Data Science UW Methods for Data Analysis

More on Hypothesis Testing, The Central Limit Theorem,
And an introduction to Regression
Lecture 4
Nick McClure

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Excellent health statistics - smokers are less likely to die of age related illnesses.'

Topics

> Review

> More on hypothesis testing

> Central Limit Theorem

> Introduction to Regression

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Review

> Sampling Methods

> Law of Large Numbers

> Hypothesis Testing

  – Normal testing

  – One tailed vs Two tailed

  – P-values

  – T-test (Student's, Welch's)

 – Chi-Squared

  – Fisher's Exact

> Outliers

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Hypothesis Testing Summary (so far)

> If data is normal,
  – If you know population mean and variance,
    > Use standard normal 'z-test'.
  – If you just know population mean,
    > Use t-test (unpaired data).
    > Use Welch's t-test (paired data).

> For categorical comparison tests,
  – If the sample/subgroup size is large enough,
    > Use Chi-squared test
  – If the sample/subgroup size is small,
    > Use Fisher's Exact test.

> How do we know the data is normal?

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Testing for Normality

> Kolmogorov-Smirnov test (K-S test).

  – Tests if two distributions are similar.

> Consider the Normal Cumulative Distribution Function (CDF).

Normal Cumulative Probability
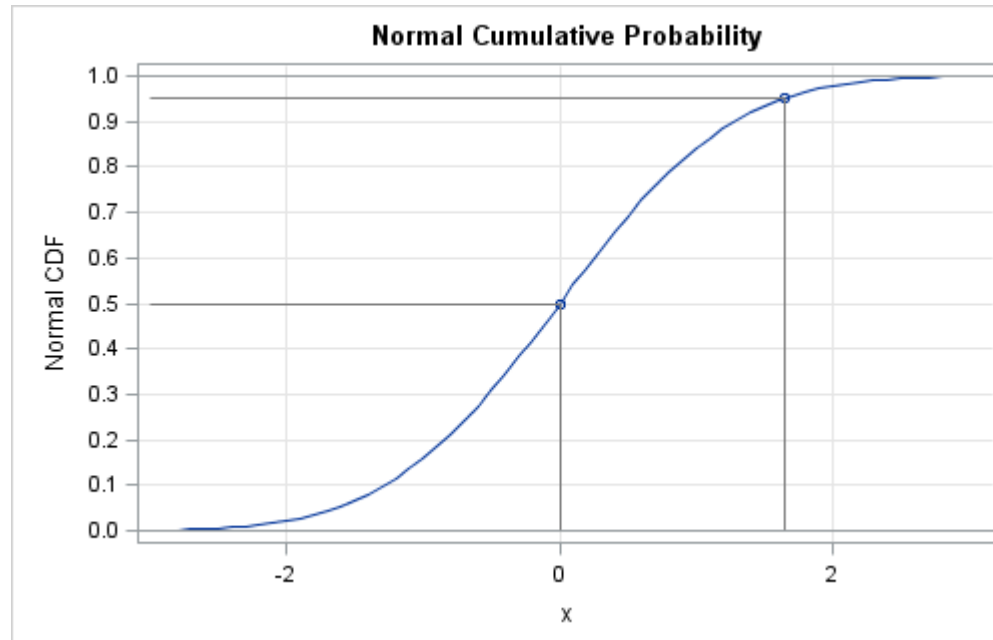
> Any similar distribution should have a similar CDF.

Testing for Normality

> The K-S statistic is just the maximum vertical distance between two CDFs.

> Note: the K-S test can test departure from any hypothetical distribution, not just normal.

> R-demo

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Testing for Normality

> Also, the Shapiro-Wilk test can tell us a test statistic for normality.

– Tests the difference in expected and sample 'moments'.

– Moments:

> 1st moment = mean

> 2nd moment = variance

> 3rd moment = skewness

> 4th moment = kurtosis

> …

– Slightly more powerful than the K-S test.

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Testing for Normality

> ALWAYS do a qq-plot to look at normality. (qqnorm())
> R-demo.

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Testing Between Multiple Groups

> What if we had multiple groups and we wanted to compare their means?

> Why can't we just do multiple two-sample t-tests for all pairs?

- Results in increased probability of accepting a false hypothesis.

- E.g., if we had 7 groups, there would be (7 Choose 2)=21 pairs to test. If our alpha cutoff is 5%, then we are likely to accept about 1 false hypothesis (21*0.05).

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Testing Between Multiple Groups

> Null Hypothesis:

– All groups are just samples from the same population.

> Alternative Hypothesis:

– At least one group has a statistically different mean.

> This type of analysis is called "Analysis of Variants", or ANOVA.

– We make data independence and normality assumptions first.

