Review Article

Pareto-Nash Reversion Strategies: Three Period Dynamic Co-operative Signalling with Sticky Efficiency Wages

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Abstract

In this paper, the Nash equilibrium reversion is used as an optimal tool for clearing dynamic prices and wages. The balanced growth path of the efficiency wage and the outcome of repeated household/firm wage bargaining decisions are determined by various *exogenous* competitive rigidities. A location model is pursued to explore the extent to which a downstream spatial co-operation agreement might affect the price equilibrium. There is also an *endogenous* hiring function and a knowledge base which is increasing in output, as is the real wage. As the article demonstrates, after accounting for real rigidities in the baseline model, the effect of wage growth on household utility through staggered bargaining can be best catered for by adopting a policy of point scoring on the mobility of skilled labour against the model's key rigidities. Finally, labour mobility is explored. Mobility point scores, which serve to encourage mobility from skilled labour within the model, not only increase the knowledge base but also place upward pressure on nominal wage growth.

Keywords: labour, signalling, mobility, wages, game theory, competition
JEL Classification: D19, D21, J31, J61, L16

1. Introduction

The progress of search theory, over the last 20 or so years, has established models which have unquestionably furthered the understanding of equilibrium labour conditions in both British and American economies (Pissarides, 1988). There have been several key papers written on the importance of the 'matching function' (Mortensen and Pissarides, 1999; Blanchard, 1991; Coles and Smith, 1996), as well as models of wage determination (Blanchard, 1982) including but not exclusive to non-cooperative bargaining games (Yellen; 1984).

There is also an extended generation of contemporary research on the relationship found between prices and wages in cross-sectional empirical data. However, there has been relatively little attempt to explain a model of *conglomerate* price competition and wage behaviour throughout the full business cycle prior to and after an output shock.

1.1 Background

Economic growth over the last 20 years has occurred at a remarkable rate, particularly in developed economies. The acclaim for such successes should no doubt be placed on the heads and shoulders of the increasing aptitude of the three or four overlapped generations who have joined and retired from the British and American labour markets and who will continue to do so. The actual number of which has increased, but not without leaving behind adequate developments in the underlying tools of their and all our productive capabilities i.e. *technology*. For good reason, this article emphasises the point that such growth may very well continue, or it may not.

Whichever scenario occurs over the course of the next century, I would have it that developed economies, with time, must enshroud their efforts to encourage the mobility of skilled labour into their domestic markets, or as Romer

(2012) terms the effect, such economies must discover 'the contribution of a mystery variable, "*the effectiveness of labour*", whose exact meaning is not specified by Romer and whose behaviour is taken as exogenous (independent of explanatory variables), although it is alluded to the paradigm of research and development.

As such, welfare should be discouraged, necessarily, as possible margins for increases in per capita output can emerge from productivity of the labour market, and a reduction in government expenditure on social security. Effective labour and the value of the knowledge economy must be prioritised. In the following baseline model, this variable equates to none other than the level of household '*effort'*, derived from a combination of 1) added work value and 2) the value of qualitative contributions – concepts that are all too often over-assumed in classical economic growth theory.

Of course, as resolution would have it, the exact relevance of this variable is its relation to the essence of this article; the mobility of the worker (or rather, the *lack* of it) and their effective level of remuneration. Many authors have taken steps to attempt to explain the irregularity created by sticky efficiency wages either by constructing new models which build on Keynesian theories concerning wage growth (Summers, 1988; Yellen and Ackerlof, 1985; Hall, 2005) or by building on newer more dynamic theories. Rapid wage growth (from $w \text{ to } w*$) is a staple of the following aggregate baseline model, and as we will come to see, depends somewhat partially on the ubiquity of skilled labour inputs. And necessarily so, due to the nature of dynamic stochastic general equilibrium (DSGE) models, where there are only a finite number of identical households and firms, each of whom are assumed to have zero switching (or menu costs) associated with job separation, destruction and creation.

1.2 Materials and Methods

The thesis begins by establishing a framework, based on the Pissarides/Mortensen matching function, which allocates values to each determinant of an equilibrium real wage, resulting in a broad and encompassing definition of the labour market, its participants, and the tools with which they negotiate with. The model in the article varies from that of Pissarides/Mortensen in terms of the proposition of a hiring rate, which replaces the conditions for job creation and destruction. The next part of the article is to prepare the conditions for analysing wage behaviour and firm competition by setting out the conditions for a general Nash wage bargaining solution.

2. Literature Review

2.1 Conglomerate Pricing: Entry Signalling

Take, for example, the case of a firm which wishes to optimise output and a real wage in a signalling and bargaining scenario, i.e. a period-by-period staggered wage and an infinitely repeated Bertrand oligopoly game, where an incumbent's profit function is diversified across industries. What is the central question. This article is seeking to present a theoretical study into the effect of *entry* signalling on the behaviour of an oligopoly, which includes at least one subsidiary firm (i.e. a 'conglomerate of firms') and the wage decisions such firms make across time i.e. dynamically. Whereas much of the previous oligopoly literature focuses on three or more single- segment firms which are competing based on price, little work has been published in relation to the effect of price competition on a corporate group or affiliate of firms operating under the same 'body corporate' and the wage minimisation/profit maximisation decisions such firms undertake. In such a context, collusion can often be a factor that determines expected future profitability. Observing the co-ordinated correspondence between two or more conglomerate firms and a market entrant presents a unique opportunity to achieve greater social output as well as optimal profit given total net demand.

2.2 Equilibrium Conditions

By converting a Bertrand-Nash equilibrium price into a co-operative equilibrium, the co-operative sub-game perfect wage equilibrium becomes a function of the optimal Pareto-efficient *argmin-max* payoff. The seminal papers on oligopoly theory (Stigler, 1964; Friedman, 1971; Harrington, 1991, Abreu, 1986; 1988) all establish a key element of this article, that is, the modelling of some sort of co-operation prior to a simultaneous (and often secretive) profit-maximising Nash-reversion, (at the expense of entry signalling). Thereafter, each firm may choose to undercut and converge to a one-shot Nash equilibrium for the remainder of the infinite game. The obvious concern with constructing barriers to entry is that a Nash-reversion creates significant increases and entry gaps that are likely to inflate discounted prices, wages and generally reduce consumer and social welfare.

