1 Cessna-172 Data

Wing Data

$$\text{wing area} \quad S = 16.1651\ m \tag{1}$$

$$\text{wingspan} \quad b = 10.9118\ m \tag{2}$$

$$\text{MAC} \quad c = 1.4935\ m \tag{3}$$

Mass Properties Data

$$\text{Mass} \quad m = 1043.3\ kg \tag{4}$$

$$\text{CG}_{\text{mac}} = 0.3 \tag{5}$$

$$I_{\text{XX}} = 1285.3\ kgm^2 \tag{6}$$

$$I_{\text{YY}} = 1824.9\ kgm^2 \tag{7}$$

$$I_{\text{ZZ}} = 2666.9\ kgm^2 \tag{8}$$

$$I_{\text{XZ}} = 0.0\ kgm^2 \tag{9}$$

$$y_{\text{CG}} = 0.0\ m \tag{10}$$

$$z_{\text{CG}} = 0.2\ m \tag{11}$$

$$g = 9.80665\ m/s^2 \tag{12}$$

Engine Data

$$v_{\text{ref}} = 51.4\ m/s \tag{13}$$

$$\rho_{\text{ref}} = 1.225\ kg/m^3 \tag{14}$$

$$n_{\text{v}} = -1.0 \tag{15}$$

$$n_{\text{æ}} = 0.75 \tag{16}$$

$$\alpha_{\text{F}} = 1.0^{\text{o}} \tag{17}$$

$$X_{\text{F}} = 1.0\ m \tag{18}$$

$$Z_{\text{F}} = 0.0\ m \tag{19}$$

$$T_{\text{max}} = 2070.0\ N \tag{20}$$

Aerodynamic Derivative: Lift Coefficient

$$C_{\text{L}_0} = 0.31 \tag{21}$$

$$C_{\text{L}_\uparrow} = 5.143\ /rad \tag{22}$$

$$C_{\text{L}_{\text{ie}}} = 0.43\ /rad \tag{23}$$

$$C_{\text{L}_\dagger} = 0.0\ s/rad \tag{24}$$

$$C_{\text{L}_{\text{q}}} = 3.9\ s/rad \tag{25}$$

Aerodynamic Derivative: Drag Coefficient

$$C_{\text{D}_0} = 0.031 \tag{26}$$

$$C_{\text{D}_\uparrow} = 0.13\ /rad \tag{27}$$

$$C_{\text{D}_{\text{ie}}} = 0.06\ /rad \tag{28}$$

Aerodynamic Derivative: Pitching Moment Coefficient

$$C_{\text{m}_0} = -0.015 \tag{29}$$

$$C_{\text{m}_\uparrow} = -0.89\ /rad \tag{30}$$

$$C_{\text{m}_{\text{ie}}} = -1.28\ /rad \tag{31}$$

$$C_{\text{m}_\dagger} = -7.27\ s/rad \tag{32}$$

$$C_{\text{m}_{\text{q}}} = -12.4\ s/rad \tag{33}$$

Aerodynamic Derivative: Side Force Coefficient

$$C_{\text{Y}_\downarrow} = -0.31\ /rad \tag{34}$$

$$C_{\text{Y}_{\text{ia}}} = 0.0\ /rad \tag{35}$$

$$C_{\text{Y}_{\text{ir}}} = 0.187\ /rad \tag{36}$$

$$C_{\text{Y}_{\text{p}}} = -0.037\ s/rad \tag{37}$$

$$C_{\text{Y}_{\text{r}}} = 0.21\ s/rad \tag{38}$$

Aerodynamic Derivative: Rolling Moment Coefficient

$$C_{\text{l}_\downarrow} = -0.089\ /rad \tag{39}$$

$$C_{\text{l}_{\text{ia}}} = -0.178\ /rad \tag{40}$$

$$C_{\text{l}_{\text{ir}}} = 0.0147\ /rad \tag{41}$$

$$C_{\text{l}_{\text{p}}} = -0.47\ s/rad \tag{42}$$

$$C_{\text{l}_{\text{r}}} = 0.096\ s/rad \tag{43}$$

Aerodynamic Derivative: Yawing Moment Coefficient

$$C_{\text{n}_\downarrow} = 0.065\ /rad \tag{44}$$

$$C_{\text{n}_{\text{ia}}} = -0.053\ /rad \tag{45}$$

$$C_{\text{n}_{\text{ir}}} = -0.0657\ /rad \tag{46}$$

$$C_{\text{n}_{\text{p}}} = -0.03\ s/rad \tag{47}$$

$$C_{\text{n}_{\text{r}}} = -0.099\ s/rad \tag{48}$$

2 Non-linear Coupled 6-DOF System of Equations

$$\mathcal{L}_{\phi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix} \tag{49}$$