– Our test statistic is based on:

$$statistic \sim \frac{between\ group\ variability}{within\ group\ variability}$$

Performing Multiple Hypothesis Tests

> For non-ANOVA methods, remember that performing many hypothesis tests increases our risk of incorrectly rejecting a null-hypothesis.

> To compensate for this we decrease the p-value cutoff.

> The most common way of doing this is with the Bonferroni Correction.

$$p' = \frac{p}{(\# \ of \ Hypotheses)}$$

> This correction is argued to be too strong and other approximations for a new-p can be used instead.
  – Tukey's Range Test

> This is VERY important in genetics/bioinformatics.

> R Example
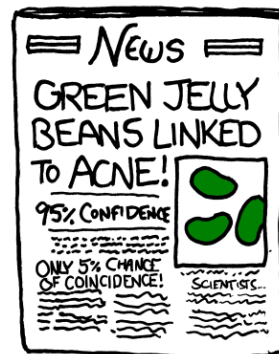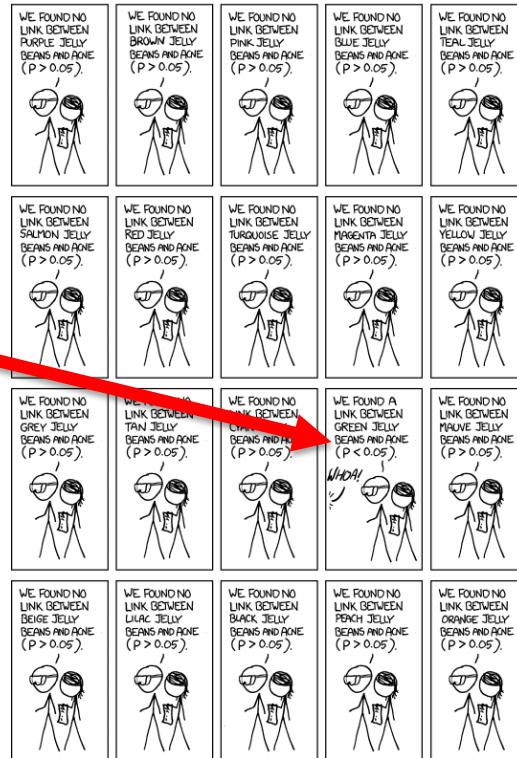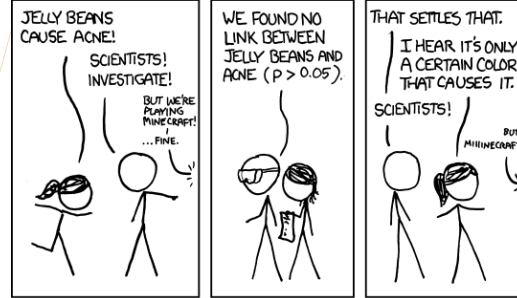
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Additional Hypothesis Tests

> Parametric test types:

– Mean comparison

– Variance comparison

– More distribution comparisons

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Central Limit Theorem

> If we sample a population over and over, the set of means of all samples are normally distributed, regardless of the population distribution.

$$\bar{X} \sim N\left(mean, \frac{variance}{\sqrt{n}}\right)$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

> Compare to Law of Large numbers ('proof' by R), shown in previous class.

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Central Limit Theorem

$$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

> We can use this central limit theorem to generate confidence intervals on expressing the population mean.

> We know the sample mean, sample variance, and number of samples.

> Then we know how our estimate of the population mean is distributed (from above formula).

> We can then generate 90%, 95%, … confidence intervals around our sample mean.

Confidence Intervals

> Confidence intervals are a way to express uncertainty in *population* parameters, as estimated by the sample.

> E.g. If we create a 95% confidence interval for the population mean, say

$$\hat{\mu} = \bar{X} = 10 \pm 5$$

– Then we say that the true population mean, μ , has a 95% chance of being between 5 and 15.

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Confidence Intervals

> To create confidence intervals for population means, we use the central limit theorem and create confidence intervals based on the normal distribution.

– Repeatedly sample from the population.

– Calculate the mean for each sample.

– Use the average of the sample means as the population estimate and create a C.I. based on the s.d. of the sample means.

– R demo
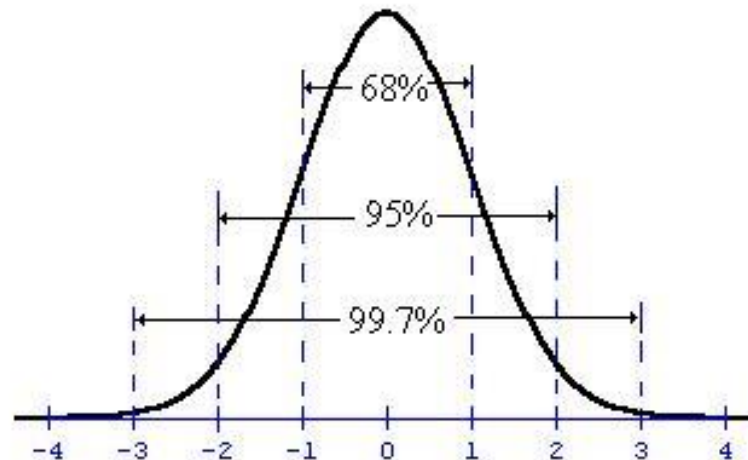
Regression Models

> The goal of regression is to produce a model that represents the best fit to some observed data.

