

5 variables x_1, x_2, \dots, x_5 each at values $\{-1, 1\}$

FFD utilised with design generators

$$x_2 x_4 x_5 = 1 \text{ and } x_1 x_2 x_5 = 1$$

maximum observations = 8

1) No. observations $m=5$ variables $t=2$ generators

$$2^{m-t} = 2^3 = 8 \text{ observations}$$

product of generators = $x_1 x_2 x_4$

Alias structure

$$1 = x_2 x_4 x_5 = x_1 x_2 x_5 = x_1 x_4$$

$$(x_1) x_1 = x_1 x_2 x_4 x_5 = x_2 x_5 = x_4$$

$$(x_2) x_2 = x_4 x_5 = x_1 x_5 = x_1 x_2 x_4$$

$$(x_3) x_3 = x_2 x_3 x_4 x_5 = x_1 x_2 x_3 x_5 = x_1 x_3 x_4$$

$$(x_4) x_4 = x_2 x_5 = x_1 x_2 x_4 x_5 = x_1$$

$$(x_5) x_5 = x_2 x_4 = x_1 x_2 = x_1 x_4 x_5$$

$$x_1 x_3 = x_1 x_2 x_3 x_4 x_5 = x_2 x_3 x_5 = x_3 x_4$$

$$x_2 x_3 = x_3 x_4 x_5 = x_1 x_3 x_5 = x_1 x_2 x_3 x_4$$

$$x_3 x_5 = x_2 x_3 x_4 = x_1 x_2 x_3 = x_1 x_4$$

Other 2nd level parameters are confounded with the intercept, main effects or the above three 2nd level parameters ($x_1 x_3$, $x_2 x_3$ or $x_3 x_5$)

The main effects parameter x_4 is confounded with the main effects parameter x_1 . A linear model cannot contain parameters that are confounded with each other at the first level that are confounded with other first level parameters. Therefore a linear model cannot contain both x_1 & x_4 and so only 4 main effects can be included in the model.

2) Design matrix

$$X = \begin{pmatrix} \beta_0 & x_1 & x_2 & x_3 & x_5 \\ 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

x_1, x_2, x_5 are constrained to equal 1.

x_3 takes two values $\{-1, 1\}$

So the design matrix for values $x_3 = -1$
is repeated for values $x_3 = 1$.

The intercept term is always 1.

Variable x_4 is dropped from the matrix as
this variable is confounded with x_1 .

3) Blocking generator : $Z = x_1 x_2 x_3$

This interaction term is not constrained by the design generators $x_1, x_2 x_5 = 1$ nor $x_2, x_4 x_5 = 1$.

Also, the blocking generator term is not included in the model

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_5 x_5$$

	β_0	$Z = x_1 x_2 x_3$	x_1	x_2	x_3	x_5
X=	1	1	1	-1	-1	-1
	1	1	-1	1	-1	1
	1	-1	-1	-1	-1	1
	1	-1	1	1	-1	1
	1	-1	1	-1	1	-1
	1	-1	-1	1	1	-1
	1	1	-1	-1	1	1
	1	1	1	1	1	1

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$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_5 x_5$$

	β_0	$Z = x_1 x_2 x_3$	x_1	x_2	x_3	x_5
X=	1	1	1	-1	-1	-1
	1	1	-1	1	-1	1
	1	-1	-1	-1	-1	1
	1	-1	1	1	-1	1
	1	-1	1	-1	1	-1
	1	-1	-1	1	1	-1
	1	1	-1	-1	1	1
	1	1	1	1	1	1