

# Estimating Bayesian Diagnostic Models with Attribute Hierarchies with the Hamiltonian-Gibbs Hybrid Sampler

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## Introduction

- Diagnostic models are psychometric models that classify respondents into latent classes based on their responses to a set of observable tasks.
- Estimation of Diagnostic models with Bayesian methods has increased, with Hamiltonian Monte Carlo (HMC) sampling demonstrating great potential for estimating DMs; however, HMC cannot directly sample discrete variable types (i.e., attributes).
- We propose a synthesis of Hamiltonian Monte Carlo and the Gibbs sampler for estimation of Bayesian diagnostic models under various attribute structure specifications.

## The Log-linear Diagnostic Classification Model

The LCDM marginal likelihood of observing respondent  $r$ 's response vector  $\mathbf{x}_r$  is

$$P(\mathbf{X}_r = \mathbf{x}_r) = \sum_{\alpha \in \{0,1\}^K} \rho_{\alpha} \prod_{i=1}^I \theta_{ri}^{x_{ri}} (1 - \theta_{ri})^{1-x_{ri}},$$

where  $\rho_{\alpha} \equiv P(\alpha)$  is marginal probability of possessing attribute profile  $\alpha$  and  $\theta_{ri}$  is the item response function given by

$$\theta_{ri} \equiv P(X_{ri} = 1 \mid \alpha_r) = \frac{\exp \{ \lambda_{i,0} + \lambda_i^T h(q_i, \alpha_r) \}}{1 + \exp \{ \lambda_{i,0} + \lambda_i^T h(q_i, \alpha_r) \}},$$

where  $\lambda_{i,0}$  denotes the item intercept and  $\lambda_i^T h(q_i, \alpha_r)$  denotes the linear combination of attributes and effects that are relevant to item  $i$ .

