# 170138286_BayesProject

The two countries investigated in this report were Pakistan (PAK) and St Lucia (LCA). PAK contained 56 data points and LCA contained 39 data points.

## 1 Question 1

My chosen priors utilised by the author in this study for the stationary model were

$\pi(\alpha) = N(\alpha|0, 1000)$

$\pi(\lambda) = Ga(\lambda|0.001, 0.001)$

For the non-stationary model, the priors utilised were

$\pi(\alpha) = N(\alpha|0, 1000)$

$\pi(\beta) = N(\beta|0, 1000)$

$\pi(\lambda) = Ga(\lambda|0.001, 0.001)$

Therefore values of the prior parameters elicted were $m_\alpha = 0$, $p_\alpha = 0.001$, $a = 0.001$ and $b = 0.001$ for both models, and $m_\beta = 0$ and $p_\beta = 0.001$ for the non-stationary model.

An explanation for my prior selection is now described.

No prior information about the distribution of $\alpha$, $\beta$ or $\lambda$ was known by myself for the stationary or non-stationary models. Weak priors were therefore selected for $\pi(\alpha)$, $\pi(\beta)$ (excluding the stationary model) and $\pi(\lambda)$ distributions to accommodate for this lack of knowledge. Both $\alpha$ and $\beta$ were assumed to be normally distributed, and that either variable could be positive or negative with equal probability. Aside from the stationary model $\beta$ with fixed parameters (2,2), a small precision (wide variance) and a mean of zero was chosen for $\pi(\alpha)$ and $\pi(\beta)$. Using the AR1_non-stationary and AR1_stationary R scripts, improper priors with precision zero could have been selected for $\pi(\alpha)$ and $\pi(\beta)$ of the non-stationary model . However, if WINBUGS was utilised instead of these scripts, a real variance value would have been required. A very small precision of 0.001 was therefore applied to $\pi(\alpha)$ and $\pi(\beta)$ where applicable.

The $\pi(\lambda)$ followed a gamma distribution. Without any prior information an approximation to the improper gamma distribution (where $a = b = 0$) was utilised. This approximation was of the form $\pi(\lambda) \sim Ga(\lambda|0.001, 0.001)$. Concerns that the posterior marginal distributions would be improper were not founded as there were at least 39 observed data points utilised in all models.

OK, but surely you could say something about alpha and beta?

### 1.1 Simulations

Simulations for the stationary and non-stationary models for both PAK and LCA were performed with 100,000 draws using the start value combinations (a=0, b=0, l=0.1), (a=-10,b=-1,l=1) and (a=10,b=1,l=1). In addition, a simulation was performed using 1,000,000 draws with starting values (a=0, b=0, l=0.1). Convergence of the chains for almost all models and starting values occurred almost immediately (Appendix: Figure 1,2,3 and 4). Convergence for the PAK stationary model took longer (around 500 draws) but was still quick. A burn-period of 4000 with thining of 5 was therefore utilised. For the stationary models, proposal variance for PAK (v=0.005) and for LCA (v=0.2) resulted in an acceptance rate within the acceptable limits between 0.2 and 0.4. Very little difference was seen in the location and spread of the posterior plots using different starting values or chain lengths for any model. As an example Table 1 displays the numerical summaries for the LCA non-stationary model.

Table 1: Numerical summaries for the LCA non-stationary model

Captions should be informative!

