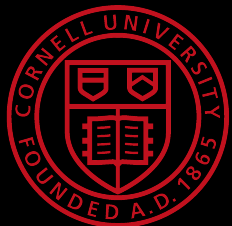


Counterfactual Learning/ Off-Policy Learning

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Batch Learning from Bandit Feedback

- Data

context

π_0 action

reward / loss

propensity

$$D_0 = ((x_1, a_1, r_1, p_1), \dots, (x_n, a_n, r_n, p_n))$$

→ Partial Information (aka “Bandit”) Feedback

- Properties

- Contexts x_i drawn i.i.d. from unknown $P(x)$
- Actions a_i selected by logging policy $\pi_0(a|x_i)$
- Feedback r_i from unknown $P(r|x_i, a_i)$
- Propensity p_i of selected action a_i under π_0

Task of Learning

Use interaction log data from logging policy π_0

$$D_0 = ((x_1, a_1, r_1, p_1), \dots, (x_n, a_n, r_n, p_n))$$

for



— Evaluation:

- Estimate online performance of some new policy π_e offline.
- Policy π_e is typically different from π_0 that generated log.



— Learning:

- Find new policy π that improves performance over π_0 .
- Do not rely on interactive experiments like in online learning.

Learning Settings

	Full-Information (Labeled) Feedback	Partial-Information (e.g. Bandit) Feedback
Online Learning	<ul style="list-style-type: none">• Perceptron• Winnow• Etc.	<ul style="list-style-type: none">• EXP3• UCB1• Etc.
Batch Learning	<ul style="list-style-type: none">• SVM• Random Forests• Etc.	?

Batch Learning
from Bandit
Feedback (BLBF)

Goal of Learning

- Given:
 - Log data $D_0 = ((x_1, a_1, r_1, p_1), \dots, (x_n, a_n, r_n, p_n))$
 - Hypothesis space H of possible policies π
- Find: Policy $\pi \in H$ that has maximum value

$$V(\pi) = \int \int \int r P(r|x, a) \pi(a|x) P(x) dx da dr$$

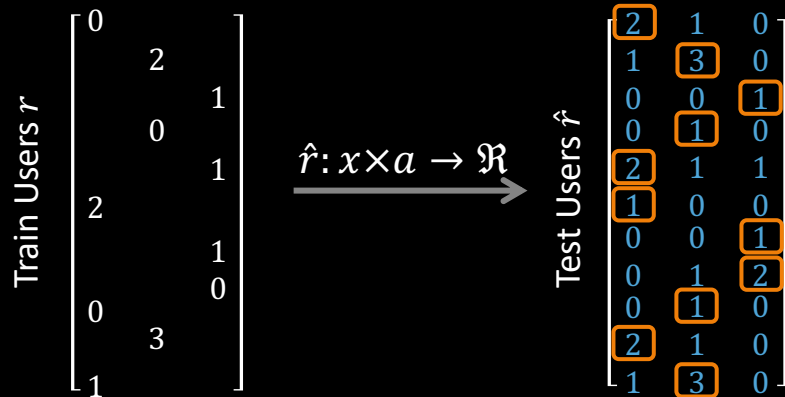
→ Optimize online metric offline.

Learning: Outline

- Goal: Optimizing online metrics offline
- • Approach 1: Model-Based Learning
 - Derive policy from predicted rewards
- Approach 2: Model-Free Learning
 - ERM via IPS: Reduction to weighted multi-class classification
- Revisiting the Variance Issue
 - CRM via IPS: Variance regularized ERM for stochastic rules (POEM)
 - CRM via SNIPS: Avoiding propensity overfitting (NormPOEM, BanditNet)
- Learning to Rank (LTR)
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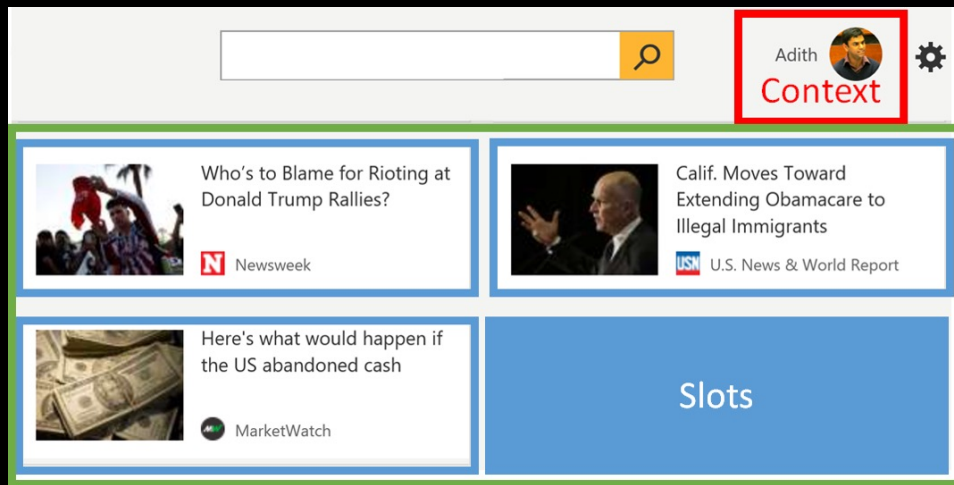
Model-Based Approach: Reward Predictor

