MAS370/6012 Sampling Theory and Design of Experiments: Exercises 2

- FOR MAS370 students this set of exercises counts towards 7.5% of your total assessment on MAS370.
- FOR MAS6012 students this set of exercises counts towards 5% of your total assessment on MAS6012.

The deadline for submission is **Thursday 28th March, 10am**. Residential students should hand in their work at the MAS370/6012 lecture on 22nd March. Distance learners should email their solutions to Kevin Walters (k.walters@sheffield.ac.uk) by the deadline. Present your work in 'exam format': you must include all your working and present your solutions clearly, but otherwise, no marks will be awarded for presentation or commentary. Hand-written solutions are fine. **Put both your name and registration number on your work**.

Your submitted solutions must be entirely your own work: do not work with anyone else on your exercises.

Description of the problem

An investigator is studying the effect of 5 variables $x_1, x_2, \ldots, x_5$, each occuring at two levels, on a response ($\mathbf{Y}$). The investigator cannot obtain more than 8 observations. The investigator decides to use a fractional factorial design with the design generators $x_2x_5x_4 = 1$ and $x_2x_5x_1 = 1$.

Question 1

Derive the alias structure for these design generators and hence explain why it is not possible to fit a linear model with all 5 main effects.

Question 2

Specify the design matrix with 8 observations with the design generators $x_2x_5x_4 = 1$ and $x_2x_5x_1 = 1$ for a statistical model that includes the intercept and as many of the main effects as possible. Do not include any interactions in the design matrix. Label the columns of the design matrix showing which variables they correspond to and say how you constructed the design matrix.

Question 3

The investigator decides to modify the design in Question 2 by including a blocking factor with 2 distinct levels. Specify an interaction term that would be a sensible choice of blocking factor and justify your answer. Write down the design matrix with this blocking factor included. Label the columns of the design matrix showing which variables they correspond to.

Questions 4

With 8 observations specify two design generators that allow the intercept, the main effects of $x_1, x_2, \ldots, x_5$, and the two pairwise interactions x_3x_2 and x_3x_4 to be included in the fractional factorial design. If there are several pairs of possible design generators you only need to specify one pair. Confirm your answer by specifying the alias structure.