Honours Degree of Bachelor of Science in Artificial Intelligence

Batch 24 - Level 1 (Semester 2)

CM 1310: Linear Algebra and Calculus

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Propositional Logic

Learning Outcomes

By the end of this chapter, students will be able to;

1. define proposition and argument.
2. recognize the basic logical connectives.
3. write an argument using logical notation.
4. explain and exemplify truth value status of a proposition.
5. explain and exemplify tautology, contradiction and contingency.
6. prove the logical equivalence of propositions.
7. identify the precedence of logical operators.
8. identify the laws of propositions.
9. prove the laws of propositions.
10. apply the laws of propositions in order to prove basic results.

Contents

1 Propositions

By a proposition we mean a declarative sentence (that is, a sentence that declares a fact), that is either true or false, but not both simultaneously.

Examples:

- All the B.Sc. in AI students follow CM 1310. $\Rightarrow$ True
- $2 + 1 = 5 \Rightarrow$ False
- $x^2 + 1 = 0$ has a solution. $\Rightarrow$ True
- $x^2 + 1 = 0$ has a real solution. $\Rightarrow$ False
- It is raining now. $\Rightarrow$ False

Non of the following is a proposition because it make no sense to ask if any of them is true or false.

Examples:

- $x + 1 = 2 \Rightarrow$ This is neither True nor False
- How are you today? $\Rightarrow$ Not a declarative sentence
- Come to our party! $\Rightarrow$ Not a declarative sentence

1.1 Propositional Variables

We use letters to denote propositional variables (or sentential variables), that is, variables that represent propositions, just as letters are used to denote numerical variables.The conventional letters used for propositional variables are $p, q, r, s, \ldots$.The truth value of a proposition is true, denoted by $\boldsymbol{T}$, if it is a true proposition, and the truth value of a proposition is false, denoted by $\boldsymbol{F}$, if it is a false proposition.The area of logic that deals with propositions is called the propositional calculus or propositional logic. It was first developed systematically by the Greek philosopher Aristotle more than 2300 years ago.

1.2 Compound Statement

The statements in examples are all simple statements. A combination of two or more simple statements is a compound statement.

Examples:
'2+1=5' and '$x^2 + 1 = 0$ has a real solution' is a compound statement.

In the study of logic we use letters such as $p, q, r, \ldots$ to represent statements. Using logical operators there are many ways of connecting statements such as $p, q, r, \ldots$ to form compound statements.

2 Logical Operators

- Negation: "NOT" symbolized by $\neg$
- Conjunction: "AND" symbolized by $\wedge$
- Disjunction: "OR" symbolized by $\vee$
- Conditional: "If ... Then ..." symbolized by $\rightarrow$
- Biconditional: "... If and Only If ..." symbolized by $\leftrightarrow$

2.1 Negation

Definition 1. *The negation of a statement p is denoted by $\neg p$.*

Examples:

p: x is 5.
$\neg p$: x is not 5.

The negation of a true statement is false. The negation of a false statement is true.

2.2 Truth Table

The truth table displays the relationship between the truth values of statements. The letters T and F stand for True and False respectively.

Table 1: Negation - Truth Table

p	$\neg p$
T	F
F	T

2.3 Conjunction

Definition 2. *Let p and q be two statements. The conjunction of the statements p and q is denoted by $p \wedge q$ and read as "p and q".*

Examples:

p: Today is Friday.
q: It is raining today.
$p \wedge q$: Today is Friday and it is raining today.

The statement $p \wedge q$ is true if and only if both p and q are true. $p \wedge q$ is false if p is false or q is false or both are false.

Table 2: Conjunction - Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2.4 Disjunction

Definition 3. *Let p and q be two statements. The disjunction of the statements p or q is denoted by $p \vee q$ and read as "p or q".*

Examples:
p: Today is Friday.
q: It is raining today.
$p \vee q$: Today is Friday or it is raining today.

The statement $p \vee q$ is true if either p or q or both p and q are true.