> Typically the model is a function describing some type of curve (lines, parabolas, etc.) that is determined by a set of parameters (e.g., slope and intercept).

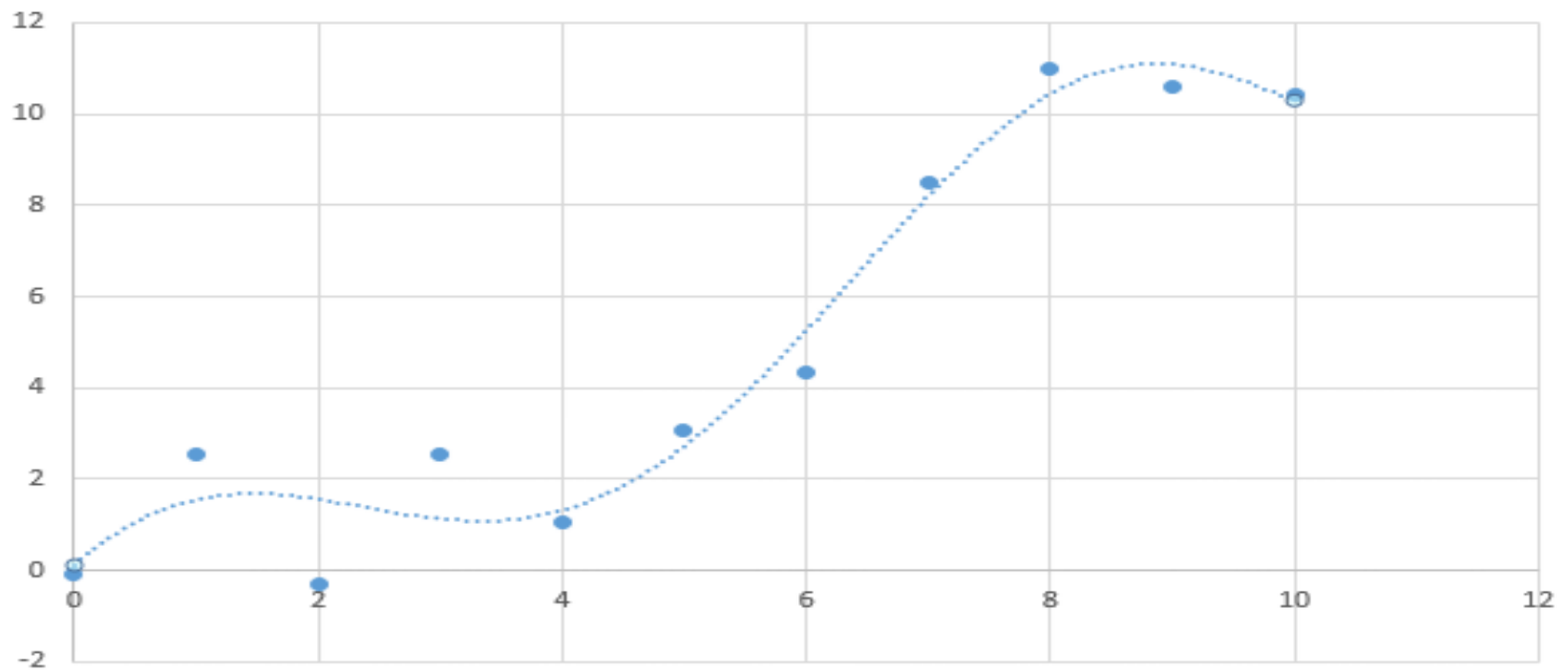> "Best fit" means that there is an optimal set of parameters according to an evaluation criteria we choose.