- Given:
 - Log data $D_0 = ((x_1, y_1, \delta_1, p_1), \dots, (x_n, y_n, \delta_n, p_n))$ from π_0
 - Design reward model $\hat{r}: x \times a \rightarrow \mathfrak{R}$ for regression
- Algorithm:
 - Train reward predictor
 $\hat{r}: x \times a \rightarrow \mathfrak{R}$ using D_0
 - Derive policy
 $\hat{\pi}(x) \equiv \operatorname{argmax}_a \{\hat{r}(x, a)\}$



News Recommender: Exp Setup

- Context x : User profile
- Action a : Slate
 - Pick from 7 candidates to place into 3 slots
- Reward r : “Revenue”
 - Complicated hidden function
- Logging policy π_0 : Non-uniform randomized logging system
 - Placket-Luce “explore around current production policy”



News Recommender: Results

- Reward Predictor:
 - Features: Stacked features of three articles
 - Regression method: selected best via CV from {Ridge, Lasso, Least Squares, Decision Trees}

Approach	True Revenue
Production policy	224.00
Randomized logging policy π_0	214.00

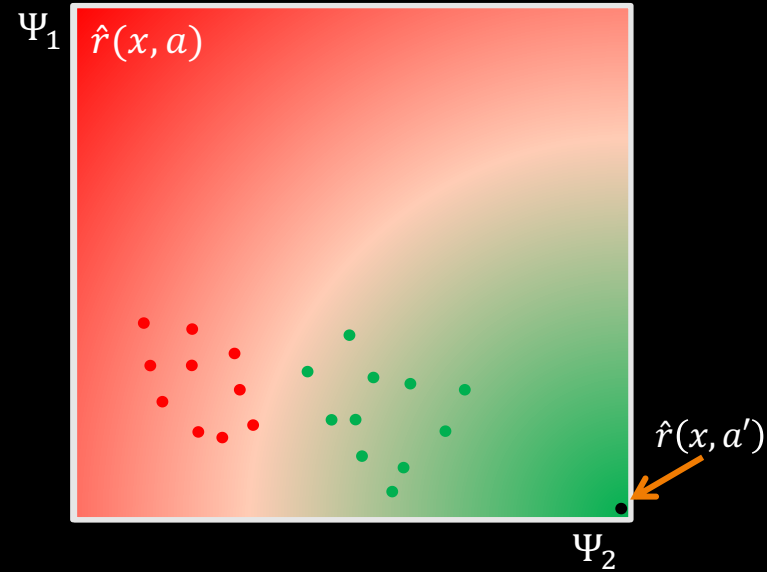
Issues with Reward Predictor

Issue 1:


- Model misspecification \rightarrow biased and not consistent

Issue 2:

- First solves hard problem (reward prediction) in order to solve easier problem (find good policy)
 - Predict correct rewards \longrightarrow optimal policy
 - Optimal policy \longrightarrow predict correct rewards



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Empirical Risk Minimization

Empirical Risk Minimization (ERM) with Regularization:

Given hypothesis space H of policies $\pi: x \rightarrow a$, find

$$\hat{\pi} = \operatorname{argmax}_{\pi \in H} [\hat{V}(\pi) - \operatorname{Reg}(\pi)]$$

→ Same as SVMs, Neural Nets, Boosted Trees, etc

Questions for learning from log data:

- What estimator to use for $\hat{V}(\pi)$?
- What regularizer $\operatorname{Reg}(\pi)$ to use?
- Deterministic vs. Stochastic policies π ?
- How to solve argmax ?