## Attribute Structures

Figure 1. Independent attribute structure

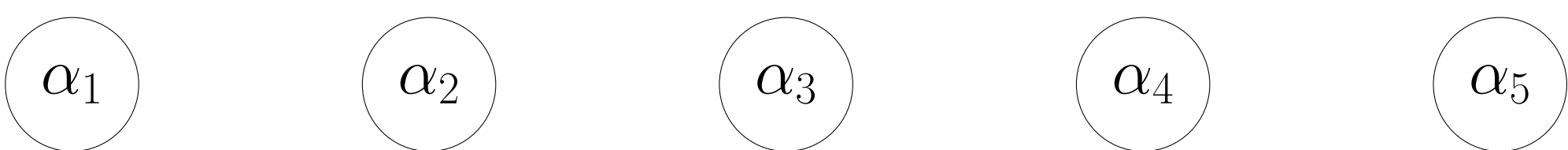


Figure 2. Linear attribute hierarchy

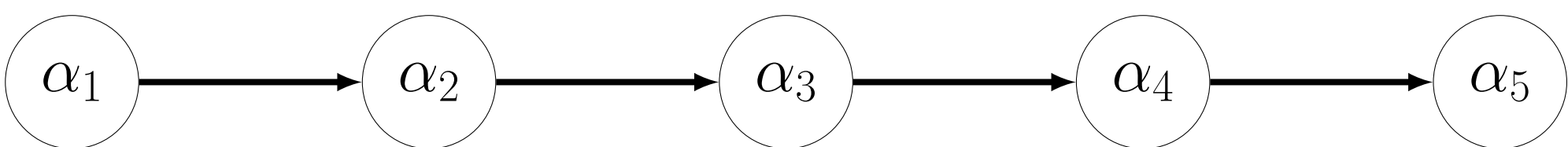


Figure 3. Divergent attribute hierarchy

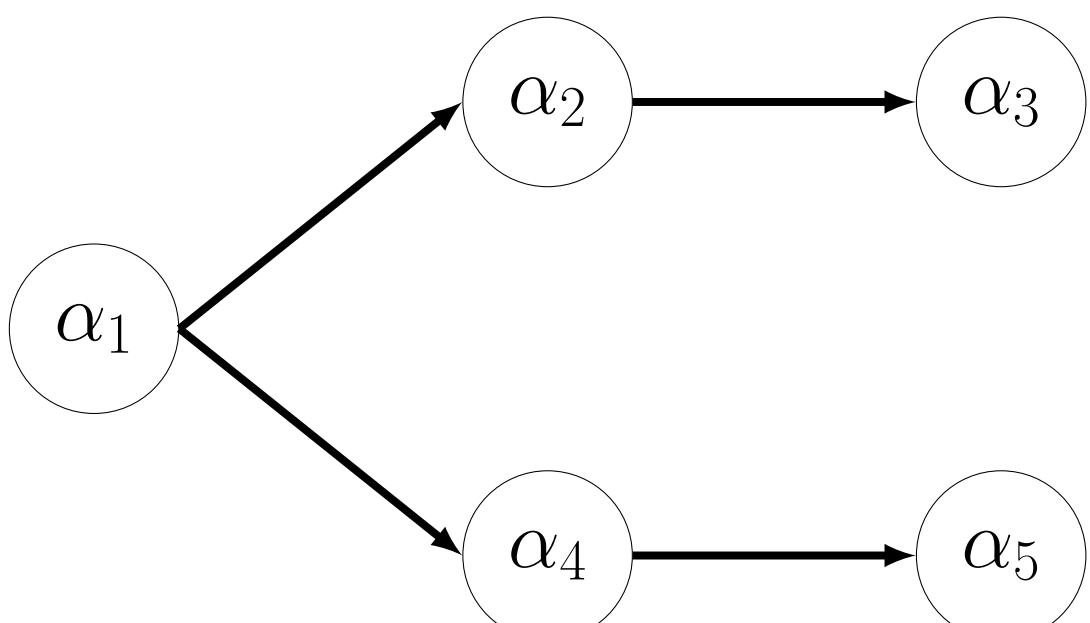
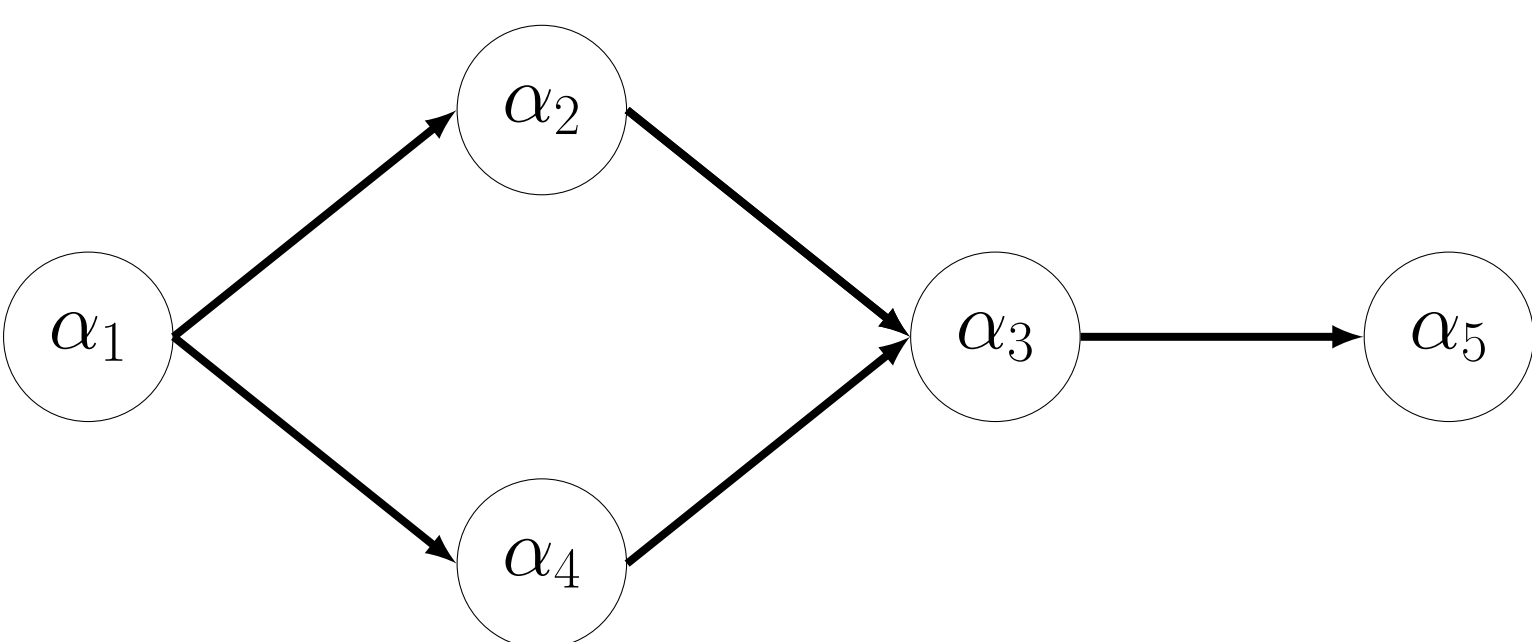


Figure 4. Convergent attribute hierarchy



## The Hamiltonian-Gibbs Hybrid Sampler

- HMC requires **Hamiltonian dynamics** which are only defined for continuous variables (Betancourt, 2019).
- HMC samplers cannot directly handle discrete/categorical variables (i.e., attribute parameters).
- The **HGHS partitions the parameter space** into discrete and continuous blocks and updates the former via a Gibbs step and the latter via an HMC step.