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Regression: Linear
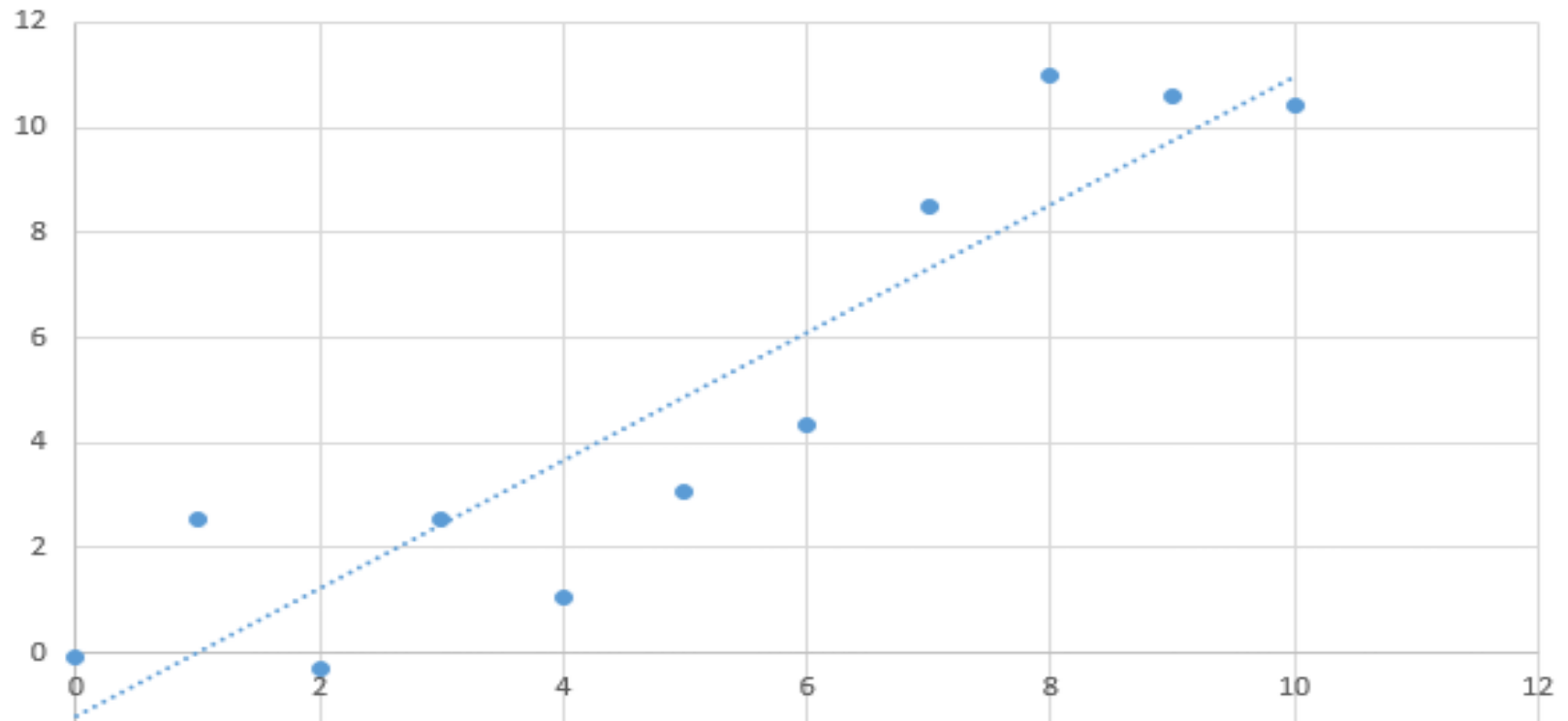