ERM with IPS Estimator

- Given:
 - Log $D_0 = ((x_1, a_1, r_1, p_1), \dots, (x_n, a_n, r_n, p_n))$ from π_0
 - Deterministic policies $\pi \in H: a = \pi(x)$
- Training:

$$\begin{aligned}\hat{\pi} &:= \operatorname{argmax}_{\pi \in H} \left\{ \frac{1}{n} \sum_i^n \frac{I\{a_i = \pi(x_i)\}}{p_i} r_i \right\} \\ &= \operatorname{argmax}_{\pi \in H} \left\{ \frac{1}{n} \sum_i^n \frac{r_i}{p_i} I\{a_i = \pi(x_i)\} \right\}\end{aligned}$$

Deterministic $\pi \rightarrow$ Multi-class ERM

- Treat π as a classifier with weighted loss
 $(x, a, r, p) \rightarrow (x, a, w); w = r/p$
- Policy utility is same as weighted accuracy!

$$V(\pi) = E_{x,a,r}[w I\{\pi(x) = a\}]$$

→ Use weighted multi-class algorithm to pick π .
(e.g., Vowpal Wabbit (VW), Open Bandit Pipeline)

Summary: ERM via IPS

- Empirical Risk Minimization (ERM) with Regularization:
 - What estimator to use for $\hat{V}(\pi)$?
 - VW: IPS or Doubly Robust
 - What regularizer $Reg(\pi)$ to use?
 - Standard regularizers to prevent overfitting
 - Deterministic vs. stochastic π ?
 - Deterministic
 - How to solve argmax?
 - Reduce to multi-class classification, use off-the-shelf algos

News Recommender: Results

- VW: Reduce to multi-class filter tree, doubly robust estimator with ridge regression, default parameters, 4 epochs via CV

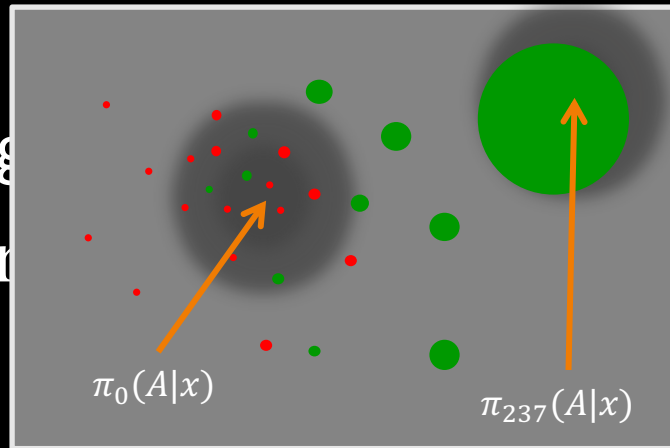
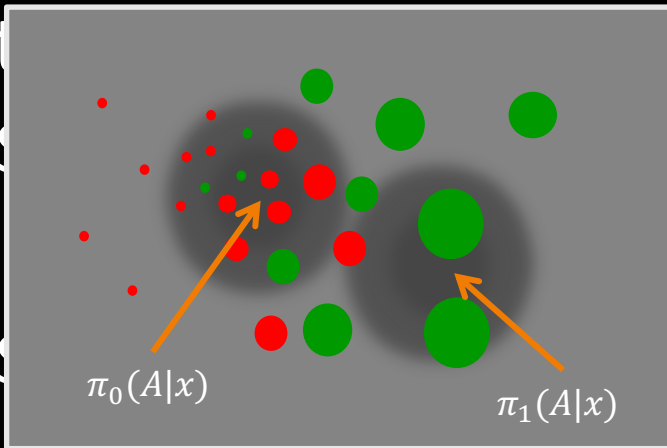
Approach	Revenue
Production ranker	224.00
Randomized π_0	214.00
Reward predictor	175.71
ERM via IPS (VW)	177.93

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Issues of ERM with IPS

- Set



- Training

$$\hat{\pi} := \operatorname{argmin}_{\pi \in H} \sum_i^n \frac{\pi(a_i|x_i)}{p_i} r_i$$

Generalization Error Bound for BLBF

Theorem [Generalization Error Bound]

