MAS6004/6062 S1 Project
Statistical analysis of economic growth

Background

Economic growth is commonly defined as an increase in the output that an economy produces over a period of time, usually a year. The most widespread measure of output is gross national product (GDP). Economic growth is one of the key drivers in public and private decision making: public policies such as wages, interest rates and spending are geared towards increasing GDP growth, while private investment strategies usually depend on the long term growth rate.

The econometrics literature suggests that,

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t \ , \tag{1}$$

is a simple but still suitable model for economic growth. Here y_t is the growth rate at time t, ε_t is an idiosyncratic error term, assumed to be Gaussian with zero mean and precision λ; β is the effect of last period's growth and α is related to the long term growth rate. A common assumption in the econometrics literature is that this is a stationary process, *i.e.* that $|\beta| < 1$ and if this is the case then the long term or steady state growth rate is $\rho = \alpha/(1-\beta)$.

Data and Statistical analysis

The file (WB_PCGR.RData) contains the annual per-capita GDP growth rates of 264 countries and regions obtained from the world bank `http://data.worldbank.org/indicator/NY.GDP.PCAP.KD.ZG/`; it can be downloaded from the MOLE webpage along with the spreadsheet `CountryCodes` with the codes for the regions/countries. **Your task is to select two different countries or regions and carry out the following statistical analysis of their economic growth**.

1. You should fit model (1) using two different priors:

   a) We impose stationarity by using a re-scaled Beta distribution —to $(-1,1)$— with parameters $(2,2)$, as a prior for the AR(1) slope, β,

   $$\pi(\beta) \propto 1-\beta^2 \ , \qquad -1 < \beta < 1 \ .$$

   This is coded in the routine `AR1_stationary.R`.

   b) If stationarity is not imposed, assume $\pi(\beta) = \mathrm{N}\left(\beta \,\middle|\, m_\beta, 1/p_\beta\right)$. This is coded in the routine `AR1_nonstat.R`.

   in both cases the priors for the other two parameters are

   $$\pi(\alpha) = \mathrm{N}(\alpha \,|\, m_\alpha, 1/p_\alpha) \quad \text{and} \quad \pi(\lambda) = \mathrm{Ga}(\lambda \,|\, a, b) \ .$$

   You should elicit values for $\{m_\alpha, p_\alpha, a, b\}$ in both models and additionally for $\{m_\beta, p_\beta\}$ for the non-stationary case. Provide a careful explanation of your parameter selection.

You should carry out informal checks for convergence by selecting different starting points and length of chains.

2. After talking to my economic consultant, my elicited prior parameters are

$$\{m_\alpha, p_\alpha, a, b\} = \{0.02, 0.1^2, 2, 0.6\} \quad \text{and} \quad \{m_\beta, p_\beta\} = \{0, 1\}$$

Describe the differences between your and my posterior distributions for $\{\alpha, \beta, \lambda\}$.

3. The model may be simplified if the data suggest that the slope is close to zero. One possible measure of model fit can then be the posterior odds,

$$O = \frac{\mathrm{P}[|\beta| < 0.01 \mid \boldsymbol{y}]}{1 - \mathrm{P}[|\beta| < 0.01 \mid \boldsymbol{y}]} \,.$$

For each country you selected, calculate O from each prior specification and use it to decide if the data provides evidence in favour of a non-zero slope.

4. You are asked to provide advice to the CEO of a large company which is considering investing in the countries you selected. She has two possible decisions:

$$d_{1i} \;:\; \text{Invest in country } i \qquad d_{2i} \;:\; \text{Do not invest in country } i$$

After elicitation, her preferences, based on the long term growth rate, are described by

$$\mathcal{L}(d_1, \alpha, \beta) = \begin{cases} 0 & \rho > 1.5\% \\ 4 & \rho \leq 1.5\% \end{cases}, \qquad \mathcal{L}(d_2, \alpha, \beta) = \begin{cases} 1 & \rho > 1.5\% \\ 0 & \rho \leq 1.5\% \end{cases}$$

where $\rho = \alpha/(1-\beta)$ is the long term growth rate. Otherwise, she will invest on the country with the best economic perspective, *i.e.* the one that will increase its growth more rapidly. As part of your submission, prepare a short executive report advising the CEO on her optimal strategy.

Use MOLE to submit your work as a PDF file. Submission deadline is **5 February 2019** 12pm. Your file name **must be** of the form `123456789_BayesProject.pdf` where `123456789` is your student ID number. Marks will be deducted if the file is submitted otherwise.

There is no specific page limit, but you should give some thought to which material is in the main text and which is in appendices, if any. Also, if you include trace plots of very long runs, please thin the output to ensure that the size of your PDF file is not excessive.