More recently however, the non-econometric micro-foundational based research literature on the topic which has been researched has become of ever-more increasing importance, with an emphasis on optimisation in co-ordinated equilibria across markets, within a variety of environments (Dal Bo and Frechette, 2011; Rees, 1993), chiefly due to the incentive each oligopolistic firm has to signal entry into an upstream market and negotiate with new labourers.

2.3 Shirking and Staggered Nash Wages

Much of the seminal work on wage behaviour found in Yellen (1984), Ackerlof (1984), Stiglitz (1984), Blanchard and Katz (1999), Mortensen and Pissarides (1999) and Diamond (1996) refers to concepts that the *nominal* wage has been used to explain, i.e. the absence of wage depreciation during aggregate shocks. This article would firstly like to highlight that the 'shirking' model of Shapiro and Stiglitz (1984) is essential in understanding the efficiency wage during recessions. The intuition that firms pay above market wages i.e. an *efficient* wage to ensure their workers do not shirk under contract, whilst maintaining an equilibrium price is conventional wisdom. Firms may however agree that this equilibrium wage is indispensable to retain their skilled labour and optimise their marginal product, in which case, the added incentive used to deter shirking disappears. The price/quantity duality will always remain constant. The unintended consequence is thus under-employment (for the agent who strictly prefers shirking to effort regarding wages).

2.4 Augmenting the G-T Model

A robust DSGE model of staggered multi-period wage contracting (Gertler and Trigari, 2009) is the first point of reference. G-T is important for two main reasons. Firstly, we are trying to understand wage effects on household utility and firm output during entry games, where households and firms renegotiate wages based on a Nash perfect sub-game, with the use of variations in hiring by firms, which is justified on the basis of studies by Hall (2005) and Shimer (2005) amongst others. The work of the G-T model, unique in the fact that it provides intuition that precedes the financial crisis, is valuable due the understanding the model makes concerning *wage* stickiness prior to an actual fluctuation in economic output vis-à-vis.

Secondly, the G-T model associates wages with hiring, where the stickiness in the wage due to staggered contract negotiations imply that the current and expected movement in the marginal product of labour (L) and capital (K) will have a greater impact on the hiring rate (h) than would have been the case otherwise. The objection here is that economists such as Barnichon (2009) have studied wage effects on the Mortensen and Pissarides (1999) matching function with more recent models, and can testify to incorporating cyclical fluctuations with job separations, using even more recent work from Shimer.

3. Households and Workers

The model's population consists of H finite households. The population grows at a constant rate (g), due to the contribution of incoming workers who supply skilled labour (Ls). The average household's problem is to choose the exact path of the efficiency wage $w(t)$ to maximise the present value of lifetime utility, given the following optimization problem. Where A represents knowledge endowments, e represents effort e.g. versus time spent on leisure, and n represents households who can form profitable alliances. Providing the integral with preferences, we obtain.

3.1 Basic Utility Function

$$U = \alpha + e(n, A, H, L, w)g + dt$$

The variable α represents a positive discrete constant integer that equates to initial household wealth. The higher this value is, the less intrinsic incentive the worker has to work. The expression for labour (L) is divided by time supplied to the labour market against leisure at time $t - 1$ by H households, where A the level of productivity (effective knowledge), which is given as exogenous.

The growth rate of A and the growth rate of $g(t)$ which is the growth rate of the population does not necessarily increase the value of the household's instantaneous utility at time $t = 0$ as the function increases in w.

3.2 Budget Constraint

Take the following evolution of the previous utility function:

$$\alpha + \frac{L(t)}{w(t)} g(t) + A(t)$$

Expanding this equation by adding the optimal level of labour for the economy, that is L (hat) less L at time t. The integral for technological '*knowledge*' growth is taken as given.

$$U = \int_t^T \sum e^{-R(t)}\, \alpha(t) + \frac{(\bar{L} - L)(t)}{w(t)H(t)} g(t) + \int_t^T A(t)\, dt$$

$$s.t.$$

$$\alpha - (1 - Ln)e)g$$

Taking the first order conditions of the household's instantaneous utility function we derive an expression which reflects the budget constraint as a function of utility:

$$U' = \int_t^T \alpha'(t) + L'(t)g'(t) + \int_t^T A'(t)\, dt$$

When the constraint is binding, total workforce effort is equal to the integral of discounted value of future lifetime earnings of the present population. The earnings of the $t + 1$ generation are given by the remaining endowments left at the end of time $t = 0$ less fiscal responsibilities carried over to the next generation. The household therefore wishes to maximise total utility by trading off its perceived leisure time with its ability to offer labour as a member of the workforce and by dissaving its initial endowments of wealth α.

4. Competitive Firms

Given that Walrasian assumptions hold for firm *f*; who wishes to hire a marginal unit of labour L^m, offering labour to a firm *f* are H / L number of workers, infinitely lived, who are invariably in and out of the market for their labour each competing for a finite number of homogenous jobs *J*. Each firm faces the following Lagrangian optimisation problem:

$$Q = p(1-q)^2 \qquad s.t.\;\; px(1-q) + wl \leq w + M$$

$$s.t.\;\; l \leq 1$$

4.1 Co-operative Games

The effect of cooperative behaviour prior to a simultaneous, (and often secretive), profit maximising Bertrand Nash-reversion, is the expense of lower barriers to entry. A co-operative sub-game perfect equilibrium of an infinitely repeated Bertrand oligopoly game is a function of the optimal Pareto-efficient payoff. This is opposed to the other theory on Cournot-Nash reversions (Abreu and Pearce, 1986) or Stackelberg competition (Green and Porter, 1984). Thereafter, an entry game converges from a dynamic equilibrium into an infinite 1-shot Nash equilibrium (Baye and Morgan, 1999). More recently, more non- econometric based research on Nash reversions6 have become of ever-increasing importance, with an emphasis on optimisation in games with co-ordinated deviation, within a variety of environments (Dal Bo and Frechette, 2011; Rees, 1993), chiefly due to increasing dynamic incentives.

