Strategic Behavior of Domestic Leaders under Market Exploration by Small Firms

Martín Alfaro*
University of Alberta

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Abstract

Some industries are neither purely oligopolistic nor monopolistic competitive; instead, they are a hybrid of both (e.g., the beer industry). In this paper, we study such industries, where a clear strategic asymmetry between types of firms exists. We consider possibly-heterogeneous domestic leaders coexisting with numerous small businesses, where the latter venture into the industry with a differentiated variety and are uncertain about their profitability. Our findings indicate that domestic leaders partially deter market exploration and never accommodate it, thereby capturing domestic sales that otherwise would go to small firms. Small domestic firms are the most adversely affected by this, given their natural home bias. Nonetheless, small firms from less efficient countries could potentially benefit by filling domestic niches left by small exporters. We also highlight that potential competition (rather than actual competition) by small firms could boost innovation, and show that partially deterring market exploration could actually be welfare-improving.

Keywords: domestic leaders, small firms, strategic investments, market exploration.
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*University of Alberta, Department of Economics. 9-08 HM Tory Building, Edmonton, AB T6G 2H4, Canada; email: malfaro@ualberta.ca. Link to personal website.

1 Introduction

Recent evidence has documented the ubiquity of industries that are neither purely oligopolistic nor monopolistic competitive; rather, they are characterized by a coexistence of a few large firms coexisting with numerous small firms.[1] Such a market structure entails a clear asymmetry between firms, where "*the fate of small businesses is often in the hands of leading firms in their markets*," in the words of Kwoka and White (2001).

The beer industry can be viewed as an example of such a hybrid market structure. It features a handful of different brands that dominate the market, where the specific leaders tend to vary by country. Simultaneously, they coexist with craft breweries, microbreweries, and brewpubs that are negligible in isolation but non-trivial as a whole. These firms tend to focus on niche consumers by offering a differentiated variety, and are quite heterogeneous in terms of performance: while most only sell their products locally, some particularly successful ones even export.[2]

This industry also provides a useful example of the strategic asymmetry between firms. In response to the success of craft breweries since the 1990s, home leaders have been exploiting their dominant local market position by engaging in entry-preemption strategies to hinder their growth. The phenomenon has been documented in several countries, including the USA, the United Kingdom, Belgium, Denmark, Italy, and Japan, among others (D'Aveni 2002; Garavaglia and Swinnen 2020).

In this paper, we investigate the strategic behavior of domestic leaders against small firms in such market structures. In Section 2, we formalize the analysis through a setup with two possibly-heterogeneous countries and an industry comprising a tradable good that is horizontally differentiated. Furthermore, we consider a market structure where a set of non-negligible firms are embedded into an otherwise monopolistic-competition framework. To make a distinction between firms in each group, we refer to the non-negligible firms as domestic leaders (DLs) and the monopolistic firms as competitive-fringe firms (CFFs).

Our characterization of CFFs follows the seminal approach by Melitz (2003) as presented in Melitz and Redding (2015). Moreover, it is in line with several patterns established by the

[1]For instance, Alfaro and Warzynski (2020b) show that half of the Danish manufacturing industries fit this description and, remarkably, they generate more than 80% of the total manufacturing revenue. Similar evidence has been provided by Gaubert and Itskhoki (2018) for French manufacturing, and Hottman et al. (2016) and Bronnenberg et al. (2011) for several consumer-goods industries in the USA.

[2]In the UK, a survey by the Society of Independent Brewers (SIBA) for 2019 indicates that around 20% of craft breweries export, whereas 54% have intentions to do so. For an analysis regarding exports and imports by small breweries, see Tremblay and Tremblay (2005) and Schnell and Reese (2014).

literature on small firms.[3] Specifically, CFFs are conceived as a pool of ex-ante homogeneous entrepreneurs that do not know their profitability in the market. Moreover, they have the possibility of becoming active in the industry by paying an entry cost. Since they are governed by free-entry rules, each CFF enters the industry as long as it expects positive profits. After they enter, they discover their profitability and are able to serve the market with a unique variety. Ultimately, CFFs become heterogeneous and face different fates: the least profitable CFFs do not survive and exit the market, more profitable ones operate locally, and the most profitable ones even export. In all cases, they end up operating at a low scale.

As for DLs, we model them as a given number of firms that are always active at home, potentially export, and are non-negligible for the industry conditions. Moreover, they are possibly heterogeneous, thus allowing each DL to have a dissimilar impact on the industry. These firms make decisions on country-specific sunk investments, which could be either cost-reducing or demand-enhancing. Additionally, they are leaders at home and choose their domestic investments to gain a better domestic position.[4]

In Section 3, we formally identify the strategic use of domestic investments. To do so, we employ the traditional two-stage approach by Fudenberg and Tirole (1984; 1991). This requires deriving the equilibrium of two games, and then compare their outcomes. In the first game, DLs decide on domestic investments prior to the CFFs' decision of exploring the industry and the market stage à la Bertrand. As a corollary, CFFs make decisions conditioning on the DLs' domestic investments. The second game constitutes a non-strategic benchmark, where rivals do not condition their choices on the DLs' domestic investments and hence cannot be used strategically. This is captured by DLs making investment choices simultaneously with prices.

In Section 4, we describe the consequences of the strategic use of domestic investments. Our main result is that each DL utilizes them to strengthen competition at home. Such a strategy has the goal of deterring industry exploration by CFFs, and allows each DL to

[3]For instance, it has been documented several times since Dunne et al. (1988) that small businesses in the USA start their operations at a reduced scale and are characterized by high rates of entry and exit (see, in particular, Decker et al. 2014). Furthermore, conditional on surviving, they tend to remain small throughout their life-cycle and, especially in Europe, tend to be quite internationalized. For instance, Alfaro and Warzynski (2020a) document for Danish manufacturing that 48% of small firms are exporters and have an export intensity of around 25% (see, also, Mayer and Ottaviano 2008 for evidence regarding other European countries).

[4]Supposing that firms decide on investments broadens the set of possible outcomes relative to "Stackelberg-type" models. For instance, more aggressive behavior in a price-leadership model necessarily requires decreasing prices. On the contrary, DLs in our setup could end up charging higher prices to strengthen competition (e.g., when investments upgrade quality). Additionally, the sunk nature of investments circumvents the issue of whether committing to some price should be credible (Tirole 1988; Vives 2001).

increase its sales at home and garner greater profit. Moreover, it triggers a reallocation of domestic market share from CFFs towards DLs, and reduces the export intensity of each DL.

A corollary of this result is that DLs never accommodate industry exploration of CFFs. This is in stark contrast with typical two-stage models with one incumbent and one entrant as in Fudenberg and Tirole (1984). In those games, it is well-known that accommodating entry could be optimal for the leader when prices exhibit strategic complementarity. Such a strategy has the goal of softening competition and keeping profits high. Nonetheless, adopting it is doomed to failure in our framework: leaving unexploited profits would attract entry of CFFs to explore the market, and therefore sabotage its original goal. By anticipating the potential entry it would trigger, DLs prefer ultimately to capture domestic sales that, otherwise, would go to CFFs.

We also establish that the more aggressive behavior by DLs is consistent with a wide range of outcomes in other respects. First, the model allows for different possibilities regarding how CFFs are affected according to their country of origin. This follows because our results only entail that there is a lower *total* mass of CFFs that explore the market in each country. However, this does not imply that there are fewer CFFs from every country testing their possibilities.

Attending to this, we begin by establishing that the strategic use of investments by DLs affects local CFFs more heavily. This is a consequence of the natural home bias that small firms exhibit, which determines that CFFs are more exposed to changes in the competitive conditions at home. Furthermore, while there is less exploration of CFFs from every country when countries are symmetric, several potential outcomes can emerge when countries are heterogeneous.

In particular, CFFs in one country could benefit from the more aggressive domestic behavior of foreign DLs when countries are highly-asymmetric. To see this, consider two countries, which we refer to as a developed and developing economy. Suppose also that the CFFs from the developed country are highly competitive, allowing them to have substantial sales in the developing country. On the contrary, the CFFs from the developing country are quite inefficient and only serve the local market. In such a scenario, the strategic behavior by DLs from the developed country would only reduce the mass of CFFs from their own country. Furthermore, if the developed country's CFFs are substantially deterred from testing the industry, the CFFs from the developing country may benefit: they could partially fill some of the niche markets that were previously served in their own country by the developed country's CFFs.

Thus, even though there is less overall industry exploration in each country, more CFFs from the developing country could be venturing into the industry.

Additionally, we study the specific strategy deployed by DLs to deter market exploration. This is because, even when DLs invest to strengthen competition at home, such an outcome could be accomplished by either investing more or less, depending on the type of investment. We establish that results are determinate when investments are cost-reducing, since making domestic competition tougher necessarily requires greater domestic investments. On the contrary, the strategy is indeterminate when investments are demand-enhancing and make demand less price elastic. This is because an increase in a DL's investment has two effects on the competitive conditions: it strengthens competition by making the DL's variety more appealing, but it also softens competition by increasing its variety's price. Thus, if the latter effect is pronounced enough, making competition at home tougher may require reducing domestic investments to lower prices. Based on this, we identify assumptions on model primitives such that DLs strategically increase their demand-enhancing investments.

The deployment of a specific strategy is especially relevant when investments take the form of process or product innovation: it would indicate when potential competition by CFFs, in contrast to actual competition, promotes innovation. In fact, the strategic behavior of DLs under these types of investments could be welfare-improving, and we show this is always the case under the standard quality-augmented variant of the CES demand.[5]

Related Literature and Contributions. Our paper contributes to different strands of the literature. First, it touches upon the vast literature on firms strategically choosing investments to gain a better market position. This encompasses models with one leader and one follower, as covered by the taxonomy of cases of Fudenberg and Tirole (1984) and the review in Shapiro (1989). As for models with one leader and followers governed by free-entry rules, Etro (2006) constitutes the main reference, while Alfaro and Lander (2020) extend his model to multiple heterogeneous leaders and more general demand-enhancing investments (see also Polo 2018 for a survey). Finally, it relates to international-trade models with large firms, such as Atkeson and Burstein (2008), Edmond et al. (2015), and Gaubert and Itskhoki (2018), among the most recent literature.

Our contribution in this respect is to study strategic behavior of DLs against small firms. This contrasts with scenarios in which large firms act as followers, as in the studies mentioned

[5]This demand is used in Hallak (2006), Baldwin and Harrigan (2011), Feenstra and Romalis (2014), Hottman et al. (2016), and Redding and Weinstein (2020), among others.

above. Furthermore, the distinction between leaders and followers in those industries is difficult. Instead, we deal with a market structure exhibiting a simultaneous presence of large and small firms. Thus, there is a clear strategic asymmetry between firms, and small firms arise naturally as followers.

We also contribute by proposing a model that tractably reflects such a market structure in a globalized industry. With this aim, we consider an industry with coexistence of non-negligible and monopolistic-competitive firms, as in Shimomura and Thisse (2012). Moreover, we incorporate firm heterogeneity, multidimensional strategies, and international trade through several assumptions. First, we follow Melitz (2003) to model heterogeneity of monopolistic-competitive firms, which parsimoniously capture stylized features of small firms in open economies. Second, we utilize the constant-expenditure demand by Vives (2001; 2008), and embed demand-enhancing investments in a way that the model becomes an "aggregative game."[6] These games are particularly convenient for environments with heterogeneity and multidimensional strategies, since they describe strategic interactions through a real-valued function aggregating all firms' strategies.[7]

2 Model Setup

We consider a world economy with a set of countries $\mathcal{C} := \{h, f\}$, and focus on an industry in isolation comprising horizontally differentiated varieties.

Throughout the paper, any subscript ij for a variable indicates a set of countries, where i corresponds to the country of origin and j to the destination. This implies that subscript ii refers to a domestic variable of country i. Additionally, we consider countries i and j such that $i, j \in \mathcal{C}$ throughout the setup description.

2.1 Generalities

In each country i, there is a set $\overline{\Omega}_i \subseteq \mathbb{R}$ of potential conceivable varieties, where each of them can be produced only by a single firm from i. Due to this, we refer to a firm or variety indistinctly.

[6] This demand includes the CES as a special case, which has been the standard demand in International Trade since Krugman (1980). Furthermore, we show in Appendix B that this demand system can be rationalized by both a random-utility model with discrete-continuous choices and by choice probabilities in the spirit of the Multinomial Logit by Luce (1959).

[7] Aggregative games have recently been put forth in the Industrial-Organization literature by Acemoglu and Jensen (2013), Nocke and Schutz (2018), and Anderson et al. (2020), among others. For a survey, see Jensen (2018).

We partition $\overline{\Omega}_i$ into a finite subset $\overline{\mathscr{L}}_i$ comprising the set of non-negligible firms from i, and an interval $\overline{\mathcal{N}}_i$ comprising negligible firms from i. Consistent with the terminology employed so far, we refer to a firm $\omega \in \overline{\mathscr{L}}_i$ as a DL, and a firm with $\omega \in \overline{\mathcal{N}}_i$ as a CFF.[8] In Appendix C, we consider an alternative partition, where we include one additional group of firms that we refer to as large followers. These firms are non-negligible, but do not act as leaders regarding domestic investments. All the results we obtain follow verbatim under this case.

Finally, we denote by Ω_{ji} the subset of varieties from j sold in i, with $\Omega_i := \cup_{k\in\mathcal{C}}\Omega_{ki}$ being the total varieties available in i. Likewise, we define $\Omega_{ji}^{\mathcal{N}} := \overline{\mathcal{S}}_j \cap \Omega_{ji}$ and $\Omega_{ji}^{\mathscr{L}} := \overline{\mathscr{L}}_j \cap \Omega_{ji}$ as, respectively, the subsets of varieties of CFFs and DLs from j that are available in i.

2.2 Supply Side

In terms of supply, CFFs are characterized as in monopolistic competition, whereas DLs as in an oligopoly. Specifically, the description of CFFs follows Melitz (2003), which has become the standard way to incorporate firm heterogeneity in monopolistic competition.

This entails that, in each country i, there is an unbounded pool of CFFs that are ex-ante identical and do not know how profitable they are. They have the possibility of entering the industry and learn this information, which eventually turns CFFs heterogeneous. While uncertainty on profitability could be due to either demand or cost aspects, we follow Melitz (2003) and suppose it is given by productivity. All the results are identical if we instead assume that CFFs are uncertain about some demand-related parameter.

Formally, each CFF has the option of paying a sunk entry cost F_i. This allows them to get a productivity draw φ and a unique variety $\omega \in \overline{\mathcal{N}}_i$ to explore their possibilities in the industry. We suppose that productivity draws come from a continuous random variable that has a non-negative support $\left[\underline{\varphi}_i, \overline{\varphi}_i\right]$ and a cdf G_i. Also, we denote the measure of CFFs from i that pay F_i by M_i^E, with $\mathbf{M}^E := \left(M_k^E\right)_{k\in\mathcal{C}}$ indicating the vector of masses in all countries that decide to venture into the industry.

In addition to this set of firms, there is an exogenous number of DLs, with each having assigned a unique variety $\omega \in \overline{\mathscr{L}}_i$ and productivity φ_ω that is common knowledge across the world. To reflect that DLs constitute the set of most profitable firms in each country, we suppose that each of them is more productive than any CFF, i.e. $\varphi_\omega > \overline{\varphi}_i$ for each $\omega \in \overline{\mathscr{L}}_i$.

