A NEW PERSPECTIVE ON

MULTIPLE INTERNAL RATES OF RETURN

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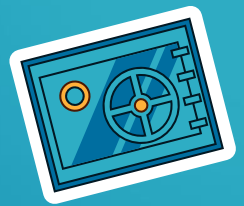


Introduction

Part 1

Property of IRR

Part 2

Computation

Part 3

Two Fatal Flaws

Part 4

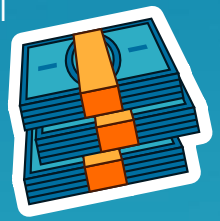
New Perspective

What's iternal rate of return(IRR)?

- A technique of assisting in decision making and capital budgeting
- The most important alternative to NPV
- the rate of return at which the net present value of a project becomes zero.
- We call it ‘internal’ because it does not take any external factor (like inflation) into consideration.







How IRR is computed?

If the IRR is greater than the hurdle rate, the project is accepted, otherwise it is rejected.

$$0 = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \frac{CF_3}{(1+IRR)^3} + \ldots + \frac{CF_n}{(1+IRR)^n}$$

Or

$$0 = NPV = \sum_{n=0}^{N} \frac{CF_n}{(1+IRR)^n}$$

Where:

CF_0 = Initial Investment / Outlay

$CF_1, CF_2, CF_3 \ldots CF_n$ = Cash flows

n = Each Period

N = Holding Period

NPV = Net Present Value

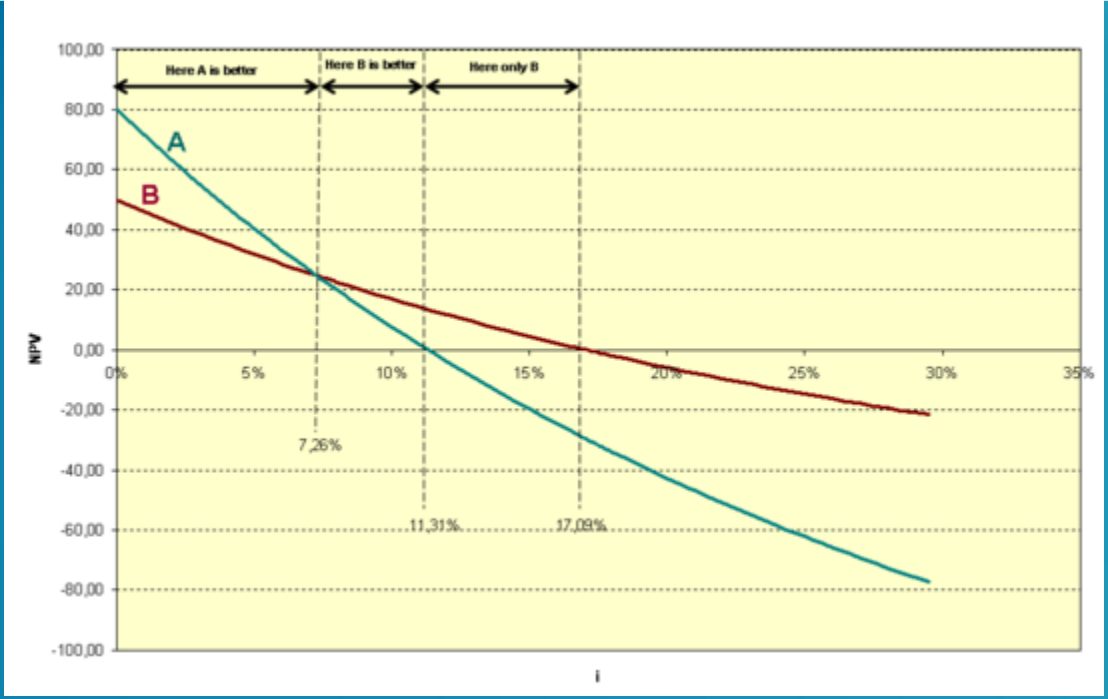
IRR = Internal Rate of Return

Two Fatal Flaws of IRR in Consensus

Flaw One: Mutually Exclusive Projects

- Mutually exclusive is a statistical term describing two or more events that cannot happen simultaneously.
- IRR cannot be used to rate mutually exclusive projects but only to decide whether a single project is worth investing in.
- In cases where one project has a higher initial investment than a second mutually exclusive project, the first project may have a lower IRR (expected return), but a higher NPV (increase in shareholders' wealth) and should thus be accepted over the second project (assuming no capital constraints).

