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Approximation Algorithms for Broadcasting and Gossiping

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Broadcasting and gossiping are two basic communication patterns which commonly occur when programming parallel and distributed systems. This paper deals with approximation algorithms for solving these problems on arbitrary topologies. We present new strategies to derive efficient broadcasting and gossiping algorithms in any networks in the telephone model. Our objective is to minimize both round complexity and step complexity. Broadcasting strategies are based on the construction of edge-disjoint spanning trees. Gossiping strategies are based on on-line computation of matchings of maximum weight. Our approximation algorithms for broadcasting offer almost optimal complexity when the number of messages to be broadcast is large. We show that our approximation algorithm for gossiping performs optimally in many cases. We also show experimentally that it performs faster than the best-known handmade algorithms in some particular cases.

1. INTRODUCTION

As is well known, intensive data communications among the nodes of a computer network can severely degrade the performance of a parallel application. Efficient communication libraries are therefore basic tools for the design of parallel and/or distributed algorithms. Among all the communication patterns appearing in parallel computation, *broadcasting* and *gossiping* are two famous information dissemination problems. Broadcasting corresponds to the problem in which a single node knows a piece of information that it wants to send to all the other nodes of the network. Gossiping corresponds to the problem in which all nodes know some piece of information, a gossip, that they want to exchange so that all the nodes will be aware of all of the information at the end of the process. We refer to [8, 10, 11] for surveys on broadcasting and gossiping in various models.

Unfortunately, but not surprisingly, the problem of finding the minimum number of rounds necessary to broadcast an atomic message from one processor of an arbitrary network to all the others is NP-hard [22] in the telephone model [10], even for particular class of graphs, see [19]. The same holds for the gossiping problem [3]. Therefore, it is of a major interest to derive approximation algorithms which allow estimation of the broadcasting and the gossiping complexities of a graph in a reasonable (i.e., polynomial) amount of time.

In this paper, we focus on two measurements: round and step complexities. We present a polynomial approximation algorithm for broadcasting large messages. This algorithm almost matches the lower bound of the problem. This result is based on the construction of λ edge-disjoint spanning trees in the considered graph (λ denotes the edge connectivity of the graph; that is, the minimum number of edges whose destruction disconnects the graph). Pieces of information move without conflict along the edges of the tree. For gossiping, we present an original strategy based on on-line computation of matchings, and gossiping proceeds by a succession of exchanges along the edges of the matchings. The matching used at round t of our approximation algorithm depends on the way the information flows during the $t-1$ previous rounds of the gossiping. Our algorithm is shown to work quite efficiently on particular graphs, and is experimentally shown to work very well on about all the usual examples of graphs used to interconnect processors of a parallel computer. We have even sometimes observed that our approximation algorithm performs faster than the best-known handmade algorithms for some particular graphs.

2. STATEMENT OF THE PROBLEM AND PREVIOUS RESULTS

Given a specific information dissemination problem $\mathcal{P}$ (as the broadcasting problem or the gossiping problem), our goal is to derive a general algorithm $\mathcal{A}$ which returns, for any graph G, an algorithm $\mathcal{A}[G]$ which solves $\mathcal{P}$ on G. The efficiency of an approximation algorithm $\mathcal{A}$ is measured by its time complexity (we are looking for polynomial approximation algorithms), and by the efficiency of the generated communication algorithm. More precisely, approximation algorithms were generally derived in the literature under the following communication constraints (usually called *telephone model*): Communications proceed by a serial–parallel sequence of synchronous *calls*. Many calls can be performed in parallel, but a given call involves exactly two neighboring vertices, and a vertex can participate in at most one call at a time. Two vertices participating in the same call can exchange all the information they are aware of at this time.

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The most usual way of estimating the efficiency of a communication algorithm in the telephone model is to count its number of *rounds*. A round is composed of the set of calls performed at a same given time. The round complexity of an information dissemination problem $\mathcal{P}$ on a graph G is the minimum number of rounds necessary to solve $\mathcal{P}$ on G. In particular, the round complexity of broadcasting from node x of graph G is denoted by $b_{\mathcal{R}}(G, x)$, and the round complexity of broadcasting in G is denoted by $b_{\mathcal{R}}(G)$, that is $b_{\mathcal{R}}(G) = \max_x b_{\mathcal{R}}(G, x)$. Similarly, the round complexity of gossiping in G is denoted by $g_{\mathcal{R}}(G)$. It is well known [8] that, for any graph of order n, $\log_2 n \le b_{\mathcal{R}}(G) \le g_{\mathcal{R}}(G) \le 2b_{\mathcal{R}}(G) - 1 \le 2n - 3$. The inequality $g_{\mathcal{R}}(G) \le 2b_{\mathcal{R}}(G) - 1$ comes from the fact that gossip can always be performed in two phases by first accumulating (a reverse broadcast) all the pieces of information in some particular vertex, and then broadcasting a single message (obtained by merging all the pieces of information) from this vertex to all the other vertices. For any broadcasting algorithm B, we denote by B^2 the corresponding two-phase algorithm for gossiping. We also have: for any graph of maximum degree Δ, and diameter D, $D \le b_{\mathcal{R}}(G) \le \Delta D$ because it is possible to broadcast in any shortest path spanning tree of G of depth D and maximum degree Δ in at most ΔD rounds.