| Variable | starting value | draws | Mean | 0.95 PI | Median | IQR |
|---|---|---|---|---|---|---|
| $\alpha$ | a=0,b=0,l=0.1 | 100,000 | 1.505 | [0.000, 3.019] | 1.508 | [1.001, 2.006] |
| $\alpha$ | a=10,b=1,l=1 | 100,000 | 1.503 | [-0.006, 3.020] | 1.502 | [1.003, 1.996] |
| $\alpha$ | a=-10,b=-1,l=1 | 100,000 | 1.501 | [-0.010, 3.031] | 1.496 | [0.995, 2.007] |
| $\alpha$ | a=0,b=0,l=0.1 | 1,000,000 | 1.497 | [-0.011, 2.998] | 1.499 | [0.996, 2.002] |
| $\beta$ | a=0,b=0,l=0.1 | 100,000 | 0.245 | [-0.059, 0.555] | 0.245 | [0.143, 0.347] |
| $\beta$ | a=10,b=1,l=1 | 100,000 | 0.245 | [-0.063, 0.553] | 0.245 | [0.142, 0.348] |
| $\beta$ | a=-10,b=-1,l=1 | 100,000 | 0.245 | [-0.062, 0.550] | 0.245 | [0.141, 0.348] |
| $\beta$ | a=0,b=0,l=0.1 | 1,000,000 | 0.244 | [-0.064, 0.550] | 0.244 | [0.141, 0.346] |
| $\lambda$ | a=0,b=0,l=0.1 | 100,000 | 0.061 | [0.036, 0.093] | 0.060 | [0.051, 0.070] |
| $\lambda$ | a=10,b=1,l=1 | 100,000 | 0.061 | [0.035, 0.093] | 0.060 | [0.051, 0.070] |
| $\lambda$ | a=-10,b=-1,l=1 | 100,000 | 0.061 | [0.036, 0.092] | 0.060 | [0.051, 0.070] |
| $\lambda$ | a=0,b=0,l=0.1 | 1,000,000 | 0.061 | [0.036, 0.093] | 0.244 | [0.051, 0.070] |

A change in starting values or increase in chain length had little effect on the posterior distributions. As such, the start values chosen were (a=0, b=0, l=0.1) and 100,000 draws were utilised for creating the posterior marginal distributions in this analysis.

Statistics for each simulation of the posterior marginal distributions were obtained using the summary command. For one simulation, the $\alpha$ statistics for the stationary model were obtained as follows

```
summary(MCout_st$alpha[keep])
quantile(MCout_st$alpha[keep],c(0.025,0.975))
```

## 2 Question 2

The differences in posterior marginal distributions using my priors compared to those of the expert were similar between the stationary and non-stationary models for each of the two countries. As an example, a graphical display of the differences for the PAK stationary model using my prior (Appendix: figure 5) and the experts prior (Appendix: figure 6) is displayed in the Appendix. The appearances of the distributions appeared similar. Both the posterior $\alpha$ and $\beta$ distributions appeared normal, while the $\lambda$ distriubutions appears positively skewed. Little visual differences could be seen for all models of the two countries. A more detailed impression of the differences was investigated from the numerical summaries.

For the stationary model the $\beta$ prior distribution had fixed parameters (2,2). The numerical summaries using the two different sets of priors for the two models for both PAK and LCA are displayed in Tables 2 to 5. From these tables it can be seen that while the mean for $\alpha$ and $\beta$ are very similar, the 95% probability intervals using the expert's prior is slightly narrower than using my prior (except for the PAK stationary model. Here, the 95% probability intervals were similar after taking into account the location parameter of both priors). The posterior $\lambda$ distribution was positively skewed. The median of the distribution using the expert's prior was slightly greater than using my prior for each of the four models investigated. In addition, the interquartile range was equal or slightly larger using the expert's prior.

The posterior marginal distributions $\alpha$, and $\beta$ for the non-stationary model, were influenced by associated posterior $\lambda$ distribution. Theoretically, increasing $\pi(\lambda)$ prior distribution was likely to decrease the variances of $\alpha$, and $\beta$ posterior distributions. The posterior $\lambda$ distribution using the expert's prior had a larger median value than using my weak $\pi(\lambda)$ prior. It is therefore not surprising that the posterior $\alpha$ distributions and the non-stationary model posterior $\beta$ distributions contained lower variances using the expert's prior. The effect on the posterior $\alpha$ distribuion using the expert's mean $\alpha$ prior of 0.02 appeared minimal. It appeared that the priors $\pi(\alpha)$, and $\pi(\beta)$ had only a small effect on the posterior distributions of $\alpha$, $\beta$ and $\lambda$ due to the large number of observed data points in the likelihood function for each country (n(PAK)=56, n(LCA)=39).

