CSandPile 0.5's Manual

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This document describes the version 0.5 of the program *CSandPile*, a program that allows to compute the sandpile group of a symmetric multidigraph G. The *CSandPile* program was programmed in C++ language using the GNU Compiler Collection and had been used over Windows XP, MAC OS 10.5.8 and Ubuntu Linux 9.04.

The *CSandPile* program version 0.5 is mainly a tool for compute the group operations of the recurrent representatives of non-negative configurations of G, that is, is the first effort to find the combinatorial structure of the group operations of the recurrent configurations that generate the sandpile group of G.

If c be a non-negative configuration, then using *CSandPile*, one may compute the following:

- the stabilization of c,
- the recurrent representative of c,
- the powers of a recurrent configuration,
- the representative of the inverse of a recurrent configuration c,
- the recurrent representative of the identity of the sandpile group of G,
- the determinant of the Laplacian matrix of G,
- the powers of the representative of the canonical base.

1 The structure of *CSandPile*

*CSandPile*consists of the file `csandpile.exe` the executable file if you are using a Windows environment, or `csandpile.out` the executable file if you are using a UNIX environment. Also, you need to have an input file `<my project>.gph` with the input data.

1.1 The input file

The input file `<my project>.gph` is structured as follows: The first line contains the order of the Laplacian matrix of G, the next lines contain the rows of the Laplacian matrix of G, the next line contain the vertex of G that will play the role of the sink, and finally the last line contains some configuration.

Example 1. *Let G be the cycle C_4 with four vertices, therefore the Laplacian matrix of C_4 is given by:*

$$L(C_4) = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

The file called "c4.gph" has the order of the Laplacian matrix of C_4, the Laplacian matrix of C_4, the vertex 4 as a sink and the vector $(2, 2, 1, 0)$ as the configuration.

```
4
2 -1 0 -1
-1 2 -1 0
0 -1 2 -1
-1 0 -1 2
4
2 2 1 0
```

If we do not write a configuration, the configuration $\sigma_{MAX} = (\deg(1)-1, \deg(2)-1, ..., \deg(n)-1)$ *will be taken by default.*

1.2 Running *CSandPile*

The files `csandpile.exe` and `csandpile.out` are the executables files in Windows and UNIX, respectively. You must run it from the console of your operating system. The syntax for calling *CSandPile* is

`csandpile <my project> [-option]`

where -option can be one of the following options:

```
-s           to obtain the stable configuration
-p           to obtain the powers of the recurrent configuration
-i           to obtain the identity
-r           to obtain the recurrent configuration
-ri          to obtain the inverse recurrent configuration
-det         to obtain the determinant of the reduced Laplacian matrix
-group       to obtain the powers of the standard base
-complete n  to create the Laplacian matrix of the complete graph of n
             vertices
-path n      to create the Laplacian matrix of the path of n vertices
-cycle n     to create the Laplacian matrix of the cycle of n vertices
```

When you type and execute `csandpile` or `./csandpile.out` if you are working in a UNIX environment on the console, the program creates a file called `<my project>.csp`.

For instance, if you want to obtain the stable configuration, you need to type

`csandpile <my project> -s` or `./csandpile.out <my project> -s`.

and *CSandPile* will create a file called `<my project>.csp`. Thus, using "c4.gph" as an input file, the file `c4.csp` contains the following information:

```
The stable configuration of
2 2 1 S
is
0 1 1 S
```

Note that we put a `S` in the coordinate of the sink.

To obtain the powers of a recurrent configuration, you need to type `-p` as option. Using "c4.gph" as an input file, the `c4.csp` file contains

```
Checking configuration: 0 1 1 S
Powers
1 - 0 1 1 S
2 - 1 1 1 S
3 - 1 1 0 S
4 - 1 0 1 S
```

To obtain the identity configuration, you need to type `-i` as an option. Using "c4.gph" as an input file, the `c4.csp` file contains

```
Identity: 1 0 1 S
```

To obtain the recurrent configuration of the configuration given in the `<my project>.gph`, you need to type `-r` as option. Using "c4.gph" as an input file, the `c4.csp` file contains

```
The recurrent configuration of 2 2 1 S
is 0 1 1 S
```

To obtain the recurrent inverse configuration of the configuration given in the `<my project>.gph`, you need to type `-ri` as option. Using "c4.gph" as an input file, the file `c4.csp` contains

```
Inverse recurrent configuration: 1 1 0 S
```

To obtain the determinant of the reduced Laplacian matrix, you need to type `-det` as option. Using "c4.gph" as an input file, the `c4.csp` file contains

```
Determinant: 4
```

To obtain the powers of the canonical base, you need to type `-group` as option. Using "c4.gph" as an input file, the `c4.csp` file contains

```
Generator 1: 1 0 0 S
Checking configuration: 0 1 1 S
Powers
1 - 0 1 1 S
2 - 1 1 1 S
3 - 1 1 0 S
4 - 1 0 1 S

Generator 2: 0 1 0 S
Checking configuration: 1 1 1 S
Powers
1 - 1 1 1 S
2 - 1 0 1 S

Generator 3: 0 0 1 S
Checking configuration: 1 1 0 S
Powers
1 - 1 1 0 S
2 - 1 1 1 S
3 - 0 1 1 S
4 - 1 0 1 S
```

1.3 Some special graphs

Also, *CSandPile* can generate the Laplacian matrix of the complete graph of n vertices, the path of n vertices and the cycle of n vertices and write it in the `<my project>.gph` file. This can be done by typing `-complete` *n*, `-cycle` *n* or `-path` *n* as an option, respectively. For instance, if you write

```
csandpile k4 -complete 4
```

you will obtain the Laplacian matrix of the complete graph of 4 vertices in the `k4.gph` file.

```
4
3 -1 -1 -1
-1 3 -1 -1
-1 -1 3 -1
-1 -1 -1 3
4
```

Note that by default, *CSandPile* will define the vertex n as the sink.