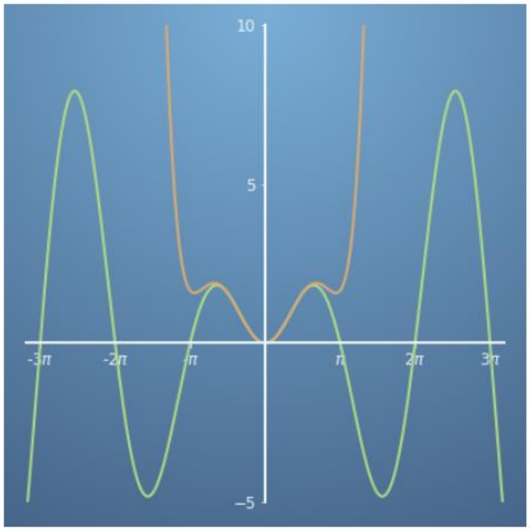
1. Now that we have completed the set of Taylor series lectures and answered all the quiz questions, we now need to test our understanding of Taylor series. We have looked at the derivation of Taylor series, broken it down into a power series approximation, explored special cases and developed the idea of multivariant Taylor series, that is required in order for us to develop a good grounding for the next chapters in this course.

1 / 1 point

For the function $f(x) = x\sin(x)$ shown below, determine what order approximation is shown by the orange curve, where the Taylor series approximation was centered about $x = 0$.

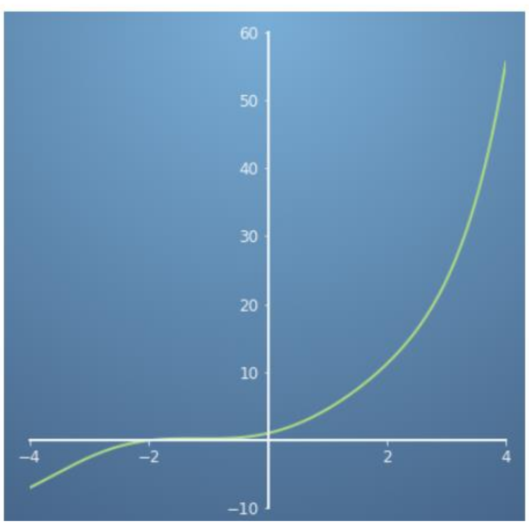


- ○ Fourth Order
- ◉ Sixth Order
- ○ None of the above

Correct
The sign of the sixth order term is positive, which dominates over the fourth order term and is particularly the reason why the approximation for $f(x)$ is always positive.

2. Find the first four non zero terms of the Taylor expansion for the function $f(x) = e^x + x + \sin(x)$ about $x = 0$. The function is shown below:

1 / 1 point



○ $$f(x) = 1 + 3x - \frac{x^2}{2} + \frac{x^4}{24} + \ldots$$

○ $$f(x) = 3x + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{720} + \ldots$$

○ $$f(x) = 1 + 3x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

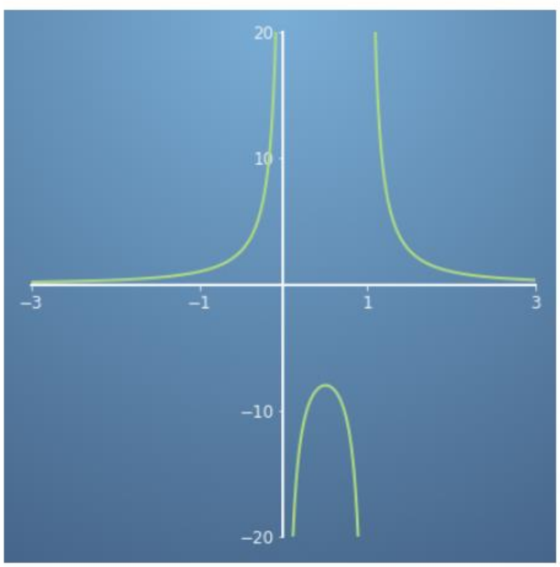
◉ $$f(x) = 1 + 3x + \frac{x^2}{2} + \frac{x^4}{24} + \ldots$$

Correct

As there is a variety of functions here i.e. $\sin(x)$ and an exponential, we are not likely to get expansions

3. The graph below shows the discontinuous function $f(x) = \frac{2}{(x^2-x)}$. Approximate the section of this function that covers the domain $0 < x < 1$. Use the Taylor series forumla and $x = 0.5$ as your starting point, find the first two non zero terms.

1 / 1 point



○ $$f(x) = -8 + 32(x - 0.5)^2 \ldots$$

◉ $$f(x) = -8 - 32(x - 0.5)^2 \ldots$$

○ $$f(x) = -8 - 32x^2 \ldots$$

○ $$f(x) = -4 - 16(x - 0.5)^2 \ldots$$

Correct
This second order approximation is only valid within the domain $0 < x < 1$, and is, therefore, a poor approximation for the entire function, but behaves well within the defined domain.

Correct
For an odd function, $-f(x) = f(-x)$. We can also determine if a function is odd by looking at its symmetry. If it has rotational symmetry with respect to the origin, it is an odd function.

5. Take the Taylor expansion of the function

1 / 1 point

$$f(x) = e^{-2x}$$

about the point $x = 2$ and subsequently linearise the function.

○ $$f(x) = \left(\frac{1}{e^2}\right)[2(x-2)] + O(\Delta x^2)$$

○ $$f(x) = \left(\frac{1}{e^4}\right)[1 - 2(x-2)] + 4(x-2)^2 + O(\Delta x^3)$$

◉ $$f(x) = \left(\frac{1}{e^4}\right)[1 - 2(x-2)] + O(\Delta x^2)$$

○ $$f(x) = \left(\frac{1}{e^4}\right)[1 + 2(x-2)]$$

Correct
Here we are taking a complicated function and simplifying it into its linear components, making sure to