Planetary Motion

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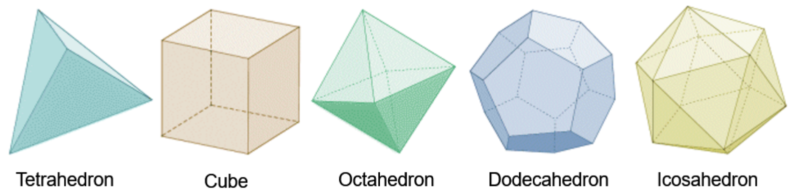
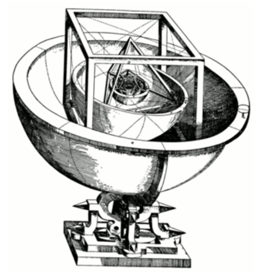
Platonic Solids

At the start of the 17^{th} century, Johannes Kepler published 3 key laws about the orbits of planets in our solar system and the universe. However, before that, he published his laws of planetary motion, he noticed that the ratios of the orbits of the six planets at the time closely matched those of the platonic solids. For those unaware, the platonic solids are the 5 three-dimensional shapes with each face being a regular polygon of equal size. The diagram below shows all five next to Kepler's representation of the planet's orbits using these 5 platonic solids.





Kepler's Model

This model was later found to be slightly inaccurate as the ratios between the planets were not exactly these shapes but due to the data available at the time this inaccuracy was unknown to Kepler. The real values for the ratios compared to the ones Kepler found are shown below. The maximum percentage difference between these values varied between 5.87% and 20.8% which is a very good approximation.

Adjacent Planets	Platonic Solid	Planet Ratio	Kepler's Ratio
Mercury - Venus	Octahedron	0.535	0.577
Venus - Earth	Icosahedron	0.723	0.795
Earth - Mars	Dodecahedron	0.658	0.795
Mars - Jupiter	Tetrahedron	0.292	0.333
Jupiter - Saturn	Cube	0.545	0.577

Kepler's First Law

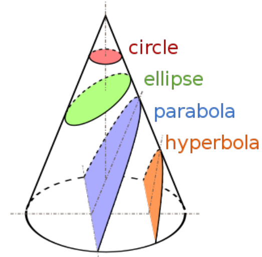
Kepler's first law states that "The orbit of a planet is an ellipse with the Sun at one of the two foci." which means that the shape of every planets orbit is either a circle (an ellipse where the two foci overlap) or an ellipse. The ellipse has a formula which can be used to plot it on a graph and this formula is shown below.

$$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta}$$

To begin with the equation is in the form $r =$ which means it is plotted using polar coordinates. For those unaware, polar coordinates use two parameters to decide where to plot a point and instead of using (x, y) it uses (r, θ) where r is the distance from the origin and θ is the angle in radians from the x-axis. This formula has two parameters, ε and a which represents the eccentricity and the semi-major axis respectively.

Eccentricity

The eccentricity of an ellipse is a measure of how far from a circle an ellipse is with 0 being a circle and 1 being a parabola. A parabola is like an ellipse however it does not have two closed ends. A diagram below demonstrates the difference.

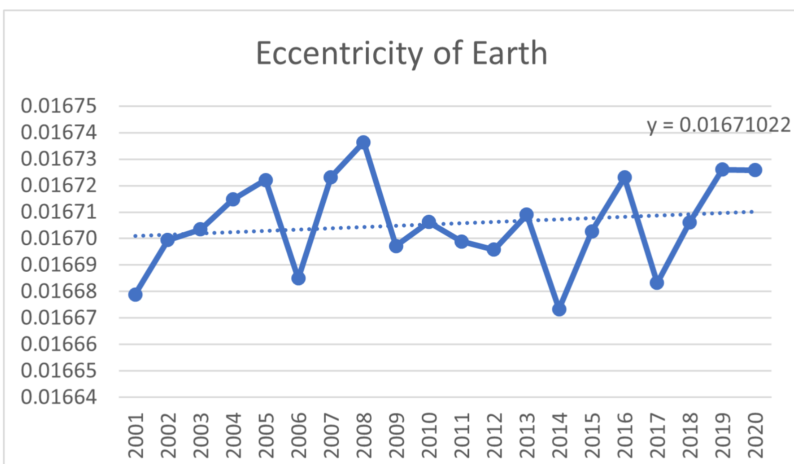


Most planets in the solar system have a very low eccentricity, almost matching a circle however because of other objects in our solar system and other external factors, the eccentricity of a planet is very rarely zero. The eccentricity can be calculated given that you know the distance when a planet is at aphelion and perihelion which are the points when the planet is furthest and closest to the sun. The formula for calculating this is the following.

$$\varepsilon = \frac{A - P}{A + P}$$

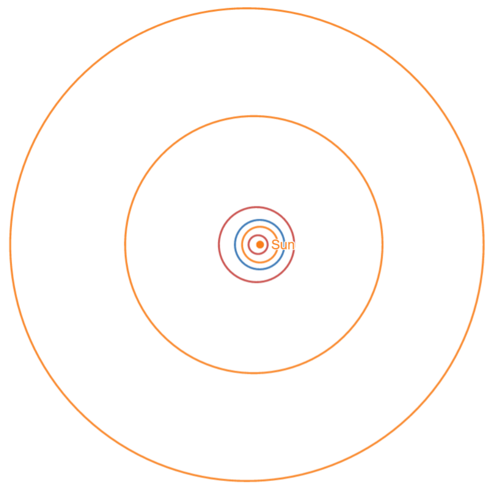
A represents aphelion which is the point where a planet is furthest away from its star whereas P represents the perihelion which is the point where the planet is closest to a star. This formula is very useful for modelling the orbits of a planet as it allows the eccentricity to be calculated based on data collected by astronomers.

