Costly Concessions: An Empirical Framework for Matching with Imperfectly Transferable Utility

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Section 1

Introduction

- The two main tools of Family Economics are **Matching Models** and **Collective Models**.
- Matching models are interested in sorting: "who matches with whom". They generally regard the question of bargaining within the household as a black box, assuming either no transfer (**Nontransferable Utility – NTU**) or additive transfers (**Transferable Utility – TU**).
- On the contrary, collective models seek to understand how utility is distributed within the household: agents bargain and reach the efficient frontier according to a "sharing rule". However, in most of these models, the **sharing rule is exogenous**.
- The goal of this talk is to bring together those two models, in order to open the black box of utility transfers in matching models, and endogenize the sharing rule. For this we'll need to go beyond TU and NTU models and work with more general models with **Imperfectly Transferable Utility (ITU)**.

- Transfers of utility (under the form of money or other exchanges):
  - Are sometimes clearly forbidden (e.g. **school choice problems**): NTU matching.
  - Are sometimes clearly allowed (e.g. **wages in the market for CEOs**): TU matching.
  - Sometimes are allowed but imperfect: **labor market** with nonquasilinear utilities (Kelso and Crawford, Hatfield and Milgrom); of taxes (Jaffe and Kominers); of risk aversion (Legros and Newman, Chade and Eckehout); of investments (Samuelson and Noeldeke): ITU matching.
- For the **marriage market**, the literature is split.
  - Indeed, there is a tradition to model the marriage market with transfers (Shapley and Shubik; Becker; Choo and Siow) and without transfers (Gale and Shapley; Dagsvik; Hitsch, Hortacsu and Ariely; Menzel).
  - It makes sense to assume that transfers whithin the couple are possible, but not efficient in the sense that the utility transfered by i to j maybe more costly to i that it is beneficial to j: ITU matching.

Outline

This talk:

1. Introduction
2. Theoretical framework
3. Empirical framework
4. Discussion
5. Application

Section 2

THE THEORETICAL FRAMEWORK

- Let $\mu_{ij} \in \{0, 1\}$ be a dummy variable that is equal to 1 if man i and woman j are matched, 0 else.
- Hence, μ_{ij} satisfies

$$\sum_j \mu_{ij} \leq 1$$
$$\sum_i \mu_{ij} \leq 1.$$

- Assume that if man i and woman j match, then i has utility α_{ij} and j has utility γ_{ij}, pre-transfer.
- Utilities of single individuals are normalized to zero.
- Assume that transfers $t_{i\leftarrow j}$ and $t_{j\leftarrow i}$ are decided, so that if matched, i and j enjoy respectively

$$\begin{aligned} u_i &= \alpha_{ij} + t_{i\leftarrow j} \\ v_j &= \gamma_{ij} + t_{j\leftarrow i}. \end{aligned}$$

- The link between $t_{i\leftarrow j}$ (what i receives) and $-t_{j\leftarrow i}$ (what j gives out) is subject to a *feasibility constraint*, which will specify whether matching is TU, NTU, or ITU.

- If $\mu_{ij} = 1$, then $t_{i \leftarrow j}$ and $t_{j \leftarrow i}$ must satisfy a feasibility constraint:
  - In the TU case $t_{i \leftarrow j} + t_{j \leftarrow i} \leq 0$
  - In the NTU case, $\max\left(t_{i \leftarrow j}, t_{j \leftarrow i}\right) \leq 0$
  - More generally, in the ITU case, $\Psi_{ij}\left(t_{i \leftarrow j}, t_{j \leftarrow i}\right) \leq 0$, where Ψ_{ij} is continuous and nondecreasing in its two variables.
- We can rewrite the feasibility constraint as

$$\mu_{ij} = 1 \Longrightarrow \Psi_{ij}\left(u_i - \alpha_{ij}, v_j - \gamma_{ij}\right) \leq 0.$$