[8] Formally, the size of each firm can be reflected by endowing $\overline{\Omega}_i$ with a measure μ such that $\mu(\cdot) := \lambda\left[\cdot \cap \left(\cup_{k\in\mathcal{C}}\overline{\mathcal{N}}_k\right)\right] + \#\left[\cdot \cap \left(\cup_{k\in\mathcal{C}}\overline{\mathscr{L}}_k\right)\right]$, where λ is the Lebesgue measure and $\#$ the counting measure. In this way, expressions such as $\int_{\omega\in\overline{\Omega}_i} f(\omega)\,\mathrm{d}\mu(\omega)$ for some function f become $\int_{\omega\in\overline{\mathcal{N}}_i} f(\omega)\,\mathrm{d}\omega + \sum_{\omega\in\overline{\mathscr{L}}_i} f(\omega)$.

At the market stage, DLs from i and the mass M_i^E of CFFs decide whether to serve country j. If any firm ω in this group decides to do so, it incurs a country-specific fixed cost $f_{ij} > 0$. Additionally, it chooses country-specific prices $p_{ij}^{\omega} \in P$ and investments $z_{ij}^{\omega} \in Z$, where $P := \left[\underline{p}, \overline{p}\right] \cup \{\infty\}$ and $Z := [0, \overline{z}]$.

Regarding costs, z_{ij}^{ω} entails sunk expenditures $f_z\left(z_{ij}^{\omega}\right)$, where f_z is smooth, $f_z' > 0$, and $f_z(0) = 0$. Furthermore, the technology of production of any CFF or DL ω from i determines constant marginal costs $c\left(\varphi, \tau_{ij}, z_{ij}^{\omega}\right)$, where τ_{ij} represents a trade cost that ω incurs when it sells to j. The function c is smooth and satisfies $\frac{\partial c\left(\varphi, \tau_{ij}, z_{ij}^{\omega}\right)}{\partial \varphi} < 0$ and $\frac{\partial c\left(\varphi, \tau_{ij}, z_{ij}^{\omega}\right)}{\partial \tau_{ij}} > 0$, with the usual convention that firms do not incur any trade cost to sell in the domestic market. To streamline notation, we denote the marginal cost of DL ω from i to serve j by $c_{ij}^{\omega}\left(z_{ij}^{\omega}\right)$. Also, we do not make any assumption regarding how z_{ij}^{ω} affects marginal costs to cover various types of investments, as we explain in further detail below.

We denote the strategy in j of a firm ω from i by $\mathbf{x}_{ij}^{\omega} := \left(p_{ij}^{\omega}, z_{ij}^{\omega}\right)$. Moreover, we suppose that it encompasses the strategy "inaction", defined by $\overline{\mathbf{x}} := (\infty, 0)$, which is used by a firm to not serve a country. This serves the purpose of representing entry of firms as decreases in prices and increases in investments, which follows since ∞ represents the highest possible price and 0 the lowest level of investments.

In turn, we denote a profile of strategies in country j for firms from i by $\mathbf{x}_{ij} := \left(\mathbf{x}_{ij}^{\omega}\right)_{\omega \in \overline{\Omega}_i}$, where any CFF ω from i not paying the entry cost is supposed to set $\mathbf{x}_{ij}^{\omega} = \overline{\mathbf{x}}$ in each j.

2.3 Demand Side

Our choice of demand system is based on two considerations. First, we require that it encompasses the CES demand as a particular case, given its prominence in the International-Trade literature and the fact that it is used in Melitz (2003). Second, we aim at having a system that displays some flexibility to embed a demand shifter that enhances demand.

Both requirements lead us to utilize the constant expenditure demand by Vives (2001; 2008), which we augment to account for a demand shifter. This is embedded in a way that a country's competitive environment can be described through a real-valued function aggregating all firms' strategies. Such a property turns our model into an aggregative game, whose tools are particularly convenient for environments with firm heterogeneity and multidimensional strategies.

Formally, we denote the demand of a variety ω produced by a firm from i and sold in j

by Q_{ij}^{ω}. This is given by

$$Q_{ij}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right]\right) := \frac{E_j}{p_{ij}^{\omega}} \frac{a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right]}, \tag{dem}$$

where E_j is country j's expenditure, and $\mathbb{A}_j$ is a real-valued function given by

$$\mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right] := \sum_{k\in\mathcal{C}}\left[\int\limits_{\omega\in\Omega_{kj}^{\mathcal{N}}} a_{kj}\left(\mathbf{x}_{kj}^{\omega}\right) \mathrm{d}\omega + \sum_{\omega\in\Omega_{kj}^{\mathcal{L}}} a_{kj}\left(\mathbf{x}_{kj}^{\omega}\right)\right]. \tag{1}$$

We suppose that $a_{ij}\left(\overline{\mathbf{x}}\right) = 0$ and $\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial p_{ij}^{\omega}} < 0$, which respectively ensure that "inaction" leads to a zero demand and that the demand is decreasing in prices. The latter in particular ensures that (dem) satisfies strategic complementarity of prices, so that greater prices of rival firms provide a firm with incentives to increase its own price.[9] Additionally, we assume that $\frac{\partial^2 \ln a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial\left(\ln p_{ij}^{\omega}\right)^2} \leq 0$, which makes the price elasticity of demand be increasing in prices. Such a property is usually referred to as Marshall's Second Law of Demand, and its main goal is to simplify the analysis. All our results hold if we relax it and establish a bound when it is positive.

2.3.1 Types of Investments

Since investments are country-specific, it is natural to think that they are demand-enhancing. The logic is that the success of a variety in a country usually requires efforts tailored to the country's idiosyncrasies (Sutton 1991; 1998). Nonetheless, we can also conceive cost-reducing investments that are country-specific, such as outlays that improve a firm's distribution network in a country.

Fortunately, the derivation of our results do not require taking a stand on this regard. Thus, we suppose that either $\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial z_{ij}^{\omega}} > 0$, or $\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial z_{ij}^{\omega}} = 0$ and $\frac{\partial c_{ij}^{\omega}\left(z_{ij}^{\omega}\right)}{\partial z_{ij}^{\omega}} < 0$. The latter case defines cost-reducing investments, since they do not affect ω's demand but reduce its marginal cost. On the other hand, the case with $\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial z_{ij}^{\omega}} > 0$ captures that investments increase demand.

Assuming $\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial z_{ij}^{\omega}} > 0$ is all we need to characterize and obtain results for demand-enhancing investments. Thus, even when it is natural to think that demand-enhancing investments make demand less price elastic or increase marginal costs, none of the results require such an assumption. In this sense, our findings are robust to less-common characterizations

[9]This follows by using the results in Alfaro and Lander (2020). They show that strategic complementarity holds under demands that depend on an additive separable aggregate, such as (dem), when $\frac{\partial \varepsilon_{ij}^{p,\omega}}{\partial \mathbb{A}_j} > 0$, where $\varepsilon_{ij}^{p,\omega}$ is the price elasticity of demand in j of a firm ω from i.

of investments. For example, they hold for investments that target price-sensitive consumers, which might make demand more elastic by changing the composition of customers. To distinguish between such cases, we denominate "investments in quality" to demand-enhancing investments that increase the consumer's willingness to pay and possibly entail a greater marginal cost.

2.3.2 Interpretations Regarding Demand

Some additional remarks on (dem) are in order. First, the term $\mathbb{A}_j$ is referred to an "aggregate" in the aggregative-games literature, and is a function of all firms' strategies in j.[10] It can be interpreted as a measure of how tough competition in country j is, since greater values of $\mathbb{A}_j$ decrease ω's demand and hence its profit.

In turn, increases in $\mathbb{A}_j$ occur through decreases in prices and increases in investments, due to the assumptions $\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial p_{ij}^{\omega}} < 0$ and $\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial z_{ij}^{\omega}} \geq 0$. Notice this also implies that entry of firms in j results in a greater $\mathbb{A}_j$. This is because any firm ω becomes active in j by setting a price $p_{ij}^{\omega} < \infty$ and choosing investments $z_{ij}^{\omega} > 0$. which represents a decrease in prices and increase in investments relative to the choice inaction $\overline{\mathbf{x}} = (\infty, 0)$.

Also, we show in Appendix B that (dem) can be microfounded in two different ways once we account for some of its particular properties. First, it can be rationalized through the approach of discrete-continuous choices by Dubin and McFadden (1984) and Hanemann (1984). This constitutes an alternative to random-utility models with discrete choices, and has been recently put forward by Nocke and Schutz (2018). Second, it can be derived axiomatically by employing the results in Bell et al. (1975). This makes (dem) be part of the so-called "attraction demand models", which are common in the Marketing and Management literature.[11]

2.3.3 Particular Cases

A demand as in (dem) includes some special cases commonly used in the literature. Next, we focus on examples with a demand shifter that is affected by a firm's investment. This should be taken instead as a parameter when investments are cost-reducing.

[10] Notice that, even though CFFs are uncertain about their productivity, the fact that there is a continuum of them vanishes any uncertainty in $\mathbb{A}_j$ and makes it deterministic. Technically, the continuum of CFFs makes it possible to apply a suitable law of large numbers, so that $\mathbb{A}_j$ becomes a scalar function. For further details, see Acemoglu and Jensen (2015).

[11] For a review of the attraction demand model and its applications, see Bernstein and Federgruen (2004), Gallego et al. (2006), and, in particular, Huang et al. (2013).

First, (dem) encompasses the quality-augmented CES commonly utilized in the International-Trade literature (e.g., Hallak 2006, Baldwin and Harrigan 2011, Feenstra and Romalis 2014, Hottman et al. 2016, and Redding and Weinstein 2020). This demand has the property of expressing a variety's value as its price per unit of quality. Specifically, it arises by defining,

$$a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right) := \left(p_{ij}^{\omega}/z_{ij}^{\omega}\right)^{1-\sigma}, \tag{ces}$$

where $\sigma > 1$.

Another case covered is given by the exponential demand used in Vives (2008), Gallego et al. (2006), and Quint (2014). By augmenting it to incorporate a non-price feature, this is given by

$$a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right) := \exp\left(\frac{z_{ij}^{\omega} - p_{ij}^{\omega}}{\beta}\right), \tag{exp}$$

where $\beta > 0$. Such a demand resembles the Multinomial Logit, with the difference that the total expenditure on each variety is constant.

3 Equilibrium in Each Scenario

Following Fudenberg and Tirole (1991), the strategic motive to invest and its corresponding gains can be isolated by comparing outcomes between a simultaneous-moves and a sequential-moves game.[12] The latter corresponds to a situation where all firms condition their decisions on the domestic investments made by DLs. On the other hand, the simultaneous-moves scenario acts as a non-strategic benchmark, where rival firms do not observe the domestic investments by DLs. Thus, by definition, these firms cannot condition on the DLs' choices.

Next, we describe and solve the model for each scenario, where we respectively denote any equilibrium variable in the simultaneous-moves and a sequential-moves game through superscripts "sim" and "seq". We assume that the equilibrium in each scenario exists, is unique, and interior.[13] Moreover, each DL's profits function evaluated at the optimal domestic prices is supposed to be strictly quasi-concave in domestic investments.

Our focus is on equilibria with active DLs and CFFs in each country. Following Melitz

[12]The approach is standard in Game Theory to measure the strategic value of a strategy. These games are generically known as the closed-loop and open-loop equilibrium in the dynamic-games literature. See Fudenberg and Tirole (1991) for more details.

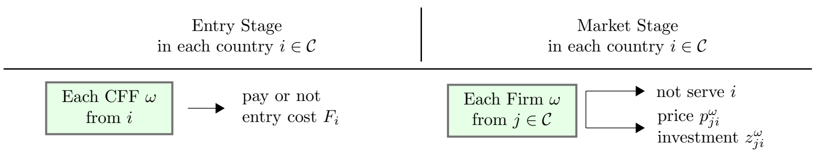
[13]The model is smooth enough to apply standard assumptions that ensure existence and uniqueness of equilibrium. This is because entry/exit occurs among a continuum of firms and the demand considered defines an aggregative game.

(2003), we also consider selection in markets among CFFs: some do not serve any country, others only sell domestically, and the most productive firms serve both the home and foreign country. We also suppose that any CFF serving both countries obtains greater revenues and profits at home than in the foreign country. This provides a representation of CFFs as small firms that mainly serve their local market, which is consistent with the typical home bias observed empirically.

3.1 Simultaneous-Moves Game

The timing of the simultaneous-moves case is presented in Figure 1. First, each CFF of country $i \in \mathcal{C}$ decides whether to pay the sunk entry cost F_i to receive a unique variety ω and a draw of productivity φ. After this, the market stage in each country takes place. This entails that each DL ω from $j \in \mathcal{C}$ and CFF ω from j that paid the entry cost decides whether to not serve $i \in \mathcal{C}$ by choosing inaction $\overline{\mathbf{x}}$, or pay f_{ji} and serve i with prices and investments satisfying $\mathbf{x}^\omega_{ji} \neq \overline{\mathbf{x}}$.

Figure 1. *Timing of the Simultaneous-Moves Case*



3.1.1 Market Stage

Let countries i and j be such that $i, j \in \mathcal{C}$. Since there is a subset of CFFs that serve both countries and DLs are more productive than any CFF, then DLs are always active at home and abroad.[14] Therefore, DL ω from i pays the fixed cost f_{ij} and chooses $\mathbf{x}^\omega_{ij} \neq \overline{\mathbf{x}}$ in each j.

Given other firms' strategies, DL ω's optimal price and investment in j are obtained by maximizing its gross profit in j. This is given by

$$\pi^\omega_{ij}\left(\mathbf{x}^\omega_{ij}, \mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right]\right) := Q_{ij}\left(\mathbf{x}^\omega_{ij}, \mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right]\right)\left[p^\omega_{ij} - c^\omega_{ij}\left(z^\omega_{ij}\right)\right] - f_z\left(z^\omega_{ij}\right),$$

[14] The fact that all DLs export reflects the empirical observation that larger firms are more likely to export. Our setup can easily incorporate that not all of them serve the foreign country through the existence of firm-specific trade costs.

and the first-order conditions are

$$\frac{\partial \pi_{ij}^{\omega}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial p_{ij}^{\omega}} + \frac{\partial \pi_{ij}^{\omega}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial \mathbb{A}_j}\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial p_{ij}^{\omega}} = 0, \tag{2}$$

$$\frac{\partial \pi_{ij}^{\omega}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial z_{ij}^{\omega}} + \frac{\partial \pi_{ij}^{\omega}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial \mathbb{A}_j}\frac{\partial a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\partial z_{ij}^{\omega}} = 0. \tag{z-sim}$$

From equations (2) and (z-sim), we can obtain a characterization of DL ω's best-response functions. Alternatively, since our model defines an aggregative game, it is possible to express optimal strategies through the so-called *backward-response functions* (Acemoglu and Jensen 2013). They express each firm's optimal strategy as a function of the aggregate, thus including not only other firms' strategies but also its own strategy. Expressing optimal solutions in this way greatly simplifies the analysis. Formally, the backward-response function of DL ω from i in j is the solution to (2) and (z-sim), and is given by

$$\mathbf{x}_{ij}^{\omega}\left(\mathbb{A}_j\right) := \left(p_{ij}^{\omega}\left(\mathbb{A}_j\right), z_{ij}^{\omega}\left(\mathbb{A}_j\right)\right). \tag{3}$$

As for CFFs from i, consider some firm ω that paid the entry cost to explore the industry. This firm has to decide whether to serve country j, along with its price and investment if it does so. Formally, it solves:

$$\max_{\left(\mathbf{x}_{ij}^{\omega}\right)_{j\in\mathcal{C}}} \sum_{j\in\mathcal{C}} \mathbb{1}_{\left(\mathbf{x}_{ij}^{\omega}\neq\overline{\mathbf{x}}\right)}\left[\pi_{ij}^{\omega}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right) - f_{ij}\right]. \tag{4}$$

Conditional on serving country j, it chooses a price and investment that satisfy the following first-order conditions:

$$\frac{\partial \pi_{ij}^{\omega}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial p_{ij}^{\omega}} = 0,$$

$$\frac{\partial \pi_{ij}^{\omega}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial z_{ij}^{\omega}} = 0,$$

which reflects that CFFs cannot influence the aggregate conditions of country j.