To a large extent, micro-foundational research focuses on two main forms of punishment strategy; Abreu-type punishment strategies (Abreu, 1988), and the Nash-reversion (Friedman, 1971) or the *grim-reaper* strategy. Recent studies have demonstrated the effect of punishment strategies on the outcome of repeated games (Wright, 2011), as well as a welcome to being effective in using case study examples to demonstrate how concentrated markets are most effective in forming collusive outcomes (Zhang and Round, 2011)?

4.2 Competive Entry and Profit Maximisation

Take for instance, the Matutes and Regibeau (MR) two-stage model, where sub-market no participation is decided in stage one, where decisions are made subsequently regarding product diversification or homogeneity standardisation. Assume for a moment that wage determination is also achieved. The conglomerate must then decide upon the use of a Pareto multi-strategy $m \in M$; either competing as part of two subsidiary markets, one accompanied with standardised goods, and a constant price P^m, and the other with non-standardised goods, and more severe competition. The entrant pays no fixed fee (Matutes and Regibeau, 1989:361). By solving the unique sub-game perfect equilibrium $\tilde{\pi}$, for the sequential M-R model, derived through backward induction, where demand for each sub-market ξ is equal to: By simultaneously undercutting the incumbent, the competition concentration ratio increases, an entrants Pareto-optimal profit function is given by: profit from undercutting incumbent less profit gained by colluding $\gamma - c$.

If $\gamma > c$ for all $p^{\Omega} > p*$, e^i then entry occurs if an entrant can control for the competition created by $p^{\Omega} - pi$, via n e^n number of entrants through limit pricing strategies. However, as the number of entrants increases, perfectly-competitive profits converge to zero. The likelihood of a collusive outcome in a cooperative oligopoly market is much more significant, in that the incentive to revert is minimised (Zhang and Round, 2011:364).

4.3 Standadisation Across Markets

Matutes and Regibeau (1989) claim that consumers bundle goods from different firms to form a coherent 'system', creating incentives for all firms that offer a part of such a system to offer as close to an undifferentiated good as possible (when the consumer has already acquired the majority of their system). The incumbent firm, pursuing entry signals, faces an optimal strategy choice; either to sell a differentiated product and risk losing consumers due to incompatibility with what may considered to be market niche 'systems', or to impose a uniformity constraint on the degree of diversification and prioritise price competition across markets. Firms may prefer incompatibility to distinguish customer incentives from that of competition, tying them in. Industry demand is raised when every firm produces all component parts of a system, and ensures they are each undifferentiated in their own respect. The incentive to undercut is reduced as is competitive.

4.4 The Pissarides/Mortensen Endogenous Hiring Rate

The hiring function is preceded by the following intuition. There are an infinite number of identical homogeneous firms (*f*), who simultaneously utilise labour inputs in order to increase their respective productive capacities. Offering labour to these firms are: 1) an infinite number of households (*H*), infinitely lived, who are invariably in and out of the market for their labour, each competing for a finite number of homogeneous jobs given by $(j)^n$. At any given time (*t*), there are consequently (e^{mc}) number of employed households and (e^u) number of unemployed households. This constitutes the labour market. The model takes the endogenous rate at which firm's hire labour (*h*) as a constant between $-1 < h < 1$. In a perfectly competitive economy, these conditions are given through the *marginal* revenue product of labour for each firm at the given period of time *t*, interpreted by:

$$x = (\Delta\pi \,/\, \Delta e^m)t$$

Firms are incentivised to participate in the labour capital and markets up until; the marginal product reaches a reservation productivity threshold, at which each firm sets accordingly. Therefore, firms willingly and invariably hire households at the hiring rate *h*. Firms can now post vacancies, if and only if:

$$x = \left({}^{\Delta\pi}/_{\Delta e^m}\right)^t + (\pi \,/\, e^m)^{t-n} \geq 0$$

This generally implies the number of vacancies, at any point in time t, is set by an aggregate hiring rate, given that the inequality constraint is binding. The constraint function is given by:

$$x = (\pi \,/\, e^m\,)^{\mathrm{t-n}}$$

which takes the average marginal product of the firm from t-n previous periods. This provides an accurate reflection of optimal labour input given the firm's revenue, the point in which firms reach the labour demand/supply equilibrium and are unwilling to change labour inputs. An important feature of the model is that:

$$h \in (0,1) \rightarrow 0, + (\Delta\pi \,/\, \Delta e^m\,)^{\mathrm{t}} \rightarrow (\pi \,/\, e^m\,)^{\mathrm{t-n}}$$

That is to say, that as the aggregate hiring rate approaches positive (signed) zero, firms are inevitably choosing to reduce the number of vacancies they post to the market in order to satisfy the reservation productivity. Furthermore, if h is an extremely low figure (e.g. 0.01), firms have either reached, or are extremely close to reaching, an optimal output. The lower h is, the higher unemployment will be. Hence the rate is also an avid indicator of structural employment. If for any reason:

$$(\,\pi \,/ e^m)^{t-n} > (\,\Delta\pi \,/ \Delta e^m)^t$$

that is to say, the reservation productivity outweighs the value of the weighted productivity, all things being even, the excess inefficiency will lead to firms taking action to reduce the period wage. That is to say, the change in MRPL is relatively low in comparison to the change in the real wage from the previous period. This will become the central theme for the discussion into wage bargaining. For a firm without a binding inequality constraint, where the MRPL is less that the reservation productivity, reducing the efficiency wage for a period has an identical effect, similar to that of job destruction. As mentioned before, job creation (creating vacancies) in the Pissarides model occurs at every point up until the optimal MRPL is reached, after which the hiring rate becomes negative, where $h \in (-1, 0)$, the condition are in place for job destruction (reducing e^m, and increasing e^u). Hence, on a very general note, the sticky efficiency wage (*w*) in this process implicitly facilitates the adjustment to a new equilibrium.

