Honours Degree of Bachelor of Science in Artificial Intelligence

Batch 24 - Level 1 (Semester 2)

CM 1310: Linear Algebra and Calculus

Set Theory

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Set Theory

Learning Outcomes

By the end of this chapter, students will be able to;

1. define a set.
2. recognize the difference forms of sets.
3. identify the universal and empty set.
4. identify some important number sets.
5. define subset and proper subset.
6. recognize different number sets.

Contents

1 Sets

Definition 1. *A set is a collection* ***well-defined*** *objects.*

Well-Defined

There are some rules, policies or certain conditions by which we can defined whether the given object does or does not belong to the given set.

The objects of a set are called the *elements* or *members* of the set.

Examples

- The set of the twelve months of the year.
- The set of all students who follow CM 1310.
- The set of all natural numbers.
- The set of all real numbers whose square is 2.

2 Notations

Sets are usually denoted by capital letters, $A, B, C, P, Q, X, Y, \ldots$

To denote elements we use $a, b, c, p, q, x, y, \ldots$. If x is an element of a set A we say x belongs to A and write $x \in A$. If x is not an element of a set A we say x does not belong to A and write $x \notin A$.

2.1 Forms of Sets

1. Statement form
   $\mathbb{N}$ is the set of natural numbers.

2. Tabular form (Listing form)
   $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

3. Set builder form
   $\mathbb{N} = \{x \mid x \text{ is a natural number}\}$

   This indicates that $\mathbb{N}$ is the set of all elements x such that x is a natural number. That is $\mathbb{N}$ is the set of all natural numbers.
   Here the symbol "|" stands for such that.
   The notation (3) can also be expressed as $\mathbb{N} = \{x : x \text{ is a natural number}\}$
   Example
   $A = \{x \mid x^2 - 3x + 2 = 0\}$ indicates that A is the set of all elements such that $x^2 - 3x + 2 = 0$. That is $A = \{1, 2\}$.

3 Finite and Infinite Sets

Definition 2. *A set consisting of a finite number of elements is called a finite set.*

Example
$\{a, b\}$, $\{1, 2, 3\}$, $\{a_1, a_2, \ldots, a_n\}$ are all *finite sets.*

Definition 3. *A set consisting of an infinite number of elements is called an infinite set.*

Example

- $\mathbb{N}, \mathbb{Q}, \mathbb{R}, \mathbb{Z}$ are all infinite sets.
- $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

4 Empty, Unit and Universal Sets

4.1 Empty Set

Definition 4. *A set consisting no elements is called a null set and is denoted by the symbol $\emptyset$. It is also called empty set or void set.*

Example

- The set of all human beings born with wings.
- The set of all integers whose square is 2.

4.2 Unit Set

Definition 5. *A set which has exactly one element is called a unit set.*

Example

- $A = \{x \mid x - 5 = 0\}$ or $A = \{5\}$
- The set of planets on which we live.

4.3 Universal Set

Definition 6. *A particular set which all objects which are to be considered during some specific discussion, is called a universal set and denoted by ξ or U.*

5 Sets of Numbers

- $\mathbb{N} = \{1, 2, 3, \dots\}$ = the set of natural numbers
- $\mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$ = the set of natural numbers with zero
- $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ = the set of integers, both positive and negative, with zero
- $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$ = the set of positive integers
- $\mathbb{Z}^- = \{-1, -2, -3, \dots\}$ = the set of negative integers
- $\mathbb{Q} = \left\{x : x = \frac{n}{m},\ \ n, m \in \mathbb{Z}\right\}$ = the set of rational numbers
  The numbers contained in $\mathbb{Q}$ are exactly those of the form $\frac{n}{m}$ where n and m are integers and $m \neq 0$.
  Eg: $\dfrac{3}{2}(= 1.5) \in \mathbb{Q}$, $\quad \dfrac{8}{4}(= 2) \in \mathbb{Q}$, $\quad \dfrac{136}{100}(= 1.36) \in \mathbb{Q}$,
  $\dfrac{-1}{1000}(= -0.001) \in \mathbb{Q}$
- $\mathbb{R} = \{x : -\infty < x < \infty\}$ = the set of real numbers
  Eg: 1.5, -12.3, 99, $\sqrt{2}$, π
- $\mathbb{I}$ = the set of imaginary numbers
- $\mathbb{C} = \{z : z = a + bi,\ \ -\infty < a < \infty,\ \ -\infty < b < \infty\}$ = the set of complex numbers