

6d. Useful Functions for Vectors

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INTRODUCTION

This section introduces a set of core functions for manipulating vectors. We focus on operations that arise frequently in data processing and numerical computing, such as sorting values, retrieving the indices that produce a sorted order, removing duplicates, counting occurrences, and computing rankings. The next section will show how these functions come together in a practical example.

SORTING VECTORS

The `sort` function arranges elements in ascending order by default, with the possibility of a descending order through the keyword argument `rev = true`. The function comes in two forms: `sort`, which returns a new sorted copy, and `sort!`, the in-place version that directly updates the vector.

SORT (ASCENDING)

```
x = [4, 5, 3, 2]
```

```
y = sort(x)
```

```
julia> y
4-element Vector{Int64}:
 2
 3
 4
 5
```

SORT (DESCENDING)

```
x = [4, 5, 3, 2]
```

```
y = sort(x, rev=true)
```

```
julia> y
4-element Vector{Int64}:
 5
 4
 3
 2
```

SORT!

```
x = [4, 5, 3, 2]
```

```
sort!(x)
```

```
julia> x
```

```
4-element Vector{Int64}:
```

```
2
```

```
3
```

```
4
```

```
5
```

Both `sort(x)` and `sort!(x)` have the option of defining the sorting order based on transformations of `x`. Specifically, given a function `foo`, the elements can be ordered by the values of `foo(x)`. Its implementation requires the keyword argument `by`.

SORT - ABSOLUTE

```
x      = [4, -5, 3]
```

```
y      = sort(x, by = abs)      # 'abs' computes the absolute value
```

```
julia> abs.(x)
```

```
3-element Vector{Int64}:
```

```
4
```

```
5
```

```
3
```

```
julia> y
```

```
3-element Vector{Int64}:
```

```
3
```

```
4
```

```
-5
```

SORT - QUADRATIC

```
x      = [4, -5, 3]
```

```
foo(a) = a^2
```

```
y      = sort(x, by = foo)      # same as sort(x, by = x -> x^2)
```

```
julia> foo.(x)
```

```
3-element Vector{Int64}:
```

```
16
```

```
25
```

```
9
```

```
julia> y
```

```
3-element Vector{Int64}:
```

```
3
```

```
4
```

```
-5
```

SORT - NEGATIVE

```
x      = [4, -5, 3]

foo(a) = -a
y      = sort(x, by = foo)      # same as sort(x, by = x -> -x)

julia> foo.(x)
3-element Vector{Int64}:
-4
5
-3

julia> y
3-element Vector{Int64}:
4
3
-5
```

RETRIEVING INDICES OF SORTED ELEMENTS

While `sort(x)` returns the ordered *values* of `x`, it's also useful to obtain the *indices* of the sorted elements. This capability is provided by the function `sortperm`, which returns the indices of `x` that would result in `sort(x)`. In other words, `x[sortperm(x)] == sort(x)` evaluates to `true`.¹

EXAMPLE 1

```
x      = [1, 2, 3, 4]

sort_index = sortperm(x)

julia> sort_index
4-element Vector{Int64}:
1
2
3
4
```

EXAMPLE 2

```
x      = [3, 4, 5, 6]

sort_index = sortperm(x)

julia> sort_index
4-element Vector{Int64}:
1
2
3
4
```

EXAMPLE 3

```
x          = [1, 3, 4, 2]
sort_index = sortperm(x)

julia> sort_index
4-element Vector{Int64}:
 1
 4
 2
 3
```

In the first two examples, the elements are already in ascending order, so `sortperm` returns the trivial permutation `[1, 2, 3, 4]`. In contrast, the last example features an unordered vector `x = [1, 3, 4, 2]`. Thus, the resulting vector `[1, 4, 2, 3]` indicates that the smallest element appears at index 1, the second smallest at index 4, the third smallest at index 2, and the largest at index 3.

Like `sort`, `sortperm` also supports retrieving indices in descending order. This requires including the keyword argument `rev = true`.

EXAMPLE 1

```
x          = [9, 3, 2, 1]
sort_index = sortperm(x, rev=true)

julia> sort_index
4-element Vector{Int64}:
 1
 2
 3
 4
```

EXAMPLE 2

```
x          = [9, 5, 3, 1]
sort_index = sortperm(x, rev=true)

julia> sort_index
4-element Vector{Int64}:
 1
 2
 3
 4
```

EXAMPLE 3

```
x      = [9, 3, 5, 1]

sort_index = sortperm(x, rev=true)

julia> sort_index
4-element Vector{Int64}:
 1
 3
 2
 4
```

Finally, `sortperm` also accepts the keyword argument `by` to define a custom transformation.