$$\mathcal{L}_{\theta} = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \tag{50}$$

$$\mathcal{L}_{\psi} = \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{51}$$

$$\mathcal{L}_{\mathrm{E}\rightarrow\mathrm{B}} = \mathcal{L}_{\phi}\,\mathcal{L}_{\theta}\,\mathcal{L}_{\psi} \tag{52}$$

$$\mathcal{L}_{\mathrm{B}\rightarrow\mathrm{E}} = \mathcal{L}_{\psi}^{-1}\,\mathcal{L}_{\theta}^{-1}\,\mathcal{L}_{\phi}^{-1} \tag{53}$$

$$\overrightarrow{v}_{\mathrm{E}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \mathcal{L}_{\mathrm{B}\rightarrow\mathrm{E}}\,\overrightarrow{v}_{\mathrm{B}} = \mathcal{L}_{\mathrm{B}\rightarrow\mathrm{E}} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{54}$$

$$\dot{x} = w\,(\sin(\phi)\,\sin(\psi) + \cos(\phi)\,\cos(\psi)\,\sin(\theta)) - v\,(\cos(\phi)\,\sin(\psi) - \cos(\psi)\,\sin(\phi)\,\sin(\theta)) + u\,\cos(\psi)\,\cos(\theta) \tag{55}$$

$$\dot{y} = v\,(\cos(\phi)\,\cos(\psi) + \sin(\phi)\,\sin(\psi)\,\sin(\theta)) - w\,(\cos(\psi)\,\sin(\phi) - \cos(\phi)\,\sin(\psi)\,\sin(\theta)) + u\,\cos(\theta)\,\sin(\psi) \tag{56}$$

$$\dot{z} = w\,\cos(\phi)\,\cos(\theta) - u\,\sin(\theta) + v\,\cos(\theta)\,\sin(\phi) \tag{57}$$

$$\overrightarrow{W}_{\mathrm{B}} = \mathcal{L}_{\mathrm{E}\rightarrow\mathrm{B}}\,\overrightarrow{W}_{\mathrm{E}} = \mathcal{L}_{\mathrm{E}\rightarrow\mathrm{B}} \begin{pmatrix} 0 \\ 0 \\ m\,g \end{pmatrix} \tag{58}$$

$$\overrightarrow{W}_{\mathrm{B}} = \begin{pmatrix} -m\,g\,\sin(\theta) \\ m\,g\,\cos(\theta)\,\sin(\phi) \\ m\,g\,\cos(\phi)\,\cos(\theta) \end{pmatrix} \tag{59}$$

$$\overrightarrow{F}_{\text{ext}} = \overrightarrow{F}_{\text{aero}} + \overrightarrow{F}_{\text{thrust}} + \overrightarrow{W}_{\text{B}} \tag{60}$$

$$\overrightarrow{F}_{\text{ext}} = \begin{pmatrix} F_{\text{ext,x}} \\ F_{\text{ext,y}} \\ F_{\text{ext,z}} \end{pmatrix} = \begin{pmatrix} F_{\text{aero,x}} + F_{\text{thrust,x}} - m\,g\,\sin(\theta) \\ F_{\text{aero,y}} + F_{\text{thrust,y}} + m\,g\,\cos(\theta)\,\sin(\phi) \\ F_{\text{aero,z}} + F_{\text{thrust,z}} + m\,g\,\cos(\phi)\,\cos(\theta) \end{pmatrix} \tag{61}$$

$$\overrightarrow{M}_{\text{ext}} = \overrightarrow{M}_{\text{aero}} + \overrightarrow{M}_{\text{thrust}} \tag{62}$$

$$\overrightarrow{M}_{\text{ext}} = \begin{pmatrix} M_{\text{ext,x}} \\ M_{\text{ext,y}} \\ M_{\text{ext,z}} \end{pmatrix} = \begin{pmatrix} M_{\text{aero,x}} + M_{\text{thrust,x}} \\ M_{\text{aero,y}} + M_{\text{thrust,y}} \\ M_{\text{aero,z}} + M_{\text{thrust,z}} \end{pmatrix} \tag{63}$$

$$\overrightarrow{\Omega}_{\text{B}} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} \dot{\phi} - \dot{\psi}\,\sin(\theta) \\ \dot{\theta}\,\cos(\phi) + \dot{\psi}\,\cos(\theta)\,\sin(\phi) \\ \dot{\psi}\,\cos(\phi)\,\cos(\theta) - \dot{\theta}\,\sin(\phi) \end{pmatrix} \tag{64}$$