Flaw One: Mutually Exclusive



Case:NPV vs discount rate comparison for two mutually exclusive projects. Project A has a higher NPV (for certain discount rates), even though its IRR (= x-axis intercept) is lower than for project B

Two Fatal Flaws of IRR in Consensus

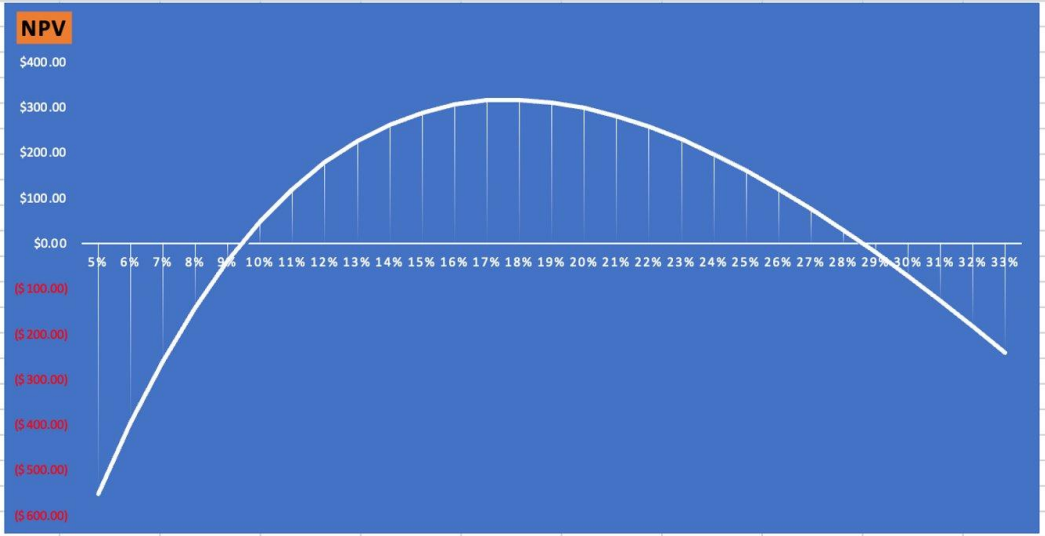
Flow Two: Multiple or Nonexistent Internal Rates

- Nonexistent IRR – IRR will not exist in situations where NPV is positive for all values of the discount rate. Thus, in these situations, the IRR rule cannot be used.
- Multiple Internal Rates – Regular cashflows are those where the outflow is only once i.e. at the time of initial investment. But in non-regular category, cash outflows more than once.

Year	0	1	2	3	4	5
Net Cash Flow	-11000	7500	7500	7500	8000	-21000

Flow Two: Multiple Internal Rates Case

NPV Profile	
Annual Required Rate of Return (RRR) Discount Rate	NPV
5%	($551.70)
6%	($395.24)
7%	($258.63)
8%	($140.26)
9%	($38.61)
10%	$47.67
11%	$119.87
12%	$179.13
13%	$226.52
14%	$263.03
15%	$289.57
16%	$306.95
17%	$315.96
18%	$317.30
19%	$311.62
20%	$299.51
21%	$281.54
22%	$258.22
23%	$230.01
24%	$197.35
25%	$160.64
26%	$120.26
27%	$76.54
28%	$29.80
29%	($19.67)
30%	($71.59)
31%	($125.73)
32%	($181.84)
33%	($239.72)



New Perspective

Cons

- Severe Drawback
- Difficult to explain or interpret properly
- Invalid and not useful

