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MASTER IN ECONOMICS: INSTRUMENTS OF ECONOMIC ANALYSIS

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GAME THEORY AND NETWORKS

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Abstract

This paper provides a theoretical introduction to game theory on networks. It presents the theoretical context and defines games on networks. Starting from such theoretical framework, the paper proposes several examples that illustrate how the structure of Nash equilibria on network changes with respect to traditional bilateral games. We show that there are more equilibria on networks than in two-player games and an equilibrium on networks does not necessarily requires all connected individuals to play mutual best responses, a necessary condition for any bilateral Nash equilibrium. Last, using few examples, we propose a rationale for an equilibrium refinement concept based on an idea of strategic dependence, in which the network structure plays a key role.

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1 Introduction.

Game theory was developed by mathematicians interested in economics, and some of its terminology reflects that history. John von Neumann is considered the father of game theory due to his book, *Theory of Games and Economic Behavior*, published in 1944 in which revolutionized the field of economics. The Nash equilibrium was described later John F. Nash (1950) and these ideas were further developed in the context of evolution by Smith and Parker (1976).

Game theory grew as an attempt to find the solution to the problems of duopoly, oligopoly, and bilateral monopoly. In all these market situations, a determinate solution is difficult to arrive at due to the conflicting interests and strategies of the individuals and organisations involved in the market.

The theory of games attempts to arrive at various equilibrium solutions based on the rational behaviour of the market participants under all conceivable situations. In a formal way, the purpose of game theory is to describe situations, in which two or more agents or entities may pursue different views about what is to be considered best by each of them. In other words, game theory —at least the non-cooperative part of it— strives to describe what the agents' rational decisions should be in such conflicting situations. The Nash equilibrium is the most commonly applied solution to a game, in which each participant considering an opponent's choice has nothing to gain by switching his strategy.

Traditional game theory typically analyzes situations, in which everybody plays against everybody. However, in real life different market participants may interact or compete against a subset of other market participants and different players make compete against different other players e.g. due to geographical distribution of market actors (Easley and Kleinberg, 2010). There have been several recent contributions to model such strategic situations using networks as a representation of who interacts with whom (see e.g. Jackson and Zenout (2015) for a review).

The interest in networks stems from the idea that humans are inherently social beings. We rely on each other for sustenance, safety, governance, information, and companionship. Production, exchange and consumption of goods and services largely take place in social settings, where the patterns and nature of interactions influence, and are influenced by, economic activity. This embeddedness of many economic transactions means that abstracting from social structure comes with the risk of severely misunderstanding behaviors and their causes. In particular, designing many economic policies requires a deep understanding of social structure. For instance, beyond such social networks, other interactions, such as international trade and political alliances, have inherent network structures that shape the impact of policies and help us understand conflict and other inefficiencies (see Jackson et al. (2016) for a recent review and other references).

A social network is a set of nodes representing people, groups, organizations, enterprises, etc. that are connected by links showing relations or flows between them. The social network analysis permits to understand patterns of behavior in a wide and variated range of situations.

Apart from that, social networks are important in many facets of our lives, including strategic situations. Most decisions that people make, from which products to buy to whom to vote for, are influenced by the choices of their friends and acquaintances. For example, the decision of an individual of whether to adopt a new technology, attend a meeting, commit a crime, find a job is often influenced by the choices of his or her friends and acquaintances (Jackson et al., 2016).

The emerging empirical evidence on these issues motivates the theoretical study of network effects. Such interactions are natural ones to model using game theory, as the payoffs that an individual receives from various choices depends on the behaviors of his or her neighbors. For example, if individuals only wish to buy a new product if a sufficient fraction of their friends do, can we say something about how segregation patterns in the network of friendships affects the purchase of the product? The theory makes predictions both about how a player's behavior relates to his/her position in a network, as well as what overall behavior patterns to expect as a function of the network structure (Jackson et al., 2016).

In this work, we follow the literature that connects game and network theories in that the network determines who interacts with whom (see Jackson and Zenout (2015), for a survey). In this study, we aim at understanding whether and how network topology affects the equilibrium structure, compared to standard bilateral games. Moreover, we will try to propose a rationale for an equilibrium refinement, attending additional criteria. Although, there is a large multiplicity of Nash equilibria theoretically, the network structure may induce certain regularities on which equilibria are more plausible than others (Easley and Kleinberg, 2010).

Our main results can be summarized as follows. Network games exhibit a large multiplicity of equilibria in comparison with bilateral games. We illustrate this via several examples. Moreover, there appear equilibria exhibiting features incompatible with behavior in bilateral Nash equilibria. In particular, a network-game equilibrium does not require all connected individuals to best-respond each other bilaterally, a necessary condition of any bilateral equilibrium.

Last, we propose a rational for an equilibrium refinement based on the idea of strategic dependence. Via several examples, we show how such a rationale can select among multiple equilibria in particular network structures for particular games. We then highlight two important features of the proposed rationale. First, the mechanism reflects the positioning in the network. Second, the strategic power —reflected mostly in the equilibrium payoffs— is not related to the overall centrality of individual agents. Rather, it stems from a relative centrality one holds in relation to the centrality of her neighbors. In other words, one has an advantage in a network game not because she is central globally but because she is more central than her neighbors.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical framework. Section 3 contains the results that are illustrated via numerous examples. Section 4 concludes.