So finally, firms are engaging in implicit or explicit behaviour both in the consumer market and in the labour market aimed at reducing the MRPL in order signal entry into a conglomerate/oligopolistic market and to reach an optimal output implicitly, by adjusting the real marginal wage being offered under a Nash bargaining game, or explicitly through job destruction, or $h < 0$ negative hiring i.e. reducing the total number of posted job vacancies (and consequently, increasing the outflows into unemployment).

4.5 Staggered Wage Bargaining

A closed-form staggered wage bargaining solution (Cheron and Langot, 2004; Gertler and Trigari, 2009) is used to understand the growth of the '*sticky*' efficiency wage. Contractual agreements take place once individual preferences have been established in order to optimise joint (worker and firm) utility. In other words, the wage of the new worker who enters the market for labour is the result of a co-operative Nash reversion between the firm (*f*) and the new worker *c*. The new worker has strict preferences which are complete, transitive and reflective. The new worker also has utility preference X such that u: $X \rightarrow R$ where $x > y$. Firms have utility preference *Y* if they agree to a high wage.

The pay-offs are reversed if they agree on a lesser wage. If they cannot agree to enter a contract, both parties end up with zero utility. The wage negotiation takes the form of a period game. Each period, wages are renegotiated by the firm and its new workers, as a function of firm output, the price level, capital and labour. We are interested in the sub-game perfect equilibrium. Assume there is perfect monitoring – i.e. past actions of all firms are known. All assumption pertaining to the histories and pure strategies of repeated game are taken as given i.e. monotonicity, non-satiation, convexity and continuity.

$$\pi_i = (p1, p2, \dots, pn) = \{\begin{array}{lll} \pi_{(pi)} & \text{if} & P_1 < P_t \\ \pi_{(pi)} & \text{if} & P_1 = P_b\} \\ 0 & \text{if} & \text{otherwise} \end{array}$$

Where there are past deviations from the Nash reversion, (i.e. if the firm chooses to relay on its agreement to pay new workers an amount that is equal to or strictly greater than the previous period wage, deviations are punished in the future by less effort and productivity). Future payoffs (*wages*) can be discounted (to *present* period of time) with a discount factor. Are there any drawbacks to an approach which calculates the minimal payoff discount factor, under which collusion can take place? Perhaps, for instance, calculating net present value of future lifetime earnings a problem for satisfying the budget constraint of the household maximisation problem stated earlier.

Let us start with the assumption that firm f then chooses to undercut other firms by negotiating a lower wage with new worker c (Nash bargaining reversion). This downward shifting effect that new workers have on an economy after average firm output has been reduced is essentially the '*sticky*' wages. The outcome of the Nash reversion is simple. The model explains two intermittent utility functions which meet at equilibrium, one which is quasi-concave and increasing in *w*, the other must be quasi-convex and decreasing in *w*.

Firms and workers discuss the possibility of entering into a contract for employment. We can calculate the simple Nash equilibria of the game, by undertaking a systematic analysis of the best responses to various strategic choices by each player.

5. Results

When hiring, each firm considers the marginal revenue product of labour, seeking to find $argmax(\Delta\pi)$, a function which maximises revenue, given wage and inventory costs. What exactly is the relevance here? Well, it means that there must be a saddle point, with respect to the linear profit and cost functions, where the revenue maximising output is set to $argmax(\Delta\pi) = argmin(w)$. On the other hand, H households can only bargain individually to maximise their wage $argmax(w)$. So for each observation on the Beveridge curve, the market wage can be said to be a process of Nash bargaining where the households $argmax(w)$ equals the firms $argmin(w)$. So, from this, it is possible to state that movements along the curve are characterised by changes in endogenous variables, under the control of households and firms; which are in-turn subject to a series of wider exogenous factors. Before and after the steady-state, the real wage for the labour market is given by the Nash solution, the outcome of $f^n(e)$ number of $argmax(w)e = argmin(w)f$ ultimatum games between each unemployed worker € and each of the corresponding firms (*f*) on the basis of wages *w*. The bargaining outcome may be two of kinds, either agreements or disagreements (often called disagreement points). Disagreements are typically defined by (Z,) where $Z = (ze, zf)$, and agreements are noted by (*W*) where $W = (we, wf)$. The Nash solution function describes the process as *n* number of *f* firms (who post vacancies) consider maximising labour productivity at a given output *q* and *n* number of *e* unemployed households bargain for each vacancy, considering the real inflation-adjusted market wage of offered by the firm, against the wage being paid to employed households. The bargaining solution can be sketched in extensive form.

Here we can derive the assumption that if co-operative optimal payoff is not affordable for the firm to agree the contract, the steady state falls back to the Nash equilibrium. The firm determines the feasibility of a payoff by calculating its net present value of discounted future payoffs.

Now, relaxing the Walsarian market assumptions, assume there are fixed number of firms and free entry condition (no-fixed fee), but there exists a fixed menu cost associated with changing an existing workers' wages. For the firm, profit maximisation occurs if the discount factor of the bargaining outcome is of a sufficiently low level. Reversion, if a collusive 'Pareto' optimum strictly dominates Nash equilibrium induces firms to revert to Pareto-inefficient Nash equilibrium. In a Co-operative game, zero utility occurs when incumbent threatens to revert from mark-up payoffs to Nash equilibrium, where firm's marginal costs are equalled. Abreu-type punishment strategies make deterrence unprofitable.

5.1 Aggregate Technology Shock to the Economy

When a median-scale aggregate technology shock affects the macro-economy, at the tn-1th period, this can be simulated by subtracting the change in output by a negative percentage for that given period. Effectively, this reduces the overall balance of the change in aggregate MRPL (the weighted productivity) in the tnth period, such that the aggregate decrease in output (a firm's gross revenue) is proportionate to the decreases in demand for goods and services, as a result of the shock. The only way to offset this negative change and maximise profits for the new output, is to either reduce the number of employed households (negative hiring), or reduce the labour market wage.