Regression: quadratic
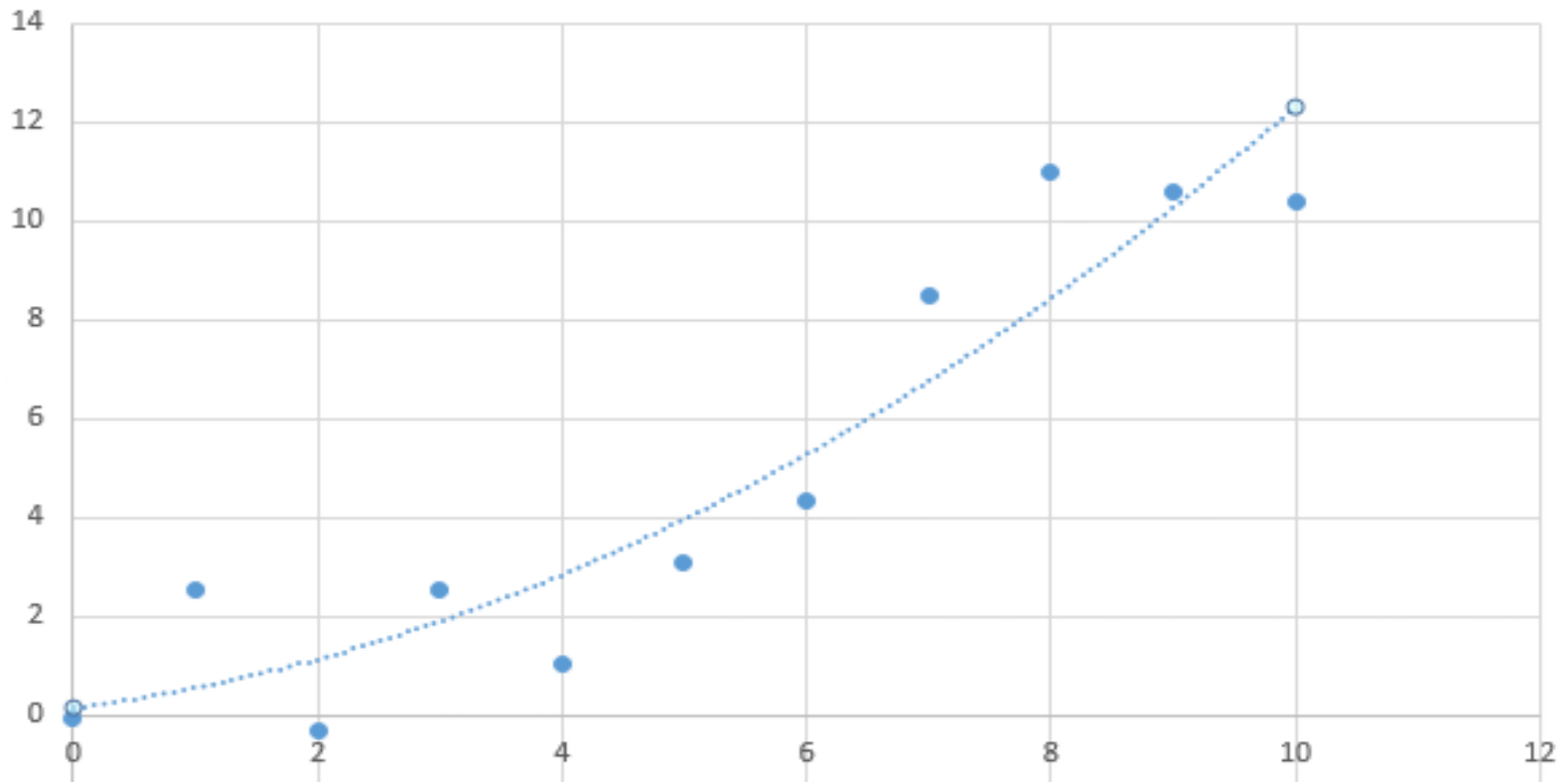
Regression: High Order