2 Definitions and Theorical Framework.

Game theory studies decisions by several agents in situations with strategic interactions. Compared to other theories of multi-person decisions, it has two distinguishing features. First, it explicitely considers each person's available strategies and the outcomes resulting from combinations of their choices; that is, a complete and detailed specification of the 'game'. Second, in non-cooperative contexts it focuses on optimal choices by each person separately (Govindan and Wilson (2008)).

John F. Nash (1950) proposed that a combination of mutually optimal strategies can be characterized mathematically as an equilibrium. According to Nash's definition, a combination of strategies is an equilibrium if each person's choice is an optimal response to others' choices. His definition assumes that a choice is optimal if it maximizes the person's expected utility of outcomes, conditional on knowing or correctly anticipating the choices of others (Govindan and Wilson (2008)).

In other words, a pair of strategies is a *Nash equilibrium* if neither player can unilaterally switch to another strategy without reducing its payoff.

Standard games specify the players, strategy spaces, and all possible payoffs, and it implicitly assumes that everybody plays against everybody. In contrast, a network game has to specify who plays against whom and such an interactions structure is describbed as a network. An important feature of such defined network game is that agents are disallowed to choose different actions for their different neighbours, if that happens there would essentially be no network effects. Moreover, we assume that each player earns the sum of the payoffs of all the bilalteral games against all her neighbors[1]. An equilibrium in a network game is obtained whenever all players choose an action that is a best response (BR_i) to whatever their neighbours choose (Kovarik et al. (2013)).

We can write that idea mathematically in the following way.

$$ij = \begin{cases} 1 & \text{if agents i and j are connected in a network} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

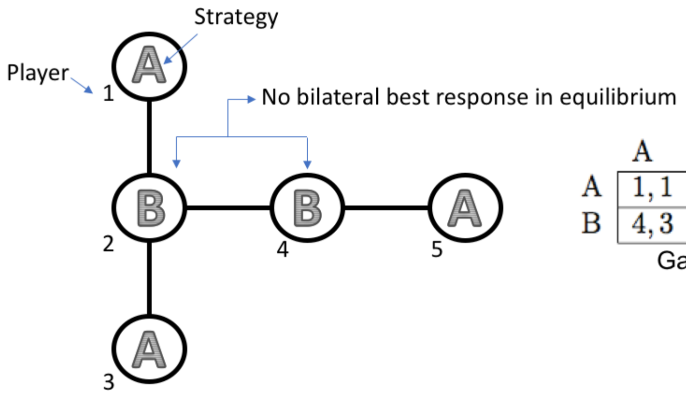
$$N.N.E = \begin{cases} N_i = \{j \mid ij = 1\} \\ s_i^* \; \in \; BR_i(s_{N_i}^*) \; \forall \; i \end{cases} \tag{2}$$

Where $s_{N_i}^*$ is the vector of strategies played by players in N_i.

As it has been described above, in Nash equilibrium of a network game each agent has to choose the best response to the distribution of her neighbours choices. Observe that such a definition does not require bilateral best responses in equilibrium.

[1] An alternative approach would be to assume that people earn an average payoff from all their interactions.

An example can be useful in order to understand this issue. Consider the network and game in Figure 1. Node 2 has three neighbors 1, 3, and 4, one of which chooses B and two A. Observe that the best response to two agents choosing A and one choosing B is to play B even though B is not the best response against node 4 playing B. Therefore, best responding to a set of neighbors does not require an agent to best respond to each of them bilaterally.



	A	B
A	1, 1	3, 4
B	4, 3	2, 2

Game

Figure 1: Example of no bilateral best response in equilibrium.

This behavior may be counterintuitive if we look at players 2 and 4 separately, but it makes sense within the context of a network because is the response against your neighbourhood what must be taken into account. In Figure 1 player-4 plays B against player-2 who is playing B, so he earns a payoff of 2 but he plays B against player-5 who is playing A (player-5 is forced to play A somehow because he has only one neighbour) so he earns 4, consequently player-4 earns a total payoff of 6. We will see this issue deeply below.

In order to place the ongoing work in his theorical context, the following definitions must be introduced (see Jackson and Zenout (2015)).

- We consider a finite set of agents $N = \{1, ..., n\}$ who are connected in a network, a strategy set S_i and a payoff function $p_i : (S_1 \times \ldots \times S_n) \to \mathbb{R}$ for each agent i. Each agent i chooses a strategy $s_i \in S_i$, hence the vector $s = \langle s_1, \ldots, s_n \rangle$ is called a **strategy profile**.

- A **network** (or graph) is a pair (**N**,**g**), where **g** is a network on the set of nodes **N**. These represent the interaction structure in the game, indicating the other agents whose actions impact a given agent's payoff.

  We abuse notation and let **g** denote the two standard ways in which networks are represented: by their adjacency matrices as well as by listing the pairs of nodes that are connected.

  Thus, **g** will sometimes be an $n \times n$ adjacency matrix, with entry $\mathbf{g}_{ij} = 1$ denoting whether i is linked to j and can also include the intensity of that relationship. We abstract from the intensity in the paper. Therefore, in our context, a link either exits or not between two individuals.

- A relationship between two nodes i and j, represented by $ij \in \mathbf{g}$, is referred to as a **link**.

- A **walk** in a network (**N**,**g**) refers to a sequence of nodes, $i_1, i_2, i_3, ..., i_{K-1}, i_K$, such that $i_k i_{k+1} \in$ **g** for each **k** from 1 to K-1. The length of the walk is the number of links in it, or K-1.