To further characterize a CFF's optimal strategy, notice that CFFs only differ by the productivity draws obtained after paying the entry cost. Therefore, if two CFFs obtain the same productivity draw, they behave in the same fashion. This enables us to express the optimal decisions by active CFFs as a function of φ, rather than ω. Formally, they can be expressed as $p_{ij}^{\mathcal{N}}\left(\mathbb{A}_j, \varphi\right)$ and $z_{ij}^{\mathcal{N}}\left(\mathbb{A}_j, \varphi\right)$.

The existence of country-specific fixed cost determines that not all of the CFFs serve country j. To characterize their decision rule, denote the optimal profit of a CFF if it does so by $\pi_{ij}^{\mathcal{N}}$. Given that optimal profits are strictly increasing in productivity, there exists a survival productivity cutoff $\varphi_{ij}\left(\mathbb{A}_j\right)$ that makes a CFF from i be indifferent between serving

j or not. Formally, this is defined by

$$\pi_{ij}^{\mathcal{N}}\left[\mathbb{A}_j, \varphi_{ij}\left(\mathbb{A}_j\right)\right] = f_{ij},$$

and determines that the optimal choice in j of a CFF from i with productivity φ is

$$\mathbf{x}_{ij}^{\mathcal{N}}\left(\mathbb{A}_j, \varphi\right) := \begin{cases} \left(p_{ij}^{\mathcal{N}}\left(\mathbb{A}_j, \varphi\right), z_{ij}^{\mathcal{N}}\left(\mathbb{A}_j, \varphi\right)\right) & \text{if } \varphi \geq \varphi_{ij}\left(\mathbb{A}_j\right) \\ \overline{\mathbf{x}} & \text{otherwise} \end{cases}. \tag{5}$$

3.1.2 Equilibrium Conditions

Given optimal strategies, it remains to specify the conditions for a Nash equilibrium at the market stage, as well as the free-entry condition of CFFs. For the former, we exploit the aggregative-game structure of the model. To do this, let $R_{ji}^{\mathcal{N}}$ and R_{ji}^{ω} respectively be the total sales in i by CFFs and DL ω from j.

The term $R_{ji}^{\mathcal{N}}$ evaluated at the optimal choices can be decomposed into the product of the total mass of varieties and the average revenue of a CFF. Formally, $R_{ji}^{\mathcal{N}} = M_j^E r_{ji}^{\mathcal{N}}$, where $r_{ji}^{\mathcal{N}}$ is given by

$$r_{ji}^{\mathcal{N}}\left(\mathbb{A}_i\right) := \int_{\varphi_{ji}\left(\mathbb{A}_i\right)}^{\overline{\varphi}_j} Q_{ji}\left[\mathbf{x}_{ji}^{\mathcal{N}}\left(\mathbb{A}_i, \varphi\right), \mathbb{A}_i\right] p_{ji}^{\mathcal{N}}\left(\mathbb{A}_i, \varphi\right) \mathrm{d}G_j\left(\varphi\right).$$

In addition, the optimal revenue of DL ω from j in i is a function $R_{ji}^{\omega}\left(\mathbb{A}_i\right)$. Thus, the optimal revenues in i of firms from j, denoted by R_{ji}, are given by

$$R_{ji}\left(\mathbb{A}_i, M_j^E\right) := M_j^E r_{ji}^{\mathcal{N}}\left(\mathbb{A}_i\right) + \sum_{\omega \in \Omega_{ji}^{\mathcal{L}}} R_{ji}^{\omega}\left(\mathbb{A}_i\right). \tag{6}$$

To identify the Nash equilibrium at the market stage, we define a function that indicates the total expenditure in i for a given $\left(\mathbb{A}_i, \mathbf{M}^E\right)$:

$$E_i^{\text{sim}}\left(\mathbb{A}_i, \mathbf{M}^E\right) := \sum_{k \in \mathcal{C}} R_{ki}\left(\mathbb{A}_i, M_k^E\right). \tag{7}$$

Thus, for a given $\mathbf{M}^E$, a Nash equilibrium at the market stage in i arises when $\mathbb{A}_i$ is a fixed point of (7):

$$E_i^{\text{sim}}\left(\mathbb{A}_i, \mathbf{M}^E\right) = E_i. \tag{MS-sim}$$

As for the free-entry condition, each CFF pays the entry cost F_i and decides to explore the industry as long as it anticipates non-negative expected profits. Given that there is a continuum of CFFs, this implies that a zero-expected-profits condition arises in each i:

$$\pi_i^{\text{expect}}\left[\left(\mathbb{A}_k\right)_{k \in \mathcal{C}}\right] := \sum_{j \in \mathcal{C}} \int_{\varphi_{ij}\left(\mathbb{A}_j\right)}^{\overline{\varphi}_i} \left\{\pi_{ij}^{\mathcal{N}}\left(\mathbb{A}_j, \varphi\right) - f_{ij}\right\} \mathrm{d}G_i\left(\varphi\right) = F_i, \tag{FE}$$

where $\pi_i^{\text{expect}}\left[(\mathbb{A}_k)_{k\in\mathcal{C}}\right]$ is the expected profits of a CFF from i and "FE" is mnemonic for "free entry".

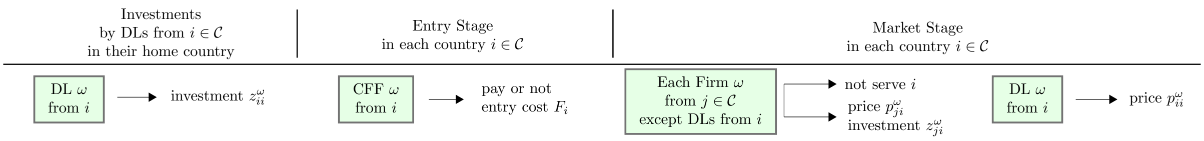
Overall, the simultaneous-moves equilibrium can be characterized through values $\mathbf{M}^{E,\text{sim}} := \left(M_k^{E*}\right)_{k\in\mathcal{C}}$ and $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$ that satisfy conditions (MS-sim) and (FE) for each $i \in \mathcal{C}$. With those values, any equilibrium variable can be pinned down, including the optimal domestic investment of a DL ω from i, $z_{ii}^{\omega,\text{sim}}$.

3.2 Sequential-Moves Game

The timing of the sequential-moves game is presented in Figure 2. It is the same as in the simultaneous-moves case, except that DLs make their domestic-investment decisions at the beginning of the game.

At the first stage, DLs from $i \in \mathcal{C}$ decide on their investments at home. All firms observe these choices, and CFFs from i decide whether to pay F_i to get a unique variety ω and a draw of productivity φ assigned. Then, the market stage in each country takes place. At this stage, each DL ω from $j \in \mathcal{C}$ decides its price in $i \in \mathcal{C}$ and investments in $i \neq j$. Moreover, each CFF ω from j that paid the entry cost decides if it does not serve i by choosing $\overline{\mathbf{x}}$, or pays f_{ji} and serves it with price and investment such that $\mathbf{x}_{ji}^{\omega} \neq \overline{\mathbf{x}}$.

Figure 2. *Timing of the Sequential Case*



3.2.1 Subgame for Given Domestic Investments

We begin by describing the subgame where domestic investments by DLs have been chosen. Let $\mathbf{z}_{ii}^{\mathscr{L}} := (z_{ii}^{\omega})_{\omega\in\Omega_{ii}^{\mathscr{L}}}$ be the domestic investments by DLs from i, $\mathbf{z}_{d}^{\mathscr{L}} := \left(\mathbf{z}_{ii}^{\mathscr{L}}\right)_{i\in\mathcal{C}}$ the vector of domestic investments by all DLs around the world, and $\mathbf{z}_{-\omega}^{\mathscr{L}}$ the domestic investments of all DLs except ω's. For the characterization of the market stage, we consider a country $i \in \mathcal{C}$.

As for CFFs from i, their optimization problem is still given by (4). This implies that their profits in $j \in \mathcal{C}$ conditional on $\mathbb{A}_j$ do not depend on $\mathbf{z}_{ii}^{\mathscr{L}}$, and so their decisions can be characterized exactly as in the simultaneous-moves scenario. This includes both the decision of whether to serve j, and the optimal price and investment when they do so. Thus, the optimal strategy in j for a CFF from i with productivity φ is $\mathbf{x}_{ij}^{\mathcal{N}}(\mathbb{A}_j, \varphi)$ and given by (5).

Additionally, the free-entry condition in each country i is the same as in the simultaneous-moves scenario and given by (FE).

Furthermore, regarding DL ω from i, its optimal price and investment in the foreign country are the same as in the simultaneous-moves equilibrium. Therefore they are given by $\mathbf{x}_{ij}^{\omega}(\mathbb{A}_j)$ with $j \neq i$ and satisfy (3). As for its domestic price, the first-order condition that characterizes it is the same as in the simultaneous-moves scenario, i.e. (2). Thus, the optimal domestic price of ω is independent of rivals' domestic investments conditional on $\mathbb{A}_i$ and z_{ii}^{ω}, which determines a function $p_{ii}^{\omega}(z_{ii}^{\omega}, \mathbb{A}_i)$.

The Nash equilibrium at the market stage can be obtained by exploiting the aggregative structure of the game, as in the simultaneous-moves case. Specifically, given $j \neq i$, we define an equation that provides the total expenditure in country i for a given $\left(\mathbb{A}_i, \mathbf{M}^E, \mathbf{z}_{ii}^{\mathscr{L}}\right)$:

$$E_i^{\text{seq}}\left(\mathbb{A}_i, \mathbf{M}^E, \mathbf{z}_{ii}^{\mathscr{L}}\right) := \sum_{k \in \mathcal{C}} M_k^E r_{ki}^{\mathcal{N}}(\mathbb{A}_i) + \sum_{\omega \in \Omega_{ji}^{\mathscr{L}}} R_{ji}^{\omega}(\mathbb{A}_i) + \sum_{\omega \in \Omega_{ii}^{\mathscr{L}}} R_{ii}^{\omega}\left[p_{ii}^{\omega}(z_{ii}^{\omega}, \mathbb{A}_i), z_{ii}^{\omega}, \mathbb{A}_i\right]. \tag{8}$$

Using this, the Nash equilibrium at the market stage in i requires that $\mathbb{A}_i$ self-generates the expenditure parameter E_i for a given $\left(\mathbf{M}^E, \mathbf{z}_{ii}^{\mathscr{L}}\right)$:

$$E_i^{\text{seq}}\left(\mathbb{A}_i, \mathbf{M}^E, \mathbf{z}_{ii}^{\mathscr{L}}\right) = E_i. \tag{MS-seq}$$

From all this, we conclude that the equilibrium of the subgame for a given $\mathbf{z}_d^{\mathscr{L}}$ can be characterized through values $\mathbb{A}_i^{\text{seq}}\left(\mathbf{z}_d^{\mathscr{L}}\right)$ and $M_i^{E,\text{seq}}\left(\mathbf{z}_d^{\mathscr{L}}\right)$ for each $i \in \mathcal{C}$, such that (FE) and (MS-seq) for each $i \in \mathcal{C}$ hold.

3.2.2 Optimal Domestic Investments

To identify the optimal domestic investment of DL ω from i, notice that ω's profit at home is independent of $\mathbf{M}^E$ and $\mathbf{z}_{-\omega}^{\mathscr{L}}$ conditional on $\mathbb{A}_i$. This is because $\mathbb{A}_i$ is a single sufficient statistic for country i's industry conditions.

In addition, inspection of (FE) and (MS-seq) reveals that the system (FE) pins down $(\mathbb{A}_k^{\text{seq}})_{k \in \mathcal{C}}$ independently of $\mathbf{M}^E$, and that $\mathbf{z}^{\mathscr{L}}$ does not affect the system (FE). Therefore, since the same system of conditions (FE) holds under both the simultaneous- and sequential-moves cases, these scenarios share the same equilibrium aggregates. We denote them by $(\mathbb{A}_k^*)_{k \in \mathcal{C}}$.

This fact also implies that variations in $\mathbf{z}_d^{\mathscr{L}}$ do not affect (FE), and hence neither the equilibrium aggregates. However, they do affect (MS-seq). Thus, any variation in optimal domestic investments translates into changes in $\mathbf{M}^E$ that make (MS-seq) hold. To illustrate the mechanism behind these effects, consider that DLs vary their investments to strengthen competition at home. Then, the expected profits of CFFs around the world would be reduced, thereby deterring these firms from exploring the industry. The crowding out of CFFs would lessen competition in each country and, ultimately, completely offset the initial tougher competitive environment created by DLs. Consequently, the aggregates would not vary.

To incorporate this mechanism into the determination of optimal investments, let $\mathbf{M}^E\left(\mathbf{z}_d^{\mathscr{L}};(\mathbb{A}_k^*)_{k\in\mathcal{C}}\right)$ be the solution to (MS-seq). This captures that, once $(\mathbb{A}_k^*)_{k\in\mathcal{C}}$ is identified through (FE), domestic investments do not affect the equilibrium aggregates due to the offsetting changes in $\mathbf{M}^E$. Thus, the domestic demand of DL ω from i becomes $Q_{ii}^\omega\left[p_{ii}^\omega\left(z_{ii}^\omega;\mathbb{A}_i^*\right),z_{ii}^\omega;\mathbb{A}_i^*\right]$, and its domestic gross profit is

$$\pi_{ii}^\omega\left[p_{ii}^\omega\left(z_{ii}^\omega;\mathbb{A}_i^*\right),z_{ii}^\omega;\mathbb{A}_i^*\right]:=Q_{ii}\left[p_{ii}^\omega\left(z_{ii}^\omega;\mathbb{A}_i^*\right),z_{ii}^\omega;\mathbb{A}_i^*\right]\left[p_{ii}^\omega\left(z_{ii}^\omega;\mathbb{A}_i^*\right)-c_{ii}^\omega\left(z_{ii}^\omega\right)\right]-f_z\left(z_{ii}^\omega\right).$$

Moreover, its optimal domestic investment satisfies the following first-order condition:

$$\frac{\partial\pi_{ii}^\omega\left[p_{ii}^\omega\left(z_{ii}^\omega;\mathbb{A}_i^*\right),z_{ii}^\omega;\mathbb{A}_i^*\right]}{\partial z_{ii}^\omega}+\frac{\partial\pi_{ii}^\omega\left[p_{ii}^\omega\left(z_{ii}^\omega;\mathbb{A}_i^*\right),z_{ii}^\omega;\mathbb{A}_i^*\right]}{\partial p_{ii}^\omega}\frac{\partial p_{ii}^\omega\left(z_{ii}^\omega;\mathbb{A}_i^*\right)}{\partial z_{ii}^\omega}=0, \tag{z-seq}$$

which characterizes $z_{ii}^{\omega,\text{seq}}$, and makes it possible to identify the solution of the sequential-moves game.