The second t_{n+1} period of the simulation assumes firms act logically and either reduce vacancies, reduce the nominal wage, or use a combination of both responses, in order to find the point in which labour costs are low enough to optimise MRPL, at the t_{n+1} period. However, households are keen to maintain the t_{n-1} period wage *w*, which may be partially sustained by a group of firms, but is not an optimal choice for the perfectly competitive firm to offer. The use of a negative hiring rate is a short-term tool for optimising MRPL and retaining the dominant bargaining position of firm *f* with respect to the market wage *w*, which firms wish to offer households at every $\boldsymbol{t_{n+m}}$ until MRPL reaches a *reservation* productivity, where *m* is also the duration of the technology shock, after which an unemployment equilibrium can be reached. At this point *h* approaches negative (signed) zero and the labour market finds a new, lower equilibrium. $+h \in (0,1)$, $-h \in (-1,0)$ where β (e_u) ↔ (e_m).

5.2 Changes to Wages and Hiring

A firm can choose either to lower *w* or to reduce vacancies and engage in job destruction. How do these changes affect wages? As the various exogenous factors impose a limiting constraint on π, employed households are paid more than firms would like, that is to say with respect to *w*. The unemployed households Nash bargaining solution is improved at each interval of time *t*. Yet firms wish to adjust this wage to the new equilibrium, which occurs at a lower wage, so they can maximise π at this equilibrium. In other words firms are operating inefficiently. Households are indeed hard done by, but such is the nature of the movement along the curve in the model. It therefore follows that when providing social security, government programmes must be more responsive to the aggregate changes in real wages.

Taking the model and changing the incentive to hire, like the Pissarides/Mortensen model, n firms choose to fill job market vacancies if what we will refer to as each firm's 'weighted productivity' satisfies this inequality constraint, which is given by the composite function

$$\frac{h(x\alpha)t}{1+r} \quad s.t. \quad (x')\alpha \geq (x)t-n$$

where the constraint is also the labour market reservation productivity. Here, *x'* the weighted productivity, is given by $x' = \pi/e_m$. Substituting for *x'* and *x*, the composite function becomes:

$$h\,(\pi^{\alpha}/em\,)\ /\ 1+r \quad s.t. \quad (x')^{\alpha} \geq (\textstyle\sum\pi\ /\ \sum em\,)^{t-n}$$

The above equation is the equilibrium job creation conditions, what it implies is that the marginal value of each firms' wage bargaining decision must be equal to or greater than the present value of their reservation productivities, over the duration of the technology shock *t*.

5.3 New Equilibirum Conditions for Wages and Prices

The three-period dynamic model consists of N downstream firms. Market share is equally distributed among these firms; hence there are benefits to colluding which are equal to $Nj + Ni = Nj{+}i$ where $j = \overline{\overline{N}} - j$ and the socially optimal merger is given by the total market share less merged market share. Downstream consumers bear a transportation cost of R for accessing the services of a competitor. If $R* < \overline{\overline{R}}(N)$ the transportation cost of the new merged entity, is less than the price of the previously acquired firms, then consumers divert consumption to $N - j$ firms. In addition to a transportation cost, a *constant* menu cost *T* is levied between each competitor's bundles of goods. Output Q is monotonic in accordance with the equilibrium price, *P*, thus $\pi = p(q) > P'(Q')$ which is a mark-up set by each dominant firm. Additionally, there are M potential entrants, who by undercutting the market equilibrium can profitably service the downstream market at each given period, paying an entry fee *E*. Firm *i* will produce at a cost of $C1(Q1)$, where the profit function for firm *i* can be written as:

The three stage model is as follows:

5.3.1 **Period 1** – *P* set to maximise profit function

5.3.2 **Period 2** – Decrease in *P* via Limit Pricing strategy (*The Nash Reversion*)

5.3.3 **Period 3** – Increase in *P* to regain any loss

5.4 Profit Maximising Location Games

In a downstream market, a competition authority is responsible for ensuring that the entrant is able to achieve a minimal profit, which can be given by the following function:

$$E(R) = Q[qPe - Ce] + E > 0$$

The above equation is an entrant's reaction function and is a profitable pricing condition for an entrant in our model. If P is significantly high, the function converges to 0, and entry is prevented. Thus, optimal spatial collusion (Huck *et. al.*, 2003) on a unit circle is profitable if the revenue of a coalition of size M is strictly greater than the expected revenue $E(R)$ from a non-cooperative equilibrium. It is possible to show that the result of the merged firm's cooperation reduces the consumer transportation cost n by the distance *D*; between the new firm and its two closest competitors. The termination fee levied by the joint incumbent on other competitors implies higher price tariffs for consumers who communicate to other competitors. Thus, the logical incentive for the marginal consumer or for those who wish to maximise consumer surplus (if prices are left unchanged) is to remain with their operator. Free-entry equilibrium for a market, which is large and dominated by several large firms, would typically yield zero-profit (Eaton and Wooders, 1985).

5.5 Conditions for Wage Growth

The matching function (see Pissarides, 2000; Shimer, 2006), taking the Cobb-Douglas form and being characterised by constant returns to scale $Y = X + I + K\alpha\ L1{-}\alpha + A$, where (Y) represents output, (X) represents total exports, (I) represents total imports, (K) represents capital, (L) represents labour, and (A) represents a mystery variable, quantified as the autonomous technology stock, research & development, which provides a succinct representation of the equilibrium as a function where the number of households looking for jobs and the proportion of capital employed by firms are strictly increasing in (Y). Green, E.J. and Porter, R.H., 1984. Noncooperative collusion under imperfect price information. Econometrica: Journal of the Econometric Society, pp.87-100.Take the basic aggregate output equation in Cobb-Douglas form:

(5.5.1) $$Y(t) = K(t)^{\alpha} + L(t)^{1-\alpha} + A(t)$$

$$For\ all\ 0 < \alpha < 1$$

Having stated the economy's production function. We derive a first order conditions:

(5.5.2) $$\frac{\partial f}{\partial Y(t)} = \frac{\partial f^{\alpha}}{\partial K(t)} + \frac{\partial f^{1-\alpha}}{\partial L(t)} + \frac{\partial f}{\partial A(t)}$$

$$= \frac{\partial f}{\partial K(t)}(\sum\nolimits_t^T K)^{\alpha} + \frac{\partial f}{\partial L(t)}(\sum\nolimits_t^T L)^{1-\alpha} + \frac{\partial f}{\partial A(t)}(\sum\nolimits_t^T \dot{A})$$

Multiplying both variables on the right by output Y and plugging in the equation for balanced wage growth; we are left with the following convex expression for output, which is increasing in Y:

(5.5.3) $$\frac{\partial f}{\partial K}\sum_t^T K\,\frac{K(t)K(t)}{Y\ K(t)}^{\alpha} + \frac{\partial f}{\partial L}\sum_t^T L\,\frac{L(t)L(t)}{Y\ L(t)}^{1-\alpha} + \frac{\partial f}{\partial A}\sum_t^T A\,\frac{A(t)A(t)}{Y\ A(t)}$$

Next, closed form expression to express a Nash bargaining solution is generated where households optimise wage maximisation decisions and firm choose profit-maximisation outputs. Here, if real wages are high, firm output is low and vice versa:

(5.5.4) $$U = (n, w, p, q)$$

$$where\ n\ and\ q\ are\ i.i.d\ constants\ 0 < x < 1$$

In forming the following Hamiltonian; aggregate output can be subjected to the following constraint faced by the perfectly competitive firm:

(5.5.5) $$Hn, q = Kq + Ln + A - nw(1 - w) + q(1 - pq)$$

The staggered aggregate growth rate of capital (*K*) and labour (L) at time *t* can be accounted for by the variables *n* & q:

(5.5.6) $$\partial Y(t)\ \partial q(t) = K + q(1 - p)$$

$$and$$

$$\partial Y(t)\ \partial n(t) = -1n + (1 - q)$$

Applying the Euler equation to the above expression:

(5.5.7) $$\frac{\partial \mathrm{Y}}{\partial \mathrm{n}} / \frac{\partial \mathrm{Y}}{\partial \mathrm{q}}$$

Everywhere along the $dY(t)/dn(t)$ slope, firms and households agree a nominal wage, such that, the expression can be said to be increasing in $q(t)$. Where the change in discounted present value of future output maximisation also changes the condition for optimal capital inputs at time *t*:

(5.5.8) $$K(t-1) + (1 - p(t))$$

$$= -1n(t) + (1 - q(t))$$

$$where\ K > 0\ and - 1 > n > 1$$

Lastly, equation 5.5.8 is simplified and rearranged to express the variable *K*, then substituted back into equation 5.5.3 with a Taylor approximation:

(5.5.9) $= \frac{\partial f}{\partial \mathrm{K(t)}} (\sum_t^T -1n(t) + (1 - q(t))^{\alpha}$

$$+ \frac{\partial f}{\partial \mathrm{A(t)}} \left(\sum_t^T A \right) + R(t)$$

$$+ \frac{\partial f}{\partial \mathrm{A(t)}} (\sum_t^T A) + R(t)$$

The model takes into consideration, the 'value of employment' and the trade-off of a switch to unemployment. Now consider the value of employment for a short period, receiving a nominal wage. The probability of remaining employed for *t* periods is e—$r(+b)$.

(5.5.10) $\int_0^T e^{-r(+b)} (\sum_t^T -1n(t)$

$$+(1 - q(t))^{\alpha} + (\sum_t^T L1 - p(t))^{1-\alpha} + \frac{1-e}{r+b} + \mathrm{R}(t)$$

5.6 Other Reversion Strategies: Grim Reaper

Now consider a two-stage oligopoly game where the removal of any barriers to entry creates a non-obligatory fixed-fee (*F*) which carries principal interest, (*r*). Supposing the incumbent plays x−314 + $\beta\mu$2 in time period 1. However, an entrant with a strictly dominant strategy is able to derive substantial profit by setting price to derive a Pareto efficient payoff, e.g. ≥ x−304 + $\beta\mu$2, which dominates the games' previous Nash equilibrium. If the discount factor is high, profit maximisation will occur, but, this requires both the incumbent and the entrant to agree to a pure strategy in order to sustain a strictly dominated payoff. Any deviation from the Pareto-efficient Nash equilibrium is a non- profit maximising strategy.

Importantly, in the interests of maximising consumer welfare; if there are avenues for galvanising demand (in the form of public, but noisy signals) a grim-trigger strategy can be maintained by both firms, with minimal secretive deviation (Aoyagi, 2002). The Pareto-efficient payoff is split evenly in the new collusive equilibrium, with the following outcome.

5.7 Other Punishment Strategies: Abreu-type

One major problem with the solution to a Pareto efficient collusive deviation is that whilst it may be optimal, a gradual increase in barriers to entry, generally increases the number of firms at the free entry equilibrium (Harrington, 1991). An incisive tool for an incumbent in a co-operative Nash equilibrium is a punishment strategy which is determined by the avoidance of retaliation, were the entrant to ever revert. To a large extent, such strategies focus on two main forms of punishment; Abreu-type punishment strategies (Abreu, 1988), and the Nash reversion (Friedman, 1971). Recent studies have demonstrated the effect of punishment strategies on the outcome of repeated games. (Wright, 2011). If, by simultaneously undercutting the incumbent firm, the competition concentration ratio increases, an entrant's Pareto-optimal profit function is given by: profit from undercutting the incumbent less profit gained by colluding $\gamma - c$. If $\gamma > c$ for all $p^{\Omega} > p*$, then entry occurs if an entrant can control for the competition created by $ei\ p^{\Omega}$ − pi, via n number of entrants through limit pricing strategies. However, as the e^n number of entrants increases, perfectly competitive profits converge to zero.

Now consider a similar market, where a specialised conglomerate operates in a single niche segment and the conglomerate owns several downstream subsidiary firms that are located within a designated geographical region as with the Hotelling-Down model of special collusion. The conglomerate must frequently allocate equity, resources and raw materials to each subsidiary, which collectively occupy a significant proportion of sales in the designated region. Assume the market segment is a highly concentrated oligopoly of firms, where direct competition gradually emerges in the form of a number of discount rivals.