- A **path** in a network (**N**,**g**) is a walk in (**N**,**g**), $i_1, i_2, i_3, ..., i_{K-1}, i_K$, such that all the nodes are distinct.

- A **cycle** in a network (**N**,**g**) is a walk in (**N**,**g**), $i_1, i_2, i_3, ..., i_{K-1}, i_K$, such that $i_1 = i_K$.

- A network (**N**,**g**) is **connected** if there is a path in (**N**,**g**) between every pair of nodes i and j.

- A **component** of a network (**N**,**g**) is a subnetwork (**N'**,**g'**) (so **N'** $\subset$ **N** and **g'** $\subset$ **g**) such that

  - There is a path in **g'** from every node $i \in$ **N'** to every node $j \in$ **N'**, $j \neq i$.

  - $i \in$ **N'** and $ij \in$ **g** implies $j \in$ **N'** and $ij \in$ **g'**.

  Thus, a component of a network is a maximal connected subnetwork with all adjacent links, so that and there is no way of expanding the set of nodes in the subnetwork and still having connected network.

- The **distance** between two nodes in the same component of a network is the length of a shortest path (also known as a **geodesic**) between them.

- The **neighbors** of a node i in a network (**N**,**g**) are denoted by $\mathbf{N}_i(\mathbf{g})$. Normally **N** is fixed (we omit dependence on the set of nodes N), and so write $\mathbf{N}_i(\mathbf{g})$ rather than $\mathbf{N}_i(\mathbf{N},\mathbf{g})$.

  $$\text{Thus, } \mathbf{N}_i(\mathbf{g}) = \{j \mid ij \in \mathbf{g}\}.$$

- The **degree** of a node i in a network (**N**,**g**) is the number of neighbors that i has in the network, so that $d_i = \mid N_i(g) \mid$.

- The $\boldsymbol{k}^{th}$ **power** $g^k = g \times ...g$ of the adjacency matrix g keeps track of indirect connections in **g**. More precisely, the coefficient g^k_{ij} in the (i,j) cell of g^k gives the ij number of walks of length **k** in **g** between i and j.

- An **independent set** relative to a network (**N**,**g**) is a subset of nodes $A \subset N$ for which no two nodes are adjacent (i.e., linked). A is a maximal independent set if there does not exist another independent set, $A' \neq A$, such that $A \subset A' \subset N$.

- A **dominating set** relative to a network (**N**,**g**) is a subset of nodes $A \subset N$ such that every node not in A is linked to at least one member of A. For example, the central node in a star forms a dominating set and also a maximal independent set, while each peripheral node is an independent set and the set of all peripheral nodes is a maximal independent set. Any set including the central node and some peripheral nodes is a dominating set, but not an independent set.

3 Results.

In this section, we first present a series of examples that shows how the structure of Nash equilibria[2] differ compared to traditional two-player games. In particular, we point out three features.

- First, there typically are more equilibria in a network game than in the same game played by two players.
- Second, we show that a network equilibrium does not require all the opponents in a network game to play mutual best responses.
- To conclude, the last subsection illustrates how the network structure itself can help to select among the multiple Nash equilibria in network games.

3.1 Multiplicity of Equilibria.

First note that theoretically it is not required for an equilibrium of the network game that two connected players choose best response bilaterally or in other words that all links are in Nash equilibrium (Kovarik et al. (2013)).

This means that if we take any two nodes from the network, between which there is a link, then in equilibrium the action choices of these two players need not be mutual best responses. This may be the case even though the network is in equilibrium. The reason is that in Nash equilibrium each player has to choose a best response to the distribution of her neighbours choices.

We now proceed to characterize the main networks features and the Nash Equilibria ($N.E$) with some games as an example.

Example 1. Considering the coordination game in Table 1:

	A	B
A	$2, 2$	$0, 0$
B	$0, 0$	$2, 2$

Table 1: Coordination Game.

[2]For illustrative purposes, we abstract from the mixed equiliria, but they exist.

If we compute now all the equilibria of this game for a two-player game, we will have two Nash Equilibrium in pure strategies and one more in mixed strategies.

$$N.E = \begin{cases} (s_1, s_1) \\ (s_2, s_2) \\ (\frac{1}{2}s_1 + \frac{1}{2}s_2), (\frac{1}{2}s_1 + \frac{1}{2}s_2) \end{cases}$$

We can create a simple network with 2 agents (like a traditional bilateral game) that represents such a strategic situation.

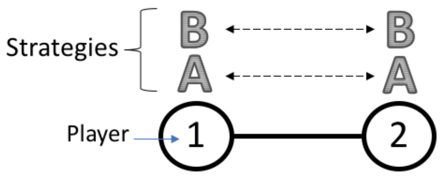


Figure 2: 2-Node Path.

The 2-node path is as simple as it gets, two people are given a game and they have to choose a strategy.

Considering now the same game in Table 1 and a more complex network in Figure 3.

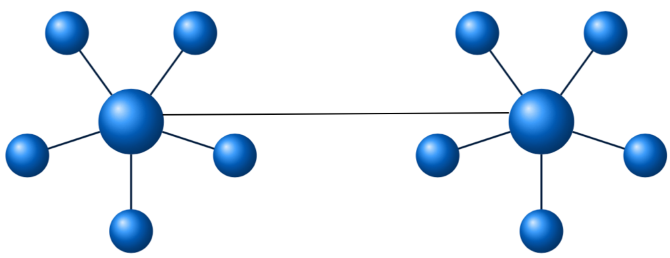


Figure 3: Network 1.