- Of particular interest is the Exponentially Transferable Utility (ETU) case, when

$$\Psi(a,b) = \tau \log\left(\frac{\exp(a/\tau) + \exp(b/\tau)}{2}\right)$$

where $\tau > 0$ is a transferability parameter.
- The ETU model interpolates between TU and NTU; indeed, $\tau \to 0$ is NTU, while $\tau \to +\infty$ is TU.
- The ETU model can be interpreted as a simple model of household consumption with logarithmic utilities: Assume that the joint budget of a household is 2, which is shared into c_i and c_j, the man and the woman's private consumptions. Assume that the utilities are $u_i = \alpha_{ij} + \tau \log c_i$ and $v_j = \gamma_{ij} + \tau \log c_j$, so that agents care about the identities of their partners and about their private consumption. Then feasibility means

$$\exp\left(\frac{u_i - \alpha_{ij}}{\tau}\right) + \exp\left(\frac{v_j - \gamma_{ij}}{\tau}\right) = c_x + c_y \leq 2.$$

- For any pair i and j, we need to rule out the case that i and j form a blocking pair, i.e. each achieve higher payoffs by rebargaining. This implies
$$\forall i, j \ \Psi_{ij}\left(u_i - \alpha_{ij}, v_j - \gamma_{ij}\right) \geq 0.$$
- In the TU case, this is the well-known stability conditions $u_i + v_j \geq \alpha_{ij} + \gamma_{ij}$,
- while in the NTU case, this reads $\max\left(u_i - \alpha_{ij}, v_j - \gamma_{ij}\right) \geq 0$.

- We are now ready to define an equilibrium in this market.
- An *outcome* (μ, u, v) is an equilibrium if
  - (i) $\mu_{ij} \in \{0, 1\}$, $\sum_j \mu_{ij} \leq 1$ and $\sum_i \mu_{ij} \leq 1$
  - (ii) $\Psi_{ij}\left(u_i - \alpha_{ij}, v_j - \gamma_{ij}\right) \geq 0$
  - (iii) $\mu_{ij} = 1$ implies $\Psi_{ij}\left(u_i - \alpha_{ij}, v_j - \gamma_{ij}\right) = 0$.

Section 3

The empirical framework

- Assume that there are groups, or clusters of men and women who share similar observable characteristics, called *types*. There are n_x men of type x, and m_y women of type y.
- Let $\mu_{xy} \geq 0$ be the number of men of type x matched to women of type y. This quantity satisfies

$$\sum_y \mu_{xy} \leq n_x$$
$$\sum_x \mu_{xy} \leq m_y$$

- We shall denote μ_{x0} and μ_{0y} the number of single men of type x and single women of type y.

- **Assumption 1**: Assume that if man i of type x and woman j of type y match, then

$$\alpha_{ij} = \alpha_{xy} + \varepsilon_{iy}$$
$$\gamma_{ij} = \gamma_{xy} + \eta_{jx}$$

Utilities of single man i and woman j are respectively ε_{i0} and η_{j0}. Recall that transfers $t_{i\leftarrow j}$ and $t_{j\leftarrow i}$ are decided, so that if matched, i and j enjoy respectively

$$u_i = \alpha_{xy} + \varepsilon_{iy} + t_{i\leftarrow j}$$
$$v_j = \gamma_{xy} + \eta_{jx} + t_{j\leftarrow i}$$

- **Assumption 2**: there are a large number of invididuals per group and the ε and the η's are i.i.d. Gumbel.

- **Assumption 3**: If i is of type x and j is of type y, then

$$\Psi_{ij}(a,b) = \Psi_{xy}(a,b)$$

(i.e. we assume that Ψ_{ij} only depends on i and j through their types).