Using data from Astropixels, the eccentricity of the Earth can be calculated for a specific year. The eccentricity varies slightly from year to year based on the position of the other planets. Below is a graph from 2001 to 2020 of the variation of the Earth's eccentricity and it can be seen that the average value is around the value given by NASA of 0.0167. The spreadsheet with the calculations and data can be found in the references.

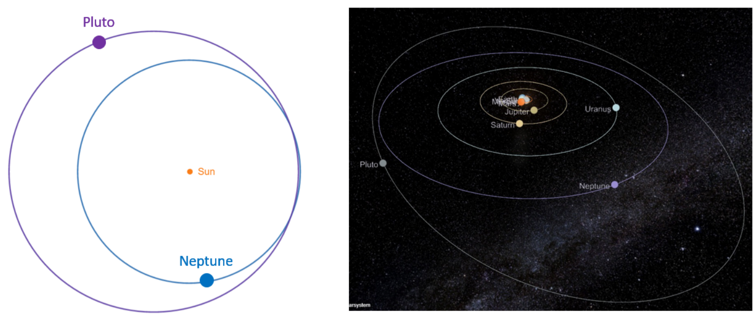


Semi-Major Axis

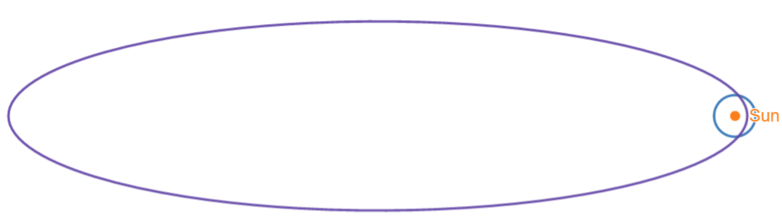
The second variable in the equation, a refers to the semi-major axis of a planet which is a measure of how far away a planet is from a focus which in the case of the solar system is the sun. This, combined with the eccentricity allows for the orbits of the planets to be plotted on a graph. This is shown below with the planets ranging from Mercury to Saturn.



Things start to get interesting if you look at the out-most planets and the dwarf planet of Pluto. The orbits of Neptune and Pluto seem to collide however this is never the case as Pluto orbits the Sun 2 times for every 3 Neptune orbits and at a different angle to Neptune. What this graph fails to show is the rotation in the three-dimensional plane that the orbits have. The diagram below shows the overlap of the two ellipses compared to the reality of their orbits in 3D space.



Out of all of the planets, Mercury is the most eccentric with a value of 0.206 but if you include the dwarf planet of Pluto, its eccentricity has the largest value at around 0.25. This value seems relatively small however if you look at other astronomical objects such as comets, larger values of eccentricity can be found. An example of an object with a large eccentricity is that of Haley's Comet which has an eccentricity of 0.967 which is why it's orbital period is about 75-76 years. Compared to the Earth below, the elliptical shape of the comet's orbit can be clearly seen.



Kepler in the Modern Day

Four centuries later and Kepler's Laws are still relevant in modern day physics. Kepler's Laws can be applied to anything in space with an orbit. This includes planets, stars, comets as well as rockets and satellites. These laws have led to many astronomical discoveries such as dark matter and examining places where Kepler's Laws do not apply has allowed for a deeper understanding of gravity and gravitational fields. Since Kepler published his work, Newton published his laws of to generalise Kepler's Laws for any scenario for any object not just planetary motion.

Along with his first law, Kepler had two others, 'The line joining planets to either focus sweeps out equal areas in equal times.' and 'The square of the period is proportional to the cube of the semi-major axis: $T^2 \propto a^3$ '. These two other laws are how the speed of a planet during its orbit is calculated and the semi-major axis or time period of a planet can be calculated. The constant which relates these two values is $\frac{4\pi^2}{GM}$ which does not change. G is the gravitational constant (figured out by Newton) and M which is the mass of the sun. Rearranging this formula gives either the semi-major axis or time period of a planet which has been very useful in better understanding the planets and other astronomical objects in the universe.

References

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Wikipedia - Platonic Solids
Nonagon - Kepler's Platonic Solids
Mechamath - Eccentricity of an Ellipse Formula
Wikipedia - Ellipse
Wikipedia - Parabola
Peel1520 - Calculating Eccentricity
Astropixels - Datasheet
Sciencepickle - Planet Eccentricity
NASA - Factsheets
Bikehike - Why Pluto won't collide with Neptune
Wikipedia - Haley's Comet
Brilliant - Applying Kepler's Laws

Links to Graphs

Google Sheets - Earth's Eccentricity
Desmos - Planet's Orbits
TheSkyLive - 3D Solar System