For any hypothesis space H with capacity C , and for all $\pi \in H$ with probability $1 - \eta$

$$V(\pi) \leq \hat{V}(\pi) + O\left(\sqrt{\widehat{Var}(\pi)/n}\right) + O(C)$$

Unbiased
Estimator

Variance
Control

Capacity
Control

$$\hat{V}(\pi) = \widehat{Mean}\left(\frac{\pi(a_i|x_i)}{p_i} r_i\right)$$

$$\widehat{Var}(\pi) = \widehat{Var}\left(\frac{\pi(a_i|x_i)}{p_i} r_i\right)$$

→ Bound accounts for the fact that variance of risk estimator can vary greatly between different $\pi \in H$

Counterfactual Risk Minimization

- Theorem [Generalization Error Bound]

$$V(\pi) \leq \hat{V}(\pi) + O\left(\sqrt{\widehat{Var}(\pi)/n}\right) + O(C)$$

→ Constructive principle for designing learning algorithms

$$\pi^{crm} = \operatorname{argmin}_{\pi \in H_i} \hat{V}(\pi) + \lambda_1 \left(\sqrt{\widehat{Var}(\hat{V}(\pi))/n} \right) + \lambda_2 C(H_i)$$

$$\hat{V}(\pi) = \frac{1}{n} \sum_i^n \frac{\pi(a_i|x_i)}{p_i} r_i$$

$$\widehat{Var}(\hat{V}(\pi)) = \frac{1}{n} \sum_i^n \left(\frac{\pi(a_i|x_i)}{p_i} r_i \right)^2 - \hat{V}(\pi)^2$$

POEM Hypothesis Space

Hypothesis Space: Stochastic policies

$$\pi_w(a|x) = \frac{1}{Z(x)} \exp(w \cdot \Phi(x, a))$$

with

- w : parameter vector to be learned
- $\Phi(x, a)$: joint feature map between context and action
- $Z(x)$: partition function

POEM Learning Method

- Policy Optimizer for Exponential Models (POEM)
 - Data: $S = ((x_1, a_1, r_1, p_1), \dots, (x_n, a_n, r_n, p_n))$
 - Hypothesis space: $\pi_w(a|x) = \exp(w \cdot \phi(x, a))/Z(x)$
 - Training objective: Let $z_i(w) = \pi_w(a_i|x_i)r_i/p_i$

$$w = \operatorname{argmin}_{w \in \mathbb{R}^N} \left[\underbrace{\frac{1}{n} \sum_{i=1}^n z_i(w)}_{\text{Unbiased Risk Estimator}} + \lambda_1 \sqrt{\underbrace{\left(\frac{1}{n} \sum_{i=1}^n z_i(w)^2 \right)}_{\text{Variance Control}} - \underbrace{\left(\frac{1}{n} \sum_{i=1}^n z_i(w) \right)^2}_{\text{Capacity Control}}} + \lambda_2 \|w\|^2 \right]$$

Unbiased Risk
Estimator

Variance
Control

Capacity
Control

Summary: CRM via IPS


- Counterfactual Risk Minimization (CRM) :
 - What estimator to use for $\hat{V}(\pi)$?
 - IPS (or Doubly Robust)
 - What regularizer $Reg(\pi)$ to use?
 - Variance regularization to control unequal IPS variance
 - Standard regularizers to prevent overfitting
 - Deterministic vs. stochastic π ?
 - Stochastic policy to have fine-grained control of variance
 - How to solve argmax?
 - Gradient descent (or SGD with repeated majorization)

Does Variance Regularization Improve Generalization?

- IPS: $w = \operatorname{argmin}_{w \in \mathcal{R}^N} [\hat{V}(w) + \lambda_2 ||w||^2]$
- POEM: $w = \operatorname{argmin}_{w \in \mathcal{R}^N} \left[\hat{V}(w) + \lambda_1 \left(\sqrt{\widehat{Var}(w)/n} \right) + \lambda_2 ||w||^2 \right]$

Hamming Loss	Scene	Yeast	TMC	LYRL
π_0	1.543	5.547	3.445	1.463
IPS	1.519	4.614	3.023	1.118
POEM	1.143	4.517	2.522	0.996
# examples	4*1211	4*1500	4*21519	4*23149
# features	294	103	30438	47236
# labels	6	14	22	4

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Problem: Propensity Overfitting

- Example

- Losses $r(x, a)$:
(blue boxes observed in training)

- Which $\pi(a|x)$ minimizes IPS?

$$\hat{V}(\pi) = \min_{\pi \in H} \frac{1}{n} \sum_i^n \frac{\pi(a_i|x_i)}{p_i} r_i$$

→ Avoid the training observations!

