Economics of Labour Markets
Royal Economic Society
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Cooperation in Sequential Games

Short Course - Class Notes
Autumn Semester

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Class Seminar Handout

(1/4) Purpose of these notes (15 mins):

- To explore *how* the various forms of the non-cooperative game resolve themselves in a static setting
- To ask: *How* coalitions bargain and negotiate optimally in a static setting
- To emphasise the importance of stylized facts in this static game of negotiation

(2/4) We will: (15 mins):

- Ask – what the mechanics of a coalitional Nash bargaining solution are?
    - Compte and Jahiel (2010)
- Ask – how do these mechanics differ from what we refer to as **'Pareto-Nash'** equilibrium?
- Define – the problem as found in Mayaki (2024)

(3/4) Finally, Class I considers (15 mins):

- What strategy profile best dominates a non-cooperative game?
- What payoff dominates when there is a cartel, grand coalition?
    - We will be solving for equilibrium payoffs for all players (i.e. *50:50, 60:40* etc)
    - Looking at closed form, matrix or extensive-form solutions for coalitions
- Do effective punishment strategies exist? (i.e. *Grim Trigger, Tit-for-Tat*) to encourage cooperation

(4/4) Any Questions? (15 mins)

The context to most labour market game theory is the principle of salary negotiation and wage bargaining. The economic workforce-via trade unions[1]-are assumed to be intricately implicated in a period game where the market wage is renegotiated periodically as prices adjust to fluctuations in demand in the real economy. Inflation is a significant determinant here.

Class I occurs in Week 2 of the **Economics of Labour Markets** short course and is entitled: Cooperation in Sequential Games. We will be exploring the theory of efficient labour markets as a complex feature of the global economy in the developed model. This is a BSc in Economics and Management optional module and is provided in your first Semester. Week 2 notes should take you 1 hour to complete and are split into four 15-minute sub-sections.

By the end of this Class, students should be able to:

1. **Differentiate** between cooperative and non-cooperative equilibria in sequential games relevant to labour market dynamics.
2. **Apply** Nash bargaining and Pareto-Nash allocation frameworks to simplified labour-related negotiation settings (e.g. wage bargaining, union agreements).
3. **Solve** for equilibrium outcomes in extensive-form and matrix games involving coalition formation and strategic dominance.
4. **Evaluate** the role of punishment strategies such as Grim Trigger and Tit-for-Tat in sustaining cooperation under repeated interaction.
5. **Interpret** the strategic implications of real-world collective bargaining examples using formal game theoretic tools.

1. *Non-Cooperative Equilibrium in Games*

We shall begin with an abstract notion of the sequential game popularised by **John Nash** (that is, a super game played over 2 or in a rare instance, 3 periods) where cooperation is either non-existent or is possible but highly discouraged and often illegal (i.e. *no* collusion or cooperative tactics). Cooperation carries an immediate penalty equal to the discounted expected payoff of all future sequences. An example of such a game might be the Prisoner's Dilemma.

Unlike cooperative equilibria where there is a collusive solution given for two or more parties in agreement, solving for *non-cooperative* equilibria involves observing the *independent* payoffs of each player. We refer to these payoffs as non-cooperative **Nash solutions**.

We refer to these outcomes as Nash solutions because no other outcome can be optimal for each player given the predetermined strategy profile, allocated resources, and level of utility observed before the game begins. In other words, the equilibrium payoff is hereby optimal for everyone where the Nash solution exists.

This solution could be staying out of prison (but at the expense of a co-defendant), finishing in a podium position in an Olympic cycling race (but behind a cyclist on the same team or from the same country), ensuring your CEO's salary is approved as a share owner in line with your company's ESG policies on executive compensation (when the matter is put to a vote) or winning a democratic election (with policies that do not cross over with opponents).

Task 1: Question & Answer

a. Provide a labour market example of a non-cooperative game? (5 Marks)
b. Given a two-player structure, identify each payoff using the correct approach, outline the utility and payoff structure of the game in *extensive form*? (5 Marks)
c. Solve for the Nash solution (5 Marks)

[1] This is referred to as what economists call "*collective bargaining*". In the United Kingdom union membership has declined since the 1970's and is roughly 22.4% of the workforce in today's economy. However, this number rises to 10.3% in France.

