Part 1-Modelling group dynamics

All 4 members of the group regularly attended the weekly online sessions. Sarah and I took the leading roles, partly because we had microphones, while the others typed their input. Sarah organised the online word document for our input (instead of the wiki) and inputted the data. Sarah and I created models independently. I came up with the calculation for maximum alpha, and also determination for trolley transit time. Sarah was often critical of my models, which was advantageous as I was forced to defend them. The other two members did obtain information for the group such as finding coefficients of friction, but their input during online sessions was limited.

Part 2- The model Report

Specify the purpose of the model

Definition of the problem

Within transport terminals, such as airports, there is often more than one floor level present. People often have luggage that is transported on baggage trolleys. One method that people and baggage trolleys can move between levels is via a ramp. The slope of the ramp is a critical factor. If the slope is too steep, then people won't be able to push their trolley up the ramp, nor will the brakes on the trolley stop it from travelling down the slope if left unattended. However, if the slope is too shallow, then people will have to walk a long way, taking excessive time to reach the next level. In addition, excessive building materials and expense will be required to build the ramp.

This report will investigate the scenario of a trolley with luggage being pushed up a ramp only in an indoor environemnt. It will not consider trolleys being moved down a ramp. The aim of the report is to suggest an optimal angle for a ramp, such that people with their baggage trolleys can move up the ramp in the shortest possible time in order to shorten their transit time within the transport terminal.

Aspects of the problem being investigated

In order to determine the optimal angle of the ramp the important elements include the ratio of push force to loaded trolley weight, the subsequent acceleration experienced by the trolley, and the resultant time taken for the trolley to move up the ramp. The frictional resistance experienced by the trolley is also an important element.

Create the Model

Outline of the approach in the first model

A simplified model of a trolley moving up an inclined plane with a constant push force will be created, allowing Newton's second law of mechanics to be utilised. The trolley will be modelled as a particle and friction will be considered. By calculating the maximum ramp angle of the inclined plane the upper limit of the optimal ramp will be known.

The minimum value of the ratio between push force and weight of trolley will be calculated from the minimum slope.

The time taken for a trolley to move up the ramp is dependent on the acceleration of the trolley and the length/angle of the ramp. This relationship will be investigated in order to find the shortest time traversed by the trolley up the ramp for a given push force and trolley weight.

Assumptions

1. The trolley is modelled as a particle
2. The ramp is flat and inclined at a constant angle α
3. The trolley experiences no lateral motion when moving up the ramp ie. motion is in one dimension
4. The initial speed of the trolley at the base of the incline is zero
5. The magnitide of the pushing force on the trolley is constant for the entire length of ramp
6. Air resistance is ignored throughout
7. Both the ramp and trolley are dry
8. There is no limit of the ramp length

Definition of variables and parameters

Symbol	Description	Units
m	Mass of the trolley and load	kg
h	Height of ramp	m
x	distance of ramp traversed by trolley	m
L	Total length of ramp	m
$\mathbf{P}$	Pushing force applied to the trolley	N
$\mathbf{N}$	Normal reaction force of ramp on trolley	N
$\mathbf{W}$	Weight of trolley and load	N
$\mathbf{Fr}$	Frictional force exerted on the trolley by the ramp	N
v_0	Initial velocity of trolley at bottom of ramp	m s^{-1}
a	Acceleration of trolley and load moving up the ramp	m s^{-2}
g	Magnitude of acceleration due to gravity	m s^{-2}
t	time taken for trolley to travel along the ramp	s
μ	coefficient of static friction between the trolley and ramp	-
μ'	coefficient of sliding friction between the trolley and ramp	-
μ_R	coefficient of rolling friction between the trolley and ramp	-
k	Constant (P/W)	-
α	angle of ramp from the horizontal	-