Given an approximation algorithm $\mathcal{A}$ for the broadcasting problem (respectively, the gossiping problem), the number of rounds necessary to complete $\mathcal{A}[G]$ on a graph G is denoted by $b_{\mathcal{R}}(\mathcal{A}[G])$ (respectively, $g_{\mathcal{R}}(\mathcal{A}[G])$). Our general objective is to derive approximation algorithms for the broadcasting and the gossiping problems such that, for any graph G, $b_{\mathcal{R}}(\mathcal{A}[G])$ (respectively, $g_{\mathcal{R}}(\mathcal{A}[G])$) differs from $b_{\mathcal{R}}(G)$ (respectively, $g_{\mathcal{R}}(G)$) by a small multiplicative or additive factor.

The problem of finding approximation algorithms for broadcasting was initiated by Scheuermann and Wu [21] who presented some heuristics to solve this problem. Then, Kortsarz and Peleg [15] presented a polynomial approximation algorithm $\mathcal{A}_{KP}$ satisfying $b_{\mathcal{R}}(\mathcal{A}_{KP}[G]) \in O(b_{\mathcal{R}}(G) + \sqrt{n})$ for any graph G of order n. However, as pointed out by the authors, the algorithm in [15] is not efficient when applied to graphs of small broadcast time, namely when $b_{\mathcal{R}}(G) \in o(\sqrt{n})$. Ravi recently presented in [20] a polynomial approximation algorithm $\mathcal{A}_R$ satisfying $b_{\mathcal{R}}(\mathcal{A}_R[G]) \in O((\log^2 n/\log\log n) b_{\mathcal{R}}(G))$ for any graph G of order n. Finally, Feige *et al.* derived in [6] a simple, but quite efficient, distributed randomized broadcast algorithm $\mathcal{A}_{RB}$. Feige *et al.* showed that the *almost sure coverage time* $\hat{b}_{\mathcal{C}}(\mathcal{A}_{RB}[G])$ of a network G of order n, maximum degree Δ, and diameter D satisfies $\hat{b}_{\mathcal{C}}(\mathcal{A}_{RB}[G]) \in O(\Delta(D + \log n))$.

For the gossiping problem, Liestman and Richards [16] have proposed techniques based on coloring the edges of the graph (a color is given to each edge so that all incident edges have different colors). The technique of Liestman and Richards allows one to derive an approximation algorithm $\mathcal{A}_\chi$ satisfying that, for any graph G of maximum degree Δ, and diameter D, $g_{\mathcal{R}}(\mathcal{A}_\chi[G]) \le \Delta D + 1$. In [12, 13], Hromkovic *et al*. consider the so-called *systolic gossiping* introduced in [16]. However, except for particular graphs as cycles and trees, no general method was introduced to find the best edge coloring (or the best systolic sequence) for arbitrary graphs.

Note that one can think that since any algorithm B for the broadcasting problem gives rise to a solution B^2 for the gossiping problem, it is useless to look for a specific gossiping algorithm. However, the main problem of B^2 is that the number of messages exchanged increases exponentially during the accumulation phase, and that the message broadcast during the broadcasting phase has size n. Allowing messages of size $\Theta(n)$ might not reflect the real complexity of the problem. To take into account this fact, one must refine the study of the complexity of information dissemination problems under the telephone model by counting not only the number of rounds, but also the sum of the maximum number of pieces of information exchanged during each successive round. During a call between two vertices x and y, if k_x pieces of information (a piece is supposed to be atomic) are sent from x to y, and k_y pieces of information from y to x, then this call requires $k = \max\{k_x, k_y\}$ *steps*. A step is therefore the operation consisting of transmitting an atomic piece of information during a call between two vertices. A round requires the maximum, over all calls of the round, of the number of steps of these calls. The rounds being synchronized, r rounds require $\sum_{i=1}^{r} k_i$ steps where k_i is the number of steps of round i, for all i. The step complexity of an information problem $\mathcal{P}$ on a graph G is the minimum number of consecutive steps necessary to solve $\mathcal{P}$ on G. In particular, the step complexity of gossiping in G is denoted by $g_{\mathcal{S}}(G)$, and, given an approximation algorithm $\mathcal{A}$ for the gossiping problem, the number of steps necessary to complete $\mathcal{A}[G]$ on a graph G is denoted by $g_{\mathcal{S}}(\mathcal{A}[G])$. Clearly, for any graph of order n, $g_{\mathcal{S}}(G) \in \Omega(n + b_{\mathcal{R}}(G))$ since every vertex must receive at least $n-1$ pieces of information and broadcast its own message. Therefore, the two-phase gossiping algorithm obtained by accumulating and broadcasting yields a quite inefficient algorithm when considering the number of steps: $g_{\mathcal{S}}(B^2[G]) \in \Omega(n \times b_{\mathcal{R}}(G))$ for any broadcasting algorithm B because each round of the second phase (broadcasting) is composed of n steps. Also, when broadcasting a message which is not an atomic piece of information but composed of k pieces of information, then, again, algorithms derived to minimize the number of rounds might be inefficient. Indeed, let $b_{\mathcal{S}}(G, x)$ be the step complexity of a broadcasting from a node x of a graph G, and let $b_{\mathcal{S}}(G) = \max_x b_{\mathcal{S}}(G, x)$ be the step complexity of a broadcasting in G. We have $b_{\mathcal{S}}(G) \in \Omega(k + b_{\mathcal{R}}(G))$ when broadcasting a message of size k. On the other hand, for any broadcasting algorithm B, we have $b_{\mathcal{S}}(B[G]) \in O(k \times b_{\mathcal{R}}(B[G]))$ because each round of B is composed of k steps.

