Theory and Practice of Humanoid Walking Control

2022 Fall semester

Homework # 6

Problem 6 Online CoM pattern generation using zmp preview control

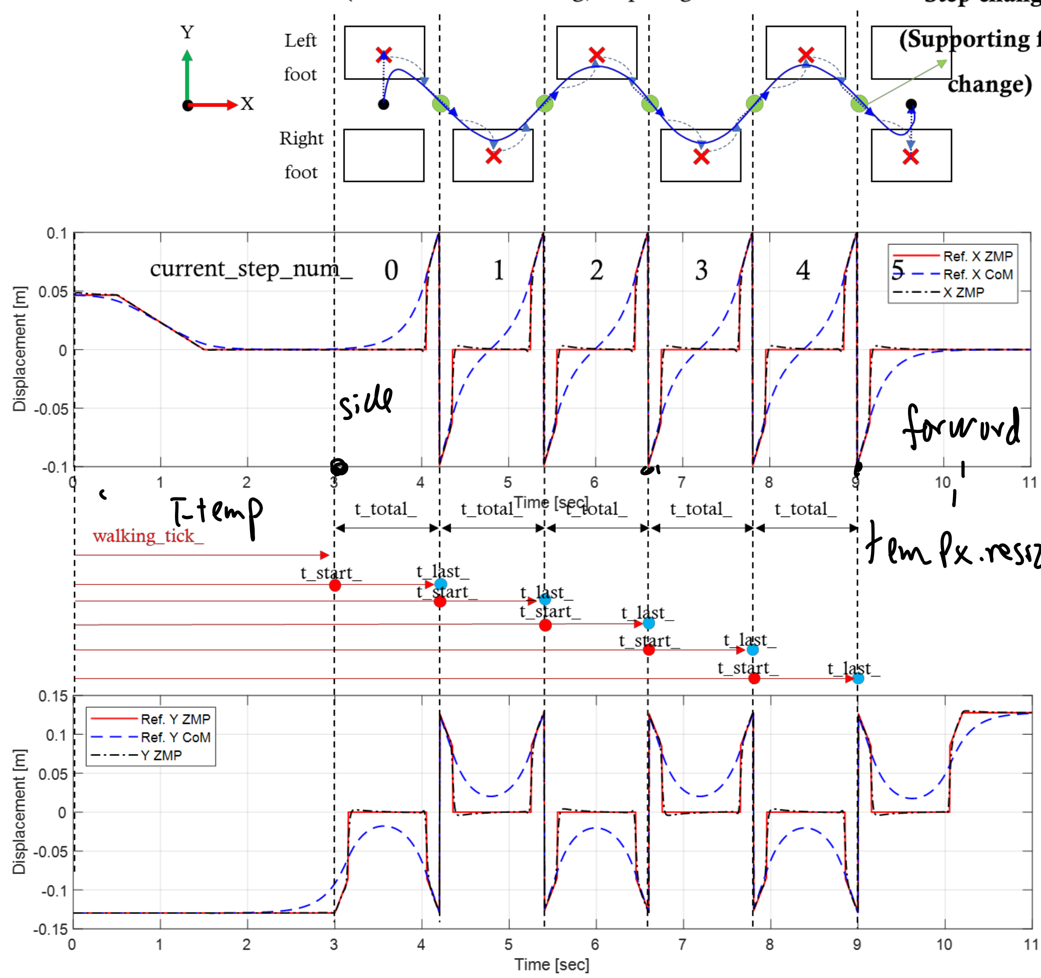
※ The first supporting foot is the left foot.

※ References:

1) Kajita, Shuuji, et al. "Biped walking pattern generation by using preview control of zero-moment point." Robotics and Automation, 2003. Proceedings. ICRA'03. IEEE International Conference on. Vol. 2. IEEE, 2003.

2) Katayama, T., Ohki, T., Inoue, T. and Kato, T., "Design of an Optimal Controller for a Discrete Time System Subject to Previewable Demand," Int. J. Control, Vo1.41, No.3, pp.677-699, 1985 (679 ~ 681p)

Reference CoM trajectory w.r.t the supporting foot frame (1.0m forward walking, step length : 0.2m



ZMP = CoM in DSP

1,2

temp_px.resize(t_total+4..)

1,2+2

9, 3,2 -12,2

✓ Find the control input gain of the plant (Cart-table model) *Refer to class PPT

1) Define the plant (Cart-table model). (A, B, C Matrix / discrete time domain)

explained class

have formula → define.

2) Define $\tilde{I}, \tilde{F}, \tilde{A}, \tilde{B}$ using A, B, C Matrix to construct the state space equation for ZMP error and state variation

3) Calculate the Riccati equation using System matrix $\tilde{A}$, Input matrix $\tilde{B}$, Weighting matrix R and $\tilde{Q}$ to find $\tilde{K}$

→ riccati solution

$$\tilde{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

✓ R = 1.0 $\times 10^{-6}$, $\tilde{Q} = \begin{bmatrix} Q_e & 0 \\ 0 & Q_x \end{bmatrix}$, $Q_e = 1.0, Q_x = Q_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

✓ In MATLAB : $\tilde{K}$ =dare($\tilde{A}, \tilde{B}, \tilde{Q}, R$) → discrete algebraic Riccati eq.

4) Calculate G_i, G_x, G_d using $\tilde{A}, \tilde{B}, R, \tilde{Q}$ and $\tilde{K}$

steps 1,2,3,4.

✓ Build the preview controller

usual time

reference signal.

