

Smirk & Relationship Between Implied Volatility And Futures Price

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DATASET



A tibble: 1,098,101 x 11

Date	Series	Market	MarketName	Commodity	CommodityName	CurrencyCode	SettlementPrice	PreviousSettlementPrice	Difference	Impliedvolatility
<date>	<chr>	<chr>	<chr>	<chr>	<chr>	<chr>	<dbl>	<dbl>	<dbl>	<dbl>
1	2020-01-02	HSIF0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	28606	28270	336	NA
2	2020-01-02	HSI20400A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	8206	7870	336	0
3	2020-01-02	HSI20600A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	8006	7670	336	0
4	2020-01-02	HSI20800A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	7806	7470	336	0
5	2020-01-02	HSI21000A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	7611	7271	340	43.7
6	2020-01-02	HSI21200A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	7406	7071	335	0
7	2020-01-02	HSI21400A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	7206	6871	335	0
8	2020-01-02	HSI21600A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	7006	6671	335	0
9	2020-01-02	HSI21800A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	6806	6471	335	0
10	2020-01-02	HSI22000A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	6606	6271	335	0
11	2020-01-02	HSI22200A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	6406	6072	334	0
12	2020-01-02	HSI22400A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	6206	5872	334	0
13	2020-01-02	HSI22600A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	6007	5672	335	29.0
14	2020-01-02	HSI22800A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	5807	5473	334	28.0
15	2020-01-02	HSI23000A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	5607	5273	334	27.0
16	2020-01-02	HSI23200A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	5407	5074	333	26.1
17	2020-01-02	HSI23400A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	5207	4874	333	25.1
18	2020-01-02	HSI23600A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	5008	4675	333	25.8
19	2020-01-02	HSI23800A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	4808	4476	332	24.7
20	2020-01-02	HSI24000A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	4609	4277	332	24.8
21	2020-01-02	HSI24200A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	4409	4079	330	23.7
22	2020-01-02	HSI24400A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	4210	3880	330	23.4
23	2020-01-02	HSI24600A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	4011	3682	329	23.0
24	2020-01-02	HSI24800A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	3812	3485	327	22.4
25	2020-01-02	HSI25000A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	3614	3288	326	22.1
26	2020-01-02	HSI25200A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	3416	3091	325	21.6
27	2020-01-02	HSI25400A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	3218	2896	322	21.0
28	2020-01-02	HSI25600A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	3021	2701	320	20.6
29	2020-01-02	HSI25800A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	2825	2508	317	20.1
30	2020-01-02	HSI26000A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	2630	2316	314	19.7
31	2020-01-02	HSI26200A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	2436	2115	321	19.2
32	2020-01-02	HSI26400A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	2230	1926	304	17.1
33	2020-01-02	HSI26600A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	2026	1741	285	15.2
34	2020-01-02	HSI26800A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	1861	1558	303	17.5
35	2020-01-02	HSI27000A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	1677	1380	297	17.1
36	2020-01-02	HSI27200A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	1490	1207	283	16.3
37	2020-01-02	HSI27400A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	1312	1056	256	15.8
38	2020-01-02	HSI27600A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	1140	893	247	15.4
39	2020-01-02	HSI27800A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	961	743	218	14.3
40	2020-01-02	HSI28000A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	816	610	206	14.4
41	2020-01-02	HSI28200A0	HSI	HANG SENG FUTURES & OPTIONS	HSI	HANG SENG INDEX HKD	673	483	190	14.0

Series : HSI20400A0

- **HSI : HANG SENG INDEX**

- **20400 : Strike Price**

- **Expiration Month Code:**

Call Option :

- January to December A to L

Put Option :