3. BROADCASTING

The following result shows that it is easy to design efficient algorithms which minimize the number of steps from efficient algorithms designed to minimize the number of rounds. This is particularly true when the size k of the message to be broadcast satisfies $k \in O(b_{\mathcal{R}}[G]/\Delta)$ where Δ is the maximum degree of the considered graph G.

THEOREM 1. *For any given polynomial time approximation algorithm B for broadcasting, there exists a polynomial time approximation algorithm $\mathcal{A}$ for broadcasting such that, for any graph G of maximum degree Δ, $b_{\mathcal{S}}(\mathcal{A}[G]) \in O(k\Delta + b_{\mathcal{R}}(B[G]))$, where k denotes the size of the message.*

Proof. Let G be any graph of maximum degree Δ. The algorithm $\mathcal{A}[G]$ proceeds as follows. Consider a broadcast tree T induced by B on G. The edges of T are labeled by the time at which they are used in B. Let $\ell(e)$ be the label of the edge e of the tree T. The i-th piece of information of the message is sent through e at time $(i-1)\Delta_T + \ell(e)$ where Δ_T is the maximum degree of T. The first piece is known by all the vertices after time $b_{\mathcal{R}}(B[G])$. Every Δ_T steps, a new piece is received by all the vertices. Thus this algorithm takes a total time of $b_{\mathcal{R}}(B[G]) + (k-1)\Delta_T$. ∎

However, for long messages ($k \gg b_{\mathcal{R}}(G)$), the result of Theorem 1 is Δ times worse than the bound $\Omega(k + b_{\mathcal{R}}(G))$. The next result shows that one can be much more efficient for long messages. Before stating this result, recall that the *chromatic index* of a graph G is the minimum number q such that it is possible to label all the edges of G with numbers in $\{0, \ldots, q-1\}$, each node having all its incident edges labeled with different numbers. Vizing's Theorem says that for any graph of maximum degree Δ, the chromatic index q of G satisfies $\Delta \le q \le \Delta + 1$. The bad news is that knowing whether $q = \Delta$ is NP-complete. The good news however is that finding a coloring of the edges of a graph G with at most $\Delta + 1$ colors is polynomial (from the proof of Vizing's Theorem). This result is the basis of Theorem 2.

The telephone model implies that when two nodes x and y of a graph G communicate during a call, they *exchange* their information. That is, links are *full-duplex*, and the underlying communication network G could also be represented by a symmetric digraph G^* where G^* denotes the digraph obtained from G by replacing every edge (x, y) of G by two symmetric arcs $\langle x, y\rangle$ and $\langle y, x\rangle$.

THEOREM 2. *There exists a polynomial time approximation algorithm $\mathcal{A}$ for broadcasting such that, for any graph G of maximum degree Δ and edge connectivity λ, $b_{\mathcal{S}}(\mathcal{A}[G]) = (\Delta+1)\lceil k/\lambda\rceil + t$, where k is the size of the message, and t is independent of k.*

Proof. Following Menger's Theorems (see [18]), Edmonds [4] (see also [17]) has shown that for any vertex r of a finite digraph H, there exists λ arc-disjoint spanning trees of H rooted at r where λ is the arc connectivity of the digraph H. If the edge connectivity of a graph G is λ, then λ is also the arc connectivity of G^*. Thus, for any vertex r of a finite digraph G^*, there exists λ arc-disjoint spanning trees of G^* rooted at r, that is, spanning trees of G where any edge (x, y) appears at most in two trees, each using (x, y) in a distinct direction. Let us call $T_1, T_2, \ldots, T_\lambda$ a family of such trees.

Assume the edges of G are colored with at most $\Delta + 1$ colors $c_1, c_2, \ldots, c_{\Delta+1}$. The broadcasting algorithm proceeds as follows: split the message of k pieces in λ packets $P_1, P_2, \ldots, P_\lambda$, each composed of at most $\lceil k/\lambda\rceil$ pieces. Packet P_i is broadcasted using T_i. Pieces of information are sent in a pipelined fashion, and edges colored c_j are used at every $\Delta + 1$ step, at time $j, j + \Delta + 1, j + 2(\Delta + 1), \cdots$.