02

Methodology

Methodology

1. Collect Cashflow: $\vec{x}$

2. Find IRR : $\vec{k}$

3. Calculate investment Stream : $\vec{c}$

4. Identify NPV(X)

Project Cash Flow: $\vec{x}$

x_0:
Amount received/paid initially

x_1:
Cash Flow in year 1

x_n:
Cash Flow in year n

Year 0 | Year 1 | ..... | Year N

Cashflow: $\vec{x}$

$$(x_0, x_1, x_2, \ldots\ldots, x_n)$$

$$\text{length}(\vec{x}) = \max(j | x_j \neq 0) - \min(i | x_i \neq 0) + 1$$

Internal Rate of Return : $\vec{k}$

```
K <- polyroot(rev(exampleDF))-1
```

$$\text{length}(\vec{k}) = \text{length}\ (\vec{x}) - 1$$

n-1 IRR solutions

$$Solving\ [a_0 + a_1 x_0 + a_2 x_1^2 \ldots + a_{n+1} x_n^{n+1} = 0]$$

polyroot() will show all real or complex solutions

Real

Complex

IRR: $\vec{k}$

$$(k_0, k_1, k_2, \ldots\ldots, k_{n-1})$$

Zero points of the complex function $F(k), k = x + iy$

Investment Stream : $\vec{c}$

Calculation:

$$C_0 = -x_0$$
$$C_1 = -((1+k)x_0 + x_1$$
$$C_2 = -((1+k)^2 x_0 + (1+k)x_1 + x_2)$$
$$\vdots$$
$$C_{n-1} = -((1+k)^{n-1}x_0 + (1+k)^{n-2}x_1 + (1+k)x_{n-2} + x_{n-1})$$
$$C_{n-1} = (1+k)^{-1}x_n$$

Based on the cashflow $\vec{x}$ and IRR $\vec{k}$, we can find the investment stream $\vec{c}$.

Each IRR (k) can generate an corresponding investment stream (c), whatever the IRR (k) is real or complex number.

Investment Stream: $\vec{c}$

$$(c_0, c_1, \ldots\ldots, c_{n-1})$$

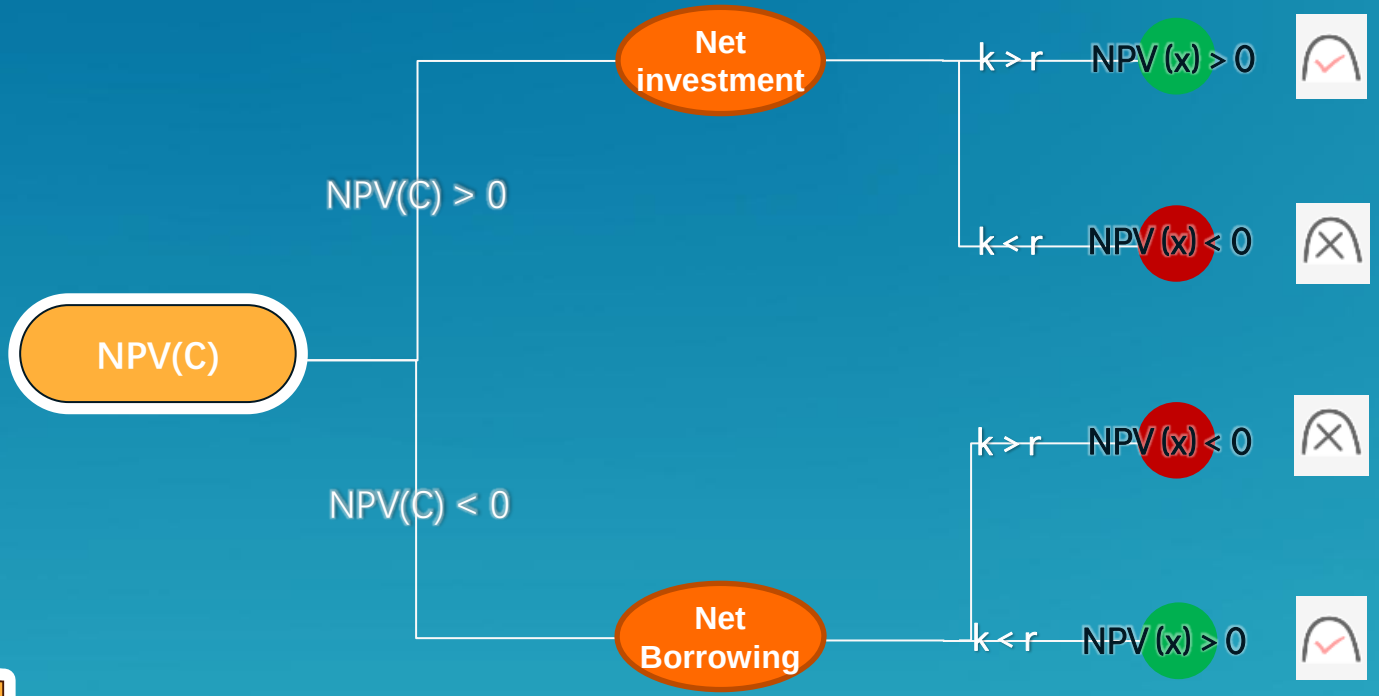
“tidyverse”