- January to December M to X

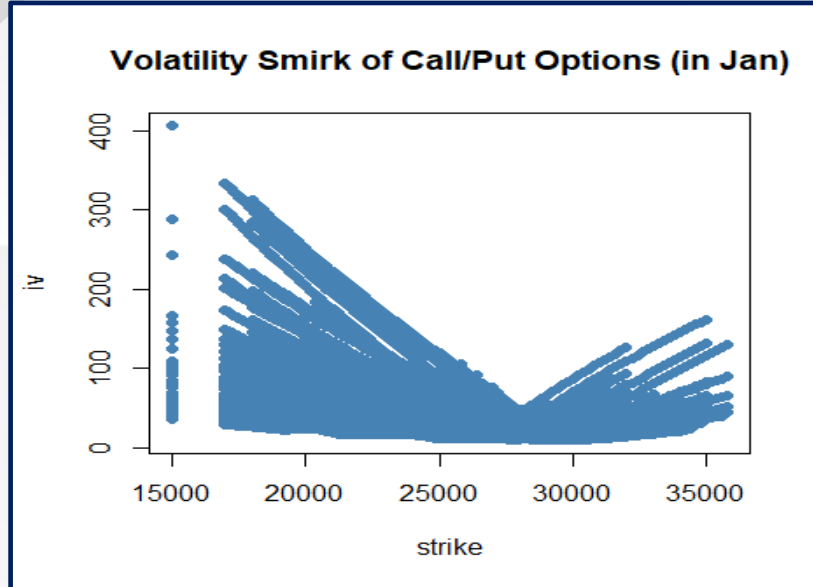
- 0 means The Last Digit of Maturity Year

Currency Code : HKD

Difference :

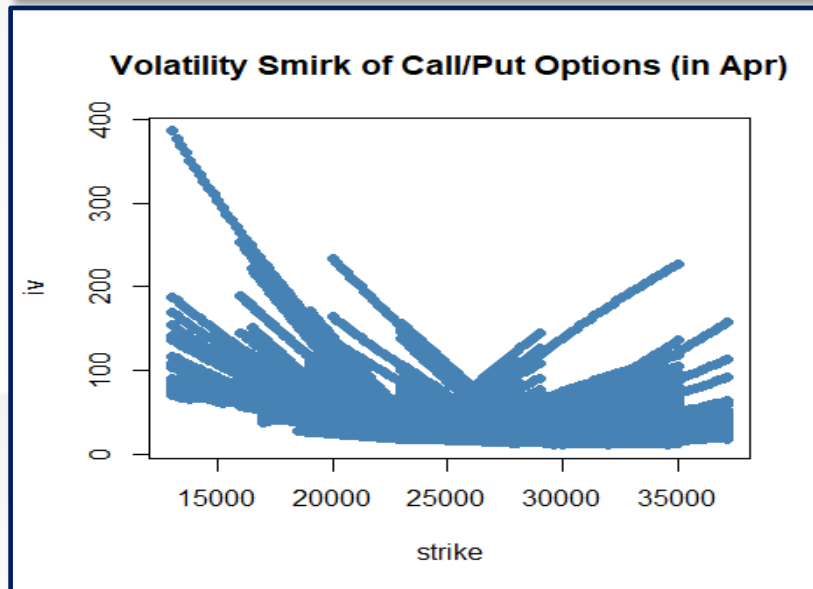
= Settlement Price – Previous Settlement Price

SMIRK



Conclusion 1 :

The Implied Volatility increases with the difference between strike price and the equity index price



Conclusion 2 :

