### **Renato Paes Leme**

**Abstract**: We investigate how complex is the class of gross substitutes as compared to both simpler classes (e.g. matroid rank functions) and more complex one (e.g. submodular valuations). To do so, we take two perspectives: (i) representation: how to construct gross substitutes from simpler classes; (ii) approximation: to what extend can gross substitutes approximate submodular functions.

**Xin Chen - Menglong Li**

**Abstract:** We propose a new concept of S-convex functions (and its variant SSQS-convex functions) to study substitute structures in economics and operations models with continuous variables. We develop a host of fundamental properties and characterizations of S-convex functions including various preservation properties, conjugate relationships with submodular and convex functions, and characterizations using Hessians. For a divisible market, we show that the utility function satisﬁes gross substitutability if and only if it is S-concave under some regularity conditions. In a parametric maximization model with a box constraint, we show that the set of optimal solutions is nonincreasing in the parameters if the objective function is (SSQS)S-concave. Furthermore, we prove that S-convexity is necessary for the property of nonincreasing optimal solutions under some conditions. The monotonicity result is applied to a classical multi-product dynamic inventory model.

**Jonathan Weinstein**

**Abstract:** I propose a notion of complementarity in consumer theory, called direct complementarity, for which I provide four equivalent definitions including an axiomatization. I point out a novel critique of the leading definition of complementarity, which is based on cross-price effects: such effects are sensitive to changes in the basis used to describe the space of available bundles. Direct complementarity, on the other hand, is defined for preferences over an abstract vector space of bundles, without reference to a particular basis (or list of ``goods''), and provides a consistent definition across all pairs of composite goods, i.e. linear combinations of standard goods. Direct complementarity better captures the intuitive notion of one good's effect on the value of another, hence the term \emph{direct}; cross-price effects are best understood as an \emph{indirect} consequence of such relationships.

**Faruk Gul**

**Abstract:** We provide conditions under which market mechanisms can be used to allocate indivisible goods efficiently and fairly. We consider pseudo markets; that is, economies in which initially the seller/designer owns all of the indivisible goods and the agents are allocated fiat money. We allow stochastic consumption and show the existence and eciency of Walrasian equilibria in this setting. We demonstrate that constraints on minimum and maximum levels of individual consumption and aggregate constraints of the kind that are relevant in combinatorial allocation problems can be accommodated by either incorporating these constraints into individual preferences or by incorporating a suitable production technology

### **Pawel Dziewulski**

**Abstract:** We investigate conditions for comparative statics in quasilinear models and show that they critically depend on an intuitive geometric property. We obtain a characterization of production functions that induce complementarities among inputs and an analogous characterization of production functions that induce substitutes among inputs. Our results unify (and, in some cases, generalize) existing conditions for comparative statics over continuous and discrete domains.

**Elizabeth Baldwin**

**Abstract:** Product-mix auctions are sealed-bid mechanisms for trading multiple units of multiple differentiated goods. They implement competitive-equilibrium allocations based on the preferences that participants express in an easy-to-use-and-understand geometric language. All concave substitutes (respectively, strong substitutes) preferences can be uniquely represented, and no other preferences can be represented, by appropriate sets of permitted bids in the corresponding version of this language. These languages thus also provide new characterizations of ordinary substitutes, and of strong substitutes. We discuss implementation of the auctions, and extensions and variants of the language, e.g., allowing for budget constraints.

**Maxime Sylvestre**

**Abstract:** We propose monotone comparative statics results for maximizers of submodular functions, as opposed to maximizers of supermodular functions as in the classical theory put forth by Veinott, Topkis, Milgrom, and Shannon among others. We introduce matrons, a natural structure that is dual to sublattices that generalizes existing structures such as matroids and polymatroids in combinatorial optimization and M-sets in discrete convex analysis. Our monotone comparative statics result is based on a natural order on matrons, which is dual in some sense to Veinott's strong set order on sublattices. As an application, we propose a deferred acceptance algorithm that operates in the case of divisible goods, and we study its convergence properties.

**Wolfgang Pesendorfer**

**Abstract:** A collective choice problem specifies a finite set of alternatives from which a group ofexpected utility maximizers must choose. We associate a public goods economy with everycollective choice problem and establish the existence and efficiency of Lindahl equilibriumallocations for that economy. We also associate a cooperative bargaining problem withevery collective choice problem and define a set-valued solution concept, theweightedNash bargaining set. We provide axioms that characterize the weighted Nash bargainingset. Our main result shows that weighted Nash bargaining set payoffs with welfare weightsωare also the Lindahl equilibrium payoffs of the corresponding economy with the sameutility functions and incomesω. Finally, we consider a general class of matching problemsand show that the set of Lindahl equilibrium payoffs and the set of Walrasian equilibriumpayoffs is the same. More generally, we show that in any discrete-goods economy, theset of Walrasian equilibrium allocations is a subset set of the set of Lindahl equilibrium allocations.