- Thus, we can rewrite the feasibility constraint as (for i in x and j in y)

$$\mu_{ij} = 1 \Longrightarrow \Psi_{xy}\left(u_i - \alpha_{xy} - \varepsilon_{iy}, v_j - \gamma_{xy} + \eta_{jx}\right) \leq 0.$$

and stability

$$\forall i \in x, j \in y,\ \Psi_{xy}\left(u_i - \alpha_{xy} - \varepsilon_{iy}, v_j - \gamma_{xy} + \eta_{jx}\right) \geq 0.$$

Theorem 1 (Galichon, Kominers and Weber). Under Assumptions 1, 2 and 3 above, equilibrium transfers $t_{i \leftarrow j}$ and $t_{j \leftarrow i}$ only depend on x and y, the observable types of i and j. Hence, let us denote these quantitites by $t_{x \leftarrow y}$ and $t_{y \leftarrow x}$.

This theorem extends to the general ITU case a result which was known in the TU case (Choo and Siow, Chiappori, Salanié and Weiss, Galichon and Salanié).

Implication of this theorem: the matching problem now embeds two sets of discrete choice problems. Indeed, man i and woman j (of types x and y) solve respectively

$$\max_y \left\{ \alpha_{xy} + t_{x \leftarrow y} + \varepsilon_{iy}, \varepsilon_{i0} \right\}$$
$$\max_x \left\{ \gamma_{xy} + t_{y \leftarrow x} + \eta_{jx}, \eta_{j0} \right\}$$

which are standard discrete choice problems; thus the log-odds ratio formula applies, and

$$\ln \frac{\mu_{xy}}{\mu_{x0}} = \alpha_{xy} + t_{x \leftarrow y}$$
$$\ln \frac{\mu_{xy}}{\mu_{0y}} = \gamma_{xy} + t_{y \leftarrow x}$$

But remember that $\Psi_{xy} \left(t_{x \leftarrow y}, t_{y \leftarrow x} \right) = 0$, thus

$$\Psi_{xy} \left(\ln \frac{\mu_{xy}}{\mu_{x0}} - \alpha_{xy}, \ln \frac{\mu_{xy}}{\mu_{0y}} - \gamma_{xy} \right) = 0.$$

Theorem 2 (GKW). Equilibrium in the ITU problem with logit heterogeneities is fully characterized by the set of nonlinear equations in μ_{xy}, μ_{x0} and μ_{0y}

$$\Psi_{xy}\left(\ln \frac{\mu_{xy}}{\mu_{x0}} - \alpha_{xy}, \ln \frac{\mu_{xy}}{\mu_{0y}} - \gamma_{xy}\right) = 0$$

$$\sum_y \mu_{xy} + \mu_{x0} = n_x$$

$$\sum_x \mu_{xy} + \mu_{0y} = m_y$$

Under very mild conditions on Ψ it exists; under mild conditions on Ψ it is also unique.

Galichon and Hsieh (in progress) extend this result to the NTU case with general stochastic utilities.

Note that first equation defines implicitely μ_{xy} as a function of μ_{x0} and μ_{0y}, which can be written as a matching function

$$\mu_{xy} = M_{xy}\left(\mu_{x0}, \mu_{0y}\right)$$

hence we can restate the previous result as:

Theorem 2' (GKW). Equilibrium in the ITU problem with logit heterogeneities is fully characterized by the set of nonlinear equations in μ_{x0} and μ_{0y}

$$\sum_y M_{xy}\left(\mu_{x0}, \mu_{0y}\right) + \mu_{x0} = n_x$$
$$\sum_x M_{xy}\left(\mu_{x0}, \mu_{0y}\right) + \mu_{0y} = m_y.$$

Section 4

DISCUSSION

- Computation of

$$\sum_y M_{xy}\left(\mu_{x0},\mu_{0y}\right)+\mu_{x0}=n_x$$
$$\sum_x M_{xy}\left(\mu_{x0},\mu_{0y}\right)+\mu_{0y}=m_y.$$

  is very easy (and computationally very efficient) by iterative fitting. Convergence is guaranteed, and in practice, very fast.
- This is a very convenient alternative to the Kelso-Crawford algorithm to get approximate equilibrium solutions when the populations are large.
- This will allows us to efficiently compute the likelihood for the purpose of maximum likelihood estimation.