$$\dot{\phi} = p + r\,\cos(\phi)\,\tan(\theta) + q\,\sin(\phi)\,\tan(\theta) \tag{65}$$

$$\dot{\theta} = q\,\cos(\phi) - r\,\sin(\phi) \tag{66}$$

$$\dot{\psi} = \frac{r\,\cos(\phi) + q\,\sin(\phi)}{\cos(\theta)} \tag{67}$$

$$\overrightarrow{F}_{\text{ext}} = m\,\left(\dot{\overrightarrow{V}}_{\text{B}} + \overrightarrow{\Omega}_{\text{B}} \times \overrightarrow{V}_{\text{B}}\right) \tag{68}$$

$$\overrightarrow{F}_{\text{ext}} = m\,\left(\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix}\right) = m\,\begin{pmatrix} \dot{u} + q\,w - r\,v \\ \dot{v} - p\,w + r\,u \\ \dot{w} + p\,v - q\,u \end{pmatrix} \tag{69}$$

$$F_{\text{aero,x}} + F_{\text{thrust,x}} - m\,g\,\sin(\theta) = m\,(\dot{u} + q\,w - r\,v) \tag{70}$$

$$F_{\text{aero,y}} + F_{\text{thrust,y}} + m\,g\,\cos(\theta)\,\sin(\phi) = m\,(\dot{v} - p\,w + r\,u) \tag{71}$$

$$F_{\text{aero,z}} + F_{\text{thrust,z}} + m\,g\,\cos(\phi)\,\cos(\theta) = m\,(\dot{w} + p\,v - q\,u) \tag{72}$$

$$\overrightarrow{M}_{\text{ext}} = I_{\text{B}}\,\dot{\overrightarrow{\Omega}}_{\text{B}} + \overrightarrow{\Omega}_{\text{B}} \times \left(I_{\text{B}}\,\overrightarrow{\Omega}_{\text{B}}\right) \tag{73}$$

$$\overrightarrow{M}_{\text{ext}} = \begin{bmatrix} I_{\text{xx}} & 0 & -I_{\text{xz}} \\ 0 & I_{\text{yy}} & 0 \\ -I_{\text{zx}} & 0 & I_{\text{zz}} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \left(\begin{bmatrix} I_{\text{xx}} & 0 & -I_{\text{xz}} \\ 0 & I_{\text{yy}} & 0 \\ -I_{\text{zx}} & 0 & I_{\text{zz}} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}\right) \tag{74}$$

$$M_{\text{aero,x}} + M_{\text{thrust,x}} = I_{\text{xx}}\,\dot{p} - I_{\text{xz}}\,\dot{r} - q\,(I_{\text{zx}}\,p - I_{\text{zz}}\,r) - I_{\text{yy}}\,q\,r \tag{75}$$

$$M_{\text{aero,y}} + M_{\text{thrust,y}} = I_{\text{yy}}\,\dot{q} + p\,(I_{\text{zx}}\,p - I_{\text{zz}}\,r) + r\,(I_{\text{xx}}\,p - I_{\text{xz}}\,r) \tag{76}$$

$$M_{\text{aero,z}} + M_{\text{thrust,z}} = I_{\text{zz}}\,\dot{r} - I_{\text{zx}}\,\dot{p} - q\,(I_{\text{xx}}\,p - I_{\text{xz}}\,r) + I_{\text{yy}}\,p\,q \tag{77}$$

$$\boldsymbol{x} = (x, y, z, \phi, \theta, \psi, p, q, r, u, v, w)^{\mathrm{T}} \tag{78}$$