Formulation of mathematical relationships

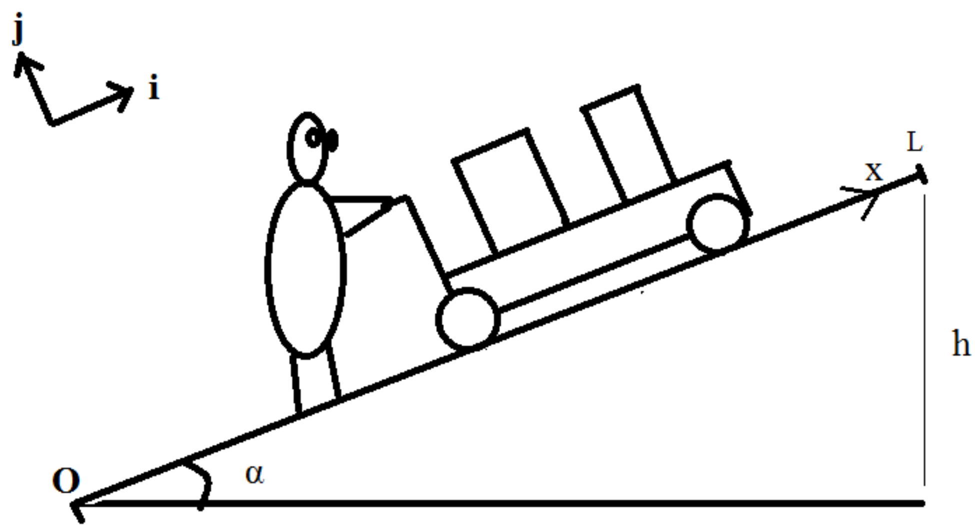

Figure 1: Positions, distances and unit vector directions of trolley moving up ramp

A diagram of the modelling situation is shown in figure 1.

By Assumption 1 the system is considered as a single particle of mass m. By assumption 3 the motion is in one direction, starting at $x = 0$ and reaching its final point at $x = L$. By assumption 2, 5, 6 and 8 the force diagram of the system is displayed in figure 2.

According to Newton's second law, the forces within the system are

$$m\mathbf{a} = \mathbf{P} + \mathbf{W} + \mathbf{N} + \mathbf{Fr}$$

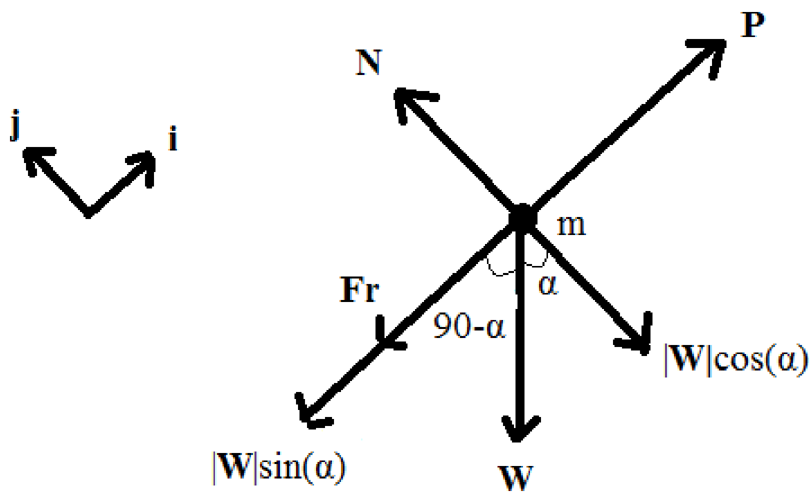

Figure 2: Force diagram of the trolley particle being pushed up the ramp

As motion is only in the positive **i** direction, Newton's second law is resolved in this direction (see figure 2), yielding

$$m\mathbf{a} = \mathbf{P} - \mu'|\mathbf{W}|\cos\alpha - |\mathbf{W}|\sin\alpha \tag{1}$$

where $|\mathbf{Fr}| = \mu'|\mathbf{N}| = \mu'|\mathbf{W}|\cos\alpha$ and $\mathbf{W} = mg$.