Regression Models

> Which one of the preceding examples is correct?

> In a sense, all of them are. They all give decent approximations to the data.

> It's hard to tell, just from looking at these plots whether any of them in fact will continue to perform well as more data is received.

> We don't know if these models will generalize

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Linear Regression

> <u>Response (Dependent) variable</u>: the variable of primary interest in a study- the one you are trying to predict or explain.

> <u>Explanatory (Independent) variable</u>: the variable that attempts to explain the observed outcomes of the response variable.

> There are two types of parameters in linear models:
  – The intercept (y-intercept).
  – The slope, rise over run, or change in Y divided by change in X

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Linear Regression

> When x = 0, then y = b.

> When x = -(b/m) then y = 0.
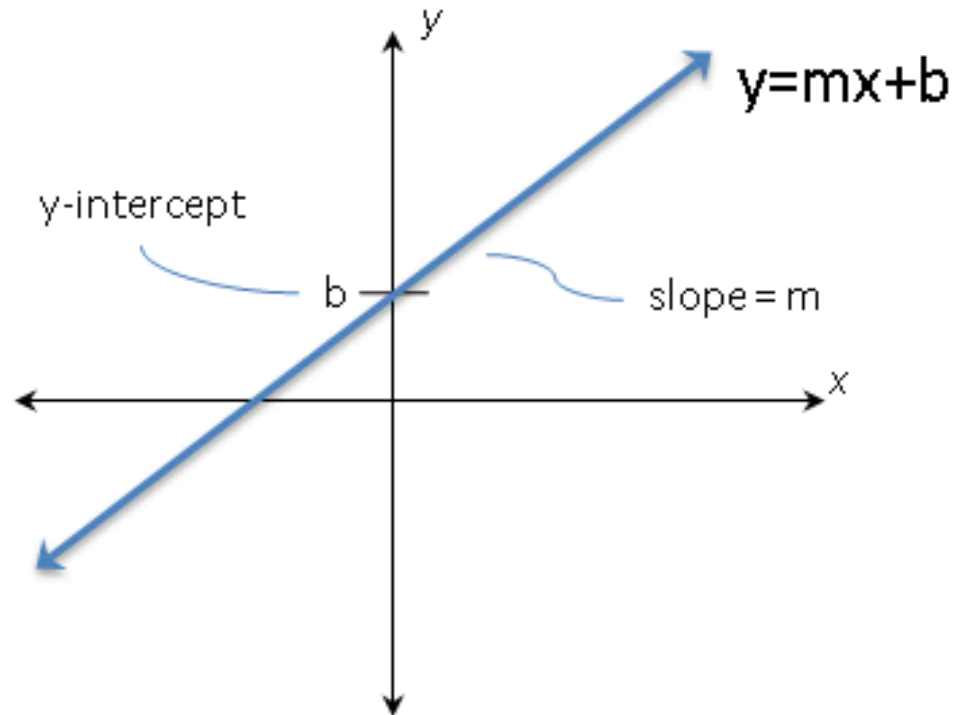
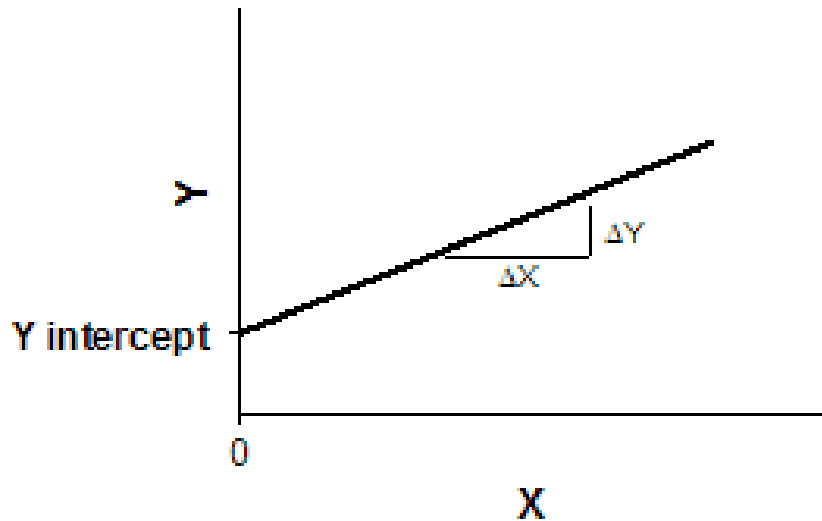# Linear Regression

> Interpret slope: $m = \dfrac{rise}{run} = \dfrac{\Delta y}{\Delta x}$

– If x changes by Δx, then y must change by Δy in order for the slope to stay the same (and it must).
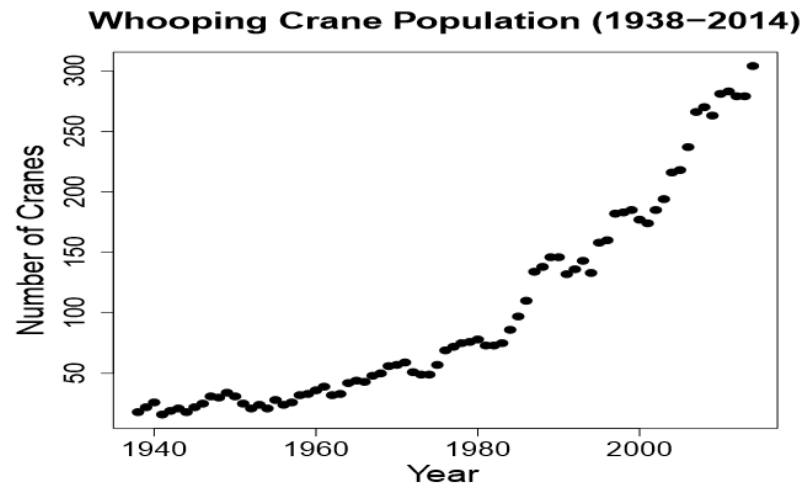

Given two points, (x1,y1), (x2,y2)

$$m = \frac{(y2 - y1)}{(x2 - x1)} = \frac{(y1 - y2)}{(x1 - x2)}$$

R example!

Linear Regression

> Consider the relationship between Whooping Crane population and year below.