Every $\Delta + 1$ rounds, at least one piece of each packet has been sent by the root. Therefore, after $(\Delta + 1)\lceil k/\lambda\rceil$ rounds, the root has completed its broadcast. In time, at most $(\Delta + 1)h$, where h is the maximum depth of the λ spanning trees, the remaining packets still in the network will reach their destinations. This additional time is independent of k. ∎

This result yields two remarks. First, if $\lambda = \Delta$ and if (by chance) the edge coloring obtained in polynomial time has Δ colors, then the approximation algorithm described in Theorem 2 is asymptotically optimal when k tends to infinity, and the lower bound $\Omega(k + b_{\mathcal{R}}(G))$ is reached (replace λ by Δ, and $\Delta + 1$ by Δ in the statement of Theorem 2). Note also that the term $t = o(k)$ in Theorem 2 is strongly dependent on the maximum depth of λ arc-disjoint spanning trees of a graph G^*. Unfortunately, minimizing this parameter has been shown to be NP-hard by Alon (see [1] and [7]).

4. GOSSIPING

Any gossiping algorithm under the telephone model can be described by a sequence of matchings (a matching is a set of pairs of neighboring vertices, i.e., a set of edges, such that no vertex belongs to more than one pair), each matching corresponding to a set of calls performed during the same round. Conversely, any sequence of matching $M_1, M_2, \cdots, M_r$ yields a communication algorithm described by: at round i, each pair of extremities of the edges of M_i exchange all the information they are aware of. It is therefore natural to describe gossiping algorithms by sequences of matchings. Our approximation algorithm for gossiping will thus proceed as follows: **repeat (1)** compute a matching M, and **(2)** exchange messages along the edges of M, **until** all the vertices get everything. This strategy depends on the way the matching M is chosen at each round. Note that, as opposed to the gossiping by edge coloring, and to the systolic gossiping, the matchings are not fixed in advance, but are computed online. We will present a solution in which the matchings are chosen to be of maximum weight. Recall that a matching of k edges is *maximum* if there does not exist any other matching with more than k edges. A matching is *maximal* if it is not possible to add an edge to this matching to get another matching. A matching is of *maximum weight* if the sum of the weights of its edges is maximum (a matching of maximum weight is not necessarily maximum, but if all the weights are positive, then a matching of maximum weight is maximal). In the following, the weight of an edge e is denoted by $w(e)$, and the matching chosen at phase i is denoted by M_i. We say that a generic communication algorithm converges to gossiping if, for any graph G, it completes a gossiping in G in a finite time.

In [9], we presented a first simple solution: at the beginning, all the weights are set to 1, and M_1 is a maximum matching. After each round, the weights evolve by multiplication by a fixed constant $\alpha > 1$. At round $i > 1$, if $e \notin M_{i-1}$ then $w(e) \leftarrow \alpha w(e)$, otherwise $w(e)$ is not modified. Although it was shown in [9] that this algorithm converges to gossiping, and although it was possible to give an upper bound on the round complexity of this algorithm, experimental results show that it is very unstable and much less efficient than the algorithm described later in this paper. In particular, making α too large implies that an edge used at round t will not be used again before a large number of rounds, and making α too small implies that an edge could be used successively many times whereas no information needs to be transferred on this edge.

Our approach here is formalized as follows. Again, during the algorithm, all matchings are chosen to be of maximum weight. At each phase $i \geq 1$, the weights are computed as follows. For every neighboring nodes x and y, let $\mathcal{I}_{x\to y}$ be the set of pieces of information that knows x but that does not know y. For any edge $e = (x, y)$, we set $w(e) = |\mathcal{I}_{x\to y}| + |\mathcal{I}_{y\to x}|$. The weight $w(e)$ represents the number of pieces of information that will cross e if e is selected in the matching of the current phase. We denote by DW (for Dynamic Weights) the corresponding approximation algorithm.

THEOREM 3. *The approximation algorithm DW converges to gossiping.*

Proof. Let $G = (V, E)$ be any connected graph. Let $\mathcal{I}_x^t$ be the set of pieces of information known by $x \in V$ after round t. Of course, $|\mathcal{I}_x^t| \leq n$, $\forall x \in V$, $\forall t \geq 1$. If there exists x such that $|\mathcal{I}_x^t| < n$ then $\sum_{x\in V} |\mathcal{I}_x^{t+1}| > \sum_{x\in V} |\mathcal{I}_x^t|$. Indeed, otherwise there is no communication at round $t+1$, that is, every vertex knows the same information as all its neighbors. This means that every vertex knows the same information ($\mathcal{I}_x^t = \mathcal{I}_{x'}^t$, $\forall x, x' \in V$). In other words, every vertex knows everything (because any vertex knows at least its gossip) which is in contradiction with $|\mathcal{I}_x^t| < n$. Therefore the total amount of information known by the vertices strictly increases at each round, and hence there exists t such that $|\mathcal{I}_x^t| = n$, $\forall x \in V$. ■

To give a flavor of the efficiency of the DW approximation algorithm, we have derived the following results (in both of these results, C_n and P_n denote the cycle and path on n vertices, respectively):

PROPERTY 1.

