Notes on: The Equity Market Model and CAPM with Markov-Switching Portfolios

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Abstract

An important insight found in the literature is that asymmetric dependence exists between cross-country and 'home-bias' investments otherwise known as domestic diversification (Garcia and Tsafack, 2007). This describes the effect created when diversification of asset returns is needed most, but they fail. Why do these portfolios usually fail? Because the correlation between diversified assets is higher when the market is in a negative downturn (Markose and Yang, 2008). Ang & Bekaert (2002) claim that asset allocation research has overlooked the implications of regime-switching of asset returns in portfolio optimisation. This essay re-visits some of the important work conducted on this topic.

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Introduction

The seminal work of Harry Markowitz in the field of quantitative finance has long been cited as constituting a fundamental contribution to the body of research responsible for today's comprehensive understanding of modern portfolio optimisation theory. However, it is also widely accepted that the traditional mean-variance portfolio choice models which Markowitz pioneered rely to stringently on the mean, variances and covariances of a stock assets historical returns when determining sufficient portfolio weights (Markowitz, 1952) i.e. between allocation to risk and risk-free assets. Current quantitative finance research models continue to reflect Markowitz's seminal work (Mahmoudi, 2023) such is its significance to the field of study. Likewise, traditional models of portfolio choice based on Markowitz concept of presently valued expected risk and return weights, or what is commonly referred to as the 2nd generation of finance literature, are much more useful to the active manager. This is because these theories direct money towards those assets with higher sample means and lower sample variance, but, under these models when a stock market observes a negative downturn or some kind of economy-wide shock, these assets will still be represented in corresponding portfolios. Thus, the traditional mean-variance approach pioneered by Harry Markowitz can be criticised as one which allows for too many *avoidable* errors. The optimal solution according to the literature is to create more robust optimisation and better mean, variance, and covariance estimates. The following review will explore this aspect in more detail.

Literature Review

Markose & Yang (2008) demonstrate an equity portfolio selection technique with regime switching - based on Hamilton (1989) and focused on UK domestic equities - their findings were significant for asymmetric correlation in the economic time series that was the focus of their research. The authors concluded that asymmetric correlations can be utilised to develop a regime switching portfolio. The current and future level of the aggregate economy and benchmark indices drive the regime switches in their model. Ntuli and Akinsomi (2017) use empirical evidence from South Africa and find evidence that REITs are a “return-enhancer” when combined with other assets in a mixed-portfolio. Moreover, Alcock and Adrlikova (2018) use empirical data from the United States and find evidence to suggest that the Beta value (which I shall notate more closely later in the paper) across the observations studied is not positive.

The Model

What I find to be true is that the regime switching method acts as a type of protection for the risk averse investor. Now, in-line with Markose and Yang (2008) in the following paper, I will consider returns drawn from a three asset portfolio with regime-switching weights (the regime-switching capital asset pricing model) and a three asset portfolio *without* regime switching weights (the single factor CAPM model). Why are we studying the asset returns of these two models in particular? Because, the presence of asymmetric correlation could significantly affect decisions of asset allocation and portfolio selection.

According to empirical evidence on regime switching, the observations indicate that an often overlooked element of the asset return profile happens to be asymmetric correlation; that is, high returns during bear markets, significantly higher than during bull markets. This is important. Why? Because the optimal portfolio in bear markets is substantially different from the optimal portfolio in bull markets. The regime switching portfolio assumes that in bull markets (positive returns) volatility is low and in bear markets (negative returns) volatility is high. Another stylized fact is asymmetry in exceedance correlation; that is, large negative returns are more correlated with large negative returns (Ang and Chen, 2002)

I will take as given the following stylized facts: I will assume that average returns are positive when they are drawn from the lower volatility and correlation regime and that average returns then become negative when returns drawn from the higher volatility and correlation regime. As noted previously, in a paper on regime-switching, this approach is better than the 'static' case, due to market timing. The authors demonstrate that there exists asymmetry between volatility and correlation not captured in portfolio weights (Markose and Yang, 2008) Also, If a market downturn occurs, asymmetric correlations may distort a global diversification strategy (Hamilton and Kim, 2000). One author suggests the importance of regime switching in asset allocation may cause investors to '*wildly* alter' portfolio holdings across regimes (Ang & Bekaert, 2002).