Table 2: Numerical summaries of PAK posterior distributions using my priors and those of an expert for the stationary model

| Variable | priors | Mean | 0.95 PI | Median | IQR |
|---|---|---|---|---|---|
| $\alpha$ | mine | 1.182 | [0.446, 1.903] | 1.184 | [0.935,1.432] |
| $\alpha$ | expert | 1.194 | [0.481, 1.895] | 1.197 | [0.954,1.438] |
| $\beta$ | mine | 0.519 | [0.369, 0.659] | 0.521 | [0.470,0.570] |
| $\beta$ | expert | 0.513 | [0.364, 0.653] | 0.514 | [0.464, 0.564] |
| $\lambda$ | mine | 0.182 | [0.118, 0.259] | 0.179 | [0.156,0.204] |
| $\lambda$ | expert | 0.195 | [0.129, 0.275] | 0.192 | [0.168,0.218] |

Table 3: Numerical summaries of LCA posterior distributions using my priors and those of an expert for the stationary model

| Variable | priors | Mean | 0.95 PI | Median | IQR |
|---|---|---|---|---|---|
| $\alpha$ | mine | 1.558 | [0.0622, 3.052] | 1.552 | [1.063,2.049] |
| $\alpha$ | expert | 1.552 | [0.121, 2.963] | 1.557 | [1.078, 2.027] |
| $\beta$ | mine | 0.224 | [-0.074, 0.521] | 0.225 | [0.125,0.325] |
| $\beta$ | expert | 0.223 | [-0.061, 0.506] | 0.223 | [0.126, 0.319] |
| $\lambda$ | mine | 0.061 | [0.0367, 0.0931] | 0.060 | [0.051,0.070] |
| $\lambda$ | expert | 0.068 | [0.041, 0.101] | 0.067 | [0.057,0.077] |

Table 4: Numerical summaries of PAK posterior distributions using my priors and those of an expert for the non-stationary model

| Variable | priors | Mean | 0.95 PI | Median | IQR |
|---|---|---|---|---|---|
| $\alpha$ | mine | 2.031 | [1.129, 2.943] | 2.032 | [1.727,2.335] |
| $\alpha$ | expert | 2.031 | [1.177, 2.892] | 2.030 | [1.738, 2.327] |
| $\beta$ | mine | 0.173 | [-0.093, 0.443] | 0.173 | [0.080,0.265] |
| $\beta$ | expert | 0.171 | [-0.088, 0.430] | 0.171 | [0.083, 0.258] |
| $\lambda$ | mine | 0.200 | [0.131, 0.283] | 0.198 | [0.172,0.225] |
| $\lambda$ | expert | 0.215 | [0.144, 0.300] | 0.212 | [0.186,0.241] |

Table 5: Numerical summaries of LCA posterior distributions using my priors and those of an expert for the non-stationary model

| Variable | priors | Mean | 0.95 PI | Median | IQR |
|---|---|---|---|---|---|
| $\alpha$ | mine | 1.505 | [0.000, 3.019] | 1.508 | [1.001, 2.006] |
| $\alpha$ | expert | 1.511 | [0.080, 2.952] | 1.508 | [1.028,1.986] |
| $\beta$ | mine | 0.245 | [-0.059, 0.555] | 0.245 | [0.143, 0.347] |
| $\beta$ | expert | 0.240 | [-0.048, 0.531] | 0.241 | [ 0.141, 0.340] |
| $\lambda$ | mine | 0.061 | [0.036, 0.093] | 0.060 | [0.051, 0.070] |
| $\lambda$ | expert | 0.068 | [0.041, 0.101] | 0.067 | [0.057,0.077] |

Quite robust. Maybe both priors encode the same informational content?