- In the TU case, $M_{xy}\left(\mu_{x0},\mu_{0y}\right)=\sqrt{\mu_{x0}\mu_{0y}}\exp\left(\frac{\alpha_{xy}+\gamma_{xy}}{2}\right)$, thus $\mu_{xy}=M_{xy}\left(\mu_{x0},\mu_{0y}\right)$ identifies

$$\alpha_{xy}+\gamma_{xy}=\ln\frac{\mu_{xy}^{2}}{\sqrt{\mu_{x0}\mu_{0y}}}$$

  which is the Choo and Siow identification formula, extended to general stochastic utilities by Galichon and Salanié. Note that α and γ are not individually identified, but $\alpha+\gamma$ is.
- In the NTU case, $M_{xy}\left(\mu_{x0},\mu_{0y}\right)=\min\left(\mu_{x0}\exp\alpha_{xy},\mu_{0y}\exp\gamma_{xy}\right)$, thus α and γ are partially identified by

$$\min\left(\alpha_{xy}+\ln\mu_{x0},\gamma_{xy}+\ln\mu_{0y}\right)=\ln\mu_{xy}.$$

- In the ETU case, α and γ are partially identified by

$$\mu_{xy}=\left(\frac{e^{-\alpha_{xy}/\tau}\mu_{x0}^{-1/\tau}+e^{-\gamma_{xy}/\tau}\mu_{0y}^{-1/\tau}}{2}\right)^{-\tau}.$$

- Comparative statics are very easy to compute in this model, thanks to the implicit function theorem.
- Of particular interest are the impact of a change in α and γ (endowments) and of n and m (number of men and women of each types) on U and V (equilibrium welfare) and μ (equilibrium matching patterns).
- Some results from the TU case extend: an increase in the number of men of type x decreases the welfare of men of type x.
- Some results don't extend from the TU case:
  - In the TU case, the Becker-Coase theorem predicts that the welfare only depends on $\alpha + \gamma$: thus, if α_{xy} is changed into $\alpha_{xy} - c_{xy}$ and γ_{xy} is changed into $\gamma_{xy} + c_{xy}$, then the welfare and matching patterns are unchanged.
  - In the ITU case, this is no longer the case. Worse, an increase in γ may result in a decrease in V (unintended/averse consequence).

Section 5

APPLICATION

- Q: How much transfer frictions are there on the marriage market? is the marriage market best described by an TU, or a NTU model?
- We would like to estimate τ, and are also interested in α_{xy} and γ_{xy}.
- However, for each market, we have one equilibrium vector μ_{xy}: **under-identification** if a single market is observed.
- This calls for a **multi-market approach**. A first guess is to assume that α_{xy}, γ_{xy}, and τ are constant across markets, and that the only source of variation is demographics n_x and m_y.

- **Data**. We use 2010 Census data to construct the population vectors.
- **Markets**. A market k is defined as a state in this application. Thus

$$k \in \mathcal{S} = \{1, ..., 51\}$$

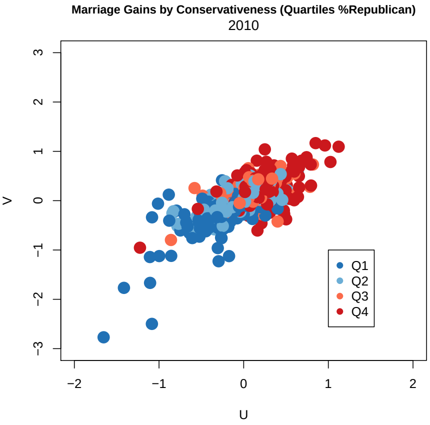
- **Types.** An individual belongs to one of the following type : Below High School or High School dropouts (grade$<$12), High School degree, Some College or more.
- **Age.** A single must be between 23 and 35 years old to be included. For couples, the head must belong to this age interval.
- **Couples.**
  - We restrict our attention to marriage, but cohabitating couples can be included.
  - One of the spouse or partner must be the head of the household (necessary to link the spouses together).
- **Singles.** Depending on the definition of a couple, cohabitating persons can be counted as singles.
- **Excluded.** Are excluded from our analysis couples in which information is missing about a spouse ; other married persons within the same household (cannot link the spouse together).