SORT - ABSOLUTE

```
x      = [4, -5, 3]

value  = sort(x, by = abs)      # 'abs' computes the absolute value
index  = sortperm(x, by = abs)

julia> abs.(x)
3-element Vector{Int64}:
 4
 5
 3

julia> value
3-element Vector{Int64}:
 3
 4
 -5

julia> index
3-element Vector{Int64}:
 3
 1
 2
```

SORT - QUADRATIC

```
x      = [4, -5, 3]

foo(a) = a^2
value  = sort(x, by = foo)      # same as sort(x, by = x -> x^2)
index  = sortperm(x, by = foo)
```

```
julia> foo.(x)
3-element Vector{Int64}:
 16
 25
  9
```

```
julia> value
3-element Vector{Int64}:
 3
 4
 -5
```

```
julia> index
3-element Vector{Int64}:
 3
 1
 2
```

SORT - NEGATIVE

```
x      = [4, -5, 3]

foo(a) = -a
value  = sort(x, by = foo)      # same as sort(x, by = x -> -x)
index  = sortperm(x, by = foo)
```

```
julia> foo.(x)
3-element Vector{Int64}:
 -4
  5
 -3
```

```
julia> value
3-element Vector{Int64}:
 4
 3
 -5
```

```
julia> index
3-element Vector{Int64}:
 1
 3
 2
```

AN EXAMPLE

One common application of `sortperm` is to reorder one variable based on the values of another. For example, suppose we want to assess the daily failures of a machine. Focusing on the first three days of the month, the following code snippet ranks these days by their corresponding failure counts.

DAYS SORTED BY LOWEST NUMBER OF FAILURES

```
days          = ["one", "two", "three"]
failures     = [8, 2, 4]

index         = sortperm(failures)
days_by_failures = days[index]      # days sorted by lowest failures
```

```
julia> index
3-element Vector{Int64}:
2
3
1

julia> days_by_earnings
3-element Vector{String}:
"two"
"three"
"one"
```

REMOVING DUPLICATES

The function `unique` removes duplicate entries from a vector, returning a new vector that contains each element exactly once. The function comes in two variants: `unique`, which produces a new copy, and `unique!`, which performs the operation in place and thus modifies the original vector.

UNIQUE

```
x = [2, 2, 3, 4]

y = unique(x)      # returns a new vector
```

```
julia> x
4-element Vector{Int64}:
2
2
3
4

julia> y
3-element Vector{Int64}:
2
3
4
```

UNIQUE!

```
x = [2, 2, 3, 4]

unique!(x)          # mutates 'x'
```

```
julia> x
3-element Vector{Int64}:
 2
 3
 4
```

The `StatsBase` package provides a related function called `countmap`, which counts the occurrences of each element in a vector. It returns a dictionary in which the unique elements act as keys, and their corresponding values represent the number of times each element appears.

By default, the keys in the resulting dictionary are unsorted. If instead sorted keys are preferred, you must apply the `sort` function to the result. This will automatically convert an ordinary dictionary into an object with type `OrderedDict`.

UNSORTED COUNT

```
using StatsBase
x           = [6, 6, 0, 5]

y           = countmap(x)          # Dict with 'element => occurrences'

elements   = collect(keys(y))
occurrences = collect(values(y))
```

```
julia> y
Dict{Int64, Int64} with 3 entries:
 0 => 1
 5 => 1
 6 => 2

julia> elements
3-element Vector{Int64}:
 0
 5
 6

julia> occurrences
3-element Vector{Int64}:
 1
 1
 2
```

SORTED COUNT

```

using StatsBase
x           = [6, 6, 0, 5]

y           = sort(countmap(x))      # OrderedDict with 'element => occurrences'

elements   = collect(keys(y))
occurrences = collect(values(y))

julia> y
OrderedCollections.OrderedDict{Int64, Int64} with 3 entries:
 0 => 1
 5 => 1
 6 => 2

julia> elements
3-element Vector{Int64}:
 0
 5
 6

julia> occurrences
3-element Vector{Int64}:
 1
 1
 2

```

ROUNDING NUMBERS

Julia provides standard functions for approximating numerical values to a specified precision:

- `round` approximates a number to its nearest integer.
- `floor` returns the greatest integer less than or equal to the given number.
- `ceil` returns the smallest integer greater than or equal to the given number.

