Multiple IRR

Group 10

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2. Pick a discount rate = min acceptable rate of return
3. Compute NPV of different alternatives

$$NPV_i = \sum_{t=0}^{n} \frac{C_{i,t}}{(1 + r_i)^t}$$

- $C_{i,t}$ is net cash inflow-outflows during a single period t
- r_i is discount rate for the *i-th* alternative investment

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4. Choose the alternative with highest NPV
   1. $NPV > 0 \rightarrow$ rate of return > discount rate
   2. $NPV < 0 \rightarrow$ no investment value

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Introduction of IRR

What Is Internal Rate of Return (IRR)

R(discount rate)=IRR $\Rightarrow$ NPV=0

$$0 = \text{NPV} = \sum_{t=1}^{T} \frac{C_t}{(1+IRR)^t} - C_0$$

where:

C_t = Net cash inflow during the period t

C_0 = Total initial investment costs

IRR = The internal rate of return

t = The number of time periods

What Is IRR Used for?

1. Comparing the profitability of two operations
2. Evaluating stock buyback programs for corporations
3. Analyzing investment returns

IRR & NPV

- $IRR >$ cost of capital $\Rightarrow NPV > 0$
- $IRR <$ cost of capital $\Rightarrow NPV < 0$
- $IRR =$ cost of capital $\Rightarrow NPV = 0$
- Accept the project if $IRR >$ required rate of return

Limitations of Internal Rate of Return

1

2

3

Picture from: https://crystalsofeconomics.wordpress.com/

Limitations of Internal Rate of Return

1. Does not extend well to situations involving uncertainty.
2.
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Limitations of Internal Rate of Return

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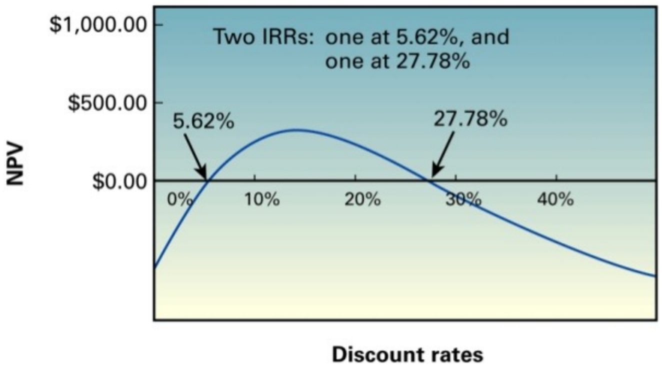
2. Cannot be used to rank cash flows for mutually exclusive projects.
3. A cash flow stream can have multiple conflicting internal rates.

$$\$0 = -\$11{,}000 + \frac{\$7{,}500}{(1+r)^1} + \frac{\$7{,}500}{(1+r)^2} + \frac{\$7{,}500}{(1+r)^3} + \frac{\$7{,}500}{(1+r)^4} - \frac{\$20{,}000}{(1+r)^5}$$



Picture from: https://crystalsofeconomics.wordpress.com/

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Concepts

Internal Rate of Return

$$PV(x|r) = \sum \frac{x_t}{(1+r)^t}$$

Assume:

$$IRR(x) \rightarrow PV(x|r) = 0$$

$$r \neq -1 \rightarrow r > -1$$

Internal Rate of Return

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Assume:

$$IRR(x) \rightarrow PV(x|r) = 0$$

$$r \neq -1 \rightarrow r > -1$$

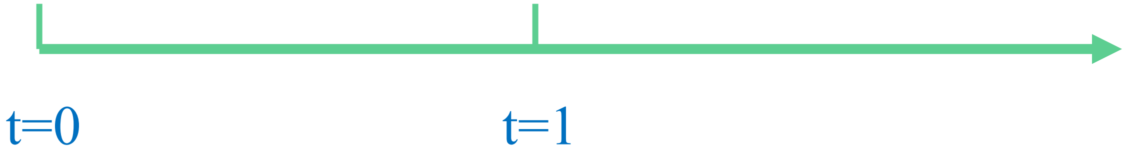
IRR exists and is unique

- Conventional cash flow

➢ -, +, +, +, +, ….