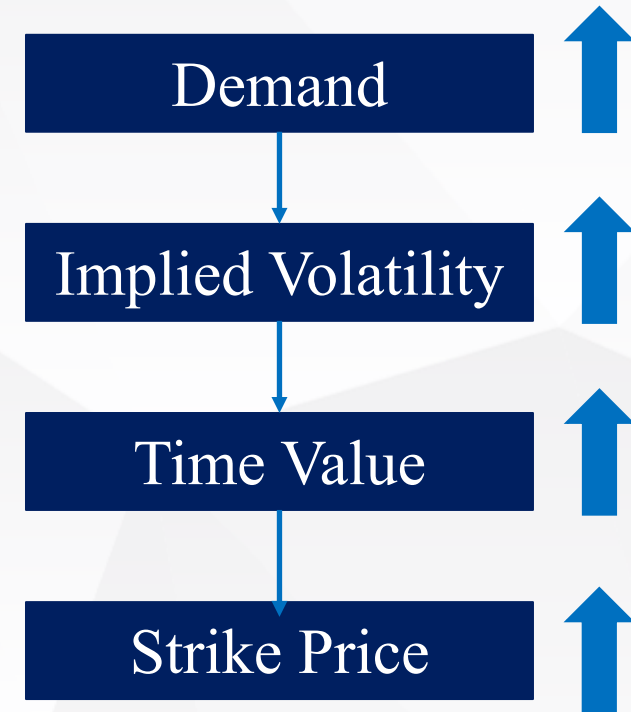
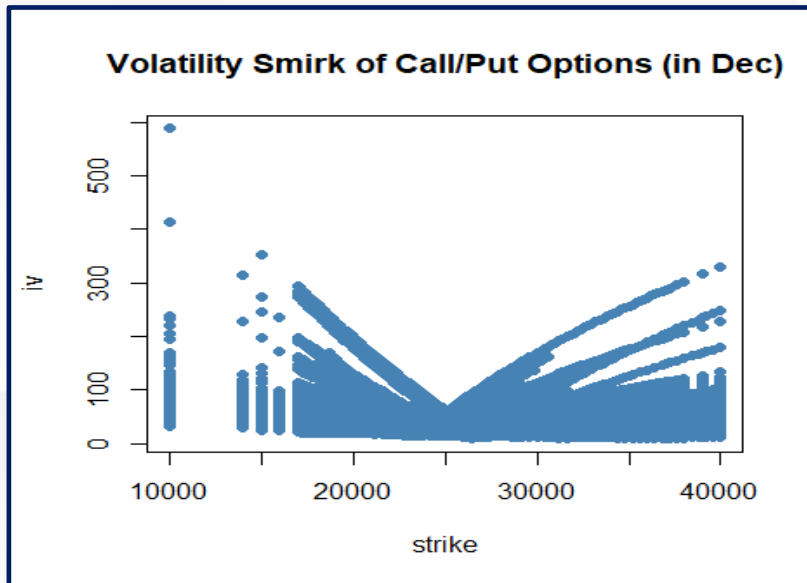
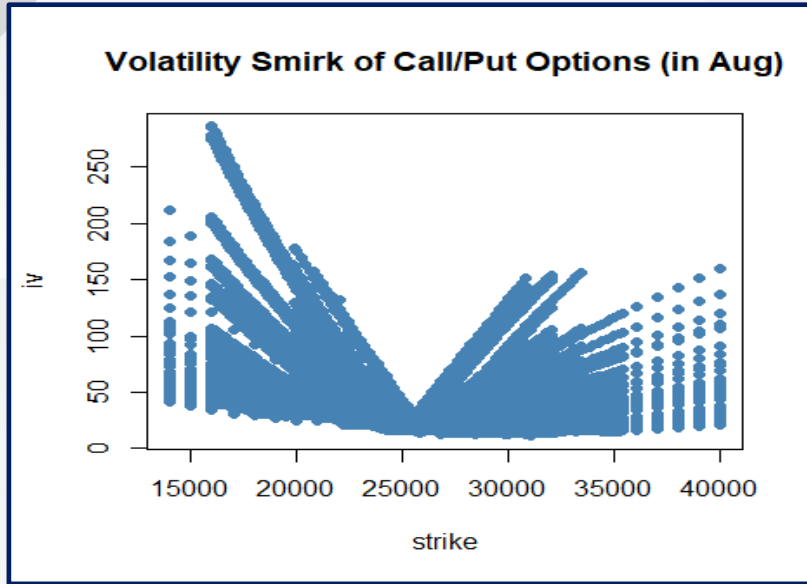
Options that are ITM and OTM by an equal amount should have roughly the same implied volatility.