$$g_{\mathcal{R}}(DW[P_n]) = g_{\mathcal{R}}(P_n) = \begin{cases} n & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even} \end{cases}$$

and

$$g_{\mathcal{R}}(DW[C_n]) = g_{\mathcal{R}}(C_n) = \begin{cases} \dfrac{n}{2} & \text{if } n \text{ is even} \\ \dfrac{n-1}{2} + 2 & \text{if } n \text{ is odd} \end{cases}$$

Proof. The complexity of the DW algorithm is easy to compute for the path of any length, and the cycle of even size. In both cases, two matchings alternate and allow to complete a gossiping in the specified time. The cycle $C_{n+1} = \{x_0, x_1, \cdots, x_n\}$ of odd size must be considered separately. Indeed, at each round, only $n/2$ edges are selected (n is even). Without loss of generality, assume that the two consecutive edges not selected at the first round are (x_0, x_1) and (x_1, x_2). At the second round, one of these two edges is selected. Again, assume without loss of generality that it is (x_0, x_1). Then (x_1, x_2) and (x_2, x_3) are the two consecutive edges that are not selected at round 2. It is not difficult to see that, at round r, (x_{r-1}, x_r) and (x_r, x_{r+1}) are the two consecutive edges that are not selected. Thus the gossiping time of $DW[C_{n+1}]$ is $(n-1)/2+2$ for n even. (The optimality of the results can be checked easily [5].) ■

Note that the chromatic approximation algorithm $\mathcal{A}_\chi$ does not match this performance as it directly follows from the results in [16] that $g_{\mathcal{R}}(\mathcal{A}_\chi[C_n]) = \frac{3}{4}n + O(1)$. Similar results as Property 1 can be obtained for the step complexity:

PROPERTY 2.

$$g_{\mathcal{S}}(DW[P_n]) = \begin{cases} 2n-2 & \text{if } n \text{ is odd} \\ 2n-3 & \text{if } n \text{ is even} \end{cases}$$

and

$$g_{\mathcal{S}}(DW[C_n]) = \begin{cases} n & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even} \end{cases}$$

In all cases (but P_n, n odd), these results are optimal.

Proof. Bermond *et al*. have shown in [2] that $g_{\mathcal{S}}(P_n) = 2n-3$. When n is even, this bound is reached by $DW[P_n]$. Indeed, as we saw in the proof of Property 1, there are two selected matchings which alternate during the gossiping: one with $n/2$ edges, and another with $n/2-1$ edges. These matchings allow gossip in $2n-3$ steps. Unfortunately, $DW[P_n]$ is not optimal when n is odd, taking one time unit more than the optimal algorithm. Indeed, if n is odd, there are two matchings, both with $(n-1)/2$ edges, which alternate. This yields a gossiping time of $2n-2$ steps.

Also, Bermond *et al*. have shown in [2] that, for any Hamiltonian graph G of n vertices, $g_{\mathcal{S}}(G) = n$ if n is odd, and $g_{\mathcal{S}}(G) = n-1$ if n is even. During the gossiping $DW[C_n]$ on the even cycle, the first round consists of exchanging one