➢ -, -, -, +, +, +, +…+

Constant Per-Period Return on Investment



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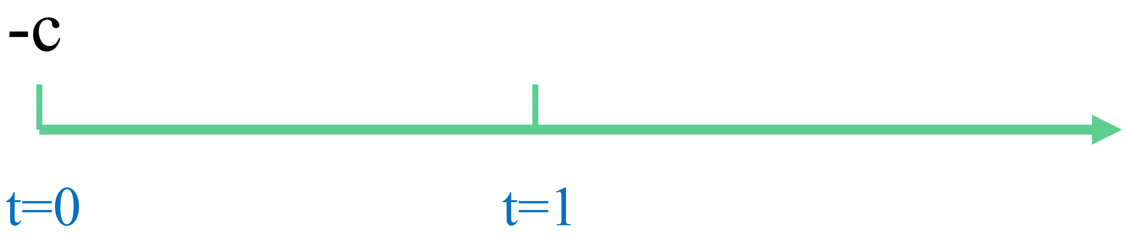

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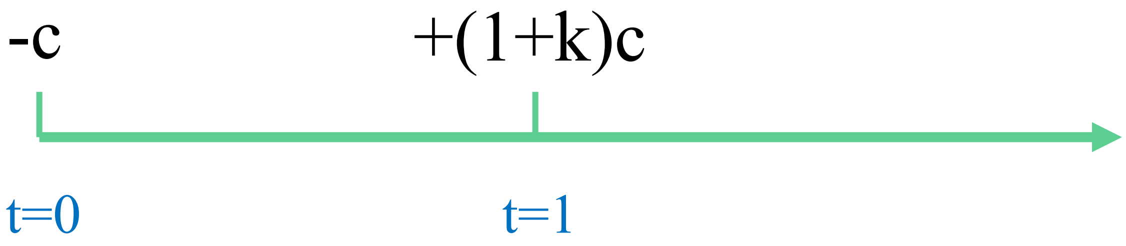

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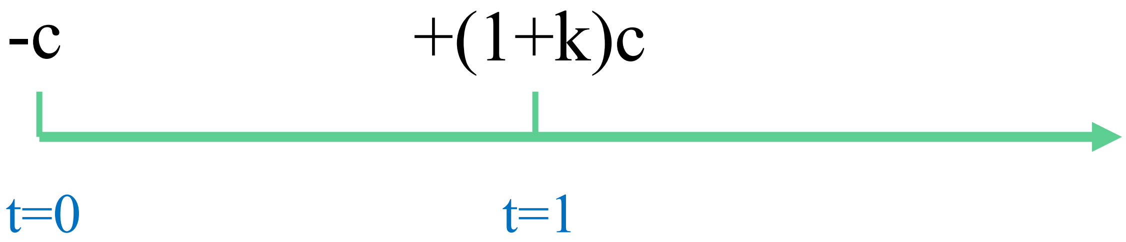


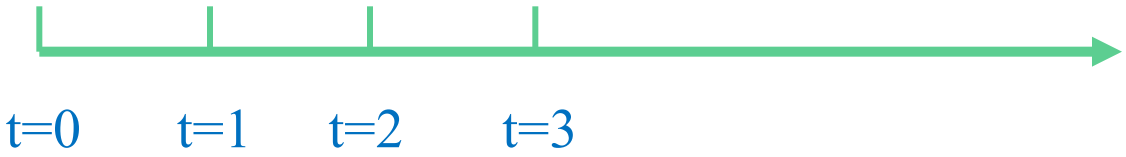
Cash flow stream x = (-c, (1+k)c)