4 Results

Next, we analyze the impact of strategic investments by DLs on several outcomes. This is accomplished by comparing the results in the sequential-moves scenario with those in the simultaneous-moves game. All the proofs of this paper are relegated to Appendix A.

For the statement of the propositions, we define the export intensity of a DL as the proportion of its export revenue relative to its total revenue (i.e., domestic sales and exports). Recall that the domestic sales of DL ω from country i are denoted by R_{ii}^ω, while its export sales by R_{ij}^ω where $i\neq j$. Thus, the export intensity of DL ω from country i is $e_i^\omega:=\frac{R_{ij}^\omega}{R_{ii}^\omega+R_{ij}^\omega}$, with domestic intensity given by $1-e_i^\omega$.

Furthermore, we refer to the (expenditure-based) market share in j of DL ω from i by s_{ij}^ω. Given (dem), this is given by the function

$$s_{ij}\left(\mathbf{x}_{ij}^\omega,\mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right]\right)=\frac{a_{ij}\left(\mathbf{x}_{ij}^\omega\right)}{\mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right]}. \tag{9}$$

4.1 Main Result

We begin by establishing the implications of strategic behavior by DLs that hold independently of any additional assumption.

Proposition 4.1. *Relative to the simultaneous equilibrium, each DL in the sequential equilibrium:*

- *behaves more aggressively at home,*
- *expands its domestic sales at the expense of CFFs, and increases its domestic market share,*
- *lowers its export intensity, and*
- *increases its total profit.*

The proposition indicates that DL ω strategically chooses its domestic investment to strengthen competition at home. Formally, this means that ω varies its domestic investment with the purpose of increasing its term a_{ii} in $\mathbb{A}_i$. The result holds irrespective of the nature of investments, i.e. whether they are cost-reducing or demand-enhancing. Furthermore, it could require deploying an under or over-investment strategy, as we show in the next section. Nonetheless, in all cases, the goal is the same: strengthening domestic competition to reduce the expected profits of CFFs and partially deter them from exploring the industry. This allows each DL to obtain domestic revenues that, otherwise, would be captured by some CFFs. Moreover, it increases each DL's domestic market share.

The fact that DLs always behave more aggressively starkly differs from typical strategic-investments models with one leader and one entrant à la Fudenberg and Tirole (1984). In such games, it is well known that the leader's optimal strategy could entail accommodating entry. In contrast, Proposition 4.1 indicates that this never occurs in our framework.

The key difference relative to those setups is that our model supposes the existence of an unbounded pool of CFFs, which are ready to explore the industry if they anticipate positive expected profits. To understand the role of this assumption, notice that an entry-accommodation strategy has the goal of softening competition to preserve the profitability of the market. Nonetheless, this strategy is doomed to failure under free entry of CFFs: it would end up attracting CFFs to explore the industry, and therefore sabotage the original attempt of softening competition at home. Ultimately, by anticipating the potential entry it would trigger, DLs prefer to capture the domestic sales that, otherwise, would be appropriated by CFFs.

Proposition 4.1 also establishes that, even where there are multiple DLs acting more aggressively in all countries, each DL sells more at home and garners greater total profits. Additionally, it can be shown that their export revenues are the same. This has two implications. First, each DL's export intensity decreases. Second, notice that exports of DLs from one country represent imports of foreign DLs for the other country. Thus, imports of DLs do not vary between scenarios, and so foreign DLs are not affected by the strategic behavior of DLs at home. Instead, the more aggressive behavior by DLs at home has exit of CFFs as a counterpart. Therefore, the deployment of strategic behavior only impacts CFFs, whereas DLs do not end up inflicting mutual damage in any country.

4.2 Market-Exploration Deterrence in Each Country

Although we have established that DLs behave more aggressively at home, Proposition 4.1 is silent regarding which CFFs are deterred from exploring the market. Specifically, we have not indicated whether this induces less industry exploration of local or foreign CFFs (or both). In the following, we show that different results can arise in this respect, and analyze its determinants.

With the aim of isolating the different mechanisms operating, we begin by studying a simple case where the set of DLs in country f is empty. Thus, differences between the simultaneous-moves and sequential-moves equilibrium arise exclusively due to the strategic investments by DLs from h. The results for this case indicate that the more aggressive domestic behavior of DLs from h induces lower industry exploration of CFFs from h, but a higher exploration of CFFs from f. The next proposition formalizes this.

Proposition 4.2. *Suppose the set of DLs in f is empty. Then, $M_h^{E,seq} < M_h^{E,sim}$ and $M_f^{E,seq} > M_f^{E,sim}$. Thus, relative to the simultaneous-moves scenario, in the sequential-moves equilibrium there is a reduction in the mass of CFFs from h and an increase in the mass of CFFs from f.*

Notice that the greater industry exploration by f's CFFs does not contradict our previous findings. Proposition 4.1 only establishes that DLs from h partially deter industry exploration by strengthening domestic competition, which lowers the total sales by CFFs in h and reallocates them towards DLs from h. Thus, consistency with Proposition 4.2 arises because the reduction in sales by CFFs from h always surpasses the increase in sales by CFFs from f.

To rationalize Proposition 4.2, it is crucial that CFFs are more impacted by changes in the market conditions at home than abroad. This occurs because CFFs obtain greater revenues at home, which is in line with the empirical evidence and reflects the hurdles associated with exporting (e.g., substantial fixed costs to enter foreign markets as in Roberts and Tybout 1997 and Das et al. 2007).[15]

Taking this into account, the expected profits of CFFs from h are substantially affected by the changes in investments by DLs from h. This deters h's CFFs from exploring the industry, thereby explaining why their mass is lower. As for the CFFs from f, their expected profits are negatively impacted by the more aggressive behavior by DLs from h, but also positively affected by the lower mass of h's CFFs serving f. Thus, even when there are tougher domestic conditions in h, CFFs from f also face better conditions to operate in their domestic country. Since the domestic competitive environment is more determinant, f's CFFs end up with greater expected profits and are induced to explore the industry, which explains why the mass of f's CFFs increases.

What does occur in the general case, where the set of DLs from f is nonempty? Then, CFFs from f would also be negatively impacted by greater domestic competition due to the more aggressive behavior of DLs from f. This implies that the result in Proposition 4.2 could be reverted, with possible lower industry exploration of CFFs from f in the sequential-moves scenario. As a corollary, the variation in the mass of CFFs from f is indeterminate.

The indeterminacy arises due to the richness of the model: depending on the features of DLs and CFFs in each country, it is possible to accommodate several scenarios. Due to this, next we derive results for two particular cases that are worthy of investigation.

The first one deals with a symmetric-countries world. This assumption is standard in the International-Trade literature, and associated with the so-called North-North or South-South type of trade (i.e., trade between two developed or two developing economies). Symmetry between countries means formally that $\tau_{hf} = \tau_{fh}$, $f_{hf} = f_{fh}$, $f_{hh} = f_{ff}$, $F_h = F_f$, $G_h = G_f$. Moreover, it entails that, for each $\omega \in \overline{\mathscr{L}}_h$, there is a firm $\omega \in \overline{\mathscr{L}}_f$ that is identical. Incorporating this, the following proposition establishes that the strategic behavior of DLs results in a lower industry exploration of CFFs from both countries.

Proposition 4.3. *Suppose that countries h and f are symmetric. Then, $M_i^{E,seq} < M_i^{E,sim}$ for each $i = h, f$. Consequently, there are less CFFs from each country exploring the market.*

Essentially, the result follows by the direct effect of the DLs' investments on the CFFs'

[15]There is an extensive literature documenting a home bias at the firm level. See, for instance, Mayer and Ottaviano (2008) for several European countries and Bernard et al. (2012) for the USA. For a study that provides evidence of a home bias in particular for small exporters, see Alfaro and Warzynski (2020a).

expected profits. Instead, indirect effects of the type highlighted in Proposition 4.2 play an ancillary role; they can dampen the direct effect, but never revert it.

The second case we investigate considers pronounced asymmetries between CFFs from each country. Specifically, we focus on a scenario where the average export sales by CFFs from h are negligible, whereas those by CFFs from f are substantial. We state the result and then proceed to explain its relevance.

Proposition 4.4. *Suppose that the expected revenues in f by CFFs from h are negligible in either the simultaneous- or sequential-moves game. Then, there always exists a sufficiently high value of average sales in h by f's CFFs in either equilibrium such that $M_h^{E,seq} > M_h^{E,sim}$ and $M_f^{E,seq} < M_f^{E,sim}$. Thus, relative to the simultaneous-moves scenario, in the sequential-moves equilibrium there is an increase in the mass of h's CFFs and a decrease in the mass of f's CFFs that explore the industry.*

To illustrate the type of situation that Proposition 4.4 can capture, consider that h is a developed economy like the USA, and f a distant developing country like Argentina. In such a scenario, CFFs from Argentina may find it hard to penetrate a highly competitive market like the USA, even more so taking into account their distance and cultural barriers. This fact is reflected by some negligible export sales by Argentine CFFs. On the other hand, CFFs from the USA could easily penetrate a developing country. The reason could lie in superior capabilities to develop goods, combined with the existence of softer competition in Argentina. This entails that the average exports by American CFFs to Argentina could be quite high.

From Proposition 4.1, we know that DLs from both the USA and Argentina strategically invest to strengthen domestic competition. Nonetheless, given the scenario depicted, the average Argentine CFF would be minimally affected by the harder competitive conditions in the USA, since they were already facing great hurdles to serve that country. Instead, they would be substantially affected by the more aggressive behavior by Argentine DLs and the reduction of imported American varieties in Argentina. The latter creates opportunities for Argentine CFFs to fill those niche markets previously served by American CFFs. In particular, Proposition 4.4 establishes that there always exists some high level of American exports that makes this effect dominate. When this occurs, the expected profits of the Argentine CFFs would increase, and induce these firms to test the industry.

4.3 Strategy Deployed by Domestic Leaders

Proposition 4.1 indicates that investments are used strategically to increase competition at home, and hence deter industry exploration. Moreover, this holds irrespective of whether investments are cost-reducing or demand-enhancing. Nonetheless, the result is silent about whether this entails an under- or over-investment strategy relative to the simultaneous-moves game. Identifying if one or the other occurs is especially relevant when investing overhauls a variety or reduces marginal cost: it would dictate whether deterring industry exploration promotes innovation in the industry.

As we show formally below, a DL always over-invests at home when investments are cost-reducing. The reason is that this type of investment only impacts the competitive environment through its effect on a DL's price. Consequently, strengthening competition requires committing to aggressive pricing, which in turn requires investing more.

On the contrary, the results for demand-enhancing investments are indeterminate. To explain why this is so, consider investments in quality. Recall that they are defined as demand-enhancing investments that increase the consumer's willingness to pay and possibly entail a greater marginal cost. Then, an increase in a DL's domestic investment triggers two effects on the domestic competitive conditions: it strengthens competition by making the DL's variety more appealing, but it also softens competition via its indirect effect on prices. Overall, if higher investments entail pronounced increases in a DL's price, it is possible that increasing competition requires investing less.

To formally identify the strategy deployed by DL ω, let $\varepsilon_{ij}^{p,\omega}$ be the price elasticity of demand in j of DL ω from i, given by a function $\varepsilon_{ii}^{p}\left(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i\right)$. We begin by establishing the indirect effect that investing at home has on domestic prices, which has a sign given by

$$\operatorname{sgn}\left(\frac{\partial p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right)}{\partial z_{ii}^{\omega}}\right)=\operatorname{sgn}\left(\frac{\partial^2 \ln a_{ii}\left[\cdot\right]}{\partial \ln p_{ii}^{\omega}\partial \ln z_{ii}^{\omega}}-\frac{\partial \ln a_{ii}\left[\cdot\right]}{\partial \ln p_{ii}^{\omega}}\left(\frac{s_{ii}^{\omega}}{1-s_{ii}^{\omega}}\frac{\partial \ln a_{ii}\left[\cdot\right]}{\partial \ln z_{ii}^{\omega}}+\frac{\varepsilon_{ii}^{p,\omega}}{1-s_{ii}^{\omega}}\frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}}\right)\right), \tag{10}$$

where $[\cdot] := \left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right), z_{ii}^{\omega}\right]$, and s_{ii}^{ω} and $\varepsilon_{ii}^{p,\omega}$ are functions of $\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right), z_{ii}^{\omega};\mathbb{A}_i^*\right]$.

Furthermore, DL ω over-invests if investing does not increase a variety's price substantially. In particular, this can be reflected by simply characterizing the effect of investments on prices in the simultaneous-moves case:

$$\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega,\mathrm{sim}};\mathbb{A}_i^*\right)}{\partial \ln z_{ii}^{\omega}}<-\left(\frac{\partial \ln a_{ii}\left(p_{ii}^{\omega,\mathrm{sim}}, z_{ii}^{\omega,\mathrm{sim}}\right)}{\partial \ln p_{ii}^{\omega}}\right)^{-1}\frac{\partial \ln a_{ii}\left(p_{ii}^{\omega,\mathrm{sim}}, z_{ii}^{\omega,\mathrm{sim}}\right)}{\partial \ln z_{ii}^{\omega}}, \tag{11}$$

where $p_{ii}^{\omega,\mathrm{sim}} := p_{ii}^{\omega}\left(z_{ii}^{\omega,\mathrm{sim}};\mathbb{A}_i^*\right)$. Instead, DL ω under-invests when the inequality in (11) is reversed.

Condition (11) makes it possible to prove that cost-reducing investments always entail greater investments in the sequential-moves game. Formally, these investments satisfy $\frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}} < 0$ and $\frac{\partial^2 \ln a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial \ln p_{ii}^{\omega} \partial \ln z_{ii}^{\omega}} = \frac{\partial \ln a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial \ln z_{ii}^{\omega}} = 0$ according to their definition in Section 2.3, which joint with (10) makes (11) always hold.