Whooping Crane Population (1938−2014)
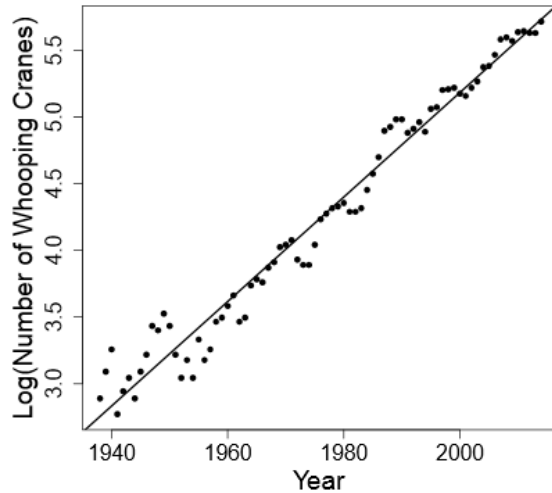

> Possible regression solutions:
  – Transform the response variable, to linearize the relationship.
  – Fit a nonlinear regression model to the data.
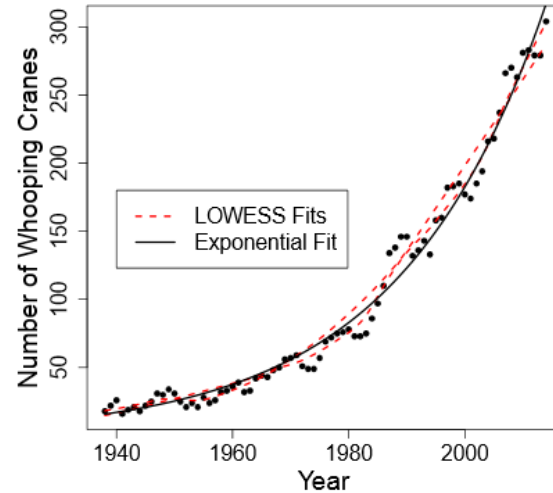
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Linear Regression

> How would we decide on a 'best' model?

Linear Regression

> We use the method of least squares to find the best fit line: $y_i = mx_i + b + \varepsilon_i$

$$\min_{m,\,b} \sum_{i=1}^{n} (\varepsilon_i)^2 = \min_{a,\,b} \sum_{i=1}^{n} (y_i - (mx_i + b))^2$$

> Explicit solutions exist (using calculus).

> Computers are really good at finding minimums of equations. We let them do this for us.

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Linear Regression

> The method of least squares finds the best fit line.

- The mean of the errors from the best fit line is zero.
- This means there is no 'bias' in our prediction.

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Linear Regression

> Linear regression is the most common.

– Fit a line (2D), plane (3D), or a hyperplane to the observed data.

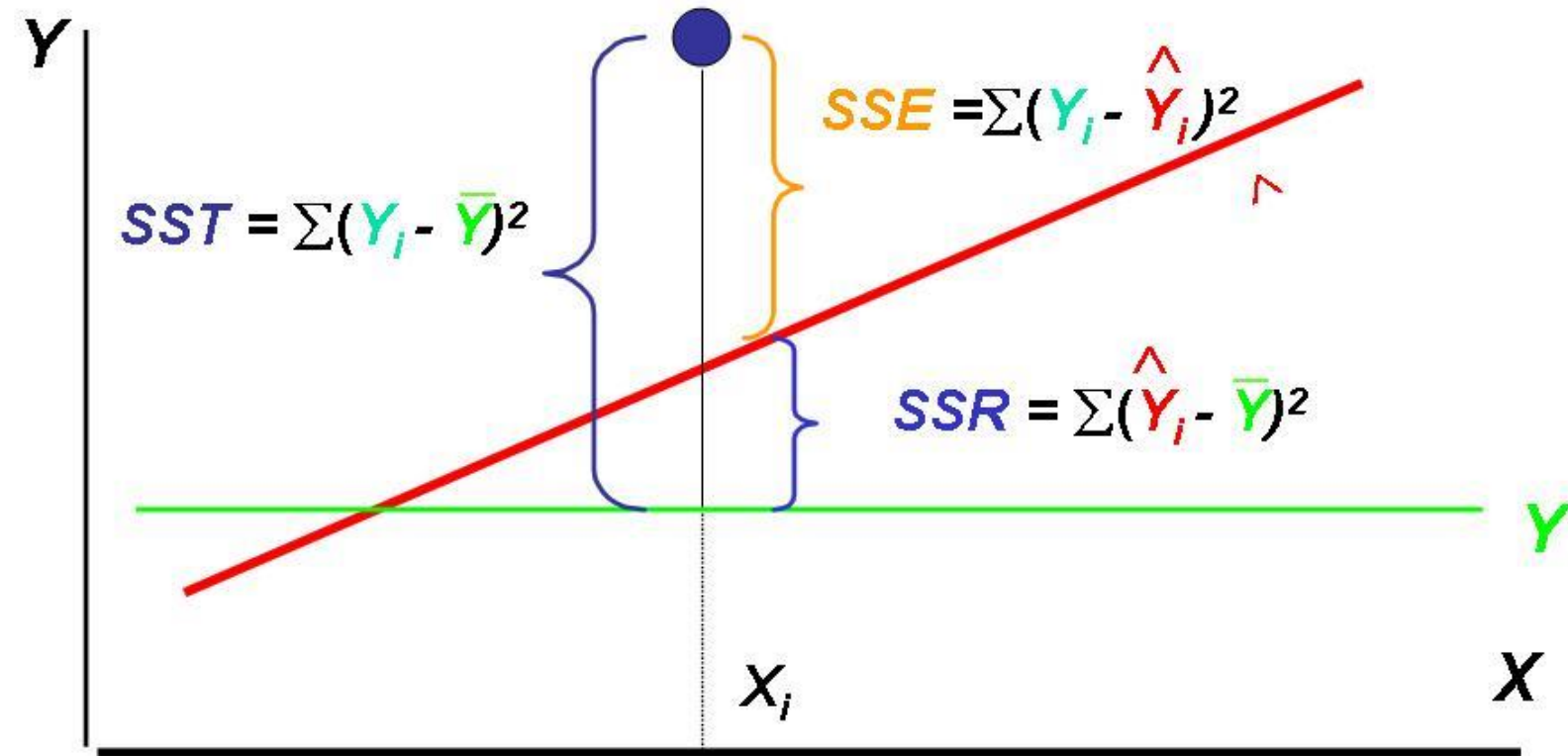> We need to define an error metric for a line through points.