We will proceed to compute and represent all the equilibria of the Figure 3.

First of all, central players hold all the bargaining power because they are linked to five individuals who are only linked to them, so when central players choose A all his neighbours are going to best respond by choosing A (and the same thing happens if they choose B instead of A). As a result, there are four pure-strategy Nash equilibria displayed in Figure 4.

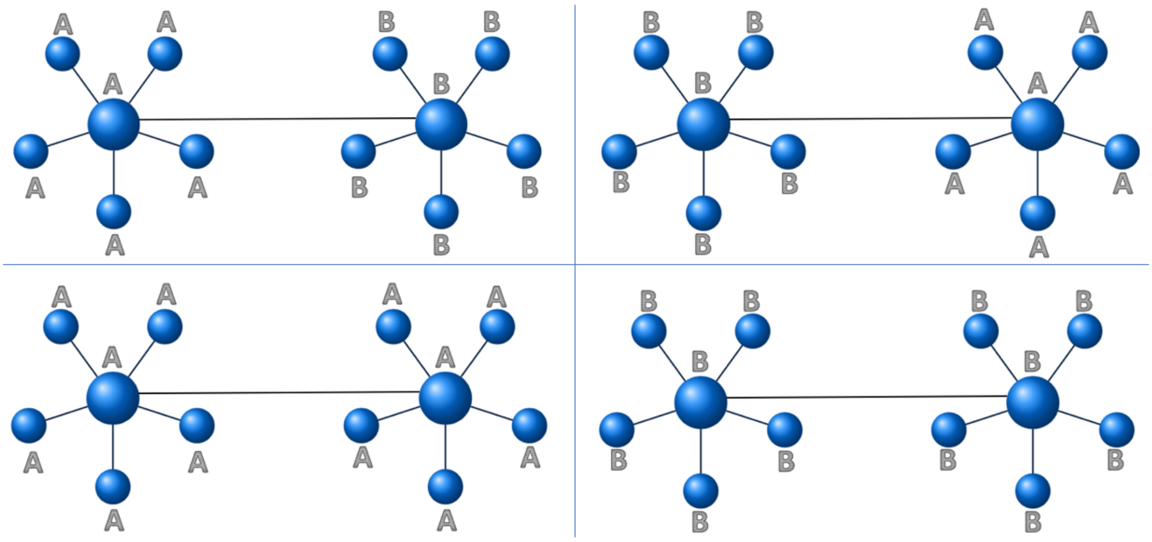


Figure 4: Four Nash Equilibria in the game from Table 1 played on the network from Figure 3.

We have already seen in Figure 4 how there is typically more equilibria in a game on network than in a simple bilateral game (in pure strategies: four Nash equilibria instead of two) and the equilibria exhibit features different from bilateral game. We are going to see these features in the following sections. Hence, there are two Nash equilibria in pure strategies in a bilateral game (whitout network) and four Nash Equilibria in pure strategies when we introduce a network.

3.2 Different Equilibria.

Another difference of network games with respect to bilateral games is that different equilibira appear, so the main features ot them are introduced separately in this section in order to have a clearer view.

3.2.1 Lack of Bilateral Coordination and Bilateral Best Response.

Lack of bilateral coordination and the presence of linked individuals who do not best respond to each other in equilbria are common features on networks. For instance, the lack of coordination in equilbrium and the fact that two opponents are not mutually best responding each other even thought the whole network is in equilibrium are showed considering the coordination game in Table 1 and the network in Figure 5.

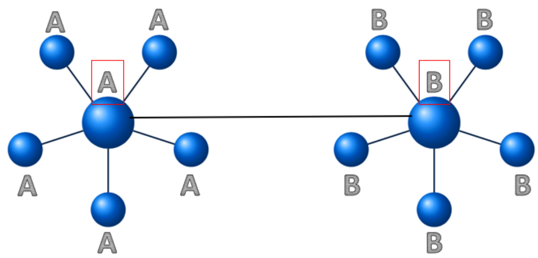


Figure 5: Lack of both bilateral coordination and bilateral best response example.

Figure 5 reproduces one of the equilibria from Figure 4. The whole network is in equilibrium. That is, the central players in each subnetwork are best responding to all their neighbors even though they are miscoordinated mutually and do not best respond to each other. This simple example illustrates that even though a network is in equilibrium, mutual best response of all connected nodes is not necessary. However, mutual best response is a necessary condition in bilateral games.

3.3 A Rationale for an Equilibrium Refinement.

The question here is whether the network itself can somehow help to select between the multiple equilibria.

To answer this question it is important to understand how a node's position in a network translates into her power in the game. In other words, the position of the agents on the network determines its bargaining power. Namely, imbalance in the relationship may be rooted in considerations of network structure, and transcend the individual characteristics of the two people involved.

When there are multiple equilibria, some of which favor one player and some of which favor another, we may need to look for additional sources of information to predict how things will turn out.

Easley and Kleinberg (2010) propose several aspects of network position that may be sources of power in strategic situations:

(i) *Dependence*: Social relations confer value, those players (or nodes) who are completely dependent on other player as a source of such value are less potent in a negotation. On the other hand, a player having multiple sources has a stronger position.