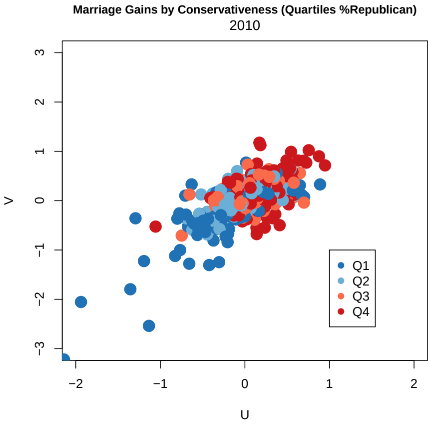
- The utilities U_{xy} and V_{xy} are computed by the standard logit inversion formula (log-odds ratio)

$$
\begin{aligned}
U_{xy} &= \log\left(\frac{\mu_{xy}}{\mu_{x0}}\right) \\
V_{xy} &= \log\left(\frac{\mu_{xy}}{\mu_{0y}}\right).
\end{aligned}
$$

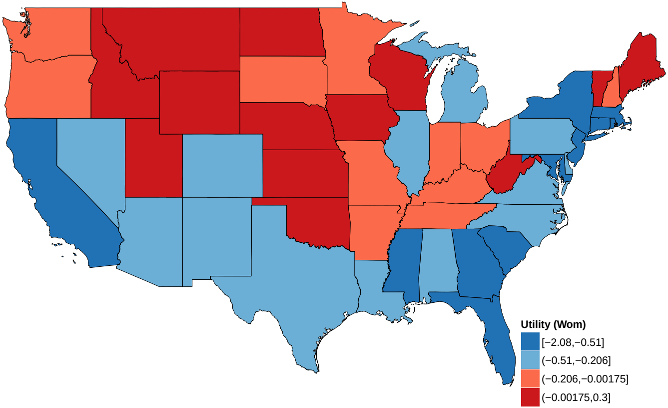
- We plot the estimated U versus the estimated V.

- We provide results for 2010. Cohabitating persons are either counted as singles (left) or married (right)
- Conservativeness is measured by %Republican vote (from 2008 elections).





- We provide estimates of marriage gains for High School degree women married to High School degree men ($= V_{22}$).



- The previous figure shows too much heterogeneity between markets.
- Let k be the index of a market (states in this application). Assume that pre-transfer value of marriage in market k is given by

$$\alpha_{xy}^{k} = \alpha_{xy}^{0} + \eta^{k}$$

for men, and

$$\gamma_{xy}^{k} = \gamma_{xy}^{0} + \nu^{k}$$

- Equilibrium implies $\Psi(U_{xy} - \alpha_{xy}, V_{xy} - \gamma_{xy}) = 0$.Then, estimation is based on seeking τ, α_{xy}^{0}, η^{k}, γ_{xy}^{0} and ν^{k} to minimize

$$\min \sum_{k} \sum_{xy} \left| \tau \log \left(\exp \left(\frac{U_{xy} - \alpha_{xy}^{k}}{\tau} \right) + \exp \left(\frac{V_{xy} - \gamma_{xy}^{k}}{\tau} \right) \right) \right|^{2}$$

in the ETU case.

- This is a $|\mathcal{X}| + |\mathcal{Y}| + 2 \times |\mathcal{S}| + 1-$dimensional minimization problem, and we have $|\mathcal{X}| \times |\mathcal{Y}| \times |\mathcal{S}|$ observations.

- **Note.** The results presented here are for the 2010 Census, with a different age interval and different types.
- In this setting, we estimate $\hat{\tau} \simeq 8$.