Least Squares Linear Regression
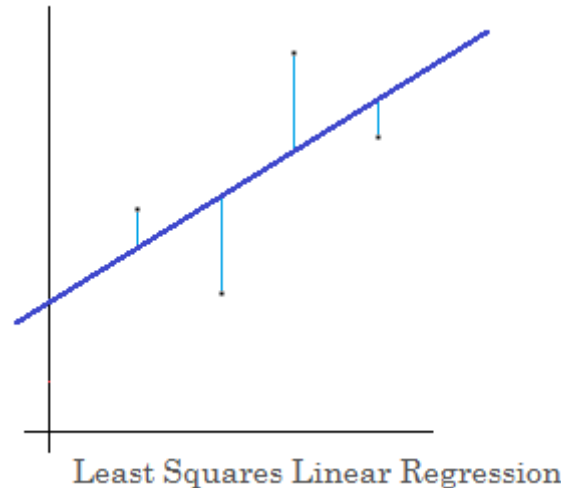
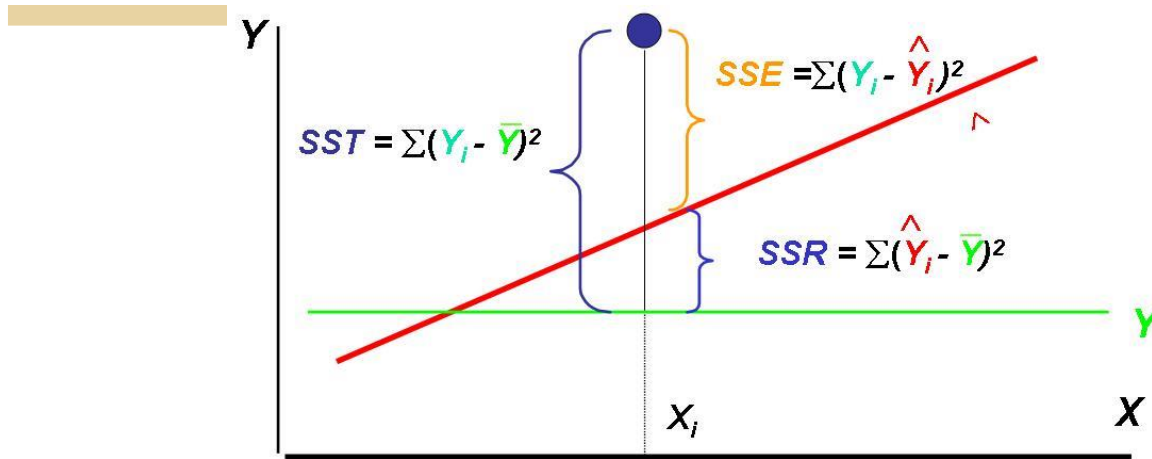> We use the sum of the squared error between the predicted and actual.

> R demo.

Linear Regression

> With modeling, we are interested in the SSE, SSR, SST.

Linear Regression

> R-squared is called the coefficient of determination.

> It indicates how well the data fits a specified model.

> For linear models, we define this as:

$$R^2 = 1 - \frac{SSE}{SST} = \frac{SSR}{SST}$$

Linear Regression

> We can also measure accuracy of the line using Root Mean Squared Error (RMSE).

– Using this as an estimate of the error means we are losing one more degree of freedom than the standard deviation, so we write the RMSE as

$$RMSE = \frac{SSE}{n-2}$$

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Assignment

> Complete Homework 4:

– Fit a linear model to the Chicago Diabetes Hospital data as follows:

> Transform the data to sum across all zip codes.

> Then plot and fit a line to the following:

– Num. Hospitalizations vs. Crude Admittance Rate

– Δ Num. Hospitalizations vs. Δ Crude Admittance Rate

> Summarize your finding of each model.

– Be sure to interpret the slope for each model (think of the units).

– You should submit:

> Just one R-script.

> Read Statistical Thinking for Programmers Chapters 6 and 7.

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