Equity Market Model

Consider the single factor capital asset pricing model (CAPM) inspired equation, where the return of the asset is given by a constant term, the slope, excess market returns of the market and a random error term. The intuition behind this equation gives us the following function form and line graph:

(1) $$E(R_p) = Alpha + Beta\,(R_m) + \sigma$$

Regime-Switching CAPM

Now, consider, a change to the conventional single factor model of CAPM:

(2) $$E(R_p) = Alpha + \beta eta\,(R_m) + \sigma$$

Whereby, excess returns on the market R_m is given by the following equation:

(3) $$R_m = \mu_m(P) + \sigma_m(Q) + \varepsilon$$

P, Q are given by the following Markov process, whereby $S_t = 1$ is the bull market characterised by high return and low volatility and $S_t = 2$ is the bear market characterised by low return and high volatility:

(4) $$P = (S_t = 1 \mid S_{t-1} = 1)$$

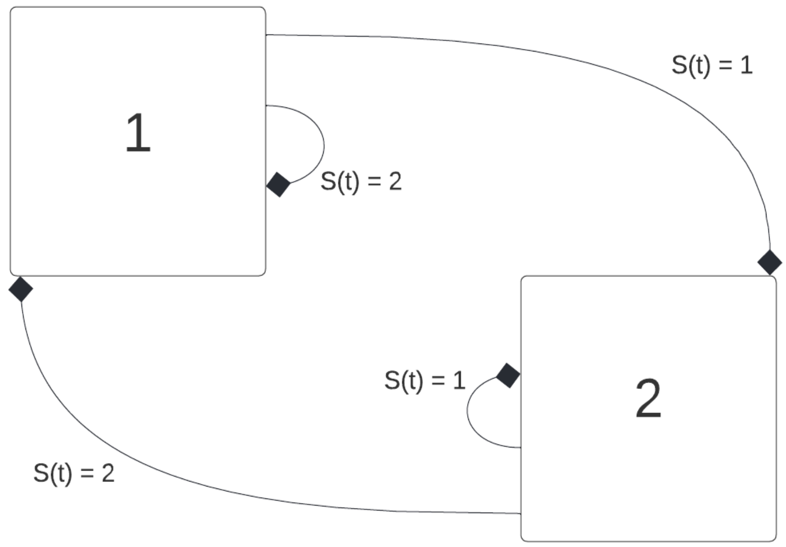
(5) $$Q = (S_t = 2 \mid S_{t-1} = 2)$$

Each governed by transitional probabilities P, Q. If we relate S_t to the Markov process P, Q, we obtain the following (Ang and Bekaert, 2004:88).

Figure 1: Transitional Probabilities

	$S_t = 1$	$S_t = 2$
$S_{t-1} = 1$	P	$(1 - P)$
$S_{t-1} = 2$	$(1 - Q)$	Q

Figure 2: The Markov Chain with Transitional Probabilities



So, to recap: The key change from equity market model to regime switching capital asset pricing model is (and I note): the inclusion of R_m which represents the excess return of the market and is subject to regime switching. The returns drawn from R_m are driven by two regimes:

(6) $$R_{it} = \alpha_t + \beta_t R_{mt} + \varepsilon_{it}$$

(7) $$R_{it} = \alpha_t + \beta_t R_{mt} + \sigma_i \varepsilon_{it}$$

Whereby:

(7) $$R_{mt} = \mu_m(S_t) + \sigma_m(S_t) + \varepsilon_{mt}$$

Where S_t is either 1 or 2 - a regime variable (of the form of Object 3). Also, $\mu_m(S_t)$ is the regime dependent mean of the all share indices and $\sigma_m(S_t)$ is the regime dependent conditional volatility (standard deviation) **not** variance. Just to recap:

how does switching occur in S_t regime variable from 1 to 2? The answer is that switches are governed by a Markov chain (which has two transitional probabilities): P, Q.

In the traditional single factor capital asset pricing model: R_m is determined by a benchmark share index (such as the FTSE). $Alpha$ and $Beta$ are determined by OLS. Whereas in the regime switching capital asset pricing model R_m is regime dependent while α and β are determined by a 2-state regime.

(8) $$R_{it} = \alpha_i + \beta_i R_{mt} + \sigma\varepsilon_{it}$$

(9) $$R_{mt} = \mu\ (S_t) + \sigma\ (S_t) + \varepsilon$$

Switching between states determined by transitional probability (Markov chain). Whereby: the variable σ_i is the idiosyncratic standard deviation of asset i and S_t depends on regime realisation (1 or 2). $\mu\ (S_t)$ is the regime dependent mean of excess returns and $\sigma\ (S_t)$ is the regime dependent conditional volatility (measured by standard deviation of FTSE). $S_t = 1$ and $S_t = 2$ are governed by transitional probabilities.

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