$$PV(x \mid r) = \sum \frac{x_t}{(1+r)^t}$$

$$= -c + \frac{(1+k)c}{(1+r)}$$

When r = k, PV(x | r) = 0

Constant Per-Period Return on Investment



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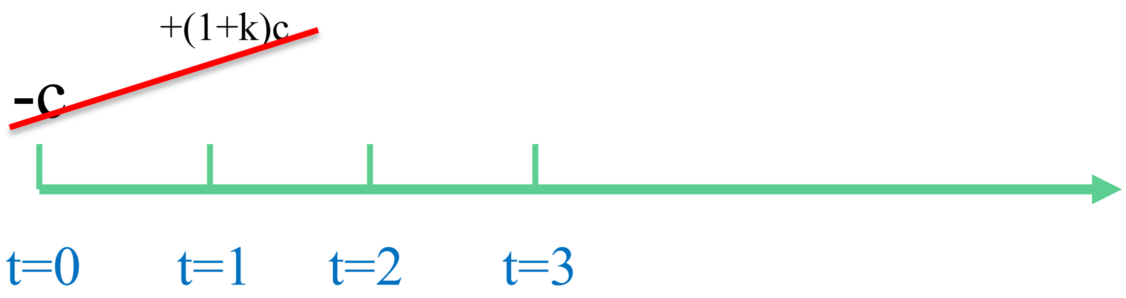

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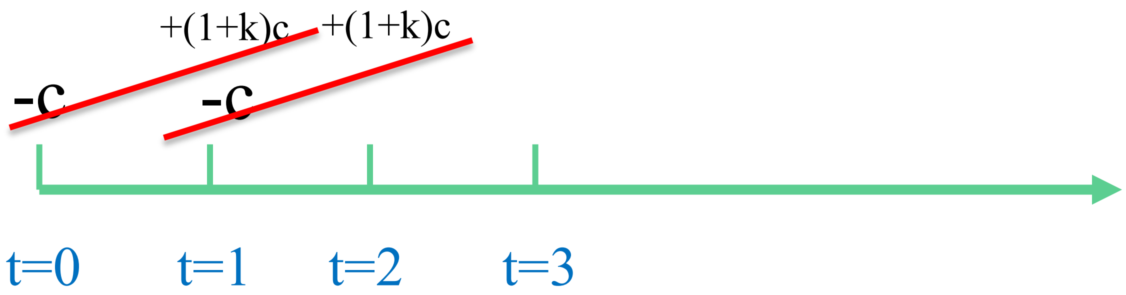

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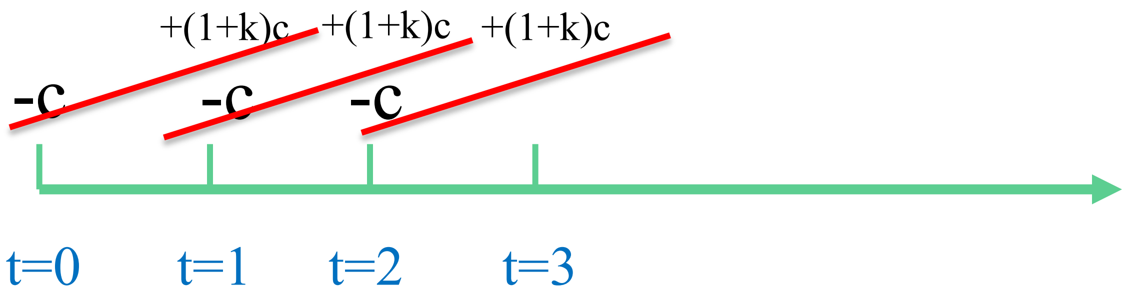