As for demand-enhancing investments, (11) does not necessarily hold. Furthermore, determining whether this occurs requires finding the equilibrium in the simultaneous-moves game, which precludes obtaining immediate conclusions. Nonetheless, it is possible to express (11) in terms of model primitives. Specifically, DL ω increases its domestic investment when, for any $(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i)$,

$$\frac{\partial^2 \ln a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial \ln p_{ii}^{\omega} \partial \ln z_{ii}^{\omega}} < \frac{\partial \ln a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial \ln z_{ii}^{\omega}} \left(\varepsilon_{ii}^{p}\left(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i\right) + \frac{\partial^2 \ln a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial \left(\ln p_{ii}^{\omega}\right)^2} \left(\frac{\partial \ln a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial \ln p_{ii}^{\omega}} \right)^{-1} \right) + \frac{\varepsilon_{ii}^{p}\left(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i\right)}{1 - s_{ii}\left(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i\right)} \frac{\partial \ln a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial \ln p_{ii}^{\omega}} \frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}}, \quad (12)$$

while DL ω decreases its investments when the inequality is reversed.

The fact that (12) is expressed in terms of model primitives allows us to easily identify cases where over-investing arises. For instance, this condition is satisfied when i) demand-enhancing investments are à la Sutton (1991; 1998), i.e. they only require incurring fixed sunk costs and have a small impact on marginal costs, and ii) demands are given by either (ces) and (exp).[16]

The next proposition formalizes all the results we have indicated.

Proposition 4.5. *For each DL ω from $i \in \mathcal{C}$:*

- *If $z_{ii}^{\omega,sim}$ satisfies (11) then $z_{ii}^{\omega,seq} > z_{ii}^{\omega,sim}$. In particular, this always occurs if (12) holds for any $(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i)$.*
- *If the inequality in (11) is reversed and holds with strict inequality, then $z_{ii}^{\omega,seq} < z_{ii}^{\omega,sim}$. In particular, this always occurs if the inequality in (12) is reversed and holds with strict inequality for $(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i)$.*

Overall, Proposition 4.5 establishes that DLs always increase their domestic investments when they are either cost-reducing, or when they are demand-enhancing and satisfy (12). The result is particularly relevant when investments entail process or product innovation: it identifies conditions under which the potential competition of CFFs (rather than actual competition) provide DLs with incentives to innovate more. Thus, the existence of CFFs could be pro-competitive, even if these firms do not end up entering the industry.

[16]The result follows immediately by noting that i) implies $\frac{\mathrm{d}\ln c_{ii}^{\omega}(z_{ii}^{\omega})}{\mathrm{d}\ln z_{ii}^{\omega}} = 0$ (or close to it), whereas the demands in ii) satisfy $\frac{\partial^2 \ln a_{ii}^{\omega}(\mathbf{x}_{ii}^{\omega})}{\partial \ln p_{ii}^{\omega} \partial \ln z_{ii}^{\omega}} = 0$.

4.4 Welfare

The fact that the DLs could deter market exploration by innovating more entails that consumers could benefit from this channel. However, an analysis of welfare involves taking into account all the mechanisms simultaneously affecting both consumers and firms. Next, we investigate this matter and show that, indeed, the strategic behavior by DLs could improve a country's welfare.

As for producer surplus, the results are determinate and establish that the strategic use of domestic investments increases them. This is because all DLs garner greater profits by Proposition 4.1 and, even though there is a simultaneous exit of CFFs, these firms have zero aggregate profits. Therefore, there is an overall increase in a country's profits.

As for consumer surplus, there are multiple factors affecting a consumer's utility. Consequently, the impact is indeterminate and depends ultimately on the weight that consumers give to each effect. Specifically, consumers are affected in the following ways.

To begin with, their welfare decreases by the exit of CFFs, since the industry good is horizontally differentiated and each CFF produces a unique variety.[17] Additionally, Proposition 4.5 identifies whether consumers benefit from the investment choices of DLs. This requires splitting the analysis according to the type of investment.

As for cost-reducing investments, consumers are always better off since DLs always invest more, and hence charge lower prices. However, matters are more complex for demand-enhancing investments. To illustrate this, suppose investments in quality. Moreover, assume that they are desirable for consumers, with the goal of avoiding any ambiguity about whether they are actually welfare-improving, as occurs with advertisements. Then, there are opposing effects acting simultaneously, and DLs could either end up investing more or less. If they invest more, consumers benefit from enhanced varieties, but are negatively impacted by the increase in the DLs' prices. In case DLs decrease their investments, the varieties of DLs would be cheaper, but also of lower quality.

Overall, when we take these effects altogether, the strategic use of domestic investments by DLs could increase or decrease consumer surplus. Thus, the impact on welfare is indeterminate. However, we can still obtain conclusions for some cases that are standard in the literature. They establish that deterring industry exploration could actually benefit countries.

To formalize this, we consider two scenarios that can be encompassed by an augmented

[17] It can also be shown that the prices set by active CFFs and the survival productivity cutoff in each country are the same in the simultaneous- and sequential-moves games. Thus, consumers are neutrally affected through these channels.

CES demand. This demand satisfies (dem) and is "integrable", i.e., it can be obtained from the utility maximization of a representative consumer. Thus, we can measure consumer welfare through the indirect utility function.

Specifically, given expenditures E_i, the utility of the differentiated good in $i \in \mathcal{C}$ is given by

$$U_i := \left\{ \sum_{k \in \mathcal{C}} \left[\int\limits_{\omega \in \Omega_{ki}^{\mathcal{N}}} \left(\lambda_{ki}^{\omega} Q_{ki}^{\omega}\right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega + \sum_{\omega \in \Omega_{ki}^{\mathcal{L}}} \left(\lambda_{ki}^{\omega} Q_{ki}^{\omega}\right)^{\frac{\sigma-1}{\sigma}} \right] \right\}^{\frac{\sigma}{\sigma-1}}, \tag{13}$$

where $\sigma > 1$ and $\lambda_{ki}^{\omega} > 0$. Routine calculations determine that the optimal demand in i of a variety ω from j is

$$Q_{ji} := E_i \left(\mathbb{P}_i\right)^{\sigma-1} \left(\lambda_{ji}^{\omega}\right)^{\sigma-1} \left(p_{ji}^{\omega}\right)^{-\sigma}, \tag{14}$$

where

$$\mathbb{P}_i := \left\{ \sum_{k \in \mathcal{C}} \left[\int\limits_{\omega \in \Omega_{ki}^{\mathcal{N}}} \left(p_{ki}^{\omega} / \lambda_{ki}^{\omega}\right)^{1-\sigma} \mathrm{d}\omega + \sum_{\omega \in \Omega_{ki}^{\mathcal{L}}} \left(p_{ki}^{\omega} / \lambda_{ki}^{\omega}\right)^{1-\sigma} \right] \right\}^{\frac{1}{1-\sigma}}.$$

Notice that this demand corresponds to (ces) once we define $z_{ij}^{\omega} := \lambda_{ij}^{\omega}$ and $\mathbb{A}_i := \mathbb{P}_i^{1-\sigma}$.[18]

Furthermore, the indirect utility function in i of the differentiated good, which we denote by $\mathbb{V}_i$, is given by

$$\mathbb{V}_i := \frac{E_i}{\mathbb{P}_i}. \tag{15}$$

Based on this demand, we can derive the following result.

Proposition 4.6. *Let $i, j \in \mathcal{C}$. Suppose that utility is given by (13) for each i. Then, consumer welfare is the same in the simultaneous- and sequential-moves games when either:*

- *investments are cost-reducing and each λ_{ij}^{ω} is taken as a parameter, or*
- *investments satisfy $z_{ij}^{\omega} = \lambda_{ij}^{\omega}$, so that they are demand-enhancing.*

Additionally, since total profits in i are greater by Proposition 4.1, country i's welfare increases.

In words, the proposition indicates that, even though the strategic use of investments by DLs has opposing effects, they are completely offset in terms of consumer welfare. Thus, consumers derive the same utility in the simultaneous-moves and sequential-moves scenario.

[18]We can also rationalize (14) in the following way. Assume a world economy comprising a homogeneous and differentiated good. Moreover, the utility in $i \in \mathcal{C}$ is given by $V_i := E_i \ln(\mathbb{Q}_i) + \mathbb{Q}_i^0$, where $\mathbb{Q}_i^0$ is the quantity of the homogeneous good, $\mathbb{Q}_i$ a quantity index for the differentiated good, and $E_i \in \mathbb{R}_{++}$. Supposing that income in i is high enough such that both goods are consumed in equilibrium and taking $\mathbb{Q}_i^0$ as the numéraire, the solution to $\mathbb{Q}_i$ is given by $\mathbb{P}_i \mathbb{Q}_i = E_i$. This determines that E_i corresponds to the expenditure on the differentiated good. Then, we obtain the same demand for each variety of the differentiated good by assuming that $\mathbb{Q}_i = U_i$.

Furthermore, since producer surplus necessarily increases because the country's profits are higher, total welfare in each country is greater.

The fact that consumer surplus does not vary follows because (14) is equivalent to (ces) by defining $\mathbb{A}_i := \mathbb{P}_i^{1-\sigma}$. Thus, since the aggregate in i is identical in the simultaneous-moves and sequential-moves equilibrium, $\mathbb{V}_i$ is the same in each scenario too.

Notice that, even when the consumer's utility is the same in each game, the channels operating might be radically different. To illustrate this, consider investments in quality. Moreover, suppose that (12) is not necessarily satisfied, so that DLs could increase or decrease their domestic investments relative to the simultaneous-moves game. Then, the proposition covers scenarios where DLs under-invest, so that consumers benefit from cheaper varieties by DLs, but are also hurt by the downgrade in the DLs' quality. Instead, when DLs over-invest, consumers benefit due to the upgrade in quality by DLs, but are negatively affected due to higher prices by DLs.

5 Conclusions

In this paper, we studied strategic behavior in industries exhibiting a coexistence of large and small firms. The focus was on the deployment of strategies by domestic leaders to gain a better position in their home market. With this goal, we considered an industry comprising a tradable horizontally-differentiated good, and a market structure with non-negligible firms embedded into a monopolistic-competition model à la Melitz (2003). This characterized small firms as businesses that decide whether to venture into the industry, and eventually discover how profitable they are.

Our findings indicate that domestic leaders always strategically invest at home to partially deter industry exploration. Instead, an accommodating strategy is never pursued, even when competition in our model is in prices that exhibit strategic complementarity. This occurs because such a strategy would simply induce further market exploration by small firms, and hence undermine its goal of softening competition.

Furthermore, we have shown that, by adopting a more aggressive behavior, each domestic leader increases its profit and captures revenues at home that otherwise would be appropriated by small firms. Additionally, this strategy can result in a wide range of outcomes. First, although total market exploration necessarily decreases in each country, we highlighted that small firms from less-efficient countries could benefit. Second, the existence of firms ready to explore the industry can promote innovation by domestic leaders. Thus, potential competition of small firms, rather than actual competition, can make desirable outcomes emerge, which echoes the results of the Contestability Theory by Baumol et al. (1982). In fact, we have shown that the aggressive behavior by domestic leaders could be welfare-improving.

A topic that we left for future work and is worthy of studying relates to strategy groups. Porter (1980) argues that firms within an industry can usually be classified into several groups displaying similar strategic behavior. In our framework, such a property would require assuming a nested demand with groups given by small and large firms. Thus, large firms would strategically use their investments taking into account how they affect small firms but also large rivals. This could provide a richer set of predictions, and in particular make accommodating entry of large firms possibly emerge. Nonetheless, it would also add a non-trivial layer of complexity to the problem.

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Online Appendix - not for publication

A Derivations and Proofs

To streamline notation, we omit the arguments of the functions when it is clear from the context. In addition, we use the compact notation a_{ij}^{ω} to denote $a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)$, which we also employ for other variables. Regarding derivatives, we follow the convention utilized in the main part of the paper, where $\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)$ is used as argument of the function if ω can influence the aggregate; instead, $\left(\mathbf{x}_{ij}^{\omega}; \mathbb{A}_j\right)$ is employed when DL ω takes $\mathbb{A}_j$ as given. Also, for any variable y_{ij}^{ω}, we keep using the notation $y_{ij}^{\omega,\text{seq}}$ and $y_{ij}^{\omega,\text{sim}}$ to refer to its equilibrium value in each scenario.

A.1 Preliminary Results

We begin by characterizing some properties of the demand given by (dem). We do this by characterizing its elasticities and market shares. Recall that we have defined the market share in j of DL ω from i by $s_{ij}^{\omega} := \frac{R_{ij}^{\omega}}{E_j}$, which is given by $s_{ij}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right) := \frac{a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\mathbb{A}_j}$. Throughout the derivations, we exploit that several terms involving firm ω can be expressed as functions of its market share.

The demand elasticities of ω are given by,

$$\varepsilon_{ij}^{p,\omega} = 1 - \left(1 - s_{ij}^{\omega}\right) \frac{\partial \ln a_{ij}^{\omega}}{\partial \ln p_{ij}^{\omega}}, \tag{A1a}$$

$$\xi_{ij}^{p,\omega} = 1 - \frac{\partial \ln a_{ij}^{\omega}}{\partial \ln p_{ij}^{\omega}}, \tag{A1b}$$

$$\varepsilon_{ij}^{z,\omega} = \left(1 - s_{ij}^{\omega}\right) \frac{\partial \ln a_{ij}^{\omega}}{\partial \ln z_{ij}^{\omega}}, \tag{A1c}$$

$$\xi_{ij}^{z,\omega} = \frac{\partial \ln a_{ij}^{\omega}}{\partial \ln z_{ij}^{\omega}}, \tag{A1d}$$

where $\varepsilon_{ij}^{\cdot,\omega}$ incorporates the changes in the aggregate, whereas $\xi_{ij}^{\cdot,\omega}$ ignores it. Formally, $\xi_{ij}^{\cdot,\omega}$ is given by functions $\xi_{ij}^{p}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right) := -\frac{\partial \ln Q_{ij}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial \ln p_{ij}^{\omega}}$ and $\xi_{ij}^{z}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right) := \frac{\partial \ln Q_{ij}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial \ln z_{ij}^{\omega}}$.

Regarding derivatives of market shares, depending on whether we take $\mathbb{A}_j$ as given or not, we obtain the following:

$$\frac{\partial \ln s_{ij}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial \ln p_{ij}^{\omega}} = \left(1 - s_{ij}^{\omega}\right) \frac{\partial \ln a_{ij}^{\omega}}{\partial \ln p_{ij}^{\omega}}, \tag{A2a}$$

$$\frac{\partial \ln s_{ij}\left(\mathbf{x}_{ij}^{\omega}; \mathbb{A}_j\right)}{\partial \ln p_{ij}^{\omega}} = \frac{\partial \ln a_{ij}^{\omega}}{\partial \ln p_{ij}^{\omega}}, \tag{A2b}$$

$$\frac{\partial \ln s_{ij}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\right)}{\partial \ln z_{ij}^{\omega}} = \left(1 - s_{ij}^{\omega}\right) \frac{\partial \ln a_{ij}^{\omega}}{\partial \ln z_{ij}^{\omega}}, \tag{A2c}$$

$$\frac{\partial \ln s_{ij}\left(\mathbf{x}_{ij}^{\omega}; \mathbb{A}_j\right)}{\partial \ln z_{ij}^{\omega}} = \frac{\partial \ln a_{ij}^{\omega}}{\partial \ln z_{ij}^{\omega}}. \tag{A2d}$$

Using these results, it is determined that, for a given $\mathbb{A}_j$,

$$\frac{\partial \ln \varepsilon_{ij}^{p}\left(\mathbf{x}_{ij}^{\omega};\mathbb{A}_j\right)}{\partial \ln p_{ij}^{\omega}} = \frac{1}{\varepsilon_{ij}^{p,\omega}}\left[s_{ij}^{\omega}\left(\frac{\partial \ln a_{ij}^{\omega}}{\partial \ln p_{ij}^{\omega}}\right)^{2} - \left(1-s_{ij}^{\omega}\right)\frac{\partial^{2}\ln a_{ij}^{\omega}}{\partial\left(\ln p_{ij}^{\omega}\right)^{2}}\right], \tag{A3a}$$

$$\frac{\partial \ln \varepsilon_{ij}^{p}\left(\mathbf{x}_{ij}^{\omega};\mathbb{A}_j\right)}{\partial \ln z_{ij}^{\omega}} = \frac{1}{\varepsilon_{ij}^{p,\omega}}\left[s_{ij}^{\omega}\frac{\partial \ln a_{ij}^{\omega}}{\partial \ln z_{ij}^{\omega}}\frac{\partial \ln a_{ij}^{\omega}}{\partial \ln p_{ij}^{\omega}} - \left(1-s_{ij}^{\omega}\right)\frac{\partial^{2}\ln a_{ij}^{\omega}}{\partial \ln p_{ij}^{\omega}\partial \ln z_{ij}^{\omega}}\right]. \tag{A3b}$$

The following lemmas refer to different algebraic derivations for expressions involving $\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right)}{\partial \ln z_{ii}^{\omega}}$.