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) = f(x, y, z, \phi, \theta, \psi, p, q, r, u, v, w) \tag{79}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} u\,C_\theta\,C_\psi + v\,(S_\phi\,S_\theta\,C_\psi - C_\phi\,S_\psi) + w\,(C_\phi\,S_\theta\,C_\psi + S_\phi\,S_\psi) \\ u\,C_\theta\,S_\psi + v\,(S_\phi\,S_\theta\,S_\psi + C_\phi\,C_\psi) + w\,(C_\phi\,S_\theta\,S_\psi - S_\phi\,C_\psi) \\ -u\,S_\theta + v\,S_\phi\,C_\theta + w\,C_\phi\,C_\theta \\ p + q\,S_\phi\,T_\theta + r\,C_\phi\,T_\theta \\ q\,C_\phi - r\,S_\phi \\ (q\,S_\phi + r\,C_\phi)\,/C_\theta \\ r\,v - q\,w + F_{\mathrm{ext},x}/m \\ p\,w - r\,u + F_{\mathrm{ext},y}/m \\ q\,u - p\,v + F_{\mathrm{ext},z}/m \\ \frac{I_{\mathrm{xz}}\,M_{\mathrm{ext},z} + I_{\mathrm{zz}}\,M_{\mathrm{ext},x} + (I_{\mathrm{xx}}\,I_{\mathrm{xz}} - I_{\mathrm{xz}}\,I_{\mathrm{yy}+I_{\mathrm{zx}}\,I_{\mathrm{zz}}})\,p\,q - \left({I_{\mathrm{xz}}}^2 + {I_{\mathrm{zz}}}^2 - I_{\mathrm{yy}}\,I_{\mathrm{zz}}\right) q\,r}{I_{\mathrm{xx}}\,I_{\mathrm{zz}} - I_{\mathrm{xz}}\,I_{\mathrm{zx}}} \\ \frac{M_{\mathrm{ext},y} - (I_{\mathrm{xx}} - I_{\mathrm{zz}})\,p\,r - I_{\mathrm{zx}}\,p^2 + I_{\mathrm{xz}}\,r^2}{I_{\mathrm{yy}}} \\ \frac{I_{\mathrm{xx}}\,M_{\mathrm{ext},z} + I_{\mathrm{zx}}\,M_{\mathrm{ext},x} + \left(I_{\mathrm{xx}}^2 + I_{\mathrm{zx}}^2 - I_{\mathrm{xx}}\,I_{\mathrm{yy}}\right) p\,q + (I_{\mathrm{zx}}\,I_{\mathrm{yy}} - I_{\mathrm{xz}}\,I_{\mathrm{xx}} - I_{\mathrm{zx}}\,I_{\mathrm{zz}})\,q\,r}{I_{\mathrm{xx}}\,I_{\mathrm{zz}} - I_{\mathrm{xz}}\,I_{\mathrm{zx}}} \end{pmatrix} \tag{80}$$

3 International Standard Atmosphere (ISA)

$$k = -0.0065 \frac{K}{m} \tag{81}$$

$$P_0 = 101325 Pa \tag{82}$$

$$T_0 = 288.15 K \tag{83}$$

$$R = 287 \frac{J}{kgK} \tag{84}$$

$$\gamma = 1.4 \tag{85}$$

$$k = -0.0065 \frac{K}{m} \tag{86}$$

$$T(h <= 11km) = T_0 + h\,k \tag{87}$$

$$T(11km < h < 20km) = T_0 + k * 11000; \tag{88}$$

$$P(h <= 11km) = P_0 \left(\frac{T}{T_0}\right)^{\frac{-g}{R\,k}} \tag{89}$$

$$P(11km < h < 20km) = P_{11}\, \mathrm{e}^{-\frac{g\,(h-h_{11})}{R\,T_{11}}} \tag{90}$$

$$\rho_{air} = \frac{P}{R\,T} \tag{91}$$

$$v_{sound} = \sqrt{\gamma\,R\,T} \tag{92}$$

$$M = \frac{v_{TAS}}{v_{sound}} \tag{93}$$

$$a_0 = \sqrt{\gamma\,R\,T_0} \tag{94}$$

$$\Delta P = P\left(\left(\frac{(\gamma-1)}{2}\,M^2 + 1\right)^{\frac{\gamma}{\gamma-1}} - 1\right) \tag{95}$$

$$v_{\mathrm{CAS}} = \sqrt{\frac{2{a_0}^2}{\gamma-1}\left(\left(\frac{\Delta_P}{P_0}+1\right)^{\frac{\gamma-1}{\gamma}} - 1\right)} \tag{96}$$

$$v_{\mathrm{TAS}} = \sqrt{u^2+v^2+w^2} \tag{97}$$

$$\alpha = \mathrm{atan2}\,(w,u) \tag{98}$$

$$\beta = \mathrm{atan2}\,(v, v_{\mathrm{TAS}}) \tag{99}$$

4 Aerodynamic Modeling

$$C_{\mathrm{L}} = C_{\mathrm{L}_0} + C_{\mathrm{L}_\alpha}\,\alpha + C_{\mathrm{L}_{\delta_\mathrm{e}}}\,\delta_\mathrm{e} + \frac{C_{\mathrm{L}_{\dot{\alpha}}}\,\dot{\alpha}\,c}{2\,v_\mathrm{TAS}} + \frac{C_{\mathrm{L}_\mathrm{q}}\,q\,c}{2\,v_\mathrm{TAS}} \tag{100}$$

$$C_{\mathrm{D}} = C_{\mathrm{D}_0} + C_{\mathrm{D}_\alpha}\,\alpha + C_{\mathrm{D}_{\delta_\mathrm{e}}}\,\delta_\mathrm{e} \tag{101}$$