(ii) *Exclusion*: Related to (i), the player who has the ability to exclude others players take advantage in a negotation. In other words, he has the unilateral power to choose his partner.

(iii) *Satiation*: Again, viewing social relations as conferring value, a powerful player will acquire value at a greater rate than the other members of the group. So the strongest player may be interested in maintaining these social relations only if he can receive an unequal share of their value.

(iv) *Betweenness*: A player has high betweenness if it lies on paths (and particularly short paths) between many pairs of other nodes. For example, it is posible to say that a player has high betweenness when he is the unique access point between multiple different pairs of nodes in the network, and this potentially confers power. More generally, betweenness is one example of a *centrality measure* that tries to find the "central" points in a network.

In game theory, refinement refers to the selection of a subset of equilibria, typically on the grounds that the selected equilibria are more plausible than other equilibria. The focus typically is on refinements that make an appeal to rationality arguments (Nachbar (2015)).

This is the traditional approach to refinement. An important alternative approach is based on dynamic equilibration: players start out of equilibrium and in one sense or another learn (or fail to learn) to play an equilibrium over time. Some examples of this strategic form refinements are defined by Nachbar (2015).

We take an alternative approach and provide several example that show how network position can help to predict which of the multiple equilibria might be more likely to be observed, using the idea of **strategic dependence**.

	A	B
A	1, 1	3, 4
B	4, 3	2, 2

Table 2: Anti-Coordination Game.

This anti-coordination game has these three equilibria if played bilaterally, showed below:

$$N.E = \begin{cases} (s_1, s_2) \\ (s_2, s_1) \\ (\frac{1}{4}s_1 + \frac{3}{4}s_2), (\frac{1}{4}s_1 + \frac{3}{4}s_2) \end{cases}$$

We are going to see in some network examples how the increase in the number of equilibria depends on the network and to conclude if the network itself can somehow help to select between the multiple equilibria.

Example 2. Considering the game in Table 2 and the Figure 3 we will compute all the equilibiria.

The game in Table 2 is an Anti-Coordination Game in which players have to choose different actions in equilibrium and it has multiple equilibria. Consequently in Figure 2 central players hold all the bargaining power because they are linked to five isolated neighbours, so when central players choose B all his neighbours are going to choose A (and vice versa). According to this the following equilibria (displayed in Figure 6) appear.

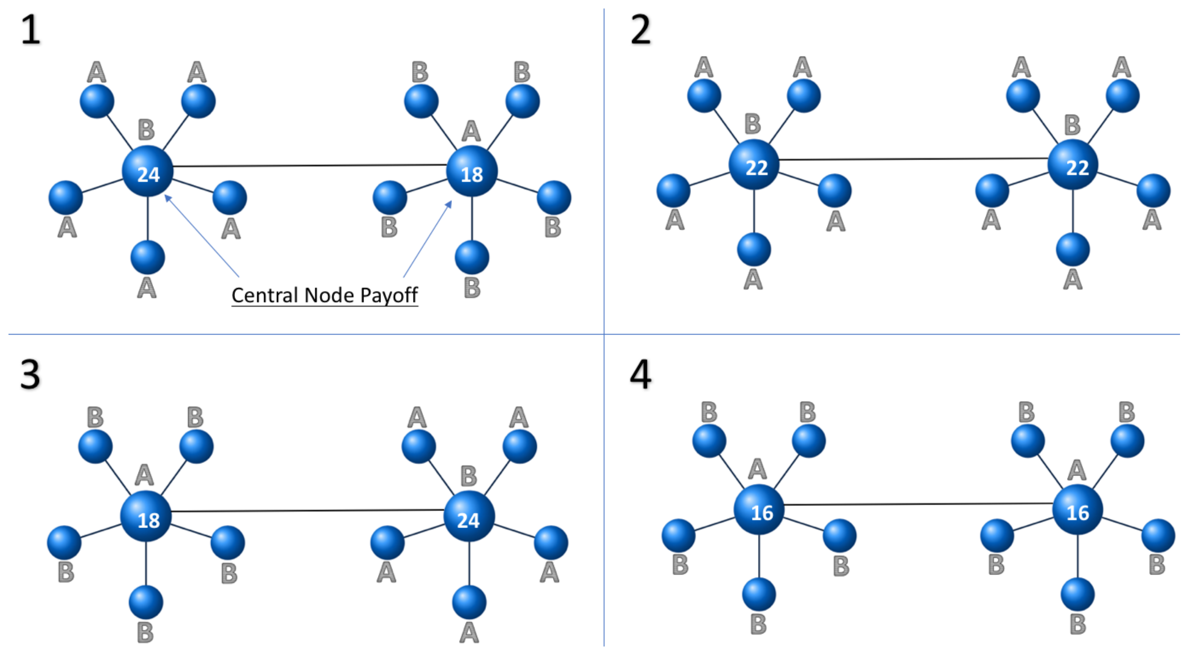


Figure 6: Four Nash Equilibria in Network 1 when we play the game in Table 2.

Based on the concept of *strategic dependence* a refinement for this network can be proposed, see Figure 7. All networks in Figure 6 are in a Nash equilibrium but the central player can get a larger payoff changing his strategy, from A to B, and the neighbors of the central players depend strategically on them.