SMIRK

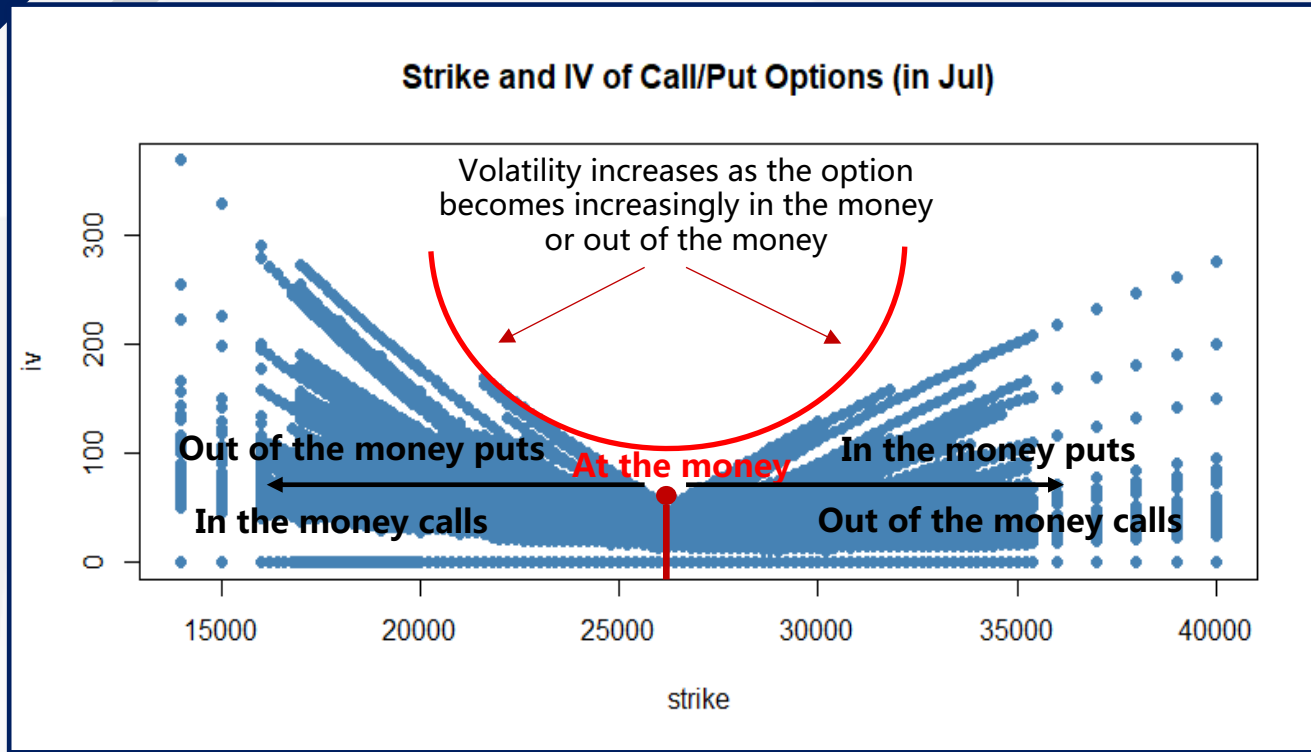


Conclusion 3 :

Compared with the ATM, Demand is greater for options that are ITM or OTM



SMIRK



$$\text{SKEW}_{i,t} = \text{VOL}_{i,t}^{\text{OTMP}} - \text{VOL}_{i,t}^{\text{ATMC}}.$$

- Main Approach is based on the option's moneyness
- We choose 1 ATM call option with its moneyness closest to 1, and 1 OTM put option with its moneyness closest to 0.95.