→ Overfitting the choices of the logging policy π_0 .

0	1	1	1	1	1	1
1	1	1	0	1	1	1
1	0	1	1	1	1	1
1	1	1	1	1	1	0
1	1	0	1	1	1	1
1	1	1	1	1	1	0
1	1	1	0	1	1	1
a						

Control Variate

- Idea: Identify propensity overfitting through control variate.

$$\hat{V}(\pi) = \frac{1}{n} \sum_i^n \frac{\pi(a_i|x_i)}{p_i} r_i \quad \hat{S}(\pi) = \frac{1}{n} \sum_i^n \frac{\pi(a_i|x_i)}{p_i} 1$$

- Correlated $\hat{S}(\pi)$ has known expectation:

$$E[\hat{S}(\pi)] = \frac{1}{n} \sum_i^n \int \frac{\pi(a_i|x_i)}{\pi_0(a_i|x_i)} \pi_0(a_i|x_i) P(x) da_i dx_i = 1$$

→ SNIPS estimator naturally corrects for propensity overfitting

$$\hat{V}^{SNIPS}(\pi) = \frac{\hat{V}(\pi)}{\hat{S}(\pi)}$$

SNIPS-POEM Learning Method

- Method:
 - Data: $D_0 = ((x_1, a_1, r_1, p_1), \dots, (x_n, a_n, r_n, p_n))$
 - Hypothesis space: $\pi_w(y|x) = \exp(w \cdot \phi(x, a))/Z(x)$
 - Training objective:

$$w = \operatorname{argmin}_{w \in \mathbb{R}^N} \left[\hat{V}^{SNIPS}(w) + \lambda_1 \sqrt{\widehat{Var}(\hat{V}^{SNIPS}(w))} + \lambda_2 ||w||^2 \right]$$

Self-Normalized
Risk Estimator

Variance
Control

Capacity
Control

SNIPS-POEM vs. IPS-POEM

Hamming Loss	Scene	Yeast	TMC	LYRL
π_0	1.511	5.577	3.442	1.459
IPS-POEM	1.200	4.520	2.152	0.914
SNIPS-POEM	1.045	3.876	2.072	0.799
Control Variate $\hat{E}[s_i]$				
IPS-POEM	1.782	5.352	2.802	1.230
SNIPS-POEM	0.981	0.840	0.941	0.945

BanditNet: Hypothesis Space

Hypothesis Space: Stochastic policies

$$\pi_w(a|x) = \frac{1}{Z(x)} \exp(\text{DeepNet}(x, a|w))$$

with

- w : parameter tensors to be learned
- $Z(x)$: partition function

Note: same form as Deep Net with softmax output

BanditNet: Learning Method

- Method:

- Data: $D_0 = ((x_1, a_1, r_1, p_1), \dots, (x_n, a_n, r_n, p_n))$
- Hypotheses: $\pi_w(a|x) = \exp(\text{DeepNet}(x, a|w))/Z(x)$
- Training objective: Optimize via SGD after reformulation

$$w = \operatorname{argmin}_{w \in \mathbb{R}^N} \left[\hat{V}^{SNIPS}(w) + \lambda_1 \sqrt{\widehat{Var}(\hat{V}^{SNIPS}(w))} + \lambda_2 ||w||^2 \right]$$