## Simulation Study Design

- Sample size: (500, 1000)
- Attribute Structure:** (no hierarchy, linear hierarchy, divergent hierarchy, convergent hierarchy)
- Attribute Correlations** (0.1, 0.7)
- Balanced  $Q$  matrix such that

$$Q^T Q = \begin{pmatrix} 10 & 5 & 5 \\ 5 & 10 & 5 \\ 5 & 5 & 10 \end{pmatrix}$$

- Intercept parameters  $\sim U(-1, -0.3)$
- Main effect parameters  $\sim U(0.3, 1.5)$
- Interaction effect parameters  $\sim U(0, 1)$
- Fully-crossed design led to 16 simulation conditions
- Outcomes:** parameter recovery (bias, RMSE, correlations), parameter posterior SD

## Selected Results

Figure 5. Bias, RMSE, and correlations for independent, linear, divergent attribute structures

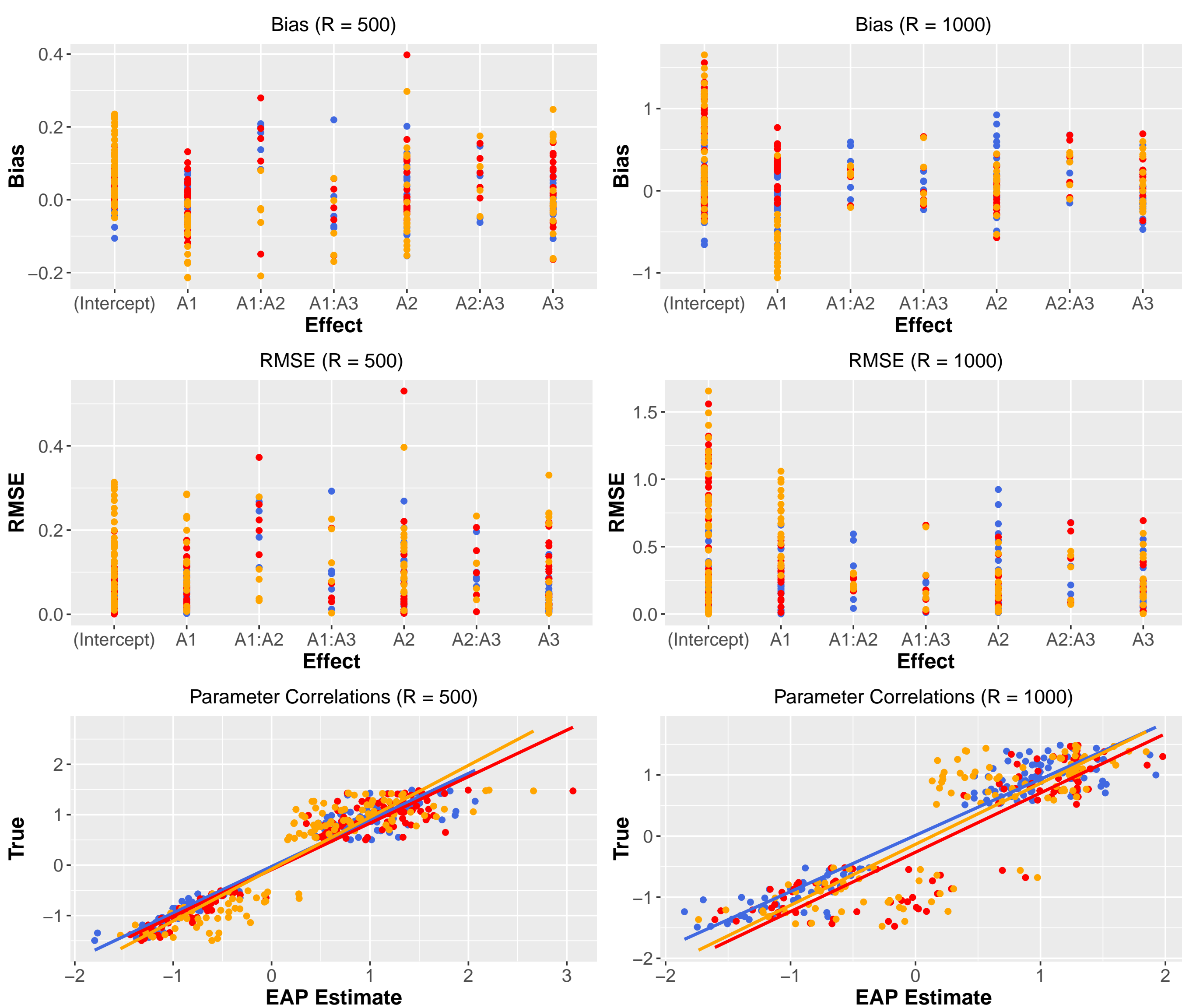
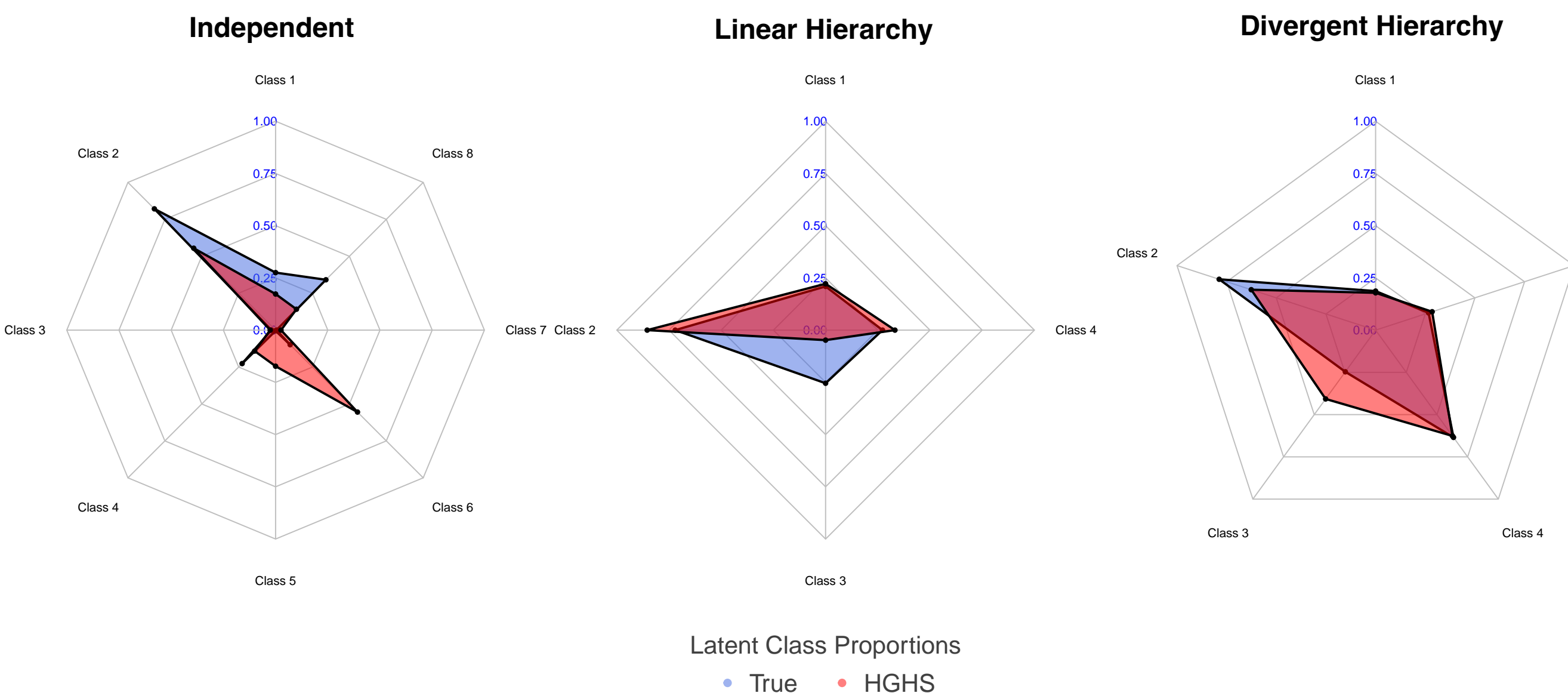
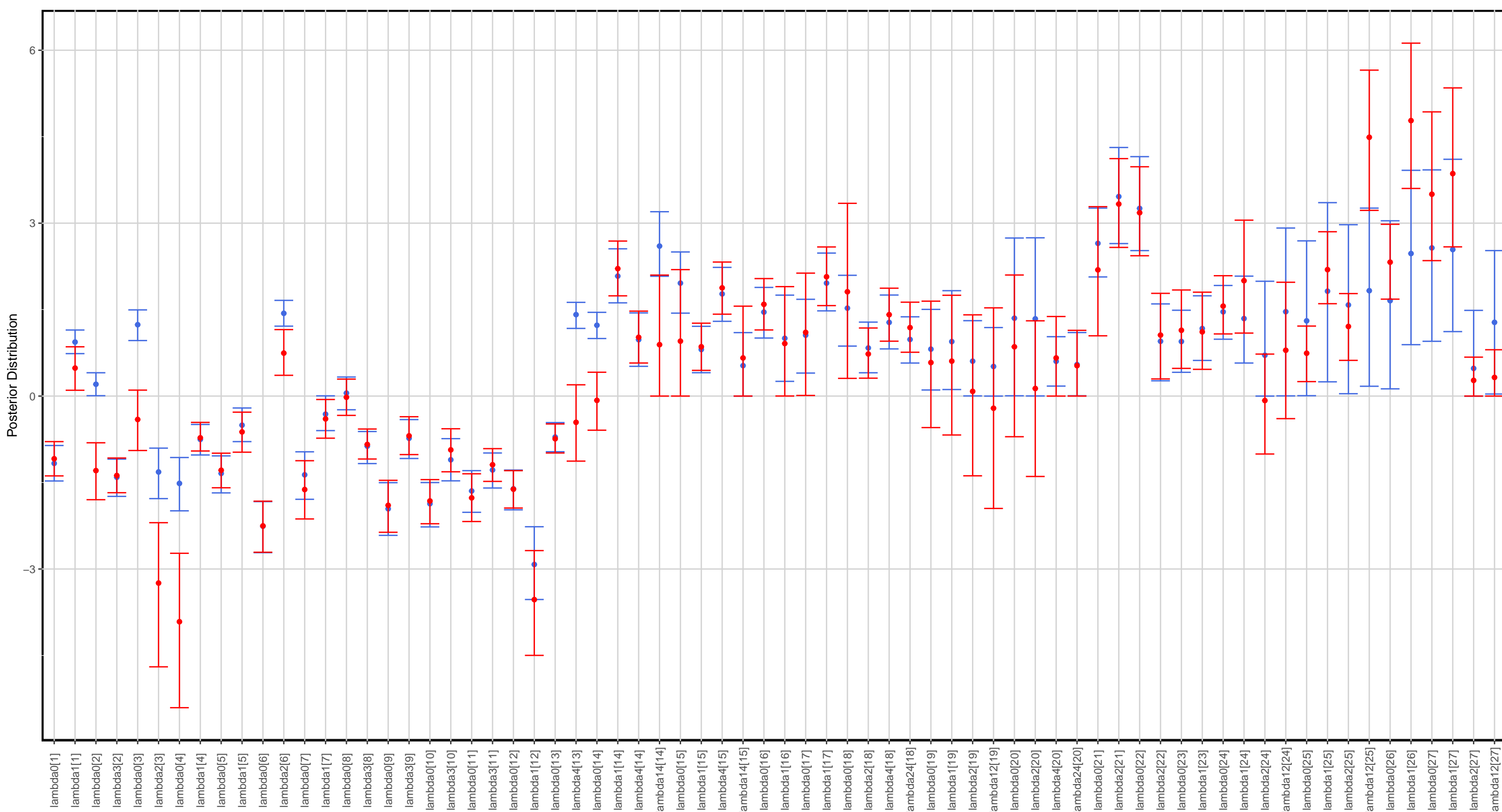


Figure 6. Latent class membership proportions



## Empirical Example: DTMR $(R, I, K) = (990, 27, 4)$

Figure 7. DTMR EAP estimates and 95% HDPI obtained from JAGS and HGHS



## Conclusions

- Preliminary results suggest **adequate parameter recovery** across multiple hierarchy specifications.
- Outcomes were more accurate for independent and linear hierarchies.
- In some instances, HGHS displayed **less variability** with respect to posterior distribution (posterior SDs and 95% HDPI) compared to Gibbs-only sampling.
- Preliminary results suggests HGHS is viable alternative to current Bayesian estimation samplers.
- Future directions: Systematic comparisons between samplers with emphasis on estimation speed and convergence rates.

## References

Betancourt, M. (2019). The convergence of Markov chain Monte Carlo methods: From the Metropolis method to Hamiltonian Monte Carlo. *Annalen der Physik*, 531(3), 1700214.