Complex number is meaningless in real world
(At least in financial field)

Only focus on the real part data

Quantum mechanics & Fluid dynamics: Are you polite?

Make Decision: NPV(x)



Demonstration

```
C <- Cal(exampleDF,k)
C <- Re(C)
```

	Case	CashFlow	r	k	InvestFlow	NPVX	NPVC	k_Larger_r	Investment	PositiveNPVX
1	Case 1	-100, 20, 20, 20, 20, 50	0.03	-0.7724+0.7976i	100, 2.76037450187949, -8...	17.4724	-11.2821	False	Net Borrowing	True
2	Case 1	-100, 20, 20, 20, 20, 50	0.03	-1.668+0.476i	100, -86.8050082418374, 1...	17.4724	-9.8275	False	Net Borrowing	True
3	Case 1	-100, 20, 20, 20, 20, 50	0.03	-1.668-0.476i	100, -86.8050082418373, 1...	17.4724	-9.8275	False	Net Borrowing	True
4	Case 1	-100, 20, 20, 20, 20, 50	0.03	-0.7724-0.7976i	100, 2.76037450187971, -8...	17.4724	-11.2821	False	Net Borrowing	True
5	Case 1	-100, 20, 20, 20, 20, 50	0.03	0.08089+0i	100, 88.0892674799155, 7...	17.4724	353.6183	True	Net Investment	True

https://app.datacamp.com/workspace/w/c4b498ba-46c7-4fca-af71-55f0ef0c6e21/edit

03

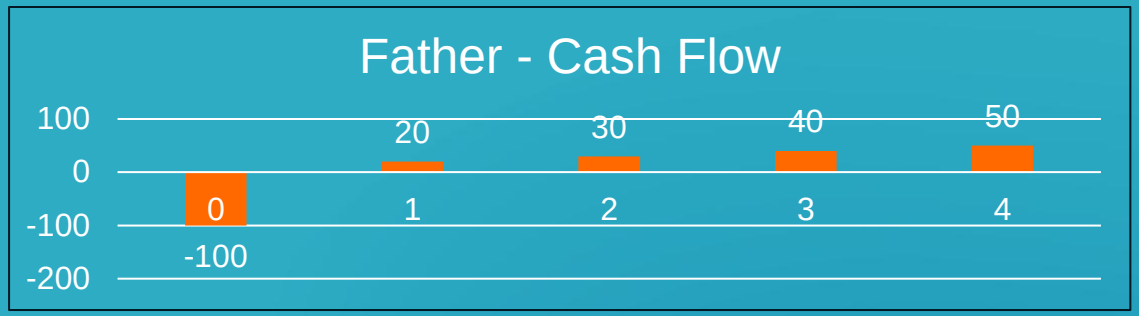
IRR Application - Investment Family Case

Case background

Father: Invest a mineral project, which required an initial investment and return constant income.
Mother: Borrow a fund from bank, lend to people in need at higher rate, and repay annually.
Brother: Invest in a risky project, whose cash flow is unstable.
Ben: A CFA, and believe he can gain through trading the stocks.