Self-Normalized
Risk Estimator

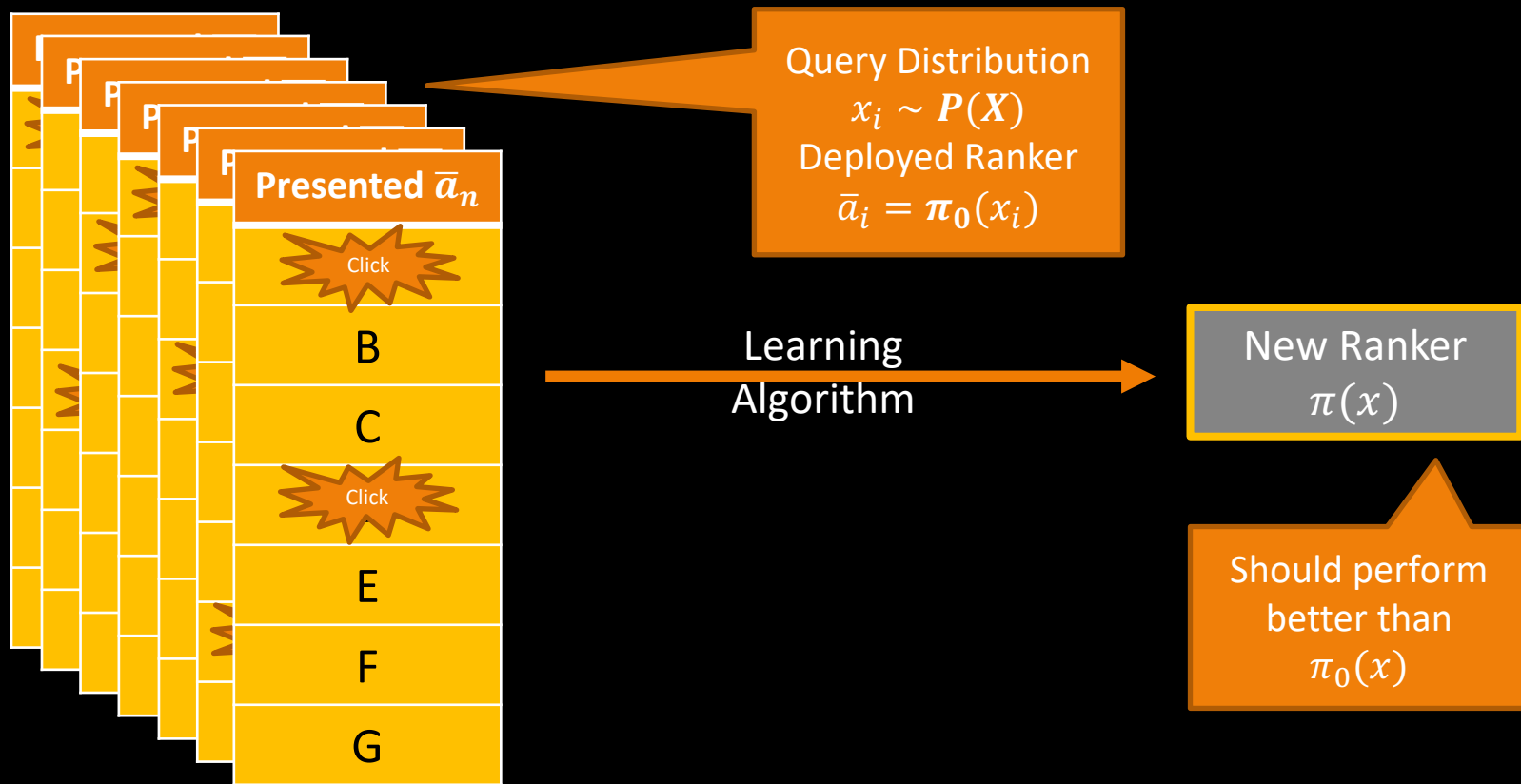
Variance
Control

Capacity
Control

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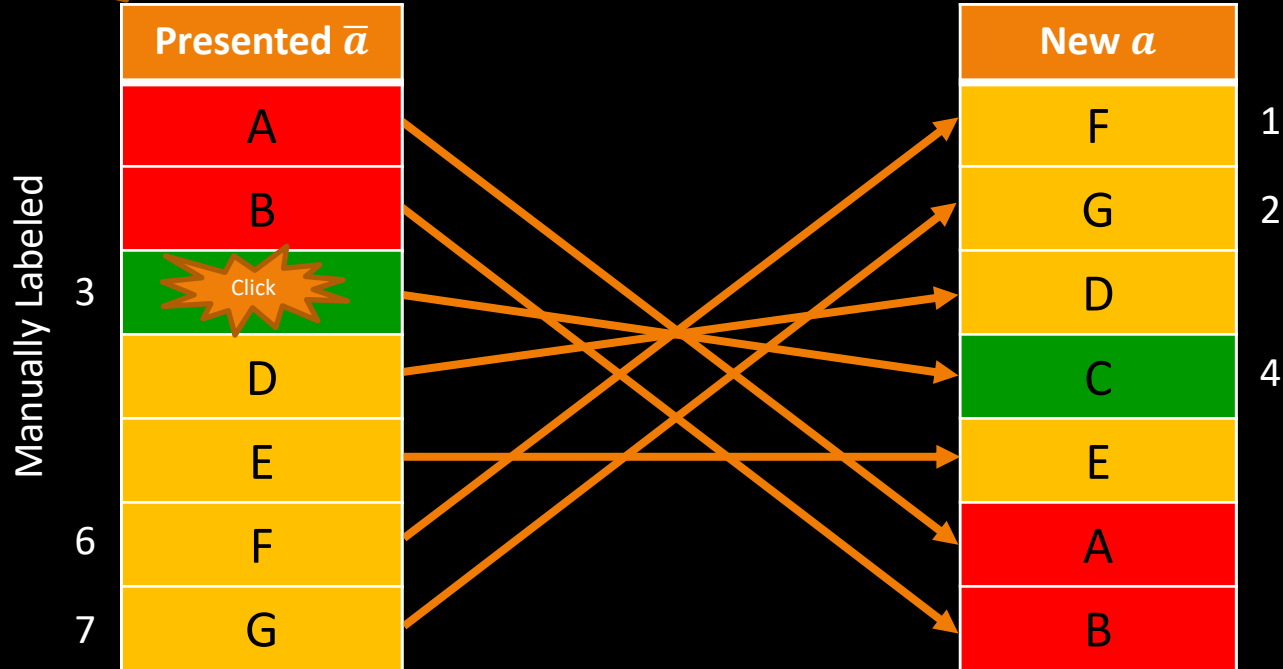
Learning-to-Rank from Clicks



Evaluating Rankings

Deployed Ranker
 $\bar{a} = \pi_0("SVM")$

New Ranker to Evaluate
 $a = \pi("SVM")$



Evaluation with Missing Judgments

- Loss: $r(x, a | r^*)$

- Relevance labels $r_d^* \in \{0, 1\}$
- This talk: rank of relevant documents

$$r(x, a | r^*) = \sum_d \text{rank}(d | a) \cdot r_d^*$$

- Assume:

- Click implies observed and relevant:

$$(c_d = 1) \leftrightarrow (o_d = 1) \wedge (r_d^* = 1)$$

- Problem:

- No click can mean not relevant OR not observed

$$(c_d = 0) \leftrightarrow (o_d = 0) \vee (r_d^* = 0)$$

- → Understand observation mechanism

Presented \bar{a}
A
B
Click
D
E
F
G

Inverse Propensity Score Estimator

- Observation Propensities $Q(o_d = 1|x, \bar{a}, r^*)$