Below, we show that these functions are quite flexible, allowing users to specify the output type (e.g., `Int64` or `Float64`), the number of decimal places via the keyword argument `digits`, and the number of significant digits.

ROUND

```

x = 456.175

round(x)                      # 456.0

round(x, digits=1)             # 456.2
round(x, digits=2)             # 456.18

round(Int, x)                  # 456

round(x, sigdigits=1)          # 500.0
round(x, sigdigits=2)          # 460.0

```

FLOOR

```
x = 456.175

floor(x)                      # 456.0

floor(x, digits=1)            # 456.1
floor(x, digits=2)            # 456.17

floor(Int, x)                 # 456

floor(x, sigdigits=1)         # 400.0
floor(x, sigdigits=2)         # 450.0
```

CEIL

```
x = 456.175

ceil(x)                       # 457.0

ceil(x, digits=1)             # 456.2
ceil(x, digits=2)             # 456.18

ceil(Int, x)                  # 457

ceil(x, sigdigits=1)          # 500.0
ceil(x, sigdigits=2)          # 460.0
```

RANKINGS

Instead of sorting a vector, you may be interested in determining the rank position of each element. The `StatsBase` package offers two functions for this purpose: `competerank` and `ordinalrank`. Their main difference them lies in how they treat tied values: `competerank` assigns the same rank to all tied elements, while `ordinalrank` assigns consecutive ranks. In both cases, a rank of 1 corresponds to the smallest value. The keyword argument `rev = true` reverses this convention, assigning a rank of 1 to the largest value.

RANK (SAME RANK FOR TIES)

```
using StatsBase
x = [6, 6, 0, 5]

y = competerank(x)
```

```
julia> y
4-element Vector{Int64}:
3
3
1
2
```

DESCENDING RANK (SAME RANK FOR TIES)

```
using StatsBase
x = [6, 6, 0, 5]

y = competerank(x, rev=true)
```

```
julia> y
4-element Vector{Int64}:
1
1
4
3
```

RANK (UNIQUE POSITIONS)

```
using StatsBase
x = [6, 6, 0, 5]

y = ordinalrank(x)
```

```
julia> y
4-element Vector{Int64}:
3
4
1
2
```

DESCENDING RANK (UNIQUE POSITIONS)

```
using StatsBase
x = [6, 6, 0, 5]

y = ordinalrank(x, rev=true)
```

```
julia> y
4-element Vector{Int64}:
1
2
4
3
```

Do not confuse `ordinalrank` and `sortperm`

The function `ordinalrank` indicates the position of each value in the *sorted* vector. Instead, `sortperm` indicates the position of each value in the *unsorted* vector.

'ORDINALRANK'

```
using StatsBase
x = [3, 1, 2]

y = ordinalrank(x)

julia> y
3-element Vector{Int64}:
 3
 1
 2
```

'SORTPERM'

```
using StatsBase
x = [3, 1, 2]

y = sortperm(x)

julia> y
3-element Vector{Int64}:
 2
 3
 1
```

EXTREMA (MAXIMUM AND MINIMUM)

We conclude by presenting a method for identifying both the indices and the values of extrema within a collection. The following examples are based on the maximum, with similar functions available for the minimum.

VALUE

```
x = [6, 6, 0, 5]

y = maximum(x)

julia> y
6
```

INDEX

```
x = [6, 6, 0, 5]

y = argmax(x)

julia> y
1
```

VALUE AND INDEX

```
x = [6, 6, 0, 5]
y = findmax(x)

julia> y
(6, 1)
```

Julia additionally provides the function `max` and `min`, which respectively return the maximum and minimum of their *arguments*. These functions will become particularly useful in procedures built on binary operations that we'll cover later (e.g., reductions).

'MAX' FUNCTION

```
x = 3
y = 4

z = max(x,y)

julia> z
4
```

FOOTNOTES

¹. The name `sortperm` originates from "sorting permutation". Although the name might seem somewhat opaque, it arises because the operation returns the permutation of indices that would sort the original vector.