	CF0	CF1	CF2	CF3	CF4
Father -++++	-100	20	30	40	50
Mother +----	100	-27	-27	-27	-27
Brother -+-+-	-100	80	-20	40	-10
Ben -----	-100	-20	-20	-20	-20

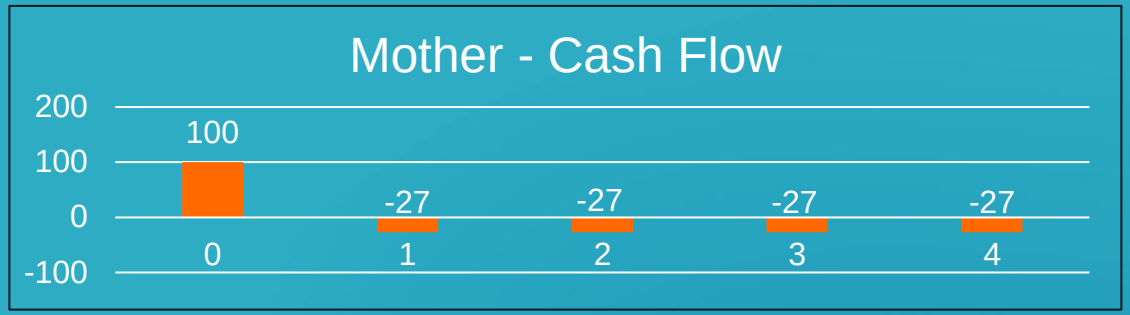
Father -++++



- Type: -++++
- Discount rate: 3%
- Consistency: Yes
- Decision: Good Investment

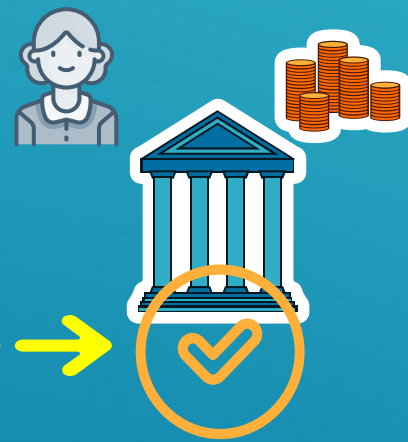
CashFlow	r	k	NPVX	NPVC	k_Larger_r	Investment	Positive NPVX
-100, 20, 30, 40, 50	0.03	-1.0974832+0.771261i	28.72537	-17.87669	FALSE	Net Borrowing	TRUE
-100, 20, 30, 40, 50	0.03	-1.7332909+0.000000i	28.72537	-16.7795	FALSE	Net Borrowing	TRUE
-100, 20, 30, 40, 50	0.03	-1.0974832-0.771261i	28.72537	-17.87669	FALSE	Net Borrowing	TRUE
-100, 20, 30, 40, 50	0.03	0.1282573+0.000000i	28.72537	301.11902	TRUE	Net Investment	TRUE

Mother +----

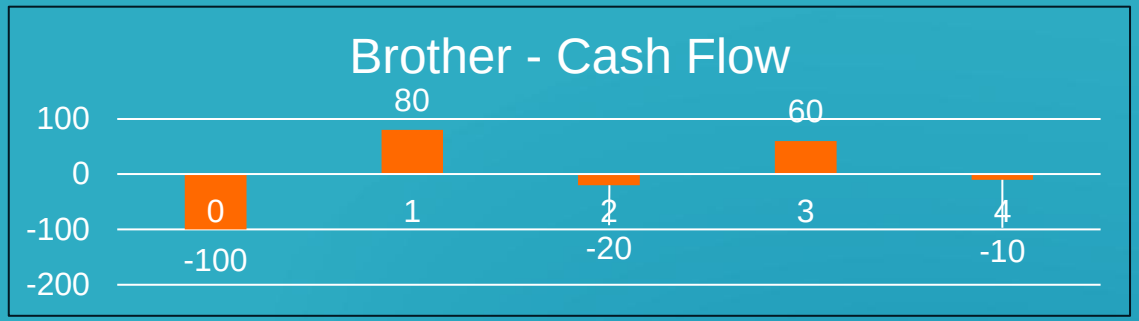


- Type: +----
- Discount rate: 12%
- Consistency: Yes
- Decision: Good Investment