SMIRK Hypothesis Test



```
> t.test((skewness(new))/2,daily_return_normalized)

Welch Two Sample t-test

data:  (skewness(new))/2 and daily_return_normalized
t = 3.9696, df = 56085, p-value = 7.208e-05
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.03988685 0.11769211
sample estimates:
mean of x mean of y
0.5780177 0.4992282
```

Conclusion :

Small p-value $7.208e^{-05}$

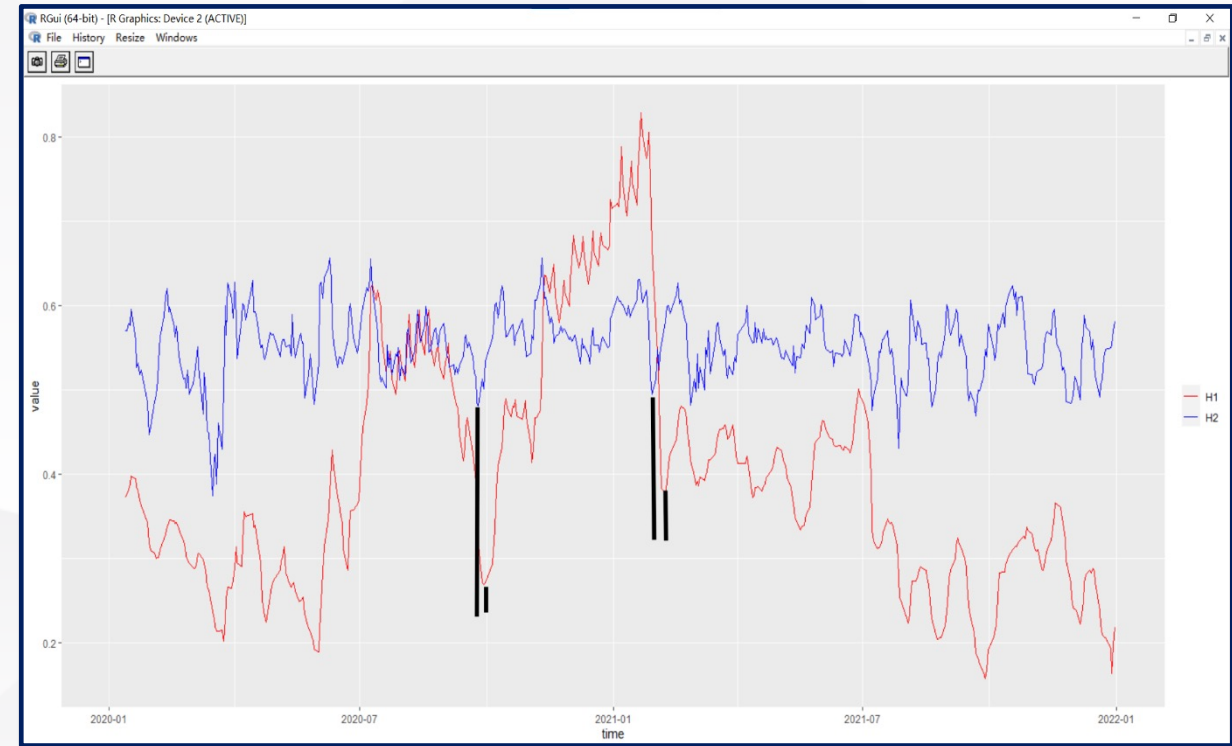
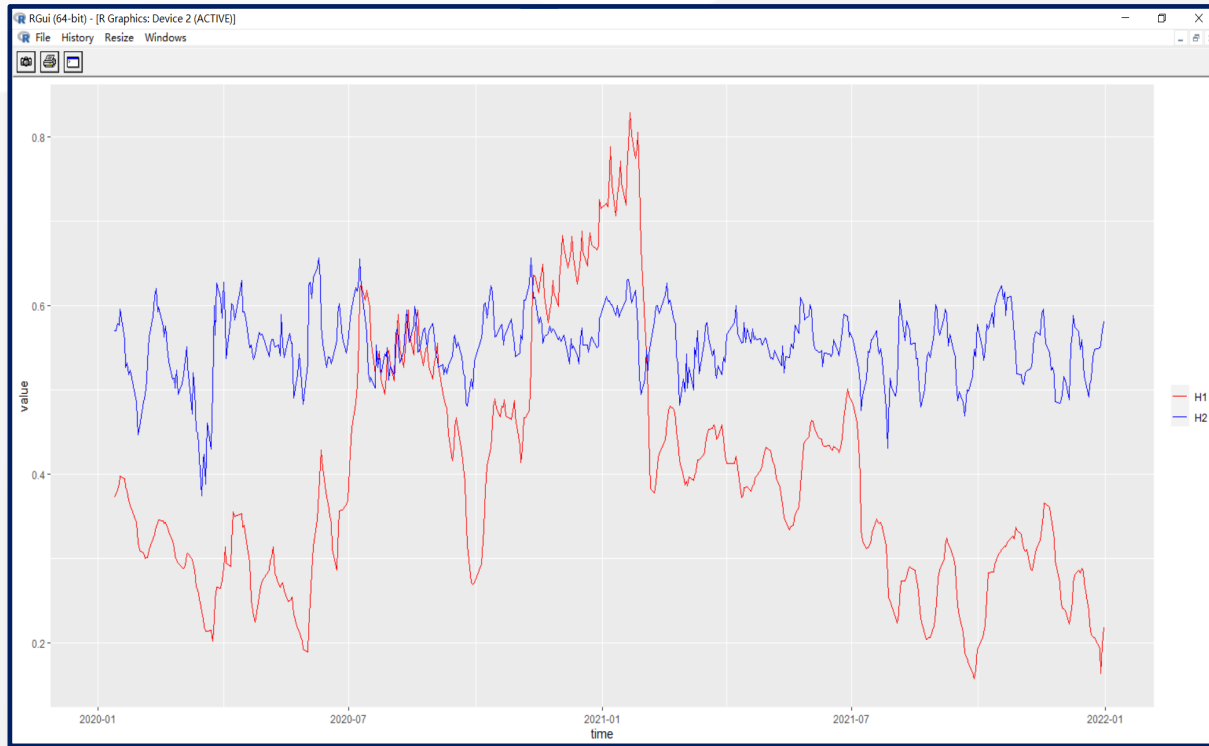