$$C_{\mathrm{Y}} = C_{\mathrm{Y}_\beta}\,\beta + C_{\mathrm{Y}_{\delta_\mathrm{a}}}\,\delta_\mathrm{a} + C_{\mathrm{Y}_{\delta_\mathrm{r}}}\,\delta_\mathrm{r} + \frac{C_{\mathrm{Y}_\mathrm{p}}\,p\,b}{2\,v_\mathrm{TAS}} + \frac{C_{\mathrm{Y}_\mathrm{r}}\,r\,b}{2\,v_\mathrm{TAS}} \tag{102}$$

$$C_{\mathrm{l}} = C_{\mathrm{l}_\beta}\,\beta + C_{\mathrm{l}_{\delta_\mathrm{a}}}\,\delta_\mathrm{a} + C_{\mathrm{l}_{\delta_\mathrm{r}}}\,\delta_\mathrm{r} + \frac{C_{\mathrm{l}_\mathrm{p}}\,p\,b}{2\,v_\mathrm{TAS}} + \frac{C_{\mathrm{l}_\mathrm{r}}\,r\,b}{2\,v_\mathrm{TAS}} \tag{103}$$

$$C_{\mathrm{m}} = C_{\mathrm{m}_0} + C_{\mathrm{m}_\alpha}\,\alpha + C_{\mathrm{m}_{\delta_\mathrm{e}}}\,\delta_\mathrm{e} + \frac{C_{\mathrm{m}_{\dot{\alpha}}}\,\dot{\alpha}\,c}{2\,v_\mathrm{TAS}} + \frac{C_{\mathrm{m}_\mathrm{q}}\,q\,c}{2\,v_\mathrm{TAS}} \tag{104}$$

$$C_{\mathrm{n}} = C_{\mathrm{n}_\beta}\,\beta + C_{\mathrm{n}_{\delta_\mathrm{a}}}\,\delta_\mathrm{a} + C_{\mathrm{n}_{\delta_\mathrm{r}}}\,\delta_\mathrm{r} + \frac{C_{\mathrm{n}_\mathrm{p}}\,p\,b}{2\,v_\mathrm{TAS}} + \frac{C_{\mathrm{n}_\mathrm{r}}\,r\,b}{2\,v_\mathrm{TAS}} \tag{105}$$

$$C_{\mathrm{X_b}} = C_{\mathrm{L}}\,\sin(\alpha) - C_{\mathrm{D}}\,\cos(\alpha) \tag{106}$$

$$C_{\mathrm{Y_b}} = C_{\mathrm{Y}} \tag{107}$$

$$C_{\mathrm{Z_b}} = -C_{\mathrm{L}}\,\cos(\alpha) - C_{\mathrm{D}}\,\sin(\alpha) \tag{108}$$

$$C_{\mathrm{l_b}} = C_{\mathrm{l}}\,\cos(\alpha) - C_{\mathrm{n}}\,\sin(\alpha) \tag{109}$$

$$C_{\mathrm{m_b}} = C_{\mathrm{m}} \tag{110}$$

$$C_{\mathrm{n_b}} = C_{\mathrm{n}}\,\cos(\alpha) + C_{\mathrm{l}}\,\sin(\alpha) \tag{111}$$

$$Q = \frac{1}{2}\rho_\mathrm{air}\,{v_\mathrm{TAS}}^2 \tag{112}$$

$$F_{\mathrm{aero,x}} = C_{\mathrm{X_b}}\,Q\,S \tag{113}$$

$$F_{\mathrm{aero,y}} = C_{\mathrm{Y_b}}\,Q\,S \tag{114}$$

$$F_{\mathrm{aero,z}} = C_{\mathrm{Z_b}}\,Q\,S \tag{115}$$

$$M_{\mathrm{aero,x}} = C_{\mathrm{l_b}}\,Q\,S\,b - F_{\mathrm{aero,y}}\,z_\mathrm{CG} - F_{\mathrm{aero,z}}\,y_\mathrm{CG} \tag{116}$$

$$M_{\mathrm{aero,y}} = C_{\mathrm{m_b}}\,Q\,S\,c + F_{\mathrm{aero,x}}\,z_\mathrm{CG} - F_{\mathrm{aero,z}}\,c\,(\mathrm{CG_{mac}} - 0.25) \tag{117}$$

$$M_{\mathrm{aero,z}} = C_{\mathrm{n_b}}\,Q\,S\,b + F_{\mathrm{aero,x}}\,y_\mathrm{CG} + F_{\mathrm{aero,y}}\,c\,(\mathrm{CG_{mac}} - 0.25) \tag{118}$$