Lemma 1. *Optimal domestic prices of DL ω from i satisfy*

$$\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right)}{\partial \ln z_{ii}^{\omega}} = \frac{\frac{\partial \ln \varepsilon_{ii}^{p}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right),z_{ii}^{\omega};\mathbb{A}_i\right]}{\partial \ln z_{ii}^{\omega}} - \left[\varepsilon_{ii}^{p}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right),z_{ii}^{\omega};\mathbb{A}_i\right]-1\right]\frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}}}{1-\varepsilon_{ii}^{p}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right),z_{ii}^{\omega};\mathbb{A}_i\right] - \frac{\partial \ln \varepsilon_{ii}^{p}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right),z_{ii}^{\omega};\mathbb{A}_i\right]}{\partial \ln p_{ii}^{\omega}}},$$

$$= \frac{\frac{s_{ii}^{\omega}}{1-s_{ii}^{\omega}}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln z_{ii}^{\omega}}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}} - \frac{\partial^{2}\ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}\partial \ln z_{ii}^{\omega}} + \frac{\varepsilon_{ii}^{p,\omega}}{1-s_{ii}^{\omega}}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}}\frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}}}{\varepsilon_{ii}^{p,\omega}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}} + \frac{\partial^{2}\ln a_{ii}^{\omega}}{\partial\left(\ln p_{ii}^{\omega}\right)^{2}} - \frac{s_{ii}^{\omega}}{1-s_{ii}^{\omega}}\left(\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}}\right)^{2}},$$

where $\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right),z_{ii}^{\omega};\mathbb{A}_i\right]$ is the argument of $\varepsilon_{ii}^{p,\omega}$, the derivatives of a_{ii}^{ω}, and the function s_{ii}^{ω}.

Proof of Lemma 1. Let $m_{ii}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right) := \frac{\varepsilon_{ii}^{p}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right)}{\varepsilon_{ii}^{p}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right)-1}$, where $\frac{\partial \ln m_{ii}^{\omega}}{\partial \ln \varepsilon_{ii}^{p,\omega}} = 1-m_{ii}^{\omega}$. Optimal domestic prices of DL ω are

$$\ln p_{ii}^{\omega} = \ln m_{ii}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right) + \ln c_{ii}^{\omega},$$

which implies that

$$\frac{\partial \ln p_{ii}^{\omega}}{\partial \ln z_{ii}^{\omega}} = \frac{\left(1-m_{ii}^{\omega}\right)\frac{\partial \ln \varepsilon_{ii}^{p,\omega}}{\partial \ln z_{ii}^{\omega}} + \frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}}}{1-\left(1-m_{ii}^{\omega}\right)\frac{\partial \ln \varepsilon_{ii}^{p,\omega}}{\partial \ln p_{ii}^{\omega}}}.$$

By using that $\left(1-m_{ii}^{\omega}\right)^{-1} = 1-\varepsilon_{ii}^{p,\omega}$, then we get

$$\frac{\partial \ln p_{ii}^{\omega}}{\partial \ln z_{ii}^{\omega}} = \frac{\frac{\partial \ln \varepsilon_{ii}^{p,\omega}}{\partial \ln z_{ii}^{\omega}} + \left(1-\varepsilon_{ii}^{p,\omega}\right)\frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}}}{1-\varepsilon_{ii}^{p,\omega} - \frac{\partial \ln \varepsilon_{ii}^{p,\omega}}{\partial \ln p_{ii}^{\omega}}}. \tag{A4}$$

Using (A1a), (A3a), and (A3b), then (A4) can be expressed as

$$\frac{\frac{1}{\varepsilon_{ii}^{p,\omega}}\left[s_{ii}^{\omega}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln z_{ii}^{\omega}}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}} - \left(1-s_{ii}^{\omega}\right)\frac{\partial^{2}\ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}\partial \ln z_{ii}^{\omega}}\right] + \left(1-s_{ij}^{\omega}\right)\frac{\partial \ln a_{ij}^{\omega}}{\partial \ln p_{ij}^{\omega}}\frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}}}{\left(1-s_{ii}^{\omega}\right)\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}} - \frac{1}{\varepsilon_{ii}^{p,\omega}}\left[s_{ii}^{\omega}\left(\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}}\right)^{2} - \left(1-s_{ii}^{\omega}\right)\frac{\partial^{2}\ln a_{ii}^{\omega}}{\partial\left(\ln p_{ii}^{\omega}\right)^{2}}\right]}.$$

By multiplying the numerator and denominator of the RHS by $\frac{\varepsilon_{ii}^{p,\omega}}{1-s_{ii}^{\omega}}$, then the result follows. ■

Lemma 2. $\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right)}{\partial \ln z_{ii}^{\omega}} < -\left(\frac{\partial \ln a_{ii}[\cdot]}{\partial \ln p_{ii}^{\omega}}\right)^{-1}\frac{\partial \ln a_{ii}[\cdot]}{\partial \ln z_{ii}^{\omega}}$ *iff* $\frac{\mathrm{d}\ln a_{ii}[\cdot]}{\mathrm{d}\ln z_{ii}^{\omega}} > 0$ *and iff*

$$\frac{\partial^{2}\ln a_{ii}[\cdot]}{\partial \ln p_{ii}^{\omega}\partial \ln z_{ii}^{\omega}} < \frac{\partial \ln a_{ii}[\cdot]}{\partial \ln z_{ii}^{\omega}}\left(\varepsilon_{ii}^{p}\left(s_{ii}^{\omega}\right) + \frac{\partial^{2}\ln a_{ii}[\cdot]}{\partial\left(\ln p_{ii}^{\omega}\right)^{2}}\left(\frac{\partial \ln a_{ii}[\cdot]}{\partial \ln p_{ii}^{\omega}}\right)^{-1}\right) + \frac{\varepsilon_{ii}^{p}\left(s_{ii}^{\omega}\right)}{1-s_{ii}^{\omega}}\frac{\partial \ln a_{ii}^{\omega}[\cdot]}{\partial \ln p_{ii}^{\omega}}\frac{\mathrm{d}\ln c_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}}, \tag{A5}$$

where $[\cdot] := \left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right),z_{ii}^{\omega}\right]$, and s_{ii}^{ω} is a function $s_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right),z_{ii}^{\omega},\mathbb{A}_i\right]$.

Proof of Lemma 2. The fact that $\frac{\mathrm{d}\ln a_{ii}^{\omega}}{\mathrm{d}\ln z_{ii}^{\omega}} > 0$ iff $\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right)}{\partial \ln z_{ii}^{\omega}} < -\left(\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}}\right)^{-1}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln z_{ii}^{\omega}}$ follows by definition. Moreover, using the expression for $\frac{\partial \ln p_{ii}^{\omega}(z_{ii}^{\omega};\mathbb{A}_i)}{\partial \ln z_{ii}^{\omega}}$ derived in Lemma 1, this occurs iff

$$\frac{\frac{s_{ii}^{\omega}}{1-s_{ii}^{\omega}}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln z_{ii}^{\omega}}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}} - \frac{\partial^2 \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}\partial \ln z_{ii}^{\omega}} + \frac{\varepsilon_{ii}^{p,\omega}}{1-s_{ii}^{\omega}}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}}\frac{\mathrm{d}\ln c_{ii}^{\omega}(z_{ii}^{\omega})}{\mathrm{d}\ln z_{ii}^{\omega}}}{\varepsilon_{ii}^{p,\omega}\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}} + \frac{\partial^2 \ln a_{ii}^{\omega}}{\partial (\ln p_{ii}^{\omega})^2} - \frac{s_{ii}^{\omega}}{1-s_{ii}^{\omega}}\left(\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}}\right)^2} < -\frac{\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln z_{ii}^{\omega}}}{\frac{\partial \ln a_{ii}^{\omega}}{\partial \ln p_{ii}^{\omega}}}. \tag{A6}$$

Using that the denominator of the LHS is negative and working out the expression, we obtain (A5). ■

Lemma 3. *Given the equilibrium aggregate $\mathbb{A}_i^*$,*

$$sgn\left\{\frac{\mathrm{d}a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega}\right]}{\mathrm{d}z_{ii}^{\omega}}\right\} = sgn\left\{\frac{\mathrm{d}R_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega};\mathbb{A}_i^*\right]}{\mathrm{d}z_{ii}^{\omega}}\right\}.$$

Proof of Lemma 3. By definition, optimal domestic revenue of DL ω as a function of its domestic investment is given by

$$R_{ii}^{\omega}\left(z_{ii}^{\omega}\right) =: R_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega};\mathbb{A}_i^*\right] := Q_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega};\mathbb{A}_i^*\right]p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right). \tag{A7}$$

Hence, by using (A1b) and (A1d),

$$\begin{aligned}\frac{\mathrm{d}\ln R_{ii}^{\omega}\left(z_{ii}^{\omega}\right)}{\mathrm{d}\ln z_{ii}^{\omega}} &= \frac{\partial \ln Q_{ii}\left[\cdot\right]}{\partial \ln z_{ii}^{\omega}} + \frac{\partial \ln Q_{ii}\left[\cdot\right]}{\partial \ln p_{ii}^{\omega}}\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right)}{\partial \ln z_{ii}^{\omega}} + \frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right)}{\partial \ln z_{ii}^{\omega}},\\ &= \frac{\partial \ln a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega}\right]}{\partial \ln z_{ii}^{\omega}} + \frac{\partial \ln a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega}\right]}{\partial \ln p_{ii}^{\omega}}\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right)}{\partial \ln z_{ii}^{\omega}},\end{aligned}$$

where $[\cdot] := \left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega};\mathbb{A}_i^*\right]$. Since the second line equals $\frac{\mathrm{d}a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega}\right]}{\mathrm{d}z_{ii}^{\omega}}$, then the result follows. ■

Lemma 4. *Let*

$$\Delta_{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right) = -\frac{\partial \pi_{ii}^{\omega}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega},\mathbb{A}_i^*\right]}{\partial \mathbb{A}_i}\left[\frac{\partial a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega}\right]}{\partial p_{ii}^{\omega}}\frac{\partial p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right)}{\partial z_{ii}^{\omega}} + \frac{\partial a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega}\right]}{\partial z_{ii}^{\omega}}\right], \tag{A8}$$

and define $p_{ii}^{\omega,sim} := p_{ii}^{\omega}\left(z_{ii}^{\omega,sim};\mathbb{A}_i^*\right)$, where $\mathbb{A}_i^*$ is the equilibrium aggregate in both scenarios. Then,

$$sgn\left\{\Delta_{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right)\right\} = sgn\left\{\frac{\mathrm{d}a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right),z_{ii}^{\omega}\right]}{\mathrm{d}z_{ii}^{\omega}}\right\}. \tag{A9}$$

In addition:

Case i) *If $z_{ii}^{\omega,sim}$ satisfies that $\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega,sim};\mathbb{A}_i^*\right)}{\partial \ln z_{ii}^{\omega}} > -\frac{\partial \ln a_{ii}\left(p_{ii}^{\omega,sim},z_{ii}^{\omega,sim}\right)}{\partial \ln z_{ii}^{\omega}}\left(\frac{\partial \ln a_{ii}\left(p_{ii}^{\omega,sim},z_{ii}^{\omega,sim}\right)}{\partial \ln p_{ii}^{\omega}}\right)^{-1}$ then $z_{ii}^{\omega,sim} > z_{ii}^{\omega,seq}$ and $\Delta_{\omega}\left(z;\mathbb{A}_i^*\right) < 0$ for any $z \in \left(z_{ii}^{\omega,seq},z_{ii}^{\omega,sim}\right)$.*

Case ii) *If $z_{ii}^{\omega,sim}$ satisfies that $\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega,sim};\mathbb{A}_i^*\right)}{\partial \ln z_{ii}^{\omega}} < -\frac{\partial \ln a_{ii}\left(p_{ii}^{\omega,sim},z_{ii}^{\omega,sim}\right)}{\partial \ln z_{ii}^{\omega}}\left(\frac{\partial \ln a_{ii}\left(p_{ii}^{\omega,sim},z_{ii}^{\omega,sim}\right)}{\partial \ln p_{ii}^{\omega}}\right)^{-1}$ then $z_{ii}^{\omega,seq} > z_{ii}^{\omega,sim}$ and $\Delta_{\omega}\left(z;\mathbb{A}_i^*\right) > 0$ for any $z \in \left(z_{ii}^{\omega,seq},z_{ii}^{\omega,sim}\right)$.*

Proof of Lemma 4. Define ω's domestic marginal profit of its domestic investment in the simultaneous and sequential case, respectively, by

$$\gamma_{\omega}^{\text{sim}}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right) := \frac{\partial \pi_{ii}^{\omega}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right)}{\partial z_{ii}^{\omega}} + \frac{\partial \pi_{ii}^{\omega}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right)}{\partial \mathbb{A}_i}\frac{\partial a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial z_{ii}^{\omega}}, \tag{A10}$$

$$\gamma_{\omega}^{\text{seq}}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right) := \frac{\partial \pi_{ii}^{\omega}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right)}{\partial z_{ii}^{\omega}} + \frac{\partial \pi_{ii}^{\omega}\left(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i\right)}{\partial p_{ii}^{\omega}}\frac{\partial p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}\right)}{\partial z_{ii}^{\omega}}. \tag{A11}$$