CashFlow	r	k	NPVX	NPVC	k_Larger_r	Investment	Positive NPVX
100, -27, -27, -27, -27	0.12	-1.07316531+0.648178i	17.99157	-13.04004	FALSE	Net Borrowing	TRUE
100, -27, -27, -27, -27	0.12	-1.61518069-0.000000i	17.99157	-11.61294	FALSE	Net Borrowing	TRUE
100, -27, -27, -27, -27	0.12	-1.07316531-0.648178i	17.99157	-13.04004	FALSE	Net Borrowing	TRUE
100, -27, -27, -27, -27	0.12	0.03151131-0.000000i	17.99157	-227.71901	FALSE	Net Borrowing	TRUE



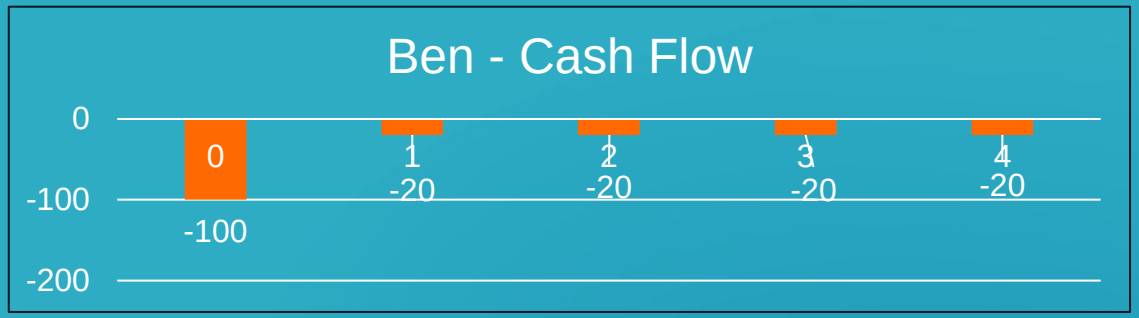
Brother -+-+-



- Type: -+-+-
- Discount rate: 3%
- Consistency: Yes
- Decision: Good Investment

CashFlow	r	k	NPVX	NPVC	k_Larger_r	Investment	Positive NPVX
-100, 80, -20, 60, -10	0.03	-0.82882284+0.000000i	4.841614	-5.806625	FALSE	Net Borrowing	TRUE
-100, 80, -20, 60, -10	0.03	-1.21601488+0.709942i	4.841614	-3.021392	FALSE	Net Borrowing	TRUE
-100, 80, -20, 60, -10	0.03	-1.21601488-0.709942i	4.841614	-3.021392	FALSE	Net Borrowing	TRUE
-100, 80, -20, 60, -10	0.03	0.06085261-0.000000i	4.841614	161.635036	TRUE	Net Investment	TRUE

Ben -----



- Type: -----
- Discount rate: 3%
- Consistency: Yes
- Decision: Bad Investment

CashFlow	r	k	NPVX	NPVC	k_Larger_r	Investment	Positive NPVX
-100, -20, -20, -20, -20	0.03	-0.6275697+0.6148704i	-174.342	145.696	FALSE	Net investment	FALSE
-100, -20, -20, -20, -20	0.03	-1.4724303+0.4047580i	-174.342	111.4336	FALSE	Net Investment	FALSE
-100, -20, -20, -20, -20	0.03	-1.4724303-0.4047580i	-174.342	111.4336	FALSE	Net Investment	FALSE
-100, -20, -20, -20, -20	0.03	-0.6275697-0.6148704i	-174.342	145.696	FALSE	Net Investment	FALSE

Kick Out From the Family

04

Limitations and Conclusion

Conclusion and Limitations

Conclusion

- Whatever cash flow looks like, we can use the same process to figure out that NPV is positive or negative.
- We have successfully solve the fatal of Multiple or Non-existent Internal Rates.

Limitation

- We still cannot solve the fatal of mutually inclusive projects.
- NPV is still superior than IRR to make making decision.
- The models is hard for operating and applying in the real case.

Thanks

Personal Recommendation.
If Unnecessary, DON'T USE IRR !!!