## 3 Question 3

Posterior odds (O) of $\beta$ for the non-stationary model was calculated from the formula

$$O = \frac{P[|\beta| < 0.01|\mathbf{y}]}{1 - P[|\beta| < 0.01|\mathbf{y}]}$$

using the following code from the output of the Project_script.R file

```
sum(abs(MCout_ns$beta[keep])<0.01)/sum(abs(MCout_ns$beta[keep])>=0.01)
```

If $O \geq 1$, this provided evidence of a non-zero slope.

Using my priors, the following odds (O) were derived for PAK and LCA

PAK: 0.028
LCA: 0.015

Using the expert's priors the following odds were derived

PAK: 0.024
LCA: 0.024

As all posterior odds were below 1, there was no evidence to suggest that $\beta$ was zero for any of the country and prior combinations. Therefore evidence favouring a non-zero slope was not observed in any of the four models.

## 4 Question 4

The cost of incorrectly investing in a country , defined by $d_1$, is four times the probability that the long-term growth rate $\rho$ of the country will be less or equal to 1.5%. The the cost of incorrectly not investing a country, defined by $d_2$, is the probability that the long-term growth rate $\rho$, of the country will be greater than 1.5%.

For each country PAK and LCA, the expected loss for $d_1$ and $d_2$ was calculated, and a decision which yielded the lowest expected loss out of $d1$ and $d2$ was taken.

The long-term growth rate data ($\rho$) was calculated from the stationary dataset using my priors. The code to generate the distribution of $\rho$ was

```
rho <- MCout_st$alpha[keep]/(1-MCout_st$beta[keep])
d1 <- 4*sum(rho<=1.5)/length(rho)
d2 <- sum(rho>1.5)/length(rho)
```

You should explain the decision to a layperson and summarise your findings.

For PAK, the following costs were calculated

d1 = 0.327
d2 = 0.918

For LCA, the costs calculated were

d1 = 1.131
d2 = 0.717

It can be seen that the cost of taking the wrong decision (d1) to invest in PAK was less than the cost not to invest in PAK (d2). I would therefore recommend investing in PAK. With regards to LCA, the cost of wrongly investing in LCA (d1) was greater than the cost of incorrectly investing in LCA. Thus, it would be pertinent to avoid investing in LCA.

## 5 Appendix

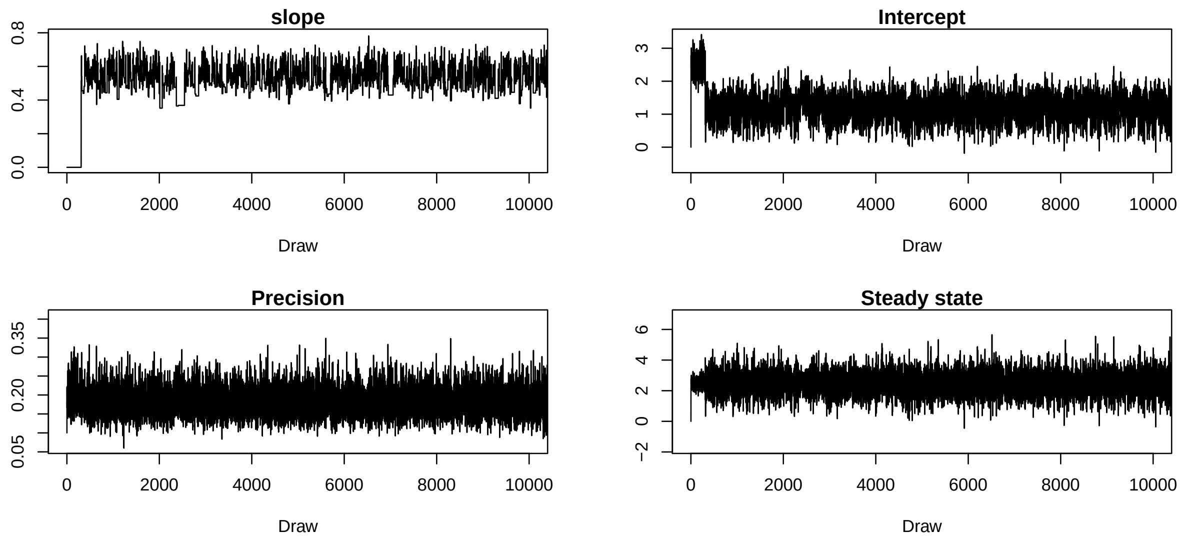


Figure 1: Trace plots for the PAK stationary model (100,000 draws and start values (a=0, b=0, l=0.1)). Convergence occured around 500 draws which was longer than for any other model. The first 10,000 draws are displayed