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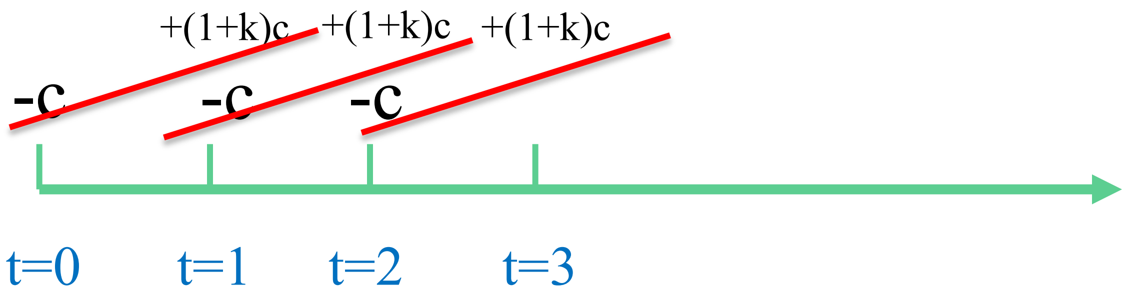
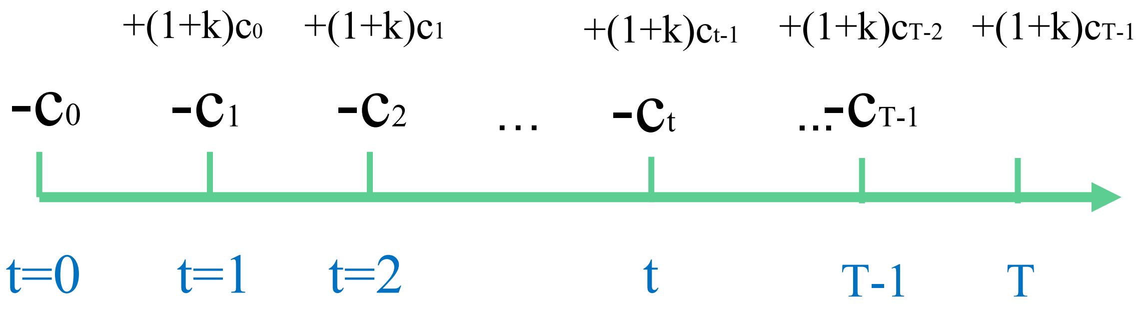


Cash flow stream $x = (-c, kc, kc, (1+k)c)$

If the constant per-period rate of return on an investment stream is k, then k will be an internal rate of return of the corresponding cash flow stream.

Investment Stream



Note:

- $t < T$
- Investment stream $c = (c_0, c_1, c_2, \dots c_{T-1})$

  c_t is the increment of capital invested at t

  c_t could be negative, $-c_t$ is the incremental of capital borrowed at t, and $(1+k)c_{t-1}$ is the increment of capital recovered at t

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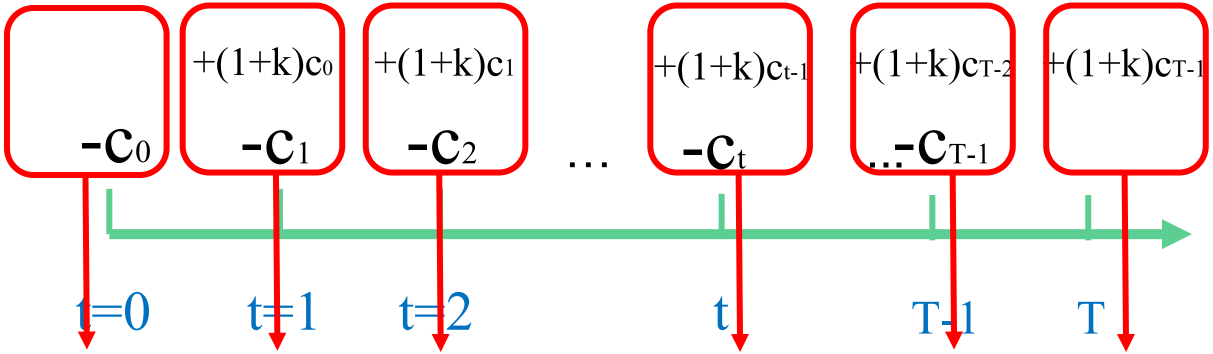
Investment Stream



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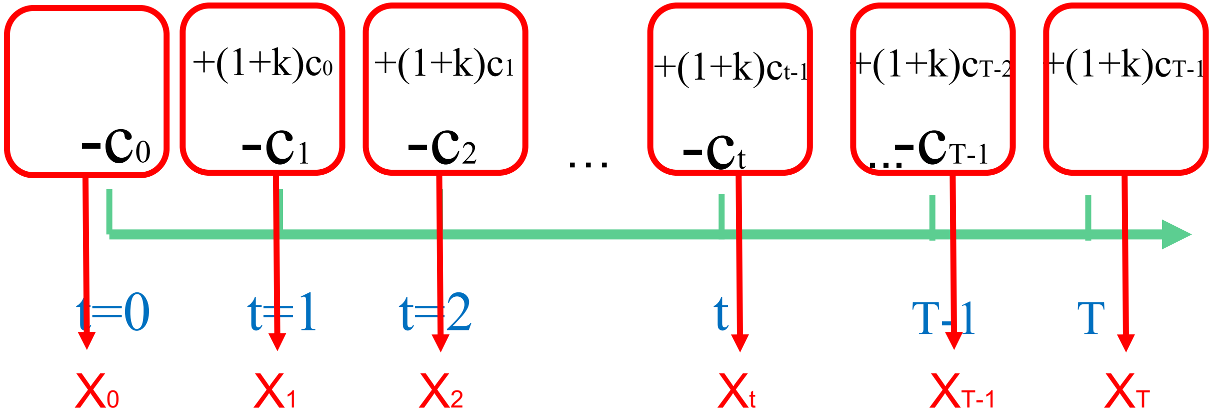

