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Heuristics for the Minimum Broadcast Time

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Abstract

The problem under study is the Minimum Broadcast Time (MBT). We are given a simple graph and a singleton that owns a message. The goal is to disseminate the message as soon as possible, where the communication takes place between neighboring-nodes in a selective fashion and each forwarding takes one time-slot. The MBT serves as an inspirational problem for the design of delay-sensitive forwarding schemes. Since the problem belongs to the $\mathcal{NP}$-Hard class, the literature offers heuristics, approximation algorithms and exact exponential-time solutions.

The contributions of this paper are two-fold. First, an ILP formulation for the problem is provided. Second, a competitive heuristic is developed. A fair comparison between TreeBlock and previous heuristics highlights the effectiveness of our proposal.

Keywords: Minimum Broadcast Time, Computational Complexity, Heuristics.

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1 Motivation

The identification of an ideal forwarding scheme is a challenging problem in telecommunications, not well understood. The scientific literature offers fluid models for massive communication systems [1], tree-like or one-to-one forwarding schemes [13], the tit-for-tat communication paradigm for incentivess [11], among others. The *Minimum Broadcast Time*, or MBT, finds original applications in telephonic services [5]. However, it serves as an inspirational problem for the design of current delay-tolerant forwarding schemes in modern communication systems like Content Delivery Networks (CDN) and Peer-to-Peer (P2P) [3] networks, or Cognitive Radio (CR) [12] networks.

A natural description of the problem is in terms of phone-calls (we can identify forwarding or gossiping, depending on the context). Suppose that a member originates a message which is to be communicated to all other members of the network. This is to be accomplished as quickly as possible by a series of calls placed over lines of the network. In the MBT, there are three strict broadcasting rules:

i Each phone-call requires one time-slot,

ii a member can participate in only one call per slot, and

iii a member can only call a neighbor member.

The problem under study is the following: given a connected graph G and a originator vertex v_0, what is the minimum number of time-slots required to complete broadcasting starting from v_0 and following the previous broadcasting rules?

There is a rich scientific literature available for this problem. Garey and Johnson include the MBT in the list of $\mathcal{NP}$-Complete problems [6]. The MBT of a complete graph $G = K_n$ is clearly $b(K_n) = \lceil log_2(n) \rceil$, yet K_n is not minimal with this property. The minimum number of links to achieve $b(K_n)$ is a relevant reverse-engineering problem with valuable progress. For instance, he hypercube Q_m has $m2^{m-1}$ links and optimum broadast time. The optimal forwarding in trees is recursively achieved in a greedy-like manner [15]. Grids accept an intuitive optimal forwarding scheme [10]. Several other particular graph families are studied, such as butterflies and Harary graphs [2]. A recent heuristic for the MBT is offered in [9]. The reader is invited to consult a literature-review in [14] and [8].

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This article is organized in the following manner. An Integer Linear Programming formulation is introduced in Section 2. *TreeBlock* heuristic is fully described in Section 3. A fair comparison with the most competitive heuristics is carried out in Section 4. Section 5 presents concluding remarks and trends for future work.

2 ILP for the MBT

For convention, we denote $V = \{0, 1, \ldots, n-1\}$ to the node-set, and we fix $v_0 = 0$ as the source-node. Denote $V(i)$ to the set of neighboring nodes of i. Let T be an upper-bound for the broadcast time ($T = n$ is a trivial one). Consider the set of binary variables x_{ij}^t that is set to 1 iff the message is transmitted from node i to node j during time-slot t. The ILP for the MBT is expressed as follows:

$$\min z \tag{1}$$

$$s.t$$

$$\sum_{j \in V(i)} x_{ij}^1 = \delta_0, \; \forall i \in \{0, \ldots, n-1\} \tag{2}$$

$$\sum_{j \in V(i)} \sum_{t=1}^{T} x_{ji}^t = 1, \; \forall i \in \{1, \ldots, n-1\} \tag{3}$$

$$\sum_{j \in V(i)} x_{ij}^\tau \leq 1, \; \forall i \in \{0, \ldots, n-1\}, t \in \{2, \ldots, T\} \tag{4}$$

$$x_{ij}^t \leq \sum_{\tau=1}^{t-1} \sum_{k \in V(i) \setminus \{j\}} x_{ki}^\tau, \; \forall (i,j) \in E, t \in \{2, \ldots, T\} \tag{5}$$

$$\sum_{t=1}^{T} t \cdot x_{ij}^t \leq z \, \forall (i,j) \in E \tag{6}$$

$$z \in \mathbb{N}; \; x_{ij}^t \in \{0,1\} \, \forall (i,j) \in E, t \in \{1, \ldots, T\} \tag{7}$$

Constraints 2 state that onle the source-node 0 sends the message when $t = 1$. Constraints 3 state that the message is received by each node exactly once. Constraints 4 state that a node transmits the message to a single neighbor. Constraints 5 state that a transmitter must receive the message first. Constraints 6 state that the broadcast time dominates any transmission. Finally, Constraints 7 set the variable domain. The computational order rests on a smart upper-bound T, which will be found using a dedicated heuristic.

3 Heuristic for the MBT

Recall that a node $v \in V(G)$ is a *cut-point* if $G - v$ has more components than G. A block is a maximal subgraph with no cut-points. Every connected graph accepts a decomposition into tree-blocks, where the blocks are linked in a tree-like structure [16].

The two main building blocks of our algorithm are Functions $BroadcastTree$ and $BroadcastBlock$. Essentially, $BroadcastTree$ is the optimal *interblock* forwarding in the tree-block structure, while $BroadcastBlock$ serves as an *intrablock* forwarding. A full forwarding strategy is obtained by a cooperation of both functions.

Algorithm 1 $F = TreeBlock(G, v_0)$

$U \leftarrow CutPoints(G)$
$U_0 \leftarrow \{v_0\} \cup U$
$d \leftarrow Distances(U_0)$
$T \leftarrow MST(U_0, d)$
$B \leftarrow Blocks(G)$
$L \leftarrow LeafBlocks(B)$
for all $B_i \in L$ **do**
  $z_i \leftarrow B_i \cap CutPoints(G)$
  $e_i \leftarrow max_{v \in B_i}\{d(w_i, v)\}$
  $w(z_i, B_i) \leftarrow e_i$
end for
$T \leftarrow T \cup \{(z_i, B_i)\}_{i=1,\ldots,|L|}$
$F \leftarrow BroadcastTree(T, v_0)$
for all $B_i