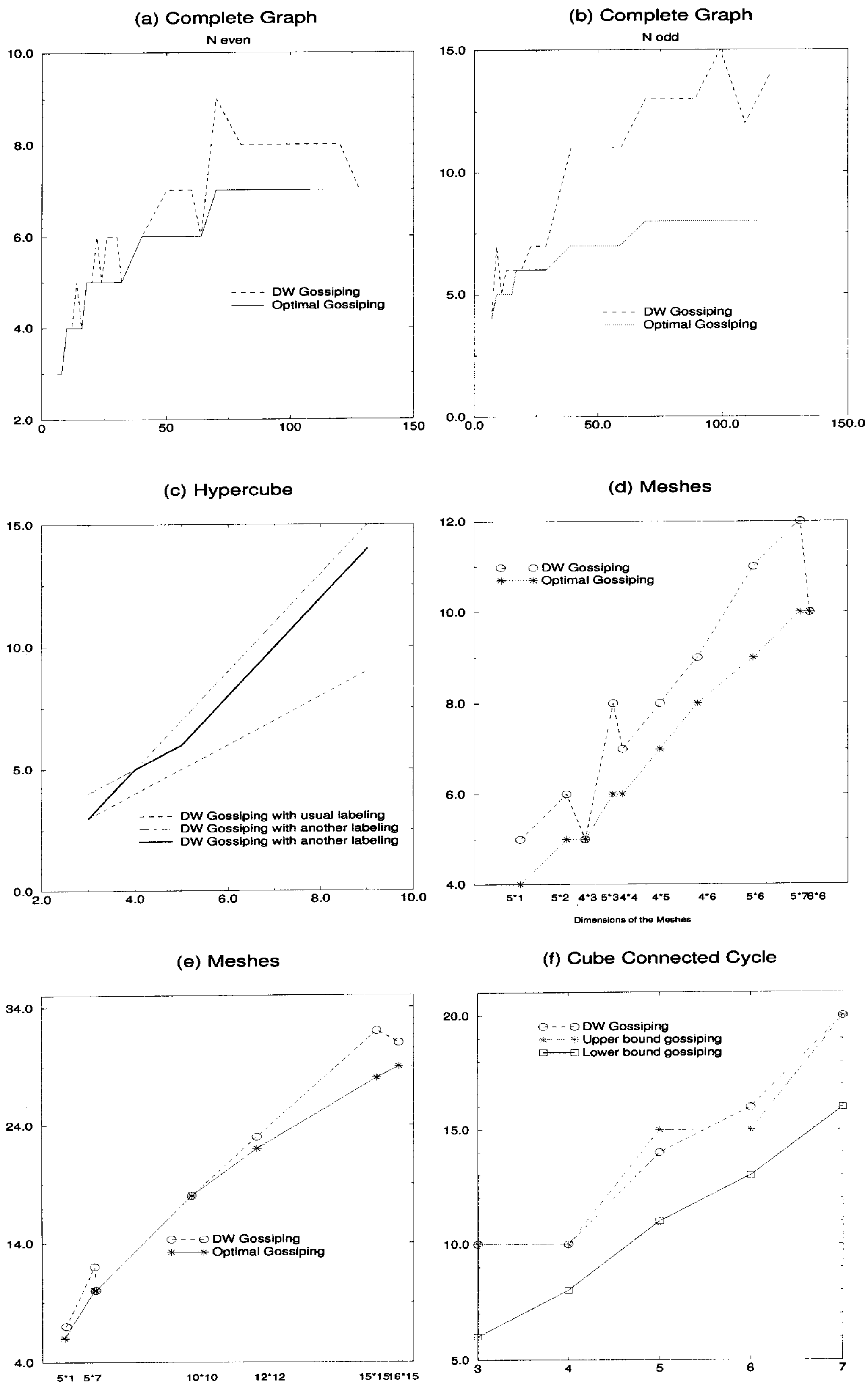

FIG. 1. Experimental round complexity of the *DW* approximation algorithm (part 1).

packet, and, during the $(n/2)-1$ other rounds, two packets are exchanged at each round. That gives $n-1$ steps in total. When n is odd, the matching chosen at each round is composed of $(n-1)/2$ edges. Moreover, at each round, two consecutive edges are not selected in the matching. It is easy to see, as in the proof of Property 1, that these pairs of edges "rotate" (say, clockwise) around the cycle. This phenomenon slows down the information flow going counterclockwise by 1 step, but does not slow down the information flow going clockwise. This yields a gossiping time of $1+2((n-1)/2) = n$ steps. ■

We have experimented with the DW approximation algorithm on several graphs as, in particular, the complete graph on n vertices K_n, the d-dimensional hypercube Q_d, the d-dimensional *cube-connected-cycle* CCC_d, the d-dimensional shuffle–exchange SE_d, the two-dimensional meshes and tori ($P_p \times P_q$ and $C_p \times C_q$), and the d-dimensional star graph S_d. All these graphs are often considered to be efficient interconnection networks for parallel computers. We present the results of our experiments below. We compare these results with those best known in the literature [8]. Of course our algorithm is not dedicated to these structures because it is often more efficient to derive a specific algorithm as soon as the topology of the network offers some regularity. However, these experiments allow us to estimate the efficiency of our algorithm by comparison with results derived analytically.

Figures 1 and 2 present results on the round complexity as a function of characteristic parameters of the graph (number of nodes, or dimension). Figure 3 presents results on the step complexity, again as a function of characteristic parameters of the graphs.

Note that the execution of the DW approximation algorithm depends on the chosen matching algorithm, and two different matching algorithms may produce different results using the DW approximation algorithm.