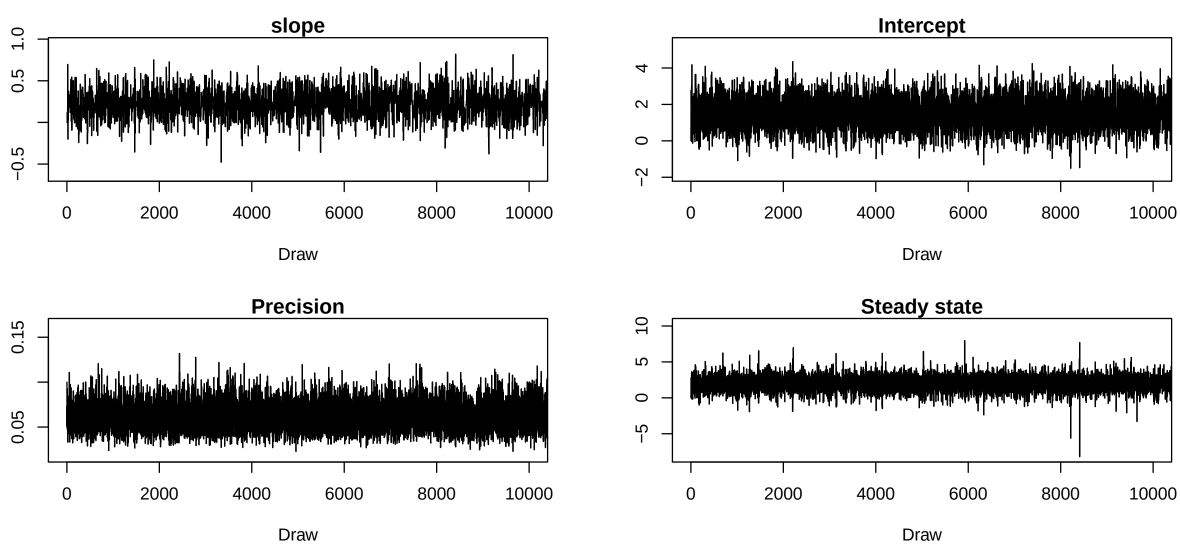


Figure 2: Trace plots for the LCA stationary model (100,000 draws and start values (a=0, b=0, l=0.1)). Convergence was almost immediate. The first 10,000 draws are displayed