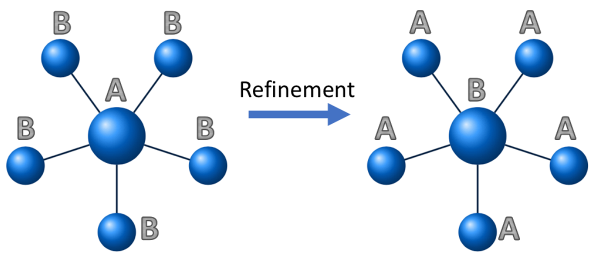


Figure 7: Network 1 Refinement Idea.

These refinements are closely linked to the power who hold certain players within networks. As a result, out of the four equilibria in Figure 6, equilibrium 2 is the most plausible because in that equilibrium both central players take advantage of their position to impose the actions that maximize their payoffs.

Example 3. Considering the game in Table 2 and the Figure 8.

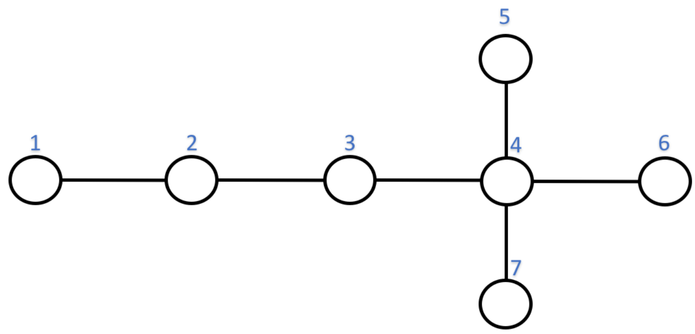


Figure 8: Network 2.

All the strict Nash equilibria of Figure 8 are collected in the following table.

Nash Equilibrium	Network 2
1	**(A,B,A,B,A,A,A)**
2	**(B,A,B,A,B,B,B)**
3	(B,A,B,B,A,A,A)
4	(A,B,B,A,B,B,B)

Table 3: Strict Nash equilibria. The format is $(s_1, ..., s_7)$, where s_i is the action of player i.

The equilibria marked in bold are those in which all the links are in bilateral Nash equilibrium. The set of additional strict Nash equilibria presented in the table above is obtained in the exhaustive way that is shown below.

· Assume that player-4 chooses A (and hence players 5-7 B). Consider candidate strings (-,-,A,A,B,B,B,B) and (-,-,B,A,B,B,B,B). A is never a best response to (A,-) and B is a best response to (B,A) yielding (A,B,B,A,B,B,B,B).

· Finally assume that player-4 chooses B (and hence players 5-7 A). Player 3 then has to choose A or B. Consider candidate strings (-,-,A,B,A,A,A,A) and (-,-,B,B,A,A,A,A). B is a strict best response to (B,A) yielding (B,A,B,B,A,A,A,A).

Figure 9 displays all the equilibria with the payoff and the strategy of each player labeled, it is very useful to have an idea of this network game at a glance.

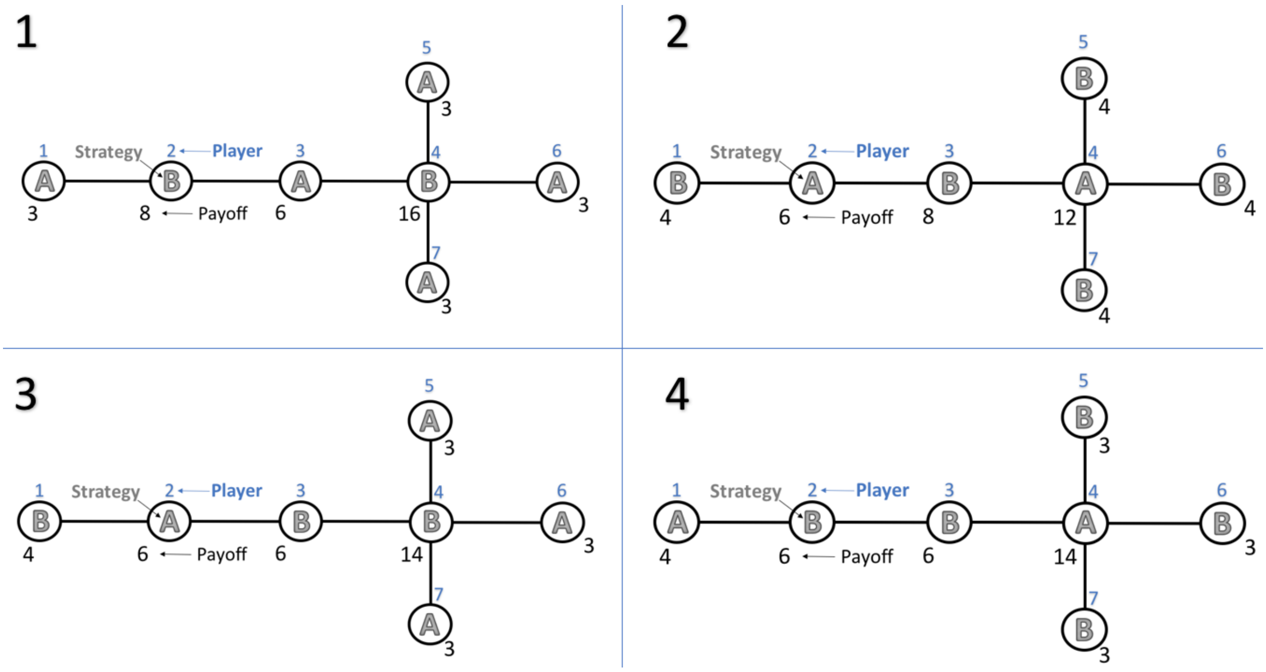


Figure 9: Four Nash Equilibria in Network-2 when we play the game in Table 2.