Subtracting **P** from the RHS of equation 1 then dividing through by $|\mathbf{W}|$ yields

$$\frac{\mathbf{P}}{|\mathbf{W}|} - \frac{\mathbf{a}}{g} = \mu' \cos\alpha + \sin\alpha$$

By assumption 5, we let

$$k = \frac{\mathbf{P}}{|\mathbf{W}|}$$

Therefore according to Newton's second law, the system is modelled as

$$k - \frac{\mathbf{a}}{g} = \mu' \cos\alpha + \sin\alpha \tag{2}$$

Expressed in terms of trolley acceleration a up the incline gives

$$\mathbf{a} = g(k - \mu' \cos\alpha + \sin\alpha\ ) \tag{3}$$

The RHS of equation 2 can be combined into the form $R\cos(\alpha - \phi)$, where $R = \sqrt{\mu' + 1}$ and $\phi = \arctan\frac{1}{\mu'}$[mathscentre]. However, $R\cos(\alpha-\phi) = R\cos(\phi-\alpha)$, depending on the interval of α values being investigated. As will be seen, for the interval $0 < \alpha < \pi/2$

$$\mu' \cos\alpha + \sin\alpha = (\sqrt{(\mu')^2 + 1}) \cos\left(\arctan\frac{1}{\mu'} - \alpha\right) \tag{4}$$

The upper bound for α beyond which upward motion is possible (α_{max}) is defined when $\mathbf{a} = 0$. Substituting $\mathbf{a} = 0$ into equation 2 and then substituting $\mu' \cos\alpha + \sin\alpha$ into 4 yields

$$k = (\sqrt{(\mu')^2 + 1}) \cos\left(\arctan\frac{1}{\mu'} - \alpha_{max}\right)$$

so

$$\alpha_{max} = \arctan\frac{1}{\mu'} - \arccos\left(\frac{k}{\sqrt{(\mu')^2 + 1}}\right) \tag{5}$$

By assumption 9 and from equation 5, the range of possible values α may take are

$$0 < \alpha < \arctan\frac{1}{\mu'} - \arccos\left(\frac{k}{\sqrt{(\mu')^2 + 1}}\right)$$

By assumption 9, and from figure 1 the length of the ramp (x=L) is defined as

$$L = \frac{h}{\sin\alpha} \tag{6}$$

Do the mathematics

Solution of the equations

The optimal ramp angle α, is that which the trolley can be moved up the ramp in the shortest time period. The journery time of the trolley is derived from the equation

$$L = \frac{1}{2}at^2 + v_0 t$$

From equation 6 this quadratic equation becomes

$$\frac{1}{2}at^2 + v_0 t - \frac{h}{\sin\alpha} = 0$$

We can solve for t in terms of the acceleration of the trolley a, the angle of the ramp α, the height of the ramp h and the initial velocity v_0 of the trolley at the origin (O), as

$$t = -\frac{v_0}{a} \pm \frac{\sqrt{v_0^2 + \frac{2ha}{\sin\alpha}}}{a}$$

From assumption 4, that $v_0 = 0$, and that t must be positive, this expression simplifies to

$$t = \sqrt{\frac{2h}{a\sin\alpha}} \tag{7}$$

Substituting the formula of trolley acceleration a (equation 3) into equation 7 results in

$$t = \sqrt{\frac{2h}{g\sin\alpha(k - (\mu'\cos\alpha + \sin\alpha))}} \tag{8}$$

We know that time (t) must be positive, therefore α must be such that the RHS of equation 8 is also positive.

Therefore for parameter values k and μ', we can use numerical methods to determine the minumum value of t and thereby determine the optimal ramp angle α.

Equation 8 is dimensionally consistant. Both α and μ' are dimensionless. We have

$$[T] = \sqrt{\frac{[M]}{[MT^{-2}]}} = 1$$

Interpret the results

Data

Quantity	Value	Source or justification
g	$9.8ms^{-2}$	Standard Value
μ'	0.75μ	Standard Value

The maximum push force an individual can perform on a trolley is dependent on the individual capacity. Generally men can apply a greater force than women, but there is considerable overlap between the sexes. A Canadian governmental organisation published recommended upper force limits for cart pushing on a horizontal plane, and this was set at 225N [CCOHS]. The maximum value of P for this report is chosen at 200N.