Reject the null hypothesis



**Option with different skewness has
significantly different average daily return**

Implied Volatility Spread & Log Return



H1: Log Return of Index

H2: Implied Volatility Spread

$$VS_{i,t} = IV_{i,t}^{\text{calls}} - IV_{i,t}^{\text{puts}} = \sum_{j=1}^{N_{i,t}} w_{j,t}^i \left(IV_{j,t}^{i,\text{call}} - IV_{j,t}^{i,\text{put}} \right),$$

Implied Volatility Spread & Log Return



```
> > summary(lm(Log_Return_normalized ~ VS_mean_normalized))

Call:
lm(formula = Log_Return_normalized ~ VS_mean_normalized)

Residuals:
    Min       1Q   Median       3Q      Max
-0.163749 -0.024828  0.001311  0.024372  0.104729

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.516145   0.005096  101.286 < 2e-16 ***
VS_mean_normalized 0.093890   0.011997   7.826 3.15e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03816 on 488 degrees of freedom
(6 observations deleted due to missingness)
Multiple R-squared:  0.1115,    Adjusted R-squared:  0.1097
F-statistic: 61.24 on 1 and 488 DF,  p-value: 3.152e-14
```

Conclusion :

The Implied Volatility Spread could forecast the movement of the trend of the index

Regression Analysis



```
## Call:
## lm(formula = df$return_10d ~ df$vs_rank)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.039442 -0.004709 -0.000155  0.004749  0.039860
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.267e-04  2.770e-05   29.85  <2e-16 ***
## df$vs_rank  -3.358e-04  9.336e-06  -35.97  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.008945 on 404520 degrees of freedom
## Multiple R-squared:  0.003188,    Adjusted R-squared:  0.003186
## F-statistic: 1294 on 1 and 404520 DF,  p-value: < 2.2e-16
```

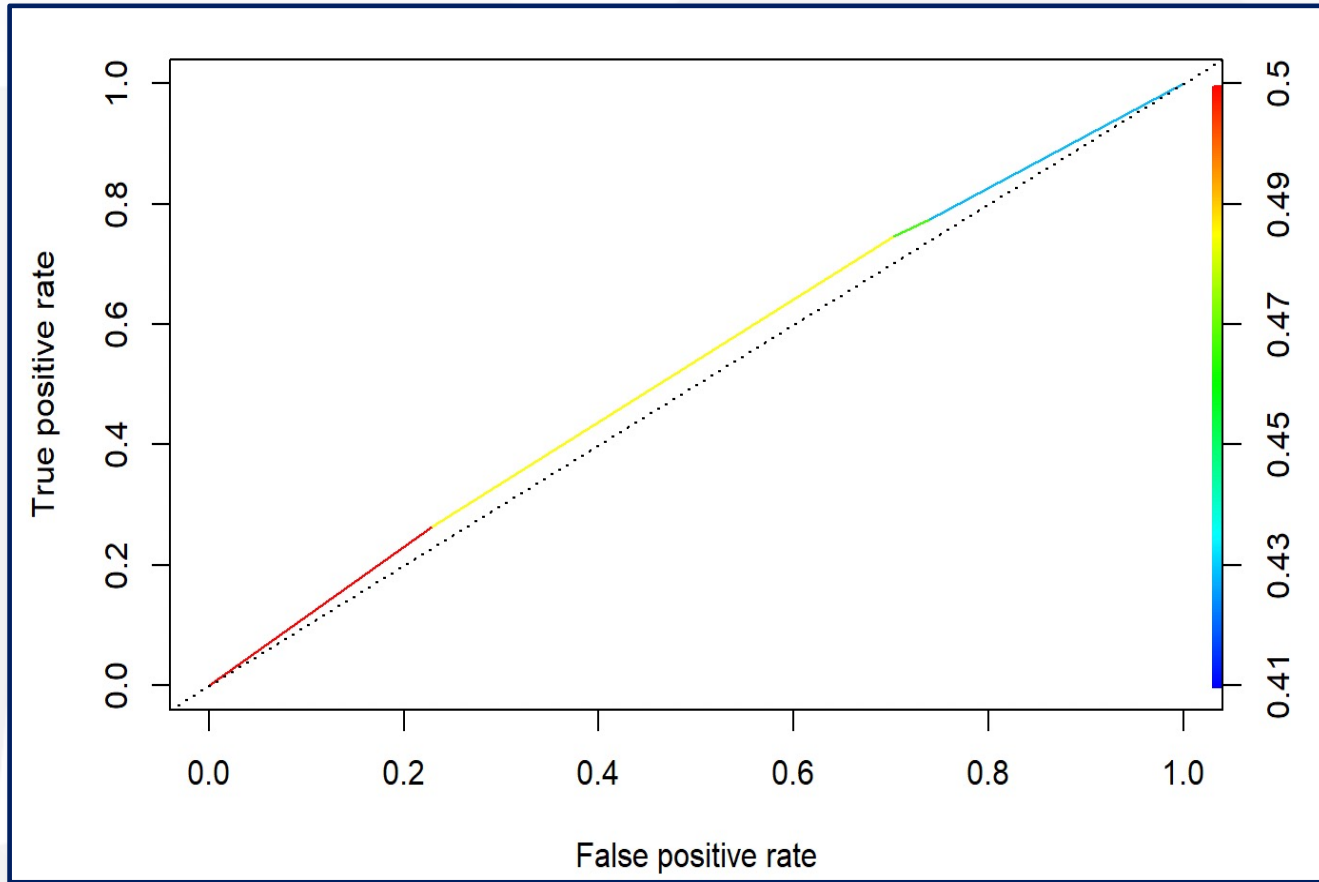
```
## Call:
## lm(formula = df$return_30d ~ df$vs_rank)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.0134086 -0.0017392 -0.0000246  0.0017422  0.0133003
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.793e-04  9.523e-06   29.33  <2e-16 ***
## df$vs_rank  -1.154e-04  3.210e-06  -35.96  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.003076 on 404520 degrees of freedom
## Multiple R-squared:  0.003187,    Adjusted R-squared:  0.003185
## F-statistic: 1293 on 1 and 404520 DF,  p-value: < 2.2e-16
```

