\partial p_F^{\max}}}{\frac{\partial \pi_{HH}^{\exp}}{\partial p_H^{\max}} \frac{\partial \pi_{FF}^{\exp}}{\partial p_F^{\max}}} < 1,$$

where the inequality is established by using (A28). ■

By using these lemmas, now we can reexpress (A24). Notice that $\frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial \tau_{FH}} = 0$ by Lemma 6. So,

$$\frac{\mathrm{d}p_H^{\max *}}{\mathrm{d}\tau_{FH}} = \frac{\frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial p_F^{* \max}} \frac{\partial p_F^{\max *}(p_H^{\max *}; \tau_{FH})}{\partial \tau_{FH}}}{1 - \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}(p_H^{\max *}, \tau_{FH})}{\partial p_H^{\max}}}. \tag{A29}$$

Moreover, $1 - \frac{\partial p_H^{\max *}(p_F^{\max *}; \tau_{FH})}{\partial p_F^{\max}} \frac{\partial p_F^{\max *}(p_H^{\max *}, \tau_{FH})}{\partial p_H^{\max}} > 0$ by Lemma 7, so that we obtain (A24).