It is possible to see how all networks are in Nash equilibrium since all players are playing the best response against their neighbours distribution.

However, if player-4 realizes that three players depends on him and will best response to his behavior he will always prefer B to A, the reason is because when he is choosing B he is willing to earn at least the same payoff than if he had choosen A but the difference here is that when he chooses B he is able to improve his payoff and otherwise would be impossible. Consequently, if he attends rational criteria he will never play A as a strategy.

In addition, for the same reason player-2 is going to play always B. Hence, **the more plausible Nash equilibrium has to be (A,B,A,B,A,A,A)**, the first represented in Table 3.

Moreover, here is a crucial idea: agent-2 will have a greater payoff than agent-3 despite of agent-3 has a greater *centrality*. The reason is that agent-3 has little influence over agent-2, whereas agent-2 dominates over agent-1. Since agent-2 and agent-4 have other nodes who depend on them, the only choice left to agent-3 is to adapt to other's choices.

Across all the examples the idea of refinement does not respect the centrality but depends on the relative centrality of each node. In other words, those agents who are linked to isolated neihgbours hold greater influence and power.

Example 4. Considering the game in Table 2 and the Figure 10 which provides a complete overview of all the paradigms previously presented.

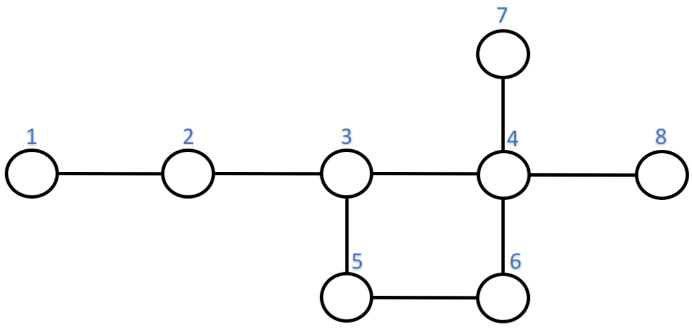


Figure 10: Network 3.

Nash Equilibrium	Network 3
1	**(A,B,A,B,B,A,A,A)**
2	**(B,A,B,A,A,B,B,B)**
3	(A,B,B,B,A,B,A,A)
4	(B,A,B,B,A,B,A,A)
5	(B,A,B,B,B,A,A,A)
6	(A,B,B,A,A,B,B,B)

Table 4: Strict Nash equilibria. The format is $(s_1, ..., s_8)$, where s_i is the action of player i.

The equilibria marked in bold in Table 4 are those in which all the links are in bilateral Nash equilibrium. However, it is known that theoretically it is not required for an equilibrium of the network game that all links are in bilateral Nash equilibrium. For example, a player with two neighbours one of which choosing A and one B will choose B as a best response in which case bilaterally he is not choosing a best response to the neighbour choosing B. The reason is that both rely more on other neighbors that depend strategically on both of them. Hence this link will not be in bilateral Nash equilibrium, while the network may well be in equilibrium.

The set of strict Nash equilibria presented in the table above is obtained in the exhaustive way that is shown below (first note that players 7 and 8 will always choose a best response to whatever player 4 chooses).

· Let us assume player-4 chooses A (and that players 7 and 8 choose B): A is a best response to (-,-,B,B) whenever players 3 and 6 choose (B,B). After checking player 5's and 6's best responses the following candidate survive: (A,B,B,A,A,B,B,B).

· Let us assume player-4 chooses B (and hence players 7 and 8 choose A): this is a strict best response to (-,-,A,A) whenever players 3 and 6 choose (A,A), (A,B), (B,B) yielding candidates (-,-,A,B,A,B,A,A), (-,-,B,B,B,A,A,A) and (-,-,B,B,A,B,A,A) where we have filled in player 5's strict response wherever possible.

Checking whether player 6 is choosing a strict best response returns candidates
(-,-,A,B,A,B,A,A), (-,-,B,B,B,A,A,A), (-,-,B,B,A,B,A,A) and (-,-,A,B,B,A,A,A).

Verifying player 3's best response in each case yields (B,A,B,B,A,B,A,A), (A,B,B,B,A,B,A,A) and (B,A,B,B,B,A,A,A) as additional equilibria.

Figure 11 displays all the equilibria with the payoff and the strategy of each player labeled, it is very useful to have an idea of this network game at a glance.