5 Propulsion Modeling

$$T_{linear} = T = \delta_\mathrm{t}\,T_\mathrm{max} \tag{119}$$

$$T_{nonlinear} = T = \delta_\mathrm{t}\,T_\mathrm{max}\left(\frac{v_\mathrm{TAS}}{v_\mathrm{ref}}\right)^{n_\mathrm{v}}\left(\frac{\rho_\mathrm{air}}{\rho_\mathrm{ref}}\right)^{n_\rho} \tag{120}$$

$$\overrightarrow{F}_{\mathrm{thrust}} = \begin{pmatrix} F_{\mathrm{thrust,x}} \\ F_{\mathrm{thrust,y}} \\ F_{\mathrm{thrust,z}} \end{pmatrix} = \begin{pmatrix} T\,\cos(\alpha_\mathrm{F}) \\ 0 \\ T\,\sin(\alpha_\mathrm{F}) \end{pmatrix} \tag{121}$$

$$\overrightarrow{M}_{\mathrm{thrust}} = \begin{pmatrix} M_{\mathrm{thrust,x}} \\ M_{\mathrm{thrust,y}} \\ M_{\mathrm{thrust,z}} \end{pmatrix} = \begin{pmatrix} 0 \\ T\,\cos(\alpha_\mathrm{F})\;Z_\mathrm{F} - T\,\sin(\alpha_\mathrm{F})\;X_\mathrm{F} \\ 0 \end{pmatrix} \tag{122}$$

6 Trimmed condition: level unaccelerated flight at 5000 ft

$$\boldsymbol{x_0} = (x_0, y_0, z_0, \phi_0, \theta_0, \psi_0, p_0, q_0, r_0, u_0, v_0, w_0)^{\mathrm{T}} \tag{123}$$

$$\dot{\boldsymbol{x}}_{\boldsymbol{0}} = f(\boldsymbol{x_0}) = f(x_0, y_0, z_0, \phi_0, \theta_0, \psi_0, p_0, q_0, r_0, u_0, v_0, w_0) = 0 \tag{124}$$

$$\boldsymbol{y_0} = (x_0, y_0, z_0, \phi_0, \theta_0, \psi_0, u_0, v_0, w_0, p_0, q_0, r_0, \alpha_0, \beta_0, \gamma_0, \zeta_0, v_{\mathrm{TAS}_0}, \rho_0)^{\mathrm{T}} \tag{125}$$

$$\boldsymbol{x_0} = (0, 0, -1524, 0, 0, 0, 62.3866, 0, 0, 0, 0, 0)^{\mathrm{T}} \tag{126}$$

$$\boldsymbol{u_0} = (\delta_{\mathrm{e}_0}, \delta_{\mathrm{a}_0}, \delta_{\mathrm{r}_0}, \delta_{\mathrm{t}_0})^{\mathrm{T}} = (-0.0032115, 0, 0, 0.6792)^{\mathrm{T}} \tag{127}$$

$$\boldsymbol{y_0} = (0, 0, -1524, 0, 0, 0, 62.3866, 0, 0, 0, 0, 0, 0, 0, 0, 62.3866, 1.0557)^{\mathrm{T}} \tag{128}$$

7 Linearized states differential equations

$$\boldsymbol{x} = \boldsymbol{x}_0 + \Delta\boldsymbol{x} = \begin{pmatrix} x_0 + \Delta x \\ y_0 + \Delta y \\ z_0 + \Delta z \\ \phi_0 + \Delta\phi \\ \theta_0 + \Delta\theta \\ \psi_0 + \Delta\psi \\ u_0 + \Delta u \\ v_0 + \Delta v \\ w_0 + \Delta w \\ p_0 + \Delta p \\ q_0 + \Delta q \\ r_0 + \Delta r \end{pmatrix} \tag{129}$$

$$\boldsymbol{u} = \boldsymbol{u}_0 + \Delta\boldsymbol{u} = \begin{pmatrix} \delta_{\mathrm{e}_0} + \Delta\delta_{\mathrm{e}} \\ \delta_{\mathrm{a}_0} + \Delta\delta_{\mathrm{a}} \\ \delta_{\mathrm{r}_0} + \Delta\delta_{\mathrm{r}} \\ \delta_{\mathrm{t}_0} + \Delta\delta_{\mathrm{t}} \end{pmatrix} \tag{130}$$