1) Set the preview time to 1.6 sec (Preview the future reference ZMP trajectory after 1.6 seconds from the current point in time/ $N_L = 320$ (200Hz control frequency))

2) The reference ZMP created in HW#2 is used. could use any IRL

3) Using the control gain calculated above, the control input variation is calculated in real time

$$\Delta u(k) = -G_i e(k) - G_x \Delta x(k) - \sum_{l=1}^{N_L} G_d(l)\, \Delta p^{ref}(k+l)$$

y_d

S

4) CoM is calculated in real time by integrating the variation of control input and putting it into the control input of the Cart table model.

$$u(k) = -G_i \sum_{i=0}^{k} e(k) - G_x x(k) - \sum_{l=1}^{N_L} G_d(l)\, p^{ref}(k+l)$$

$$x(k+1) = Ax(k) + Bu(k), x(k) = \begin{bmatrix} x_c(k) \\ \dot{x}_c(k) \\ \ddot{x}_c(k) \end{bmatrix}$$

Posit
vel
accele

u previous states

5) The calculated $x_c(k)$ becomes the reference CoM position.

✓ Run it after programming

x_c → joint angles → joint angles.

1) rosrun dyros_jet_gui dyros_jet_gui → X: 1.0m, Step length : 0.2m → START walking button click!!

2) Plot the reference ZMP, CoM and calculated ZMP.

Pos
vel
acc
$(\vec{X})$ $\vec{y}$

3) Record the walking simulation video.

* Hint

↗ 400

Simulation time → walking_tick_ (1tick : 0.005sec)

1 step time (1.2sec) → t_total_

Start time of each step → t_start_

End time of each step → t_last_

First DSP and last DSP time in one step → t_double1_ (0.15 sec), t_double2_ (0.15 sec)

The total number of steps to reach the target point. (It is automatically calculated when you click the start walking button.) → total_step_num_

Current number of steps → current_step_num_

Initial X, Y, Z CoM position w.r.t the support foot → xi_, yi_, zc_

Real pelvis position w.r.t the supporting foot frame → pelv_support_current_.translation()(n), n = 0, 1, 2 (X, Y, Z respectively.)

Initial pelvis height w.r.t the supporting foot frame → pelv_support_start_.translation()(2)

Real CoM position w.r.t the supporting foot frame → com_support_current_(n), n = 0, 1, 2 (X, Y, Z respectively.)

Foot step position w.r.t the current support foot frame

→ foot_step_support_frame_(n,0), foot_step_support_frame_(n,1)

→ The first element n of the variable means sequence, and the second elements 0 and 1 mean the positions of X and Y, respectively.

Measured joint angle → current_motor_q_leg_ (Vector12d)

CoM was created offline → now online

COM, ZMP

input to control ZMP as we want

Assume

* periodic sampling

* control vector constant btw steps

1)

$$\dot{x} = Ax + Bu \rightarrow x(i+1) = Ax(i) + Bu(i)$$

$$y = Cx(t) \qquad y(i) = Cx(i) \quad (1\times3 \cancel{\times 3\times1}) = 1\times1$$

then solving state space

$$A = e^{\begin{bmatrix} 0 & T & 0 \\ 0 & 0 & T \\ 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} e^0 & t & \frac{t^2}{2} \\ 0 & e^0 & t \\ 0 & e^0 & e \end{bmatrix} = \begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{tA} = I + tA + \frac{t^2}{2}A^2$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \int_0^T e^{A\tau} B \, d\tau = \int_0^t \begin{bmatrix} \frac{\tau^2}{2} \\ \tau \\ 1 \end{bmatrix} d\tau = \begin{bmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & -\frac{z_c}{g} \end{bmatrix}$$

$$CA \quad 1\times3 \cancel{\times 3}\times3$$

2) find $\tilde{I}, \tilde{F}, \tilde{A}, \tilde{B}$ using A,B,C to construct state space eq. for ZMP error, state var.

$$1\times\cancel{3}\times\cancel{3}\times1$$

$$\tilde{B} = \begin{bmatrix} CB \\ B \end{bmatrix} = \begin{bmatrix} [1 \; 0 \; -\frac{z_c}{g}] \begin{bmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{bmatrix} \\ \begin{bmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{bmatrix} \end{bmatrix} = \left[\frac{T^3}{6} - \frac{z_c}{g} T, \frac{T^3}{6}, \frac{T^2}{2}, T \right]^T$$

$$\tilde{I} = \begin{bmatrix} I_p \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow A_d = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \tilde{A} = [\tilde{I} \; \tilde{F}]$$

$$\tilde{F} = \begin{bmatrix} CA \\ A \end{bmatrix} = \begin{bmatrix} 1 & t & \frac{t^2}{2} - \frac{z_c}{g} \\ 1 & t & t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix} \qquad \tilde{A} = \begin{bmatrix} 1 & 1 & t & \frac{t^2}{2} - \frac{z_c}{g} \\ 0 & 1 & t & t^2 \\ 0 & 0 & 1 & t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e_{(i+1)} = e_{(i)} + CA\Delta x_{(i)} + CB\Delta u_{(i)} - [I_p \; 0 \; 0 \ldots](y_{total,d}(i+1) - y_d(i))$$

$$= y_{(i+1)} - y_d(i+1) = \text{calc\_x\_desired}$$

$$\Delta u(k) = -G \sum_{i=0}^{k} e(k) - G_x \Delta x_{(k)} - \sum_{1}^{N_L} G_d(l) y_d(k+l)$$

$$x(k+1) = Ax(k) + Bu(k)$$