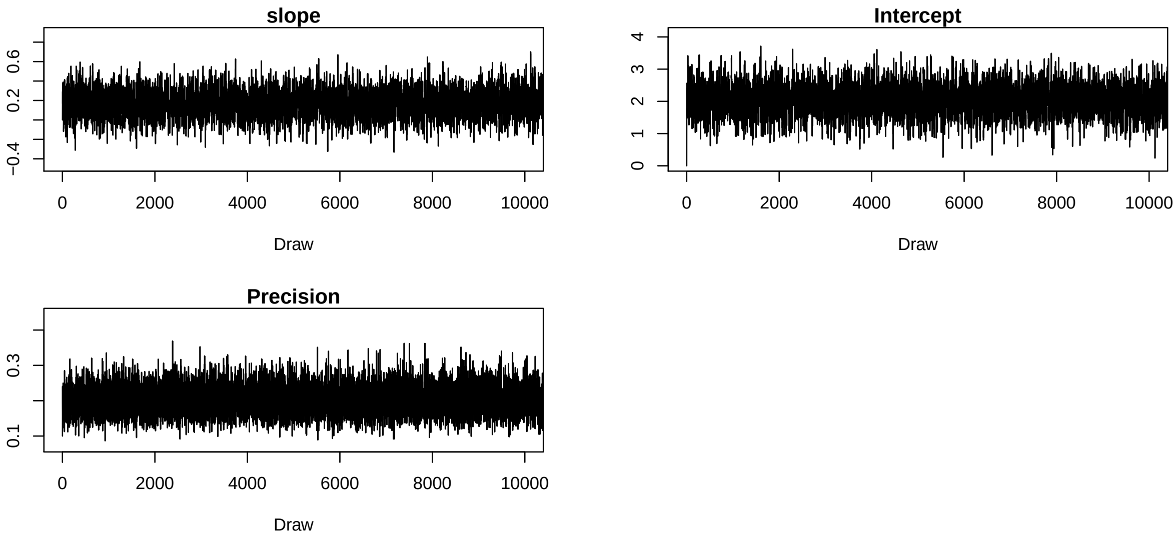


Figure 3: Trace plots for the PAK non-stationary model (100,000 draws and start values (a=0, b=0, l=0.1)). Convergence was almost immediate. The first 10,000 draws are displayed

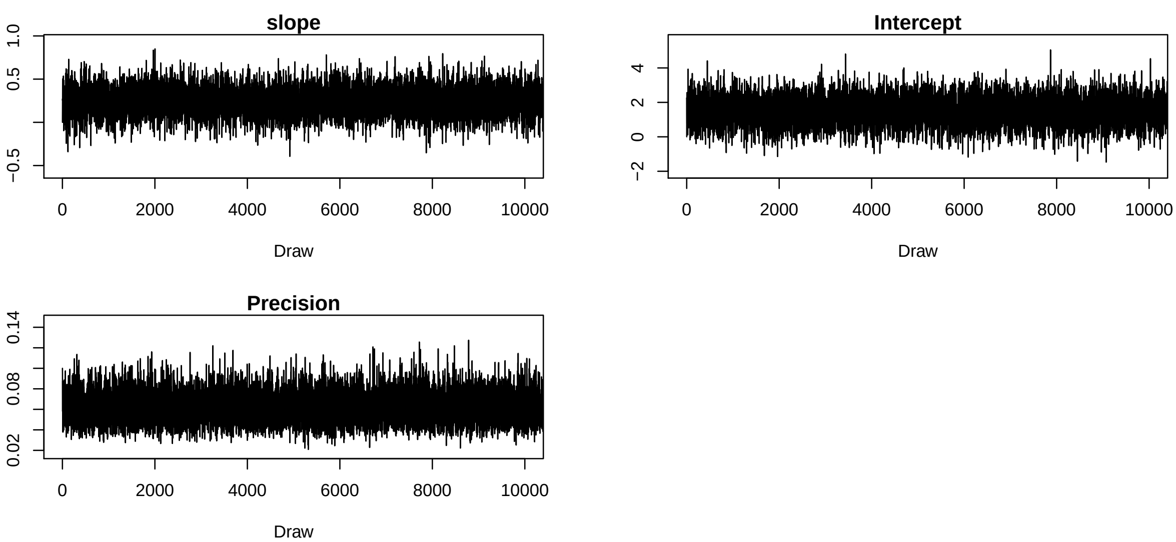


Figure 4: Trace plots for the LCA non-stationary model (100,000 draws and start values (a=0, b=0, l=0.1)). Convergence was almost immediate. The first 10,000 draws are displayed