$$\boldsymbol{y} = \boldsymbol{y}_0 + \Delta\boldsymbol{y} = \begin{pmatrix} x_0 + \Delta x \\ y_0 + \Delta y \\ z_0 + \Delta z \\ \phi_0 + \Delta\phi \\ \theta_0 + \Delta\theta \\ \psi_0 + \Delta\psi \\ u_0 + \Delta u \\ v_0 + \Delta v \\ w_0 + \Delta w \\ p_0 + \Delta p \\ q_0 + \Delta q \\ r_0 + \Delta r \\ \alpha_0 + \Delta\alpha \\ \beta_0 + \Delta\beta \\ \gamma_0 + \Delta\gamma \\ \zeta_0 + \Delta\zeta \\ v_{\mathrm{TAS}_0} + \Delta v_{\mathrm{TAS}} \\ \rho_0 + \Delta\rho \end{pmatrix} \tag{131}$$

$$\Delta \dot{\boldsymbol{x}} = A\,\Delta \boldsymbol{x} + B\,\Delta \boldsymbol{u} \tag{132}$$

$$\Delta \boldsymbol{y} = C\,\Delta \boldsymbol{x} + D\,\Delta \boldsymbol{u} \tag{133}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 62.39 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -62.39 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\ 0 & 0 & -0.0001 & 0 & -9.807 & 0 & -0.0477 & 0 & 0.2238 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.807 & 0 & 0 & 0 & -0.1582 & 0 & -0.103 & 0 & -61.8 \\ 0 & 0 & -0.0022 & 0 & 0 & 0 & -0.3152 & 0 & -2.64 & 0 & 60.9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3765 & 0 & -11.57 & 0 & 2.272 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0005 & 0 & -0.2494 & 0 & -3.971 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.137 & 0 & -0.3595 & 0 & -1.159 \end{pmatrix} \tag{134}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.91 & 0 & 0 & 1.462 \\ 0 & 0 & 5.953 & 0 \\ -13.69 & 0 & 0 & 0.0255 \\ 0 & -50.19 & 3.178 & 0 \\ -33.99 & 0 & 0 & -0.0146 \\ 0 & -7.202 & -8.754 & 0 \end{pmatrix} \tag{135}$$

$$C = \begin{pmatrix} 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.016 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.016 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & -0.016 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0.016 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0002 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{136}$$

$$D = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}} \tag{137}$$

$$
\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \Delta u \\ 62.39\,\Delta\psi + \Delta v \\ \Delta w - 62.39\,\Delta\theta \\ \Delta p \\ \Delta q \\ \Delta r \\ 1.91\,\Delta\delta_{\mathrm{e}} + 1.462\,\Delta\delta_{\mathrm{t}} - 9.807\,\Delta\theta - 0.0477\,\Delta u + 0.2238\,\Delta w - 0.0001\,\Delta z \\ 5.953\,\Delta\delta_{\mathrm{r}} - 0.103\,\Delta p + 9.807\,\Delta\phi - 61.8\,\Delta r - 0.1582\,\Delta v \\ 0.0255\,\Delta\delta_{\mathrm{t}} - 13.69\,\Delta\delta_{\mathrm{e}} + 60.9\,\Delta q - 0.3152\,\Delta u - 2.64\,\Delta w - 0.0022\,\Delta z \\ 3.178\,\Delta\delta_{\mathrm{r}} - 50.19\,\Delta\delta_{\mathrm{a}} - 11.57\,\Delta p + 2.272\,\Delta r - 0.3765\,\Delta v \\ 0.0005\,\Delta u - 0.0146\,\Delta\delta_{\mathrm{t}} - 3.971\,\Delta q - 33.99\,\Delta\delta_{\mathrm{e}} - 0.2494\,\Delta w \\ 0.137\,\Delta v - 8.754\,\Delta\delta_{\mathrm{r}} - 0.3595\,\Delta p - 1.159\,\Delta r - 7.202\,\Delta\delta_{\mathrm{a}} \end{pmatrix} \tag{138}
$$

8 Aircraft Longitudinal Dynamics

$$\Delta\dot{x} = \Delta u \tag{139}$$

$$\Delta\dot{z} = \Delta w - 62.39\,\Delta\theta \tag{140}$$

$$\Delta\dot{\theta} = \Delta q \tag{141}$$

$$\Delta\dot{u} = -0.0001\,\Delta z - 9.807\,\Delta\theta - 0.0477\,\Delta u + 0.2388\,\Delta w + 1.91\,\Delta\delta_{\mathrm{e}} + 1.462\,\Delta\delta_{\mathrm{t}} \tag{142}$$

$$\Delta\dot{w} = -0.0022\,\Delta z - 0.3152\,\Delta u - 2.64\,\Delta w + 60.9\,\Delta q - 13.69\,\Delta\delta_{\mathrm{e}} + 0.0255\,\Delta\delta_{\mathrm{t}} \tag{143}$$

$$\Delta\dot{q} = 0.0005\,\Delta u - 0.2494\,\Delta w - 3.971\,\Delta q - 33.99\,\Delta\delta_{\mathrm{e}} - 0.0146\,\Delta\delta_{\mathrm{t}} \tag{144}$$