6. Discussion

6.1 Labour Migration Controls

As the Stark and Bloom (1985) labour migration model implies, employment is not the only determinant of life chance and not everything can be explained by what an employment classification program directly measures. However, for the social planner, the point scoring is adequate for the purpose of facilitating the mobility of workers who are sufficiently skilled but who are external labourers in relation to the domestic market. To explain the relevance of labour migration; points are given as a function of the finite form 0 < s < 1. The variable s is mechanism for optimising research and development or 'knowledge' endogenously, although, the actual variable is taken as given and exogenous. Each firm who wishes to signal entry into a competitive market, hire a labourer into their domestic market and are required to use the system.

Firm (*f*) observes the productivity of household (a) who is employed and calculates a point score for (H / a) workers. Point scores are effectively socio-economic wage bands which act to categorise skilled individuals who are exceptionally qualified for remuneration within a particular sector. Two variables can internalise the concept of the point scores. The first is aggregate (a) productivity (i.e. qualifications/experience) and the second is (b) the nominal offered wage. We assume all households have the ability to freely invest in improving their skills and productivity. In a more advanced model, point scores can be assigned to job vacancies as pre-requisites which must be met by empployer upon entry into the labour market. Assume a potential worker (c), with points score (s*), wishes to enter market. Assume household (b) also has a point score of (s*). The new worker (c) can enter the market if vacancies with points score [0 < s < s*] exist within the labour market. If not, the new worker remains structurally unemployed. Such is the importance of the labour migration scores.

6.2 Other Policy Implications: Labour Mobility

Optimal labour mobility systems must offer a criterion for adopting a point-scoring system but must be governed by the perfectly competitive firm (f), where the effects are decentralised and the onus is placed on the perfectly-competitive firm to negotiate and maintain balanced wage growth in the short run. As suggested previously, each firm must allocate a socio-economic classification to each matched job vacancy. Households are then allocated 'mobility' point scores, based on their skills. For instance, points between $s0$ and $s1$ equate to a specific socio-economic classification. Vacancy bands are then matched with mobility points. All labour supply that has joined the workforce in the last time period, subsequently become incumbents, who can supply their labour without the criterion of a 'mobility' point-score. So, in essence all households and workers within the labour market will be of adequate productivity, given the aggregate point score level. As suggested previously, this is the exogenous framework within which the real wage is derived. The underlying issue here is that, as stated in the introduction, when labour supply is not fully mobile across markets; there are real wage rigidities that exist. Specifically, the less mobility the labour market affords to workers in terms of point scores, the higher the nominal wage will be.

The solution to this is that, if to propose that firms be allowed to designate point scores, which will prevent direct control of industry wide labour mobility vis-à-vis. This is opposed to the less appealing centrally-controlled system where labour market mobility can be restricted or encouraged accordingly by government, which causes wages to inflate, and/or deflate explained by the model.

Price increases are an altogether overlooked paradigm concerning the catalysts for labour mobility in the short run. Whilst price increases are a reason for increasing the real wage, the high real wage is not itself a reason for increasing labour mobility. However, the low real wage is. Thus is the price level is high and the real wage is low, there is an argument for increasing the mobility of labour. As wage growth inflates over time, the social planner is tasked with optimising both social surplus and producer surplus by maintaining a Pareto-efficient wage level. Real household income can be controlled for using central government legislation. One policy tool that is at the social planner's discretion is that extension of job protection for existing workers. What are the exact details of job protection laws? They consist of legislation which are aimed to reduce the job destruction rate. In other words, while wages are high, efforts can be made to ensure firms do not over utilise the counter mechanism for reducing labour costs. For example, firms can be discouraged from terminating contracts for existing workers who have worked a period of t^{1+n} periods, where $t - n > 0$. The outcome of such legislation is simple. The existing wage equilibrium is prevented from becoming a sticky wage in effect, as firms can neither reduce employees that are hired, em or and as such cannot renegotiate lower wages with new workers e^{n}.

Lastly, What exactly comes to pass in those less fortunate economies when all but the upper echelon of their skilled labour migrate to the more developed economies? What needs to be done in order to plug such a gap? I make no attempt in convincing the audience of my gravitas in understanding such a dilemma, but I have thought of the most obvious of solutions to ease the burden of the model, should such a scenario come to pass. Efforts need to be made, where possible, to increase the ubiquitous access to further and higher education for women in developing economies. This is at the heart of any such resolution. Why? For the simple fact that skilled labourers in industries such as healthcare, pharmaceuticals and medicine are highly transferable and can leave large cyclical gaps in employment once acquired. Where necessary, agriculturally inspired education and that which involves the understanding of farming and developmental but primary sector know how, must be prioritised. Not because these industries are systemically important, and as such, pose no threat to the developed labour market, but because of the pre- industrialised nature of capital and technology in such economies, there is no value in investing in unsustainable capital markets prior to any establishment of a bottom-up growth model where all workers can viably contribute to the production possibilities of such economies.

7. Conclusion

Using a formula for the extensive form outcome for an oligopoly seeking to protect cartel profits from entry, this paper has attempted to solve a similar case; for a conglomerate firm. The social planner must always give due notice to the underlying intent of the pricing strategy of the firm. It has been shown how price reductions in spatial location games can affect the transportation costs of reaching the service of a newly merged firm, when the price is strictly less than the cost of reaching either of the pre-merged firms, an effect which can encourage market participants to pursue co-operative '*code-sharing'* agreements.

The price effect of a coalition reduces consumer welfare by a proportional amount, because by undercutting an incumbent's 1st period price, the change in a generic consumer's marginal surplus over time must be strictly negative. Taking this further, there is also a negative limit price effect experienced by potential entrants, as the equilibrium plus the entrance fee exceeds marginal revenue. This effect can be shown by demonstrating that a merged firm can simply impose a higher fixed entry cost on potential entrants, which is greater than the cost of an outside option, i.e. higher than the revenue gained from participating in the market.