2. Equilibria in Cooperative Games

There exists another type of sequential game that economists have grappled with for over half a century. One where cooperation *is* permitted and furthermore where co-ops are generally encouraged. In such games, wages are re-negotiated in each period given perfect information on all potential strategic profiles and player histories. There are two win-win perspectives to consider:

- The Nash solution – where all players maximise outcomes
- The Pareto-Nash solution, where efficiency is maximised alongside outcomes

3. The Concept of Pareto-Nash

The above win-win scenario reflects a desired outcome. A Nash solution or Nash equilibrium is a strategy profile in which no player has an incentive to unilaterally deviate given the strategies of others—it reflects mutual best responses. However, this equilibrium might not be efficient: there could be other outcomes where all players would be better off. A *Pareto*-Nash allocation, by contrast, named jointly after **Vilfredo Pareto**, and **John Nash** refines the Nash concept by imposing a Pareto efficiency condition on top of the Nash stability.

That means it is not only a stable outcome (no unilateral deviations), but also one where no player can be made better off without making another worse off. In labour market bargaining, for example, a Nash solution might represent a stable but inefficient wage agreement, whereas a Pareto-Nash allocation would represent a stable and collectively optimal agreement—often requiring some cooperative mechanism (e.g. side payments or credible long-term commitments) to be sustained.

Table 3.1 Summary of Key Differences

Sequential Game Feature	*Non-Cooperative Equilibrium*	*Cooperative (Competitive) Equilibrium*
Player Behavior	Independent decision-making	Joint decisions through negotiation
Binding Agreements	Not allowed	Allowed or assumed
Equilibrium Concept	Nash Equilibrium	Core, Bargaining Solution, Pareto Core
Strategy Type	Individual strategies	Coalition strategies
Typical Use Case	Conflict, competition	Negotiation, cartels, unions

Assume an employer is profitable and systemically important, it has payoff 10 from a highly productive workforce. The wages it offers may be defined as low or high. Collective productivity or representative union effort has payoffs which are given by a maximum utility of 10. In each sequential game period[2]:

Player 1 (Union) chooses:

- Low Effort (L)
- High Effort (H)

Player 2 (Employer) chooses:

- Low Wage (W/L)
- High Wage (W/H)

These strategic profiles are compiled under conditions of uncertainty. The stylized fact is information asymmetry on each employer's profitability. The table below shows the normal form (strategic form) matrix for the Union–Employer game. Payoffs are in the format (Union, Employer). The strategy combination (High Effort, High Wage) is both a Nash equilibrium and Pareto-efficient.

[2] We shall assume varying payoffs (Bayesian SPNE) from utility in each sequential game, given discounted probabilities.

Table 3.2 Collective Bargaining in Closed Form

Union \ Employer	*Low Wage (W/L)*	*High Wage (W/H)*
Low Effort (L)	(0, 8)	(2, 6)
High Effort (H)	(5, 5)	(10^*, 10^*)

4. *Subgame Perfect Nash Equilibrium (SPNE)*

A Subgame Perfect Nash Equilibrium (SPNE) is a refinement of the Nash equilibrium that eliminates non-credible threats by ensuring that players' strategies constitute a Nash equilibrium after every possible history (i.e., in every subgame) of the game.

5. *Bayesian Subgame Perfect Nash Equilibrium (Bayesian SPNE)*

In labour market models (e.g., wage bargaining with asymmetric information):

- A union may **not know** the firm's true profitability ("type").
- The firm's wage offer reveals **partial information** about its type.
- The union updates its beliefs and decides whether to accept or strike.

The equilibrium requires both players to act **optimally** at each decision point, and for those beliefs to be **consistent** with observed actions.

Here's the payoff matrix (6.1) for the Bayesian Subgame Perfect Nash Equilibrium (Bayesian SPNE) in a union–employer wage bargaining scenario with incomplete information.

Each path represents:

- The employer's type (Random Walk)
- The employer's wage offer (High or Low),
- The union’s response (Accept or Strike).

6. *Comparison of Pooling vs Separating Equilibria in Wage Bargaining*

This table compares the expected payoffs for the Union and Employer under two types of Bayesian Subgame Perfect Nash Equilibria: Pooling and Separating. In the **pooling equilibrium**, both types of employers offer the same wage, making it impossible for the union to distinguish between them. In the **separating equilibrium**, employer types reveal themselves through different wage offers, allowing the union to condition its response.