$$\Delta\boldsymbol{x} = (\Delta x, \Delta z, \Delta\theta, \Delta u, \Delta w, \Delta q)^{\mathrm{T}} \tag{145}$$

$$\Delta\boldsymbol{u} = (\Delta\delta_{\mathrm{e}}, \Delta\delta_{\mathrm{t}})^{\mathrm{T}} \tag{146}$$

$$\Delta y = \Delta\theta \tag{147}$$

$$\Delta\dot{\boldsymbol{x}} = A_{\mathrm{long}}\,\Delta\boldsymbol{x} + B_{\mathrm{long}}\,\Delta\boldsymbol{u} \tag{148}$$

$$\Delta y = C_{\mathrm{long}}\,\Delta\boldsymbol{x} + D_{\mathrm{long}}\,\Delta\boldsymbol{u} \tag{149}$$

$$A_{\mathrm{long}} = \begin{pmatrix} 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & -62.39 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \\ 0 & -0.0001 & -9.807 & -0.0477 & 0.2388 & 0 \\ 0 & -0.0022 & 0 & -0.3152 & -2.64 & 60.9 \\ 0 & 0 & 0 & 0.0005 & -0.2494 & -3.971 \end{pmatrix} \tag{150}$$

$$B_{\mathrm{long}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1.91 & 1.462 \\ -13.69 & 0.0255 \\ -33.99 & -0.0146 \end{pmatrix} \tag{151}$$

$$C_{\mathrm{long}} = \begin{pmatrix} 0 & 0 & 1.0 & 0 & 0 & 0 \end{pmatrix} \tag{152}$$

$$D_{\mathrm{long}} = \begin{pmatrix} 0 & 0 \end{pmatrix} \tag{153}$$

9 Aircraft Lateral Dynamics

$$\Delta\dot{y} = 62.39\,\Delta\psi + \Delta v \tag{154}$$

$$\Delta\dot{\phi} = \Delta p \tag{155}$$

$$\Delta\dot{\psi} = \Delta r \tag{156}$$

$$\Delta\dot{v} = 9.807\,\Delta\phi - 0.1582\,\Delta v - 0.103\,\Delta p - 61.8\,\Delta r - 5.953\,\Delta\delta_{\mathrm{r}} \tag{157}$$

$$\Delta\dot{p} = -0.3765\,\Delta v - 11.57\,\Delta p + 2.272\,\Delta r - 50.19\,\Delta\delta_{\mathrm{a}} + 3.178\,\Delta\delta_{\mathrm{r}} \tag{158}$$

$$\Delta\dot{r} = 0.137\,\Delta v - 0.3595\,\Delta p - 1.159\,\Delta r - 7.202\,\Delta\delta_{\mathrm{a}} - 8.754\,\Delta\delta_{\mathrm{r}} \tag{159}$$

$$\Delta\boldsymbol{x} = (\Delta y, \Delta\phi, \Delta\psi, \Delta v, \Delta p, \Delta r)^{\mathrm{T}} \tag{160}$$

$$\Delta\boldsymbol{u} = (\Delta\delta_{\mathrm{a}}, \Delta\delta_{\mathrm{r}})^{\mathrm{T}} \tag{161}$$

$$\Delta y = \Delta\psi \tag{162}$$

$$\Delta\dot{\boldsymbol{x}} = A_{\mathrm{lat}}\,\Delta\boldsymbol{x} + B_{\mathrm{lat}}\,\Delta\boldsymbol{u} \tag{163}$$

$$\Delta y = C_{\mathrm{lat}}\,\Delta\boldsymbol{x} + D_{\mathrm{lat}}\,\Delta\boldsymbol{u} \tag{164}$$

$$A_{\mathrm{lat}} = \begin{pmatrix} 0 & 0 & 62.39 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \\ 0 & 9.807 & 0 & -0.1582 & -0.103 & -61.8 \\ 0 & 0 & 0 & -0.3765 & -11.57 & 2.272 \\ 0 & 0 & 0 & 0.137 & -0.3595 & -1.159 \end{pmatrix} \tag{165}$$

$$B_{\mathrm{lat}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -5.953 \\ -50.19 & 3.178 \\ -7.202 & -8.754 \end{pmatrix} \tag{166}$$

$$C_{\mathrm{lat}} = \begin{pmatrix} 0 & 0 & 1.0 & 0 & 0 & 0 \end{pmatrix} \tag{167}$$

$$D_{\mathrm{lat}} = \begin{pmatrix} 0 & 0 \end{pmatrix} \tag{168}$$