- Y: 10-day and 30-day daily average return of the Hang Seng Index forecast
- X: Implied Volatility Spread

Conclusion :

P-value are equal to $<2.2e^{-16}$ and $<2.2e^{-16}$ respectively, we can conclude that it exists a relationship between return and the implied volatility spread

Regression Analysis



Conclusion :

The curve is lying over the 45-degree diagonal of the ROC space, which mean that the model has certain accuracy.

Regression Analysis



```
## Call:
## lm(formula = df$return_10d ~ df$vs_10ma_factor)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.039038 -0.004720  0.000029  0.004740  0.039227
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -4.567e-05  1.407e-05  -3.245  0.00118 **
## df$vs_10ma_factor  4.289e-04  1.294e-05  33.146 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.008948 on 404520 degrees of freedom
## Multiple R-squared:  0.002709, Adjusted R-squared:  0.002706
## F-statistic: 1099 on 1 and 404520 DF, p-value: < 2.2e-16
```

```
## Call:
## lm(formula = df$return_30d ~ df$vs_30ma_factor)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.0133319 -0.0017352  0.0000212  0.0017427  0.0130387
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -2.003e-05  4.837e-06  -4.141 3.46e-05 ***
## df$vs_30ma_factor  1.538e-04  4.046e-06  38.005 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.003075 on 404520 degrees of freedom
## Multiple R-squared:  0.003558, Adjusted R-squared:  0.003555
## F-statistic: 1444 on 1 and 404520 DF, p-value: < 2.2e-16
```

- Y: 10-day and 30-day daily average return of the Hang Seng Index forecast
- X: The difference between Implied Volatility Spread and the mean of which in last 10 and 30 days

Conclusion :

P-value are equal to to $<2.2e^{-16}$ and $<2.2e^{-16}$ respectively, we can conclude that it exists a stronger relationship between return and the difference between Implied Volatility Spread and the mean of which in time period

Regression Analysis



```
## Call:
## lm(formula = t$return_5d ~ t$cross30)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.071017 -0.007089 -0.000311  0.007501  0.078113
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 8.494e-05  6.538e-05   1.299   0.194
## t$cross30    1.960e-03  6.538e-05  29.984 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01687 on 67309 degrees of freedom
## Multiple R-squared:  0.01318,    Adjusted R-squared:  0.01317
## F-statistic: 899.1 on 1 and 67309 DF,  p-value: < 2.2e-16
```

- Y: 5-day daily average return of the Hang Seng Index forecast
- X: If current between Implied Volatility Spread positively or negatively exceed the 80% confidence interval of the mean of which in last 30 days

Conclusion :

P-value is equal to $<2.2e^{-16}$, we can conclude that it exists a stronger relationship between return and the constructed indicator. Adjusted R-squared is also significantly higher

Limitation



- Data Missing : Missing Value
- Factor Inadequate : Model needs more factors for Regression
- Small Amount of Noise : There is still an individual difference between the groups (Implied Volatility Spread), not every group will react the same way
- The Scale of Measurement : The t test we conducted can only indicate the difference between two groups of data. But the scale of the data cannot be compared in the testing.

Q & A



Thank you