Table 6.1 Pooling vs Separating Equilibrium

Equilibrium Type	*Employer Type*	*Wage Offered*	*Union Response*	*Union Payoff*	*Employer Payoff*
Pooling	E_1 (Profitable)	High	Accept	5	5
Pooling	E_2 (Struggling)	High	Accept	5	-2
Separating	E_1 (Profitable)	High	Accept	10	10
Separating	E_2 (Struggling)	Low	Strike	-1	1

7. **Solving Bayesian SPNE explicitly:**

Let’s solve step-by-step for the Bayesian SPNE in a **wage bargaining game** with incomplete information using the previous game structure:

Stage 1: Assume the employer type is given by:

- A profitable employer (E_1) which has probability p
- A struggling employer (E_2) which has probability $1 - p$

Stage 2: Employer chooses the wage: High (*H*) or Low (*L*)
Stage 3: Union observes the wage and decides by voting to Accept (*A*) or Strike (*S*)

8. *Maximal Utility Function*

Ergo, the union is said to maximise its **type-dependent utility** given by:

(8.1) $$\mathrm{E}(U) = p \cdot u(E_1) + (1 - p) \cdot u(E_2)$$

Table 8.1 Payoff Matrix

Employer Type	*Wage*	*Union Action*	*Payoff (Employer)*	*Payoff (Union)*
E_1	*H*	*A*	10	10
E_1	*L*	*A*	2	7
E_1	*H*	*S*	0	-1
E_1	*L*	*S*	-1	2
E_2	*H*	*A*	5	-2
E_2	*L*	*A*	2	4
E_2	*H*	*S*	0	-5
E_2	*L*	*S*	-1	1

Task 2: What is the Nash equilibrium payoff?

The above table shows the payoff matrix for each employer type, where the union observes a high wage or low wage offer and responds with an **acceptance or strike**. Notice how the employer's strategy set (payoff) is consistent across all types, but the union's payoff varies, even when the union takes the same action.

Task 3: Read Compte and Jahiel's (2010) Core Nash bargaining model for multiplayer coalitions.

a. Define in your own words the meaning of what the authors refer to as 'core allocation'?
b. How would you derive a Nash equilibrium ensuring Pareto-efficiency *and* 'core' allocation?
c. Solve explicitly for the utility maximisation problem in (8.1)?

Answers: a) If you can recall, earlier in our Class session we noted that cooperative equilibria involve so-called 'no collusion' rules, which forbid players from forming agreements. However, the solution concept in Compte and Jahiel (2010) **bridges cooperative** and **non-cooperative** game theory by providing a strategic justification for a *particular* core allocation of payoffs.

b) Let $N = \{1,2, \ldots n\}$ be a finite set of individual players in a **cooperative game** with characteristic function v. Where we define $2^N \to \mathbb{R}$, and $v(M)$ denotes the surplus that coalition $M \subseteq N$.

The **core** is defined as a set of payoff vectors $x \in \mathbb{R}^2$ satisfying:

$$\sum_{i \in M} x_i \geq v(M) \quad \forall M \subseteq N, \;\; and \;\; \sum_{i \in N} x_i = v(N)$$

Then the **Coalitional Nash Bargaining Solution** is defined by:

$$x^* = arg \max_{x \in Core} \prod_{i \in N} x_i$$

Where:

- $x^* \in Core$: the solution is coalitionally stable – no group of players can block the outcome.
- x^* is Pareto efficient: player 1 cannot be made better off without making player n worse off.
- There exists a **non-cooperative bargaining game** (Compte and Jahiel, 2010) such that:

    - Players sequentially propose coalitions and surplus divisions
    - Bargaining continues until agreement is reached
    - As players become arbitrarily patient, discount factor δ converges to 1

c) As per 8.1 where the structure of the game is such that: There are 2 players, an employer and a union, who reserve the right accept a wage offer or strike. The employer offers a wage which is high or low. Payoffs are as below:

Table 8.2 Recap: Payoffs for Wage Bargaining Game

Union/Employer	*Low Wage (W/L)*	*High Wage (W/H)*
Low Effort (L)	(0,8)	(2,6)
High Effort (H)	(5,5)	(10^*,10^*)

9. Bayesian Subgame Perfect 'Coalitional' Nash Equilibrium

Consider now a **non-cooperative sequential game** with a cooperative subgame with perfect information, where unions bargain for a maximal utility function and employers offer a wage given an observed type (the employer is either profitable or struggling). In the subgame, unions consider whether to **accept or decline** an offer given political interests and a majority (we call this '**majority rule**') required for votes to pass a union ballot. This voting process becomes a subgame.