 - Random variable $o_d \in \{0,1\}$ indicates whether relevance label r_d^* for is observed

- Inverse Propensity Score (IPS) Estimator:

$$\hat{r}(x, a|r^*, o) = \sum_{d: c_d=1} \frac{\text{rank}(d|a)}{Q(o_d = 1|x, \bar{a}, r^*)}$$

New Ranking

- Unbiasedness: $E_o[\hat{r}(x, a | r^*, o)] = r(x, a|r^*)$



Presented \bar{a}	Q
A	1.0
B	0.8
C	0.5
D	0.2
E	0.2
F	0.2
G	0.1

ERM for Partial-Information LTR

- Unbiased Empirical Risk:

$$\hat{V}_{IPS}(\pi) = \frac{1}{N} \sum_{(x,a,c) \in S} \sum_{d:c_d=1} \frac{\text{rank}(d|\pi(x))}{Q(o_d = 1|x, \bar{a}, r^*)}$$

Consistent
Estimator of
True
Performance

- ERM Learning:

$$\hat{\pi} = \underset{\pi}{\operatorname{argmin}} [\hat{V}_{IPS}(\pi)]$$

Consistent
ERM
Learning

- Questions:
 - How do we optimize this empirical risk in a practical learning algorithm?
 - How do we define and estimate the propensity model $Q(o_d = 1|x, \bar{a}, r^*)$?

