

Part 1 - Efficient Frontier

- Download adjusted closing prices for 10 Eurozone-listed stocks over the last ten years (monthly frequency: 120 observations) from Bloomberg or Yahoo Finance. Compute their returns, variance-covariance matrix, and correlation matrix. When selecting stocks, consider diversification across sectors.
- Select 5 out of the 10 stocks and determine the efficient frontier (mean-standard deviation) in at least 7 points without constraints (short selling allowed). Choose the stocks to achieve the "best efficient frontier" and explain the rules and insights guiding your selection.
- Using the same 5 stocks, recalculate the efficient frontier with the non-negativity constraint (no short selling). Discuss how investment opportunities change with this constraint.
- Choose a proxy for the market portfolio and evaluate whether it is efficient relative to the frontiers you identified.

Calculate the following portfolios:

Global Minimum Variance Portfolio (GMV)

Equally Weighted Portfolio with the 5 selected stocks (EW5)

Equally Weighted Portfolio with the initial 10 stocks (EW10)

For EW5, EW10, and the market portfolio, compute summary statistics (expected return and standard deviation) and compare their expected returns with those on the efficient frontier at the same standard deviation. Comment on the results.

- Identify a proxy for the risk-free asset and recalculate the efficient frontier by including it. Find the tangency portfolio and provide: portfolio weights, expected return e standard deviation
- Bonus (1 point): Based on the previous answer, find the optimal portfolio along the efficient frontier assuming a quadratic utility function. Determine this portfolio for two levels of risk aversion: A=3 and A=5. Plot the efficient frontier along with the indifference curves tangent to it at the two optimal portfolios. Comment on the results.

Part 2 - Beta Estimation and CAPM Validation

- Estimate the beta coefficients of your 10 stocks relative to your market portfolio using the most recent 60 observations. Test the significance of the coefficients, including the intercept (alpha). Discuss the results in light of CAPM predictions.
- Calculate adjusted betas using Blume's technique.
- Analyze and graphically illustrate how beta estimates change over time using all available observations. Apply a rolling window of 60 observations, shifting by 3-month intervals. Comment on the findings.