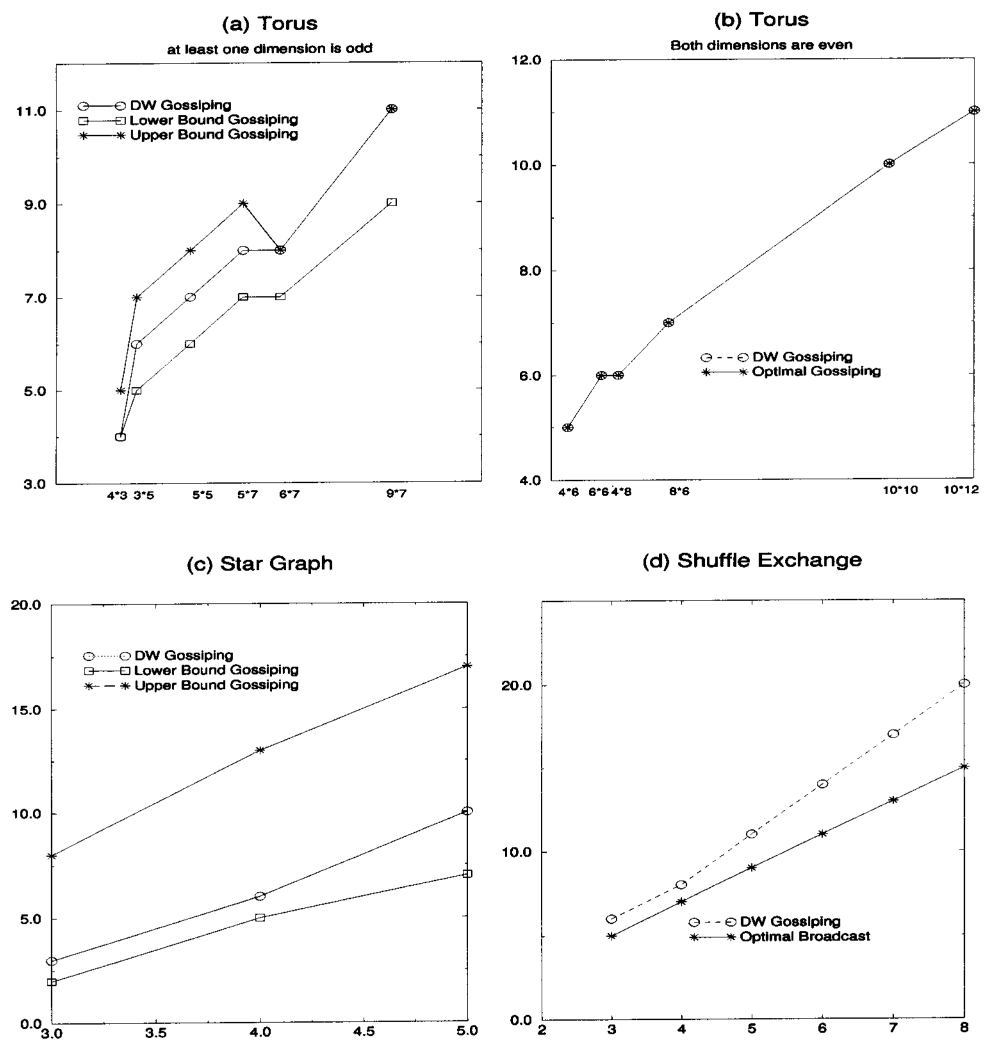

FIG. 2. Experimental round complexity of the DW approximation algorithm (part 2).

(a) *The complete graph K_n.* Figures 1a, 1b, and 3a give the results obtained on complete graphs. The DW algorithm performs quite well. The first even value of n for which the $DW[K_n]$ is not round optimal is 14 (Knodel [14] has shown that for n even, it is possible to gossip in $\lceil \log_2 n \rceil$ rounds). Our experiment shows that the DW algorithm performs well on K_{14} during the 3 first rounds (that is the global amount of information known by each vertex double at each round). Unfortunately, the way in which the information flows in the fourth round is not enough to complete the gossiping in K_{14}. For n odd, the general structure of the $DW[K_n]$ consists of a gossiping among $n - 1$ vertices plus a broadcast from the uninformed vertex to all the others. This at least doubles the usual gossiping time of $\lceil \log_2 n \rceil + 1$ for n odd [14] (this is what happened for $n = 9$).

(b) *The hypercube Q_d.* Figure 1c gives the experimental results obtained on the d-dimensional hypercube Q_d. These results strongly depend on the labeling of the vertices. For instance, for the "usual" labeling $\mathcal{L}$ (two nodes are linked by an edge if and only if their binary representations differ in exactly one bit), the DW gossiping algorithm performed optimally in terms of both round and step. However, Figure 1c shows that, for other labelings, the DW gossiping algorithm may be less efficient, even with a slight modification of the labels: $\mathcal{L}'(x) = (\mathcal{L}(x) + 13) \bmod n$ and $\mathcal{L}''(x) = (\mathcal{L}(x) + 7) \bmod n$. Of course, there is no reason why we should use such labelings for the hypercube, this is just to show that, given an arbitrary graph, the way we label its vertices matters. This might be a problem since there is often not any natural labeling of the vertices of an arbitrary graph.