Under both the simultaneous or sequential case, optimal price is characterized through the first-order condition, which is given by $\frac{\partial \pi_{ii}^{\omega}(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i)}{\partial p_{ii}^{\omega}} = -\frac{\partial \pi_{ii}^{\omega}(\mathbf{x}_{ii}^{\omega},\mathbb{A}_i)}{\partial \mathbb{A}_i}\frac{\partial a_{ii}(\mathbf{x}_{ii}^{\omega})}{\partial p_{ii}^{\omega}}$. Thus, we can reexpress (A11) as

$$\gamma_{\omega}^{\text{seq}}\left(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i\right) := \frac{\partial \pi_{ii}^{\omega}\left(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_{ii}\right)}{\partial z_{ii}^{\omega}} - \frac{\partial \pi_{ii}^{\omega}\left(\mathbf{x}_{ii}^{\omega}, \mathbb{A}_i\right)}{\partial \mathbb{A}_i}\frac{\partial a_{ii}\left(\mathbf{x}_{ii}^{\omega}\right)}{\partial p_{ii}^{\omega}}\frac{\partial p_{ii}^{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i\right)}{\partial z_{ii}^{\omega}}. \tag{A12}$$

Moreover, we know that the same equilibrium aggregate holds in the simultaneous-moves and sequential-moves game. This is denoted by $\mathbb{A}_i^*$ in country i. Define the difference in domestic marginal profits as a function of domestic investments by

$$\Delta_{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right) := \gamma_{\omega}^{\text{seq}}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right), z_{ii}^{\omega}; \mathbb{A}_i^*\right] - \gamma_{\omega}^{\text{sim}}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right), z_{ii}^{\omega}; \mathbb{A}_i^*\right]. \tag{A13}$$

Then, substituting in by (A10) and (A12),

$$\Delta_{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right) = -\frac{\partial \pi_{ii}^{\omega}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right), z_{ii}^{\omega}, \mathbb{A}_i^*\right]}{\partial \mathbb{A}_i}\left[\frac{\partial a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right), z_{ii}^{\omega}\right]}{\partial p_{ii}^{\omega}}\frac{\partial p_{ii}^{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right)}{\partial z_{ii}^{\omega}} + \frac{\partial a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right), z_{ii}^{\omega}\right]}{\partial z_{ii}^{\omega}}\right].$$

Since $\frac{\partial \pi_{ii}^{\omega}}{\partial \mathbb{A}_i} < 0$, then

$$\operatorname{sgn}\left\{\Delta_{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right)\right\} = \operatorname{sgn}\left\{\frac{\mathrm{d}a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega}; \mathbb{A}_i^*\right), z_{ii}^{\omega}\right]}{\mathrm{d}z_{ii}^{\omega}}\right\}.$$

Define $\Delta_{\omega}^{\text{sim}} := \Delta_{\omega}\left(z_{ii}^{\omega,\text{sim}}; \mathbb{A}_i^*\right)$. The strict quasi-concavity of profits evaluated at optimal prices establishes that $\gamma_{\omega}^{\text{sim}}$ and $\gamma_{\omega}^{\text{seq}}$ are single-peaked as a function of z_{ii}^{ω}. Consequently, if $\Delta_{\omega}^{\text{sim}} > 0$ then DL ω over-invests, while it under-invests if $\Delta_{\omega}^{\text{sim}} < 0$. Next, we consider the two cases stated in the information of the lemma separately.

Case i) Suppose that $\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}};\mathbb{A}\right)}{\partial \ln z_{ii}^{\omega}} > -\frac{\partial \ln a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}};\mathbb{A}_i^*\right),z_{ii}^{\omega,\text{sim}}\right]}{\partial \ln z_{ii}^{\omega}}\left(\frac{\partial \ln a_{ii}\left(p_{ii}^{\omega,\text{sim}},z_{ii}^{\omega,\text{sim}}\right)}{\partial \ln p_{ii}^{\omega}}\right)^{-1}$, which is equivalent to $\frac{\mathrm{d}\ln a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}};\mathbb{A}_i^*\right),z_{ii}^{\omega,\text{sim}}\right]}{\mathrm{d}\ln z_{ii}^{\omega}} < 0$. Then, $\Delta_{\omega}^{\text{sim}} < 0$ by Lemma 2, and so $z_{ii}^{\omega,\text{sim}} > z_{ii}^{\omega,\text{seq}}$. Next, we show that $\Delta_{\omega}(z) < 0$ for any $z \in \left(z_{ii}^{\omega,\text{seq}}, z_{ii}^{\omega,\text{sim}}\right)$. By using (A10) and (A12), we have that $\gamma_{\omega}^{\text{sim}}\left(z_{ii}^{\omega,\text{sim}}, \mathbb{A}_i^*\right) = 0$, $\gamma_{\omega}^{\text{seq}}\left(z_{ii}^{\omega,\text{seq}}, \mathbb{A}_i^*\right) = 0$, and $\gamma_{\omega}^{\text{seq}}\left(z_{ii}^{\omega,\text{sim}}, \mathbb{A}_i^*\right) < 0$. Besides, by the strict quasiconcavity of profits evaluated at optimal prices concavity and taking z such that $z_{ii}^{\omega,\text{sim}} > z > z_{ii}^{\omega,\text{seq}}$, we know that $\gamma_{\omega}^{\text{seq}}\left(z; \mathbb{A}_i^*\right) < 0$ and $\gamma_{\omega}^{\text{sim}}\left(z; \mathbb{A}_i^*\right) > 0$. Therefore, $\Delta_{\omega}\left(z; \mathbb{A}_i^*\right) < 0$ for any $z \in \left(z_{ii}^{\omega,\text{seq}}, z_{ii}^{\omega,\text{sim}}\right)$.

Case ii) Suppose that $\frac{\partial \ln p_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}};\mathbb{A}_i^*\right)}{\partial \ln z_{ii}^{\omega}} < -\frac{\partial \ln a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}};\mathbb{A}_i^*\right),z_{ii}^{\omega,\text{sim}}\right]}{\partial \ln z_{ii}^{\omega}}\left(\frac{\partial \ln a_{ii}\left(p_{ii}^{\omega,\text{sim}},z_{ii}^{\omega,\text{sim}}\right)}{\partial \ln p_{ii}^{\omega}}\right)^{-1}$, which establishes that $\frac{\mathrm{d}\ln a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}};\mathbb{A}_i^*\right),z_{ii}^{\omega,\text{sim}}\right]}{\mathrm{d}\ln z_{ii}^{\omega}} > 0$. Consequently, $\Delta_{\omega}^{\text{sim}} > 0$ by Lemma 2, and so $z_{ii}^{\omega,\text{seq}} > z_{ii}^{\omega,\text{sim}}$. Next, we show that $\Delta_{\omega}(z) > 0$ for any $z \in \left(z_{ii}^{\omega,\text{sim}}, z_{ii}^{\omega,\text{seq}}\right)$ Using (A10), (A12), and the strict quasi-concavity of profits evaluated at optimal prices, we have that $\gamma_{\omega}^{\text{sim}}\left(z_{ii}^{\omega,\text{sim}}, \mathbb{A}_i^*\right) = 0$, $\gamma_{\omega}^{\text{seq}}\left(z_{ii}^{\omega,\text{seq}}, \mathbb{A}_i^*\right) = 0$, and $\gamma_{\omega}^{\text{seq}}\left(z_{ii}^{\omega,\text{sim}}, \mathbb{A}_i^*\right) > 0$. Moreover, for z such that $z_{ii}^{\omega,\text{seq}} > z > z_{ii}^{\omega,\text{sim}}$, then $\gamma_{\omega}^{\text{seq}}\left(z; \mathbb{A}_i^*\right) > 0$ and $\gamma_{\omega}^{\text{sim}}\left(z; \mathbb{A}_i^*\right) < 0$. All this implies that $\Delta_{\omega}(z) > 0$ for any $z \in \left(z_{ii}^{\omega,\text{seq}}, z_{ii}^{\omega,\text{sim}}\right)$. ■

A.2 Proofs

Proof of Proposition 4.1. First, notice that $\mathbb{A}_i^*$ holds in both the simultaneous-moves and sequential-moves game for each country $i \in \mathcal{C}$. We show that DL ω from i obtains greater profit in the sequential-moves game. This follows by a revealed-preference argument, since the same aggregate $\mathbb{A}_i^*$ holds in both scenarios and ω can choose $z_{ii}^{\omega,\text{sim}}$ in both scenarios. Thus, ω can always secure at least the simultaneous-move game's profit.

To show that DL ω behaves more aggressively at home, define $a_{ii}^{\omega}(z) := a_{ii}\left[p_{ii}^{\omega}(z;\mathbb{A}_i), z\right]$. Thus,

$$a_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) - a_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right) = \int_{z_{ii}^{\omega,\text{sim}}}^{z_{ii}^{\omega,\text{seq}}} \frac{\mathrm{d}a_{ii}^{\omega}(z)}{\mathrm{d}z}\mathrm{d}z. \tag{A14}$$

To determine the sign of this expression, we consider the two cases of Lemma 4.

Case i) implies that $z_{ii}^{\omega,\text{sim}} > z_{ii}^{\omega,\text{seq}}$ and $\Delta_{\omega}(z;\mathbb{A}_i^*) < 0$ for any $z \in \left(z_{ii}^{\omega,\text{seq}}, z_{ii}^{\omega,\text{sim}}\right)$, where Δ_{ω} is defined as in (A8). By Lemma 2, this also implies $\frac{\mathrm{d}\ln a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right), z_{ii}^{\omega}\right]}{\mathrm{d}\ln z_{ii}^{\omega}} < 0$. Therefore, we can reexpress the RHS of (A14) by $\int_{z_{ii}^{\omega,\text{sim}}}^{z_{ii}^{\omega,\text{seq}}} \frac{\mathrm{d}a_{ii}^{\omega}(z)}{\mathrm{d}z}\mathrm{d}z = -\int_{z_{ii}^{\omega,\text{seq}}}^{z_{ii}^{\omega,\text{sim}}} \frac{\mathrm{d}a_{ii}^{\omega}(z)}{\mathrm{d}z}\mathrm{d}z > 0$, which implies that $a_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) > a_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right)$.

As for Case ii), it implies that $z_{ii}^{\omega,\text{seq}} > z_{ii}^{\omega,\text{sim}}$ and $\Delta_{\omega}(z;\mathbb{A}_i^*) > 0$ for any $z \in \left(z_{ii}^{\omega,\text{sim}}, z_{ii}^{\omega,\text{seq}}\right)$. Moreover, by Lemma 2, we have that $\frac{\mathrm{d}\ln a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i\right), z_{ii}^{\omega}\right]}{\mathrm{d}\ln z_{ii}^{\omega}} > 0$. By using (A14), then $\int_{z_{ii}^{\omega,\text{sim}}}^{z_{ii}^{\omega,\text{seq}}} \frac{\mathrm{d}a_{ii}^{\omega}(z)}{\mathrm{d}z}\mathrm{d}z > 0$, which determines that $a_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) > a_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right)$.

Regarding domestic revenues of DL ω, they are given by (A7). Given the compact domain of investments and that functions are smooth, we can apply the Fundamental Theorem of Calculus. By using the expression of optimal revenues given by (A7), this determines that

$$R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) - R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right) = \int_{z_{ii}^{\omega,\text{sim}}}^{z_{ii}^{\omega,\text{seq}}} \frac{\mathrm{d}R_{ii}^{\omega}(z)}{\mathrm{d}z}\mathrm{d}z. \tag{A15}$$

By Lemma (3), we have that $\operatorname{sgn}\left\{\frac{\mathrm{d}a_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right), z_{ii}^{\omega}\right]}{\mathrm{d}z_{ii}^{\omega}}\right\} = \operatorname{sgn}\left\{\frac{\mathrm{d}R_{ii}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega};\mathbb{A}_i^*\right), z_{ii}^{\omega}\right]}{\mathrm{d}z_{ii}^{\omega}}\right\}$. Thus, by using (A15) and (A14), it is established that $R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) > R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right)$ iff $a_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) > a_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right)$ and, so, the result follows.

Finally, the fact that the increase in revenues is at the expense of CFFs follows by using that (MS-sim) and (MS-seq) for country i.

$$E_i = \sum_{k\in\mathcal{C}} M_k^{E,\text{sim}} r_{ki}^{\mathcal{N}}\left(\mathbb{A}_i^*\right) + \sum_{\omega\in\Omega_{ii}^{\mathscr{L}}} R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right) + \sum_{\omega\in\Omega_{ji}^{\mathscr{L}}} R_{ji}\left[\mathbf{x}_{ji}^{\omega}\left(\mathbb{A}_i^*\right)\right],$$

$$E_i = \sum_{k\in\mathcal{C}} M_k^{E,\text{seq}} r_{ki}^{\mathcal{N}}\left(\mathbb{A}_i^*\right) + \sum_{\omega\in\Omega_{ii}^{\mathscr{L}}} R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) + \sum_{\omega\in\Omega_{ji}^{\mathscr{L}}} R_{ji}\left[\mathbf{x}_{ji}^{\omega}\left(\mathbb{A}_i^*\right)\right],$$

where $j \neq i$. Since the equilibrium aggregate is the same in each equilibrium, $R_{ji}^{\omega}\left[\mathbf{x}_{ji}^{\omega}\left(\mathbb{A}_i^*\right)\right]$ for DL ω from $j \neq i$ and $r_{ij}^{\mathcal{N}}\left(\mathbb{A}_j^*\right)$ are the same in both the simultaneous-moves and sequential-moves game. Moreover, (dem) implies constant expenditures in each country, which are given by the parameter E_i. Therefore, taking the difference of (MS-sim) and (MS-seq) for country i,

$$0 = \sum_{k\in\mathcal{C}} \left[M_k^{E,\text{seq}} - M_k^{E,\text{sim}}\right] r_{ki}^{\mathcal{N}}\left(\mathbb{A}_i^*\right) + \sum_{\omega\in\Omega_{ii}^{\mathscr{L}}} \left[R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) - R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right)\right].$$

Since $R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) > R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right)$ for each $\omega \in \Omega_{ii}^{\mathscr{L}}$, the variation in revenues in i of CFFs necessarily have to be negative, which can only takes place through variations in $\mathbf{M}^E$. ■

To prove the propositions stated in Section 4.2, we use the notation $\Delta y := y^{\text{seq}} - y^{\text{sim}}$ for any variable y. Furthermore, we streamline notation by defining $\Delta R_{ii}^{\omega} := R_{ii}\left(z_{ii}^{\omega,\text{seq}}\right) - R_{ii}\left(z_{ii}^{\omega,\text{sim}}\right)$ for any $i \in \mathcal{C}$, where we have used the optimal revenues given by (A7). Also, we refer to $r_{ij}^{\mathcal{N}}\left(\mathbb{A}_j^*\right)$ for any $i, j \in \mathcal{C}$ by $r_{ij}^{\mathcal{N}}$.

Throughout the proofs, we make use of the following lemma.

Lemma 5.

$$\Delta M_i^E = \frac{\left(\sum_{\omega \in \Omega_{jj}^{\mathscr{L}}} \Delta R_{jj}^{\omega}\right) r_{ji}^{\mathcal{N}} - \left(\sum_{\omega \in \Omega_{ii}^{\mathscr{L}}} \Delta R_{ii}^{\omega}\right) r_{jj}^{\mathcal{N}}}{\det J}, \tag{A16}$$

where

$$J := \begin{pmatrix} r_{hh}^{\mathcal{N}} & r_{fh}^{\mathcal{N}} \\ r_{hf}^{\mathcal{N}} & r_{ff}^{\mathcal{N}} \end{pmatrix},$$

which satisfies that $\det J > 0$.