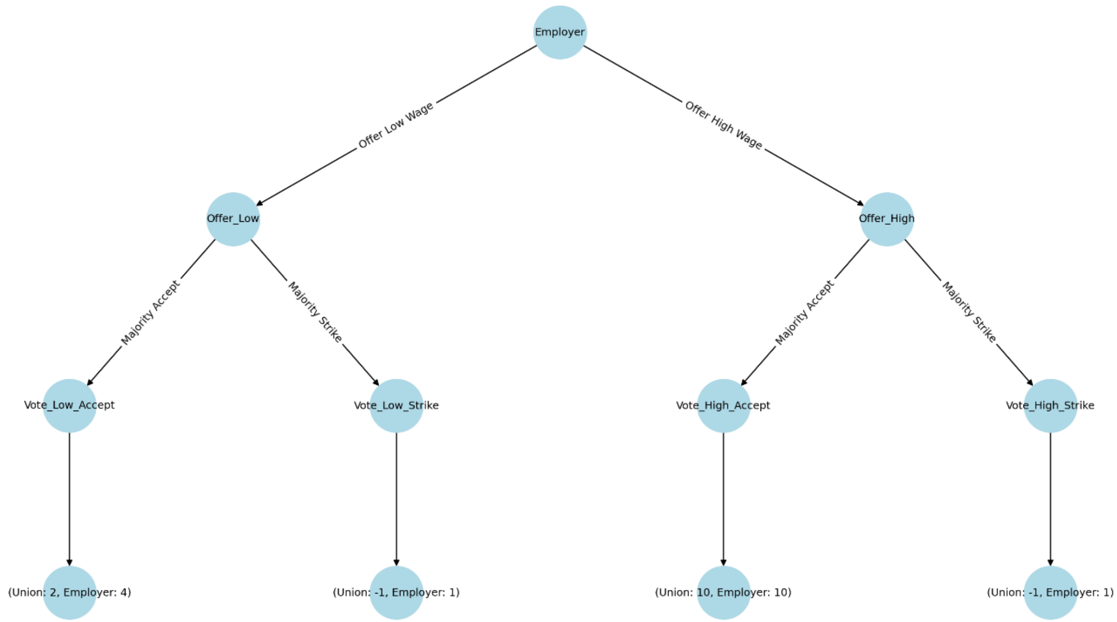
Assume:

- There are $i = 1,2, \ldots N$ union members
- Each union member has utility u_i from the offered wage $u_i(w)$
- Each unionist compares:

    - A payoff from **accepting** $u_i(w) \cdot A$
    - A payoff from **striking** $u_i(w) \cdot S = -1$ (*loss of income, stress from picketing*)

- The subgame outcome is determined by a majority vote:
    - $i \neq u_i(w)A > u_i(w)S > {}^N\!/_2 \rightarrow (w)A$
    - Else $\rightarrow (w)S$

Figure 9.1 Union/Employer Nash Equilibrium



- The **employer moves first**, choosing either a **low wage** or **high wage** offer.
- Then the **union votes**, deciding to **accept** or **strike** by majority rule.
- Each terminal node shows the corresponding **payoffs** for the union and employer.

Q: Do effective punishment strategies exist (i.e. *Grim Trigger*, *Tit-for-Tat*) to encourage cooperation from union members before a vote takes place?

A: Before the union-wide vote:

- Each **individual union member** could choose to vote **strategically or selfishly** (e.g., vote to strike when it's personally optimal, even if collective bargaining power is harmed).
- This creates a **social dilemma** — short-term gain from individual deviation vs. long-term collective benefit from coordination.

The union's challenge is to decide **how to sustain cooperative voting behaviour** that supports negotiation strength and credibility? In other words, can the union credibly enforce a Grim Trigger punishment strategy to ensure co-operation?

- Each member votes with the **collective majority** (e.g., accepts a fair wage if it maintains unity).
- If a member **defects** (votes selfishly or leaks strategy), the rest **permanently refuse to cooperate** with them in future negotiations or votes.

Ergo:

- The **threat of permanent exclusion or marginalisation** keeps members in line.
- Effective when:
    - The union has a **long-term future** (δ close to 1)
    - Members value **reputation**, or **solidarity**

10. Class I Extended Notes

In our Class I session, we spent the best part of 1-hour introducing the principles of **Bayesian Subgame Perfect '*Coalitional*' Nash Equilibria** (BSPCNE) and associated payoffs. In the final section of our notes, we apply an extensive form game found in Compte and Jahiel (2010) to an industrial action dispute between unions and employers. We noted how the associated payoffs are contextually specific to two player sequential games (spanning 2 or often 3 time periods) characterised by **incomplete information** – hence the necessity for pooled or separating equilibrium. We solved for optimality where multi-player and coalitional games have payoffs which are Pareto-efficient along a constraint of a majority rule vote. This session will cover:

- 15 mins on Pissarides-Mortensen's search and matching model with frictions
- 15 mins on the unemployment and vacancies (henceforth the 'U/V' or the 'Beveridge') curve
- 15 mins on fiscal and monetary policy implications

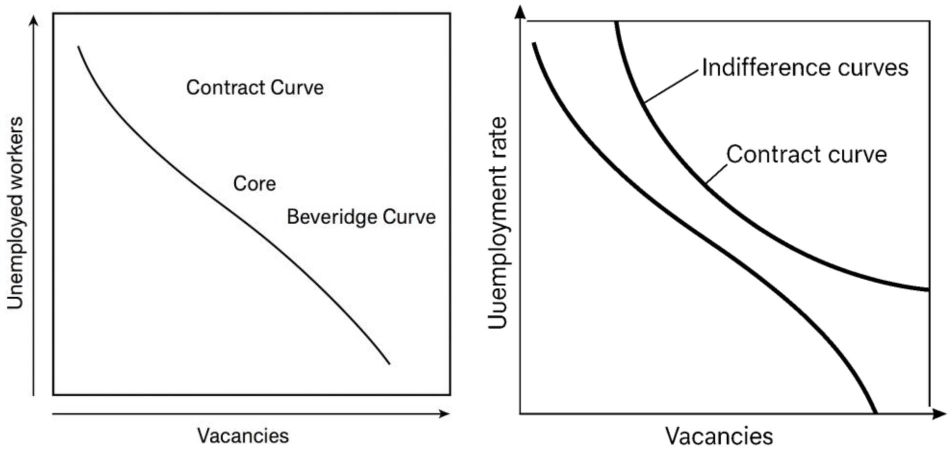
Class II: In Class I, we also mentioned the mechanics of the *Pareto-Nash* solution–a game theoretic concept–addressing wage re-negotiation for the *employed* worker and the employer. Class II extends this notion of game theory in labour markets by introducing a basic form of the Pissarides-Mortensen search and matching model which upends our prior **Nash bargaining solution** with search frictions appearing between the *unemployed* worker and the prospective employer.

As wages adjust, employers engage in job creation (*expansionary actions*), job seekers are hired marginally at the previous period's wage at a constant hiring rate. However, along this relationship, rigidities appear, or frictions occur discounting job creation and limiting perfect matches. Note: In search and matching theory, we consider a model where the market for work is constituted by inefficiencies in what Nobel Laureate **Prof. Christopher Pissarides** refers to in his paper with **Dale Mortensen** as a 'real rigidity' or 'friction'. This leads to something we call *structural* unemployment in our closed economy model.

Workers are thus unemployed, structurally, first and foremost, and by way of the employer's *contractionary actions* in the second order. The employer, who in ideal circumstances wishes to engage in job *destruction* which takes place when firms demonstrate strong incentive to reduce their balance sheets amidst fluctuations in the business cycle. The U/V curves below shows the inverse relationship between unemployment (U) and vacancies (V) when U is high, V is typically low, and vice versa. It also shows the contract curve which optimises the trade-off. Such a negative correlation arises partly due to matching inefficiencies and frictions in the labour market.

If you can recall, the union vote is a decision that could propose picket line action in an extended-form super game when, one of two employer '*types*', are *observable* in Bayesian SPNE.

Figure 10.1 Beveridge, Indifference and Contract Curves



11. Where and Why Firm Entry Occurs

In Class I, we assumed each employer was a player in a game of sequential negotiation. This is problematic for several reasons. Firstly, because employer utility is an estimated concept in the absence of **a profit motive**, or a function expressed as a payoff variable of each player in the game and secondly because it prevents firms from outperforming (assuming a firm obtains high-effort workers via an offer acceptance) or in other words, firms become **dynamic variables**. Here's a more detailed breakdown of how the Mortensen/Pissarides model treats firms in a generalised search model (Acemoglu and Hawkins, 2007).

12. Empirical Evidence on the Beveridge Curve

Consider: $M_{it} = \alpha + \beta_1 U_{it} + \beta_2 V_{it} + \varepsilon$ as an arbitrary regression model expressing **a matching function** (M) as a function of both: "unemployment" (U) and "vacancies" (V) respectively. Using data from the main statistical agencies on three continents (Europe, Africa and Asia) we test an empirical fixed effects panel regression.

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