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References

Abreu, D., Pearce, D. and Stacchetti, E. (1986) 'Optimal cartel equilibria with imperfect monitoring' *Journal of Economic Theory*, 39(1), pp.251-269

Abreu, D. (1988) 'On the theory of infinitely repeated games with discounting', *Econometrica: Journal of the Econometric Society*, pp.383-396

Akerlof, G.A. and Yellen, J.L. (1985) 'A Near-National Model of the Business Cycle, with Wage and Price Inertia', *Quarterly Journal of Economics*, 100(1), pp.823-838

Aoyagi, M. (2002) 'Collusion in Dynamic Bertrand Oligopoly with Correlated Private Signals and Communication', *Journal of Economic Theory*, 102(1), pp. 229–248

Barnichon, R. (2009). 'The Shimer puzzle and the identification of productivity shocks', *SSRN Working Paper*

Baye, M.R. and Morgan, J. (1999) 'A folk theorem for one-shot Bertrand games', *Economics Letters*, 65(1), pp.59-65.

Bester, H. *et. al*. (1996) 'A Non-Cooperative Analysis of Hoteling's Location Game', *Games and Economic Behaviour*, 12(2), pp.165-186

Blanchard, O. J. (1982) 'Price asynchronization and price level inertia*', NBER Working Paper Series*

Blanchard, O. J. (1991) 'Wage bargaining and unemployment persistence', *NBER Working Paper Series*

Blanchard, O. J. and Katz, L. (1999) 'Wage Dynamics: Reconciling Theory and Evidence', *American Economic Review*, 89(1), pp.69-74

Cheron, A. and Langot, F. (2004) 'Labor Market Search and Real Business cycles: Reconciling Nash Bargaining with the Real Wage Dynamics', *Review of Economic Dynamics*, 7(2), pp.476-493

Coles, M. and Smith E. (1996) 'Cross-Section Estimation of the Matching Function: Evidence from England and Wales', *Economica*, 63(252), pp.589-597

Dal Bo, P. and Frechette, G.R. (2011) 'The Evolution of Cooperation in Infinitely Repeated Games: Experimental Evidence', *American Economic Review*, 101(1), pp.411-429

Diamond, P. A. (1996) 'Proposals to restructure social security', *Journal of Economic Perspectives*, 10(3), 67-88

Eaton, B.C. and Wooders, M.H. (1985) 'Sophisticated Entry in a Model of Spatial Competition', *Rand Journal of Economics*, 18(2), pp.282-297

Friedman, J.W. (1971) 'A Non-Cooperative Equilibrium For Super Games', *The Review of Economic Studies*, 38(1), pp.1-12

Gertler, M. and Trigari, A. (2009) 'Unemployment Fluctuations with Staggered Nash Wage Bargaining', *Journal of Political Economy*, 117(1), pp.38-86

Green, E.J. and Porter, R.H. (1984) 'Noncooperative collusion under imperfect price information', Econometrica: *Journal of the Econometric Society*, pp.87-100.

Hall, R. (2005) 'Employment Efficiency and Sticky Wages: Evidence From Flows in the Labour Market', *Review of Economics and Statistics*, 87(3), pp.397-907

Harrington, J. E. (1991). 'The Determination of Price and Output Quotas in a Heterogeneous Cartel', *International Economic Review*, 32(4), pp.767–792

Huck, S. *et. al*.(2003) 'On the Profitability of Collusion in Location Games', *Journal of Urban Economics*, 54(3), pp.499-510

Matutes, C. and Regibeau, P. (1989) ‘Standardisation Across Markets and Entry’, *Journal of Industrial Economics*, 37(4), pp.359-371

Mortensen, D. T., and Pissarides, C. A. (1999) ‘New Developments in Models of Search in the Labor Market’, *Handbook of Labor Economics,* 3, 2567-2627

Nickell, S. and Quintini, G. (2001) ‘The Beveridge Curve, Unemployment and Wages in the OECD from the 1960s to the 1990s’ in Aghion, *et. al*. (2003) “*Knowledge, Information and Expectations in Modern Macroeconomics*”, Oxford: Princeton University Press, pp.394-431

Pissarides, C. A. (1988) ‘The Search Equilibrium Approach to Fluctuations in Employment’, *The American Economic Review*, 78(2), 363-368

Pissarides, C. A. (2000) ‘*Equilibrium Unemployment Theory*’, MIT Press, Cambridge Massachusetts

Rees, R. (1993) ‘Collusive Equilibrium in the Great Salt Duopoly’, *Economic Journal*, 103(419), pp.833-842

Romer, D. (2012) ‘*Advanced macroeconomics*’, McGraw-Hill Companies, Inc.

Shimer, R. (2005) ‘The Cyclical Behaviour of Equilibrium Unemployment’, *American Economic Association*, 95(1), pp.25-49

Shapiro, C. and Stiglitz, J. (1984) ‘Equilibrium Unemployment as a Worker Discipline Device’, *American Economic Review*, 74(3), pp.433-444

Stark, O. and Bloom, D.E. (1985) ‘The New Economics of Labor Migration’, *American Economic Review,* 75(2), pp.173-178

Stiglitz, J. (1976) ‘The Efficiency Wage Hypothesis, Surplus Labour, and the Distribution of Income in LDCs, Oxford Economic Papers, 28(2), pp.185-207

Summers, L. (1988) ‘Relative Wages, Efficiency Wages and Keynesian Unemployment’, *American Economic Review*, 78(2) pp.383-388

Wright, J. (2011) ‘Punishment Strategies in Repeated Games: Evidence from Experimental Markets, *Games and Economic Behaviour*, 82(1), pp.92-102

Yellen, J.L. (1984) ‘Efficiency Wage Models of Unemployment’, *American Economic Review*, 74(2), pp.200-205

Zhang, Y., and Round, D. K. (2011). ‘Price wars and price collusion in China's airline markets’, *International Journal of Industrial Organization*, 29(4), pp. 361-372

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