The weight of an airport trolley is approximately $20kg$ [made-in-china.com]. Usually a trolley can store two cases plus hand luggage. Emirates Ecomony Class passangers may take up to 30kg per suitcase. I have considered the trolley and luggage to weigh a maximum of 1000N. Therefore the value of $k = 0.2$

The value for μ is dependent on the material of trolley wheel and of the floor. Values of static friction for nylon on nylon on a dry surface (Assumption 7) are from 0.15 to 0.25 [The-engineering-toolbox]. Therefore μ' was taken to be $\frac{3}{4}$ of this lower limit, that is 0.1.

The parameter h does not impact on α (assumptions 5 and 8). A value of $h = 2m$ was chosen for the model.

Interpretation of results

The program wxMaxima was used to calculate values of α, t from variables k and frictional coefficients (See Appendix 1, figure 5). Figure 3 displays the relationship between travel time and α where $k = 0.2$, $\mu' = 0.1$ and $h = 2m$. By numerical analysis, t is minimum at 12.7 seconds (to 1dp) and $\alpha = 0.050$ radians (to 3dp). Hence, assuming the trolley and its contents of 1000N can be pushed on the horizontal with a force of 200N, with a frictional resistance component characterised by μ' of 0.1, the optimal angle α_{opt} would be 0.05 radians, or 2.86°.

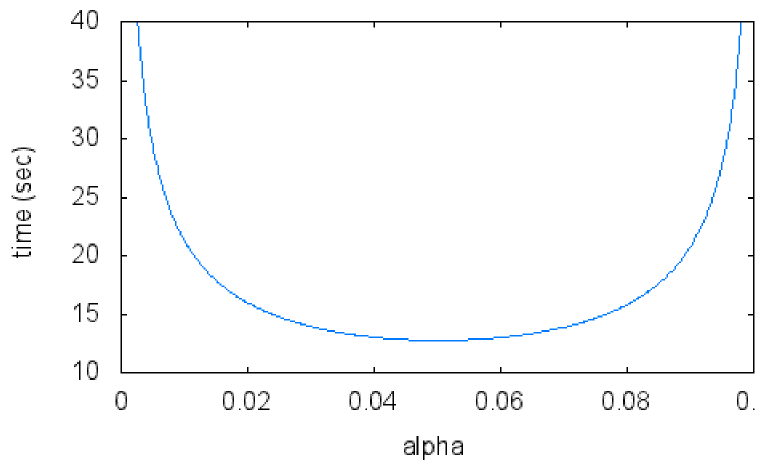

Figure 3: Transit time up ramp, $k = 0.2$, $\mu' = 0.1$ $h = 2m$

Substituting $k = 0.2$ and $\mu' = 0.1$ into equation 5 yields α_{max} as 5.77° (to 2dp), greater than α_{opt} as expected.

According to the British Standards the maximum slope of ramps in dwellings is 1 in 12 (4.76°) [Department of Finance and Personnel]. The optimal α falls below this value and therefore satisfies this standard.

The effect of altering values of k and μ' on α are shown in Tables 1 and 2 respectively. These values were calculated using wxMaxima software. A copy of the wxMaxima file is displayed in figure 5 (Appendix 1), where the values of k and the frictional coefficient were altered as appropriate. Values of time and α_{opt} were obtained from numerical analysis.

k	$\alpha_{opt}{}^{\circ}$	time (sec)
0.1	undefined	-
0.15	1.44	79.9
0.20	2.87	39.9
0.25	4.35	26.6
0.3	5.81	19.9
0.35	7.27	16
0.40	8.73	13.3
0.45	10.2	11.4
0.5	11.75	9.9

Table 1: Dependency of α_{opt} (degrees 2dp) and time on k where $\mu' = 0.1$ and $h = 2$

μ'	$\alpha_{opt}{}^{\circ}$	time (sec)
0	5.76	20.0
0.02	5.2	22.4
0.04	4.6	25
0.06	4.06	28.5
0.08	3.45	33.3
0.1	2.89	39.9
0.12	2.3	49.9
0.14	1.72	66.6
0.16	1.15	99.9
0.18	0.57	200
0.2	undefined	-