Proof of Lemma 5. For each country $i \in \mathcal{C}$ and given $j \neq i$, (MS-sim) and (MS-seq) are given, respectively, by

$$E_i = \sum_{k \in \mathcal{C}} M_k^{E,\text{sim}} r_{ki}^{\mathcal{N}}\left(\mathbb{A}_i^*\right) + \sum_{\omega \in \Omega_{ii}^{\mathscr{L}}} R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{sim}}\right) + \sum_{\omega \in \Omega_{ji}^{\mathscr{L}}} R_{ji}^{\omega}\left(\mathbb{A}_i^*\right),$$

$$E_i = \sum_{k \in \mathcal{C}} M_k^{E,\text{seq}} r_{ki}^{\mathcal{N}}\left(\mathbb{A}_i^*\right) + \sum_{\omega \in \Omega_{ii}^{\mathscr{L}}} R_{ii}^{\omega}\left(z_{ii}^{\omega,\text{seq}}\right) + \sum_{\omega \in \Omega_{ji}^{\mathscr{L}}} R_{ji}^{\omega}\left(\mathbb{A}_i^*\right).$$

Taking the difference of these equations, it is determined that

$$0 = \Delta M_i^E r_{ii}^{\mathcal{N}} + \Delta M_j^E r_{ji}^{\mathcal{N}} + \sum_{\omega \in \Omega_{ii}^{\mathscr{L}}} \Delta R_{ii}^{\omega},$$

which determines the following system:

$$\begin{pmatrix} r_{hh}^{\mathcal{N}} & r_{fh}^{\mathcal{N}} \\ r_{hf}^{\mathcal{N}} & r_{ff}^{\mathcal{N}} \end{pmatrix} \begin{pmatrix} \Delta M_h^E \\ \Delta M_f^E \end{pmatrix} = - \begin{pmatrix} \sum_{\omega \in \Omega_{hh}^{\mathscr{L}}} \Delta R_{hh}^{\omega} \\ \sum_{\omega \in \Omega_{ff}^{\mathscr{L}}} \Delta R_{ff}^{\omega} \end{pmatrix}. \tag{A17}$$

Since the domestic revenues of any CFF are greater than those obtained by exporting, then $r_{ii}^{\mathcal{N}} > r_{ij}^{\mathcal{N}}$ for $i \in \mathcal{C}$ and $j \neq i$. This determines that $\det J > 0$. Finally, the result follows by solving (A17). ■

Proof of Proposition 4.2. Suppose that the set of DLs in f is empty. Then, $\Delta R_{ff}^{\omega} = 0$ for any DL ω from f. Thus, by using (A16), it is determined that

$$\begin{aligned} \Delta M_h^E &= \frac{-\left(\sum_{\omega \in \Omega_{hh}^{\mathscr{L}}} \Delta R_{hh}^{\omega}\right) r_{ff}^{\mathcal{N}}}{\det J} < 0, \\ \Delta M_f^E &= \frac{\left(\sum_{\omega \in \Omega_{hh}^{\mathscr{L}}} \Delta R_{hh}^{\omega}\right) r_{hf}^{\mathcal{N}}}{\det J} > 0, \end{aligned}$$

where the signs follow since $\det J > 0$ by Lemma 5 and since $\Delta R_{hh}^{\omega} > 0$ by Proposition 4.1. ■

Proof of Proposition 4.3. When countries are symmetric, (FE) pins down the same aggregate in each country, which we denote by $\mathbb{A}^*$. Also, (7) and (8) define the same functions in each country given the same expenditure for both countries, E. Thus, the Nash equilibrium at the market stage in each scenario defines the same $M^{E,\text{sim}}$ and $M^{E,\text{seq}}$ in each country too.

Given $i, j \in \mathcal{C}$ with $j \neq i$, and using (7) and (8), this implies that

$$E = M^{E,\text{sim}} \left(\sum_{k \in \mathcal{C}} r_{ik}^{\mathcal{N}} \left(\mathbb{A}^* \right) \right) + \sum_{j \neq i} \sum_{\omega \in \Omega_{ji}^{\mathcal{L}}} R_{ji}^{\omega} \left(\mathbb{A}^* \right) + \sum_{\omega \in \Omega_{ii}^{\mathcal{L}}} R_{ii}^{\omega} \left(z_{ii}^{\omega,\text{sim}} \right),$$

$$E = M^{E,\text{seq}} \left(\sum_{k \in \mathcal{C}} r_{ik}^{\mathcal{N}} \left(\mathbb{A}^* \right) \right) + \sum_{j \neq i} \sum_{\omega \in \Omega_{ji}^{\mathcal{L}}} R_{ji}^{\omega} \left(\mathbb{A}^* \right) + \sum_{\omega \in \Omega_{ii}^{\mathcal{L}}} R_{ii}^{\omega} \left(z_{ii}^{\omega,\text{seq}} \right).$$

Taking the difference of both equations,

$$0 = \Delta M^E \left(\sum_{k \in \mathcal{C}} r_{ik}^{\mathcal{N}} \right) + \sum_{\omega \in \Omega_{ii}^{\mathcal{L}}} \Delta R_{ii}^{\omega},$$

which determines that

$$\Delta M^E = -\frac{\sum_{\omega \in \Omega_{ii}^{\mathcal{L}}} \Delta R_{ii}^{\omega}}{\sum_{k \in \mathcal{C}} r_{ik}^{\mathcal{N}}}. \tag{A18}$$

Given that $\Delta R_{ii}^{\omega} > 0$ by Proposition 4.1, then $\Delta M^E < 0$. ∎

Proof of Proposition 4.4. By using (A16) when $r_{hf} \to 0$,

$$\Delta M_h^E = \frac{-\left(\sum_{\omega \in \Omega_{hh}^{\mathcal{L}}} \Delta R_{hh}^{\omega} \right) r_{ff}^{\mathcal{N}} + \left(\sum_{\omega \in \Omega_{ff}^{\mathcal{L}}} \Delta R_{ff}^{\omega} \right) r_{fh}^{\mathcal{N}}}{r_{hh}^{\mathcal{N}} r_{ff}^{\mathcal{N}}}.$$

We know that $\Delta R_{ii}^{\omega} > 0$ for any $i \in \mathcal{C}$ by Proposition 4.1. Then, if $r_{fh}^{\mathcal{N}} > r_{ff}^{\mathcal{N}} \frac{\sum_{\omega \in \Omega_{hh}^{\mathcal{L}}} \Delta R_{hh}^{\omega}}{\sum_{\omega \in \Omega_{ff}^{\mathcal{L}}} \Delta R_{ff}^{\omega}}$ then $\Delta M_h^E > 0$. ∎

Proof of Proposition 4.5. The fact that DL ω over-invest at home when (11), and under-invest when the inequality is strict and reversed follows directly by Lemma 4. Moreover, Lemma 2 shows that condition (11) is equivalent to (12). Thus, the result follows. ∎

Proof of Proposition 4.6. We use that the demand (14) is equivalent to (ces) once we define $z_{ij}^{\omega} := \lambda_{ij}^{\omega}$ and $\mathbb{A}_i := \mathbb{P}_i^{1-\sigma}$. The result regarding consumer welfare follows because the same $\mathbb{A}_i^*$ holds in both the simultaneous-moves and sequential-moves game for each country $i \in \mathcal{C}$. Thus, $\mathbb{P}_i^*$ is the same in both scenarios, and so the indirect utility given by (15) is the same in each scenario too.

As for producer surplus, CFFs satisfy zero expected profits and there is a continuum of them, so that their total profits are zero by applying a suitable law of large numbers. Furthermore, each DL garners greater profits by Proposition 4.1. Therefore, producer surplus increases. ∎

B Microfoundations of the Demand System

There are two approaches that rationalize (dem). The first one is related to the discrete-continuous choices by Dubin and McFadden (1984) and Hanemann (1984), which constitute an alternative to discrete-choice models. In this framework, agents consume only one variety, but are not restricted to consume either one or zero units of the good; instead, they can consume as much as they want of the variety chosen. To rationalize (dem), we sketch an argument based on Thisse and Ushchev (2016) for an industry in isolation among a continuum. The interested reader is referred to this paper and Nocke and Schutz (2018) for further details.

Consider a continuum of consumers in country j with mass normalized to unity, where each of them consumes only one variety and has a random utility. Moreover, conditional on a variety being chosen, suppose she allocates a fixed share of her income to the industry under analysis. This entails that, in case the variety ω is chosen, she consumes $\frac{E_j}{p_{ij}^{\omega}}$ units of it.

In order to generate our demand system based on this decision process, we define the (random) indirect utility for each variety in a way that the probability of choosing the variety ω ends up being

$$\Pr_{ij}(\omega) := \frac{a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right]}. \tag{B1}$$

Given that there is a continuum of agents, $\Pr_{ij}(\omega)$ also corresponds to the proportion of consumers from j that buy ω. Due to this, the aggregate demand of the variety ω is given by the total quantity consumed by each consumer, given by $\frac{E_j}{p_{ij}^{\omega}}$, times the proportion of consumers that buy ω, which is given by (B1). Formally,

$$Q_{ij}^{\omega} = \frac{E_j}{p_{ij}^{\omega}} \Pr_{ij}(\omega),$$

that coincides with (dem).

There is a second approach to obtain a functional form as in (dem). This corresponds to the axiomatic derivation by Bell et al. (1975) to model market shares. It gives rise to what is known as "attraction demand models", which are commonly used in the Marketing and Management literature.[19]

Bell et al. (1975) establish that there is only one functional form that market shares can take if they satisfy some intuitive axioms. Remarkably, the result is derived irrespective of whether market shares are expenditure- or quantity-based. Moreover, their result encompasses the choice probabilities derived by Luce (1959), which include the Multinomial Logit as a special case under quantity-based market shares.

To illustrate how this approach can generate our demand system, consider some variety ω produced in i and consumed in j. Moreover, suppose that the market share is defined in terms of expenditures, so that $s_{ij}^{\omega} := \frac{p_{ij}^{\omega} Q_{ij}^{\omega}}{E_i}$. Within this framework, a_{ij} is understood as an "attraction" function, and determines an allocation of market share for each variety. This is done by quantifying how appealing variety ω is according to its price and non-price features. Based on this, Bell et al. (1975) establishes that there is only one functional form that market shares can take if they satisfy some intuitive axioms. This is given by

$$s_{ij}^{\omega}\left(\mathbf{x}_{ij}^{\omega}, \mathbb{A}_j\left[(\mathbf{x}_{kj})_{k\in\mathcal{C}}\right]\right) = \frac{a_{ij}\left(\mathbf{x}_{ij}^{\omega}\right)}{\sum_{k\in\mathcal{C}}\left[\int\limits_{\omega\in\Omega_{kj}^{\mathcal{N}}} a_{kj}\left(\mathbf{x}_{kj}^{\omega}\right) \mathrm{d}\omega + \sum_{\omega\in\Omega_{kj}^{\mathcal{L}}} a_{kj}\left(\mathbf{x}_{kj}^{\omega}\right)\right]}. \tag{B2}$$

Inspecting (dem) shows that this coincides with the expenditure-based market share of our demand system, given by (9).

[19]See, for instance, Bernstein and Federgruen (2004), Gallego et al. (2006), and Huang et al. (2013).

C Adding Non-Negligible Followers

In the main part of the paper, we have considered a model with two types of firms: DLs and CFFs. However, it is possible to conceive industries with a third type of firms, which are non-negligible but do not behave as leaders at home. We refer to them as "large followers" (henceforth, LFs).

Next, we show that all our findings follow verbatim if we incorporate such a type of firm. Thus, DLs strategically use their domestic investments to strengthen competition at home and partially deter market exploration by CFFs. Furthermore, the increase in the DLs' domestic revenues is entirely at the expense of CFFs. In other words, the strategic use of investments only affects CFFs, and so LFs are not impacted by it.

With this goal, consider countries $i, j \in \mathcal{C}$. We modify the baseline setup by partitioning the set of potential conceivable varieties $\overline{\Omega}_i$ into three subsets: a finite subset $\overline{\mathscr{L}}_i$ comprising DLs from i, a finite subset $\overline{\mathcal{F}}_i$ comprising LFs, and a real interval $\overline{\mathcal{N}}_i$ comprising the negligible firms from i.

Moreover, we suppose that each LF produces a unique variety $\omega \in \overline{\mathcal{F}}_i$ and that each of them is more productive than any CFF, so that $\varphi_\omega > \overline{\varphi}_i$ for each $\omega \in \overline{\mathcal{F}}_i$. As in the case of DLs, this characterization of the LFs' productivity determines that they are active in both countries. Finally, although intuitively LFs should be modeled as less productive than DLs, assuming this is not necessary for deriving the results.

The characterization of equilibrium in the simultaneous-moves games is similar to the baseline model. It only requires incorporating that LFs make decisions as DLs. Thus, their decisions are characterized by (2) and (z-sim), which in turn determine that their backward-response functions are given by (3). As for the decisions of DLs and CFFs, they are identical to the baseline case. Furthermore, (FE) and (MS-sim) also hold, with the only difference that total revenues in i of firms from j are given by

$$R_{ji}\left(\mathbb{A}_i, M_j^E\right) := M_j^E r_{ji}^{\mathcal{N}}\left(\mathbb{A}_i\right) + \sum_{\omega \in \Omega_{ji}^{\mathscr{L}}} R_{ji}^{\omega}\left(\mathbb{A}_i\right) + \sum_{\omega \in \Omega_{ji}^{\mathcal{F}}} R_{ji}^{\omega}\left(\mathbb{A}_i\right).$$

Regarding the equilibrium in the sequential-moves game, the LFs' decisions are characterized exactly as in the simultaneous-moves games. They are given by (2) and (z-sim), with backward-response function given by (3).

The rest of the equilibrium conditions are the same, with the only difference that (8) has to be substituted by

$$E_i^{\text{seq}}\left(\mathbb{A}_i, \mathbf{M}^E, \mathbf{z}_{ii}^{\mathscr{L}}\right) := \sum_{k \in \mathcal{C}} M_k^E r_{ki}^{\mathcal{N}}\left(\mathbb{A}_i\right) + \sum_{\omega \in \Omega_{ji}^{\mathscr{L}}} R_{ji}^{\omega}\left(\mathbb{A}_i\right) + \sum_{\omega \in \Omega_{ii}^{\mathscr{L}}} R_{ii}^{\omega}\left[p_{ii}^{\omega}\left(z_{ii}^{\omega}, \mathbb{A}_i\right), z_{ii}^{\omega}, \mathbb{A}_i\right] + \sum_{k \in \mathcal{C}} \sum_{\omega \in \Omega_{ki}^{\mathcal{F}}} R_{ki}^{\omega}\left(\mathbb{A}_i\right), \tag{C1}$$

where $j \neq i$. Once we incorporate these modifications, all the proofs of the propositions follow verbatim. This is because, basically, LFs in each country play a similar role to that of foreign DLs at home. Thus, all their equilibrium variables end up being the same in the simultaneous- and sequential-moves games, including their prices, quantities, and revenues. We formally state this in the following proposition.

Proposition C.1. *Suppose a setup where there are DLs, CFFs, and LFs. Then, Propositions 4.1, 4.2, 4.3, 4.4, 4.5, and 4.6 hold.*