Propensity-Weighted SVM Rank

- Data: $D = (x_j, d_j, D_j, q_j)^n$
 - Query
 - Clicked
 - Others
 - Propensity
- Training QP:

$$w^* = \operatorname{argmin}_{w, \xi \geq 0} \frac{1}{2} w \cdot w + \frac{C}{n} \sum_j \frac{1}{q_j} \sum_i \xi_j^i$$
$$\forall \bar{d}^i \in D_1: w \cdot [\phi(x_1, d_1) - \phi(x_1, \bar{d}^i)] \geq 1 - \xi_1^i$$
$$\vdots$$
$$\forall \bar{d}^i \in D_n: w \cdot [\phi(x_n, d_n) - \phi(x_n, \bar{d}^i)] \geq 1 - \xi_n^i$$
- Loss Bound: $\forall w: \operatorname{rank}(d, \operatorname{sort}(w \cdot \phi(x, d))) \leq \sum_i \xi^i + 1$
- Analogous method with Deep Nets [Agarwal et al., 2019b]

Optimizes convex upper bound on unbiased IPS risk estimate!

Position-Based Propensity Model

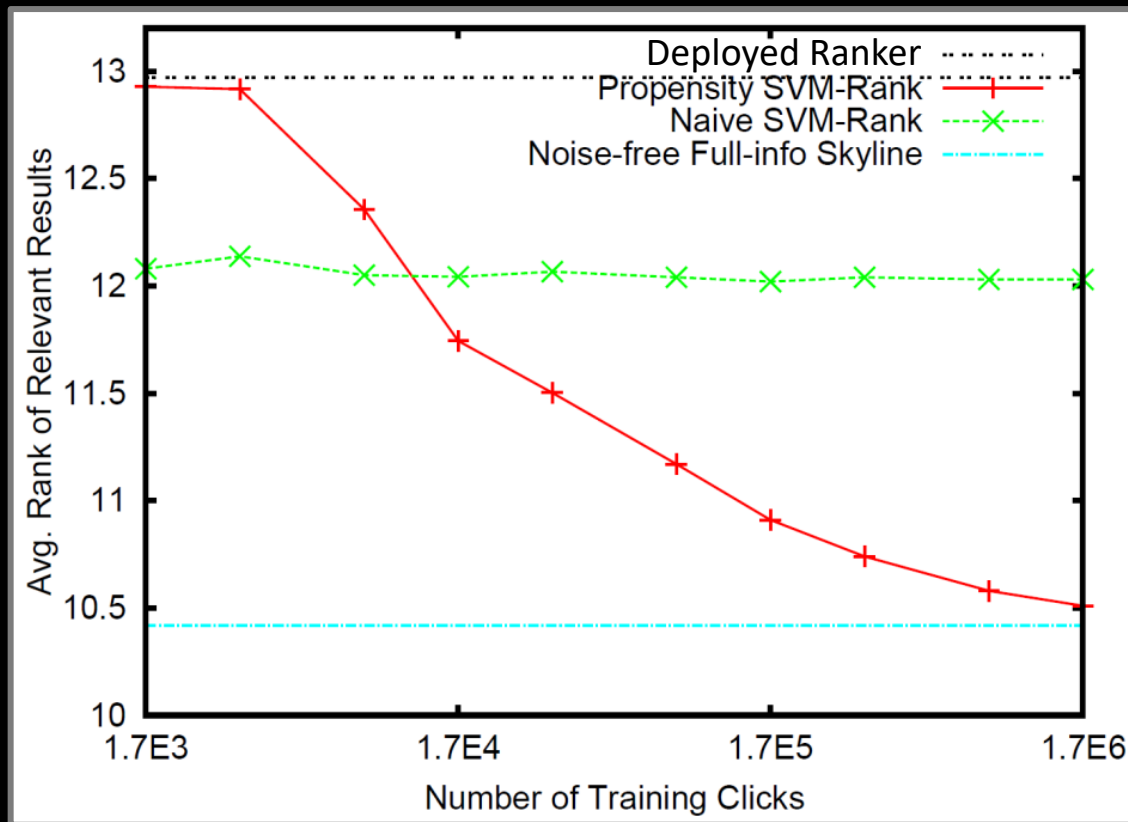
- Model:

$$P(c_d = 1 | r_d^*, \text{rank}(d | \bar{a})) = q_{\text{rank}(d | \bar{a})} \cdot [r_i^* = 1]$$

- Assumptions
 - Examination only depends on rank
 - Click reveals relevance if rank is examined
- Estimation
 - Estimate q_1, \dots, q_k via small intervention experiments
 - See [Joachims et al., 2017] [Agarwal et al., 2019a] [Fang et al., 2019] [Chandar & Carterette, 2018]

Presented \bar{a}	q
A	q_1
B	q_2
C	q_3
D	q_4
E	q_5
F	q_6
G	q_7

Ranking Accuracy vs. Training Data



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