Table 2: Dependency of α_{opt} (degrees 2dp) and time on μ' where $k = 0.2$ and $h = 2$

Inspection of these tables indicate that there is a linear relationship between k and α_{opt} and between μ' and α_{opt} for the ranges specified. By increasing k by 0.1 increases α_{opt} by 1.44-1.46 degrees. By increasing μ' by 0.02 decreases α_{opt} by 0.55-0.6 degrees. It can be seen from both tables that the value of k must be greater than for μ' otherwise assumption 5 is not met and the trolley will not accelerate up the incline.
It is noted that both k and μ' are not proportional to time.

The major difficulty in practice calculating an optimal ramp angle is to identify values of k that the trolley will experience. As noted previously, that depends on individual capability and the trolley load. I have taken what I think to be 'the average" value for my model.

In conclusion, the optimal angle for my "average" value model is 2.86°, well below the british standard of 4.76°.

Evaluate the model

Comparison with reality

The experimental setup displayed in Appendix 2 was used to check the value of the coefficient of sliding friction used in equation 8. It was found that the angle which the scooter rolled down the slope from a static position with a small push was 1.4° (see Appendix 2, figure 6). It was assumed that the scooter was rolling at constant velocity. Because the scooter was rolling rather than sliding the frictional coefficient in terms of rolling resistance μ_R was calculated as follows

$$\mu_R = \frac{\sin\alpha}{\cos\alpha} = 0.024$$

see [Pellissippi State Community College]

The experimental setup to acquire the transit time of the scooter along the ramp with various weights and experiencing various push forces is displayed in Appendix 3. Substituting μ_R for μ' the theoretical value for transit time using the model described in equation 8 was compared with the experimental values, shown in Table 3 and figure 4. (see Appendix 5 for theoretical calculation .wxm file)

W	P	k	avg time	predicted time
19	2	0.105	2.5	3.6
26.6	3	0.113	3.3	2.8
35.6	4	0.112	3.6	2.9
50.6	5	0.099	4.5	5.5
64	6	0.094	5.7	54

Table 3: Experimental vs theoretical values of scooter transit time (seconds). $\mu_R = 0.024$, l=0.75m, $\alpha = 4°$.

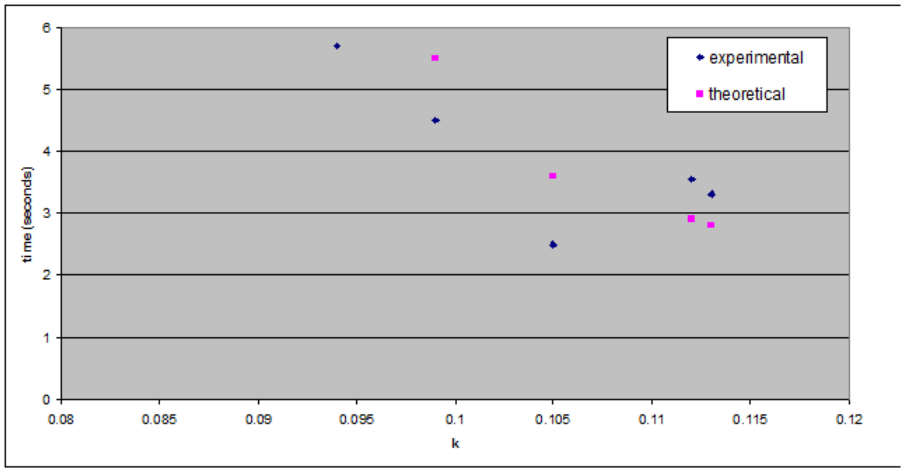

Figure 4: Experimental vs theoretical values of scooter transit times for various values of k. Note that the theortical time value (54 seconds) for $k = 0.094$ is not displayed.