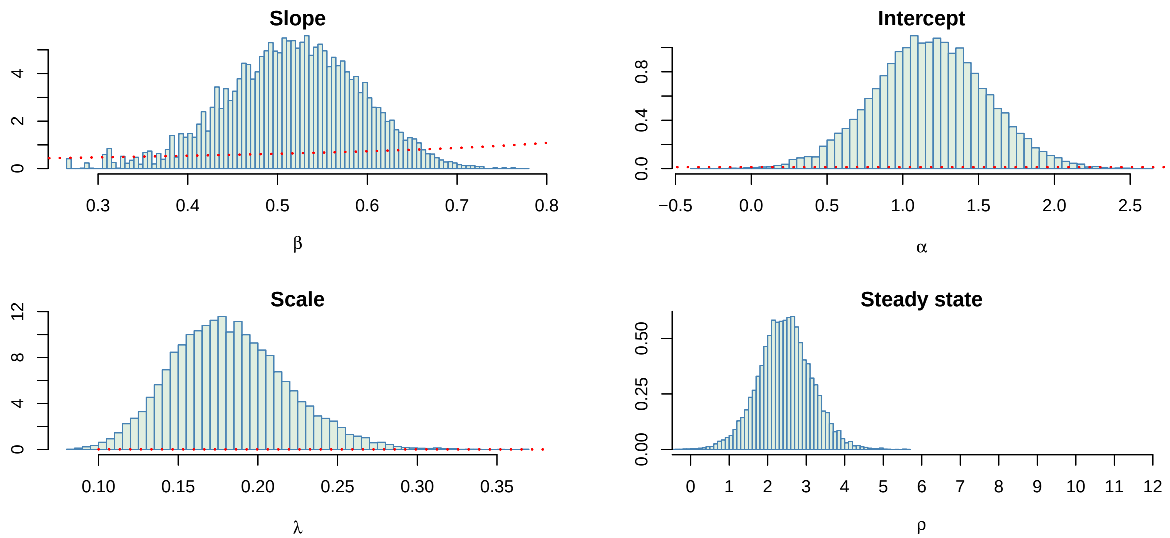


Figure 5: Stationary model posterior plots for PAK using my priors.

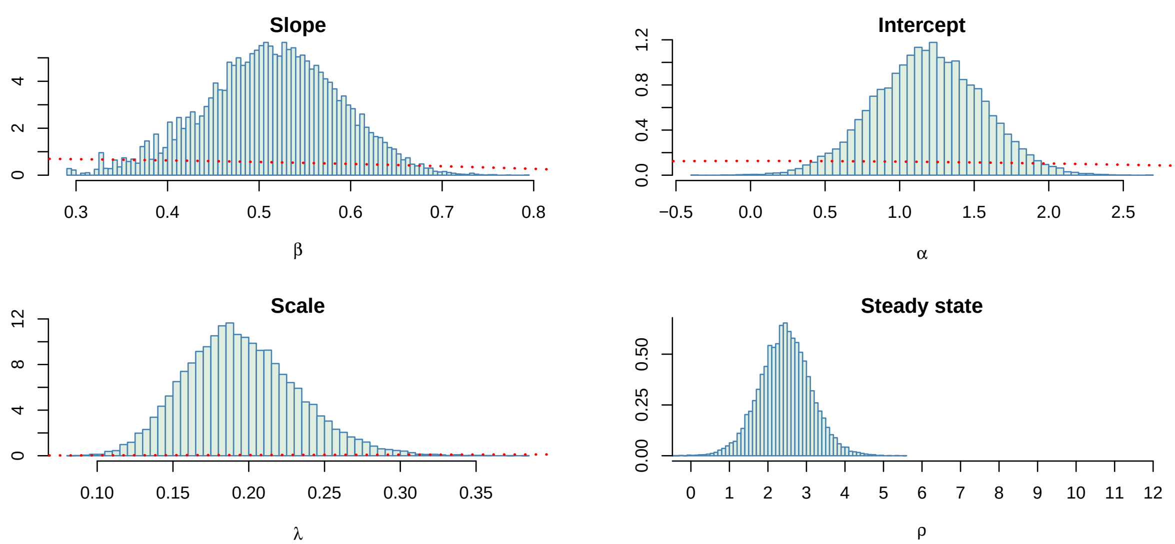


Figure 6: Stationary model posterior plots for PAK using the experts priors. Very little difference can be seen from the equivalent plots using my prior (figure 5).