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- Cash flow stream $x = (x_0, x_1, x_2, \ldots x_T)$

Theorem

Net cash flows:

$X_0 = - C_0$

$X_1 = - C_1 + (1+k)*C_0$

.

.

$X_t = - C_t + (1+k)*C_{t-1}$

.

.

$X_T = (1+k)*C_{T-1}$

Vector form: $x = -(c, 0) + (1 + k)(0, c)$

$$PV(x|r) = -PV(c, 0|r) + (1 + k)PV(0, c|r)$$

$$= -PV(c|r) + \frac{1+k}{1+r}PV(c|r)$$

$$= (-1 + \frac{1+k}{1+r})PV(c|r)$$

$$= \frac{k-r}{1+r}PV(c|r)$$

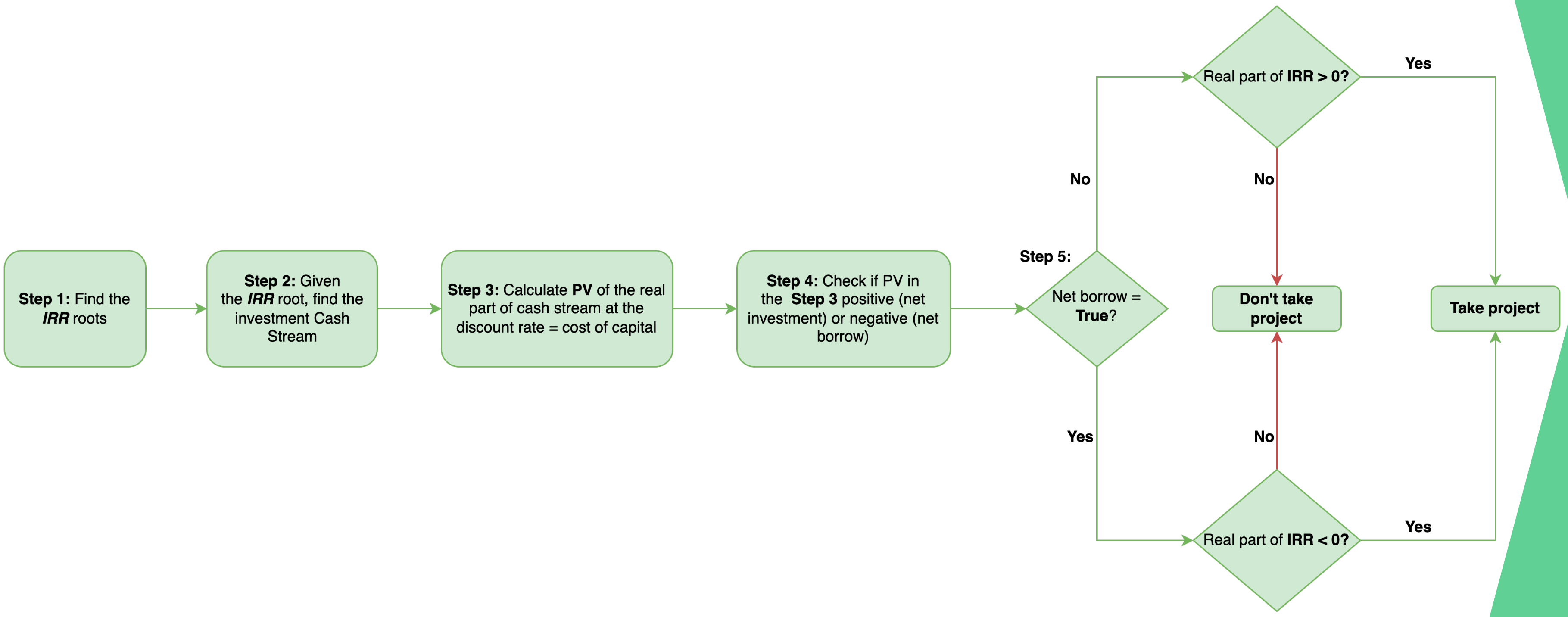
$$PV(x|r) = \frac{k-r}{1+r} PV(c|r)$$

a. If $PV(c|r) > 0$ (c is net investment) then $PV(x|r) \geq 0$ if and only if $k \geq r$

b. If $PV(c|r) < 0$ (c is net borrowing) then $PV(x|r) \geq 0$ if and only if $k \leq r$

c. If $PV(c|r) = 0$ then $PV(x|r) = 0$

Methodology of Multiple IRR Decision Tool



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Implementation

Code

```
generalIRR <- function(x,rate){
  # since PV is a polynomial in (1/(1+rate))
  invroots <- polyroot(x)

  # After obtaining solution, we find IRR
  roots <- 1/invroots-1

  # Function that generates investment cash stream given k=IRR, and CF=y
  cashstream <- function(k,y){
    c <- rep(0,length(y)-1)
    c[1] <- -y[1]
    for(i in 2:length(c)){
      c[i] <- (1+k)*c[i-1]-y[i]
    }
    return(c)
  }

  # Function that calculates the PV of any stream of cash flow
  PV <- function(y,r){
    return(sum(y/((1+r)^(0:(length(y)-1)))))
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where $y = \frac{1}{1+r}$

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where $y = \frac{1}{1+r}$

$$x_0 = -c_0$$

$$x_t = (1+k)\cdot_{t-1} - c_t \quad t = 1,\cdots,T-1$$

Code

```
  # Generate a list of investment streams for each IRR
  investmentstreams <- lapply(roots,function(j){cashstream(j,x)})