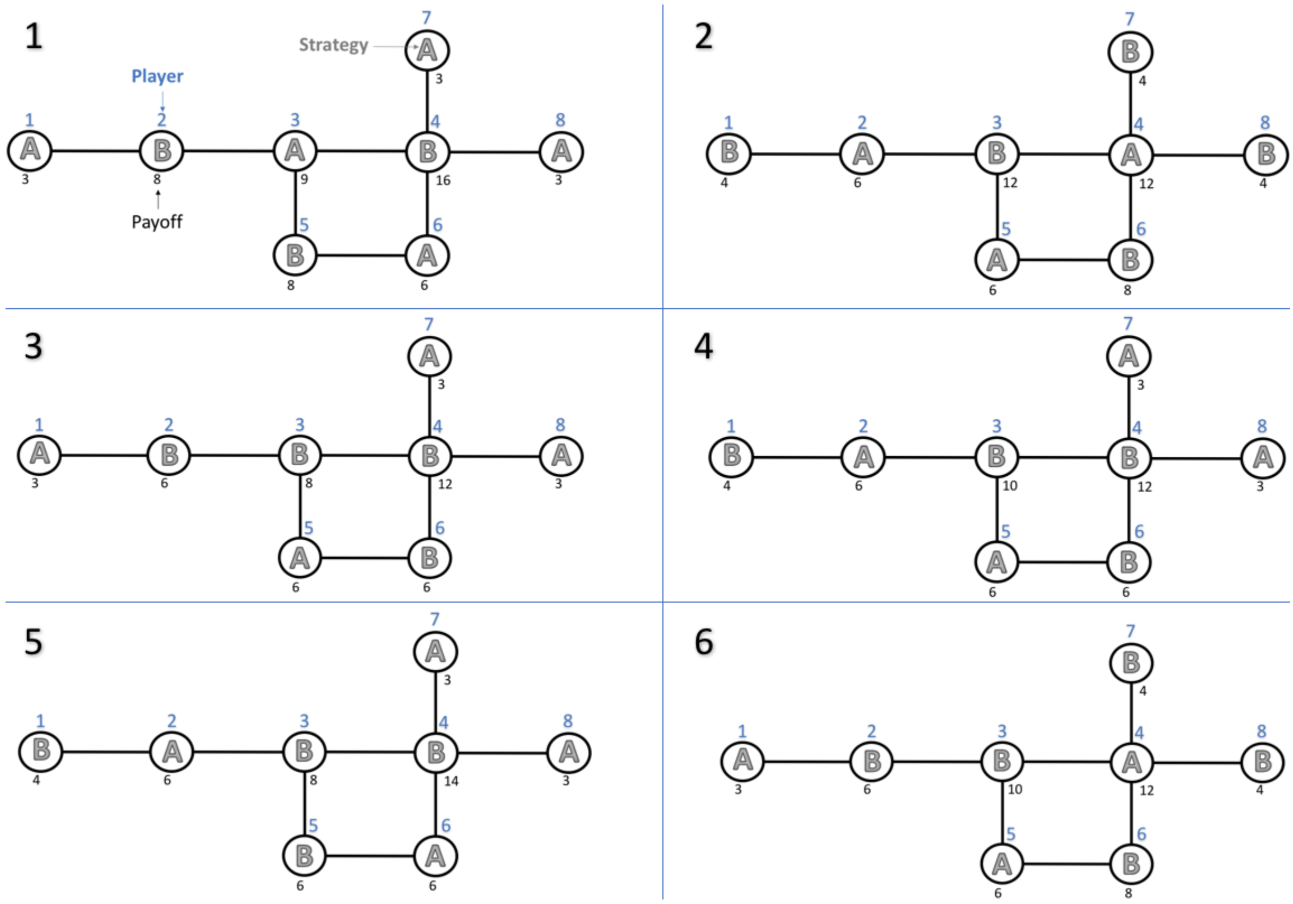


Figure 11: Six Nash Equilibria in Network-3 when we play the game in Table 2.

Following the idea of refinement introduced above: as long as playing a strategy B provides a payoff greater or equal than playing an alternative strategy A, it has sense to play always B.

With regard to Figure 11 in which player-4 has the chance to improve his payoff playing B (and at least he will earn the maximum payoff corresponding to the alternative strategy A), **the more plausible equilibrium has to be (A,B,A,B,B,A,A,A)** (equilibrium number one in Table 4) since player-2 and player-5 are willing to play B because the same reasoning.

Out of the 6 pure-strategy equlibria, we observe that only one seems to be plausible, given players position in networks and the incentives.

4 Conclusions and Further Work.

This paper provides a theorical introduction to game theory on networks and shows few examples on how network may affect the way in which individuals play. Specifically, we have obtained an increase in the Nash equilibria which is the main difference from traditional game theory in which networks are not taken into account. Furthermore, the subjects, who are distributed on a fixed network and matched with all their neighbours to play a game, are not necessarily in a Nash equilibrium bilaterally in order to achieve an equilibrium of the network game.

Besides, we have described ways of how the definition of an equilibrium among players' strategies in a game can be sharpened by invoking additional criteria, understanding how the selection of a more efficient subset of equilibria comes from those which are more plausible than others. Hence refinements make an appeal to rationality arguments.

All this work has been effectuated with some practical examples in order to help readers understand games on networks.

To sum up, the main conclusions are as follows.

- Games on Networks provide a larger number of Nash equilibria in comparison with traditional bilateral games.
- Network structure has a strong impact on equilibrium selection. Although there is a large multiplicity of Nash equilibria theoretically, one equilibrium is selected like more plausible than the others.
- Network position has a strong impact on the idea of strategic dependence and consequently on individual's behaviour.

These findings have implications for modeling social structures and serve as analysis of human behavior since we have highlighted the importance that social networks have in determining economic behaviors. For instance, leadership can emerge in a natural way from the learning process of individuals who differ only in their position in the social network and this is a significant consequence.

Future research could aim at developing a formal definition of an equilibrium refinement concept, based on the ideas introduced in this paper. The ideas brought forward in Section 3 of this article might be a useful step in this direction. Furthermore, it could be interesting to test these proposals in an empirical research.

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