Both Table 3 and Figure 4 qualitatively demonstrate good agreement between experimental and theoretical values of time when using coefficient of rolling friction μ_R (except for the points where k=0.094, where the theoretical upper bound of the slope angle is being approached). From figure 4 it appears that $\alpha = 4°$ when $k = 0.105$, although more data points are required to confirm this. If the sliding frictional coefficient of nylon on nylon would have been used for the theoretical calculation $\mu' = 0.1$ then no real values of time would have been obtained because μ' would have been too large in the theoretical calculation. There is therefore considerable support to incorporate rolling resistance into the model and exclude sliding friction.

Criticim of the model

The model proposed is simple and through experiental results we have found that it does not characterised reality very well. The main problem with the model is incorporating the sliding frictional coefficient. This results in non-real transit time predictions. When replaced by the rolling resistance coefficient, the model usually makes real transit time predictions that are appear close to reality.

The assumptions at the start of the report are considered in turn.

1. Modelling the trolley as a particle incorporates the sliding frictional coefficient in the model as a particle cannot roll. Therefore we can no longer

assume the trolley is a particle. The parameter μ' should be exchanged for the parameter μ_R.

2. Most ramps are designed with a constant slope, and this is incorporated into the model

3. A trolley being pushed up a ramp may experience some lateral motion. However, such lateral motion should hardly affect the prediction of the model.

4. The initial speed of the trolley at the base of the incline is unlikely to be zero. This assumption improved the accuracy of the experiment but in reality most trolleys will enter upon the ramp travelling at some speed. Therefore the model should incorporate an initial velocity. There are times where people stop the trolley on the ramp and need to restart pushing the trolley from rest.

5. It is unreasonable to assume the pushing force on the trolley is constant along the ramp. It is more likely that a specific trolley velocity is obtained and then maintained for the duration of the ramp. This assumption therefore requires revision

6. It is unlikely that air resistance would have an impact indoors

7. It would be expected that ramp floors are dry indoors. It would be a health and safety issue if the floors are wet.

8. The length of ramp is limited according to its surrounds. However this should not affect the calculation of the optimal angle, as strategies are available to overcome limited space.

The main candidate for revision is Assumption 1, as this will allow real relatively accurate results to be produced. This will be reviewed in the following section. Both Assumptions 4 and 5 also need to be reviewed and are linked together.

Revise the model

As has previously been explained, the initial model to find the optimal angle of the ramp contained the parameter of the sliding friction coefficient, whose value was found to be too high. In the revised model, this parameter has been exchanged for that of rolling friction coefficient μ_R. Equation 8, used to determine the transit time of the trolley up the ramp becomes

$$t = \sqrt{\frac{2h}{g\sin\alpha(k - (\mu_R\cos\alpha + \sin\alpha))}} \tag{9}$$

Using equation 9 the minimum transit time and hence optimal angle can be calculated. One method to estimate μ_R is

$$\mu_R = \sqrt{\frac{z}{d}}$$

where z is the sinkage depth of a slow rigid wheel on a perfectly elastic surface and d is the diameter of the rigid wheel [wikipedia].

Further experimental work is required to determine if this new model is accurate. Ideally, a greater range of k values would be measured at different ramp angles, in order to determine optimal angles for specific values of the parameters used. The model should also incorporate a non-zero initial velocity.

Now that the trolley is not modelled as a particle, we may also need to incorporate the centre of mass into the model. Then the general motion of the trolley would be described in terms of the centre of mass.

Conclusion

The first model incorporating μ' performed poorly. Predicted values of transit times using μ' were non-real. Revising the model by using μ_R instead of μ' resulted in relatively good agreement between the model and experimental results. Incorporating the centre of mass of the trolley into the model may produce further improvements.

The modelling process was logical and understandible, but there was a tendency to overcomplicate the first model. Keeping the model simple is an important, but paradoxically not simple to perform.

Appendices

Appendix 1-wxMaxima .wmm file to calculate optimal ramp angle

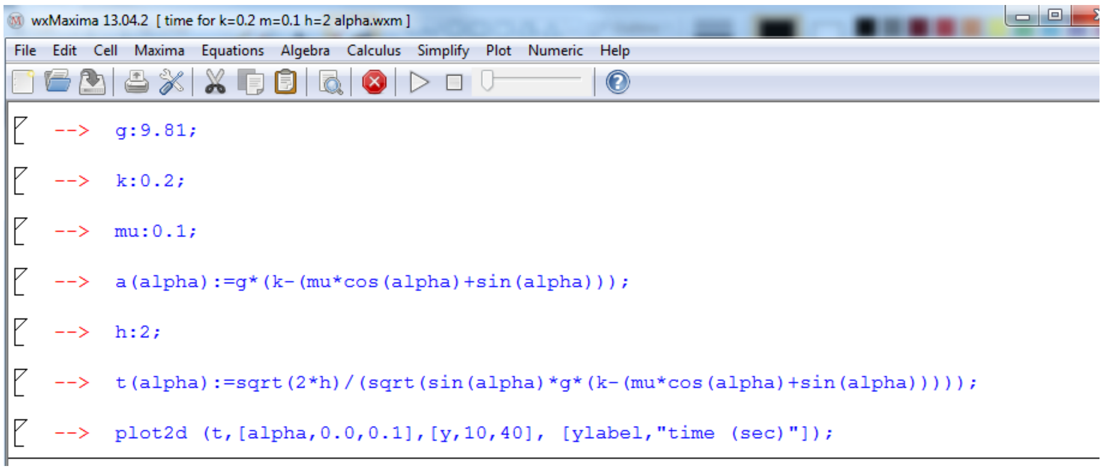

Figure 5: .wxm file to determine optimal angle for $k = 2$, $\mu' = 0.1$. The minimal time and hence optimal angle was determined numerically

Appendix 2-Sliding Friction coefficient

Figure 6: Photo displaying experimental determination of μ_R. The slope of the incline was found whereby the trolley would begin to roll down the slope at constant velocity with an initial minor push

Determination of sliding/rolling friction using an inclined slope has been described elsewhere [Pellissippi State Community College]. The experimental setup is shown in Figure 6. Essentially, if the object on the slope is static, but then given a small push, and the object travels down the slope at a constant velocity, then

$$\mu'/\mu_R = \frac{\sin\alpha}{\cos\alpha}$$

Appendix 3-Experimental setup to determine trolley transit time

Figure 7: The experimental setup.

Figure 7 demonstrates the experimental setup. The trolley used in the experiment was a three wheel scooter. Bags of flour were added onto the scooter to increase the value of k. The trolley was pushed using a push-force hand meter. A constant push force was attempted to be applied to the trolley during each experiment. A mobile phone with a spirit level application was utilised to determine α.

Appendix 4-wxMaxima .wmm file to calculate theoretical trolley transit time

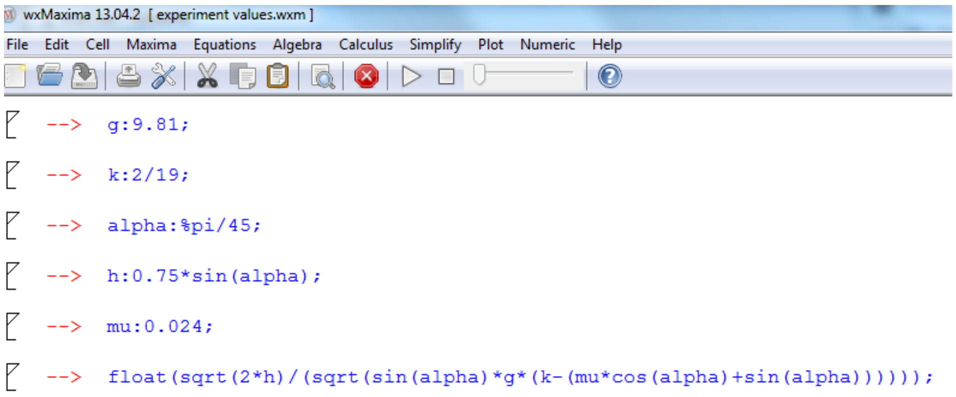

Figure 8: .wxm file to mathematically determine trolley transit time for various values of k.

Appendix 5

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