  # Calculate the PV of the Real part of the investment streams at r = rate
  PresentV <- sapply(investmentstreams,function(j){PV(Re(j),rate)})

  # Check if investment stream is net-borrowing/investing
  netborrow <- (PresentV < 0)

  # Check if each IRR, the project is viable, ideally for any IRR, it should
  # have the same outcome
  feasible <- (Re(roots) < rate)*netborrow + (Re(roots) > rate)*(1-netborrow)

  ans <- list(IRRs = roots, CashFlows = investmentstreams, RealPV = PresentV,
              TakeProject = ifelse(feasible==1,TRUE,FALSE),
              NetBorrow = netborrow, NPV=PV(x,rate))
  return(ans)
}
```

Result

```r
set.seed(17)
x <- runif(6,min=-1000,1000)
x

## [1] -689.89833  936.75762  -63.47383  553.63930 -184.22852   77.59430

# Check IRR against oppurtunity cost of 10%
test2 <- generalIRR(x,0.1)
test2$NPV

## [1] 447.5486

# Check to make sure only 1 unique conclusion from the multiple IRR and what
# the outcome is
unique(test2$TakeProject)

## [1] TRUE
```

Result

```
test3 <- generalIRR(x,0.7)
test3$NPV

## [1] -64.73187

unique(test3$TakeProject)

## [1] FALSE

# Get the IRRs
test3$IRRs

## [1]  0.5733246+0.000000i -1.3016644-0.662740i -1.3016644+0.662740i
## [4] -0.8060880+0.311804i -0.8060880-0.311804i

# Get the Investment Streams for each IRR
test3$CashFlows[[1]]

## [1] 689.89833+0i 148.67638+0i 297.39004+0i -85.74825+0i  49.31868+0i

# Get the PV of the investment streams discounted at opportunity cost
test3$RealPV

## [1] 868.70976  49.54504  49.54504  70.06325  70.06325
```

Conclusion

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- It’s better to just stick to the NPV decision rule