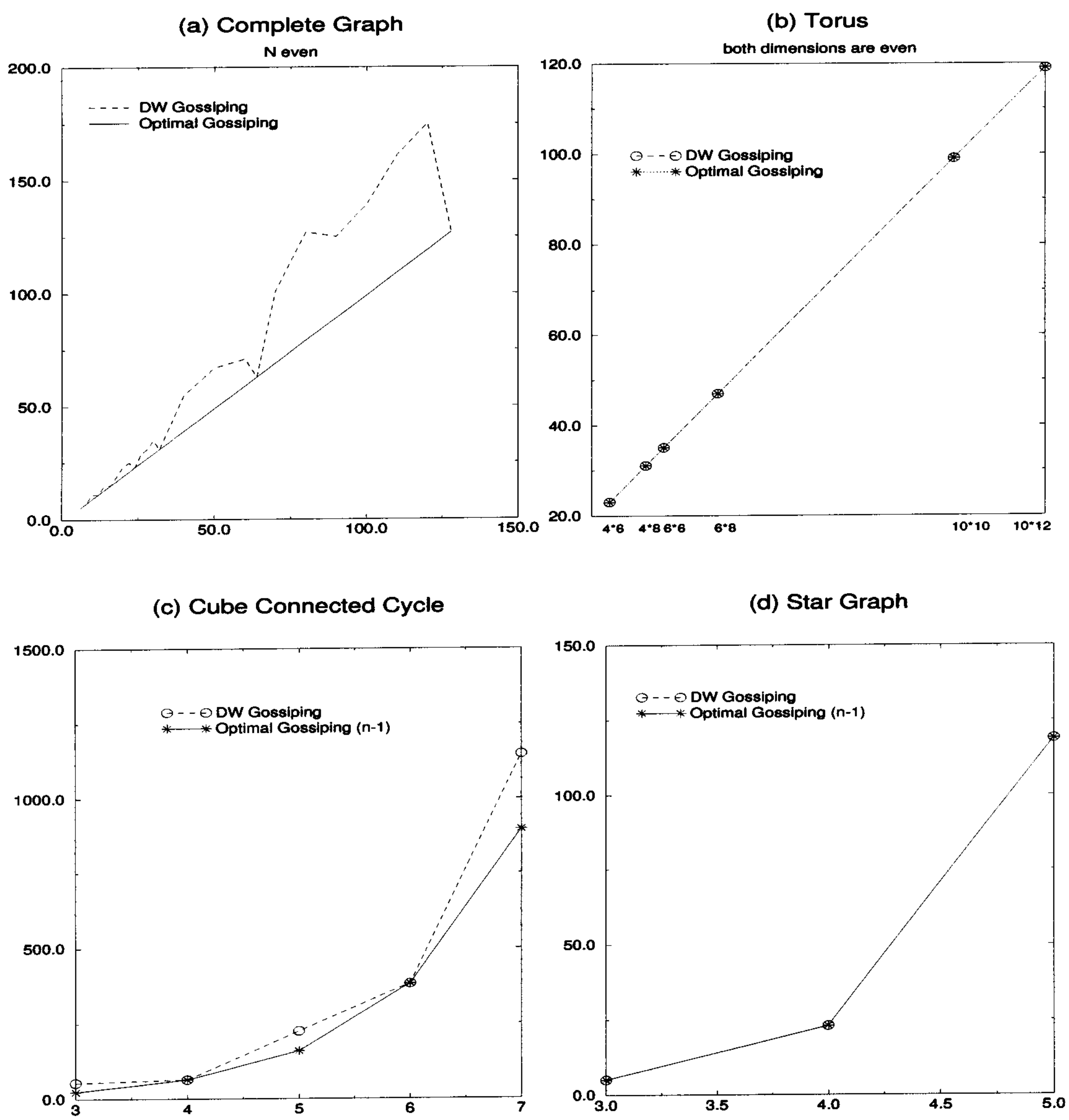

FIG. 3. Experimental step complexity of the DW approximation algorithm.

(c) *Meshes* $P_p \times P_q$. Figures 1d and 1e present experimental results on two-dimensional meshes. Compared to the optimal, the performances (in terms of rounds) are quite good: always less than the optimal time plus 2 in our experiments.

(d) *Torus* $C_p \times C_q$. Figures 2a, 2b, and 3b present experimental results on two-dimensional tori. In our experiments (which may have been helped by the usual labeling of the torus), we have always obtained optimal results (in term of both number of rounds and steps) when p and q are both even. In the general case, the round complexity of $DW[C_p \times C_q]$ is always between the known lower and upper bounds on the round complexity of the gossiping in $C_p \times C_q$.

Cube-connected-cycle CCC_d. Figures 1f and 3c present experimental results on d-dimensional CCC. The round complexity of $DW[CCC_d]$ is about the same as the best-known upper bound for this graph (it even performs faster in some cases!). The step complexity of $DW[CCC_d]$ is close to the general lower bounds of $n-1$ for even order, and n for odd order.

Star graph S_d. Figures 2c and 3d present experimental results on star graphs. The round complexity of $DW[S_d]$, $d = 3, 4, 5$, is much closer to the optimal than the best-known upper bound! Results on step complexities are also quite good. In fact, they are optimal for $d = 3, 4$, and 5.

Shuffle–exchange SE_d. Figure 2d presents experimental results on shuffle–exchange graphs. Determining the optimal gossip time for this graph is still an open problem. However, our results show that gossiping and broadcasting in the shuffle–exchange must have similar round complexities. Indeed, the round complexity of $DW[SE_d]$ is quite close to the round complexity of broadcasting in SE_d.

5. EXTENSIONS AND FURTHER RESEARCH

In this paper, we have presented approximation algorithms for broadcasting and gossiping. They can be applied practically for the design of efficient communication algorithms mainly in two cases: when the interconnection network is not fixed (for instance for "homemade" parallel computers constructed in assembling computation and routing chips), and when the physical or logical topology of the network can change dynamically (for instance for multi-users systems, or in case of possible faulty elements).

We can extend our results in many directions (see [9]). Indeed, they apply also to the store-and-forward routing model, and to the telegraph model (1-way mode). Moreover, they can also be generalized to the multi-scattering problem. Finally, our study gives rise to interesting questions and problems: first of all, it would be nice to derive tight analytic bounds on the step and round complexities of the DW-algorithm. Also, is there any more efficient way to choose the matching at each round of the gossiping? More generally, is it possible to improve the result of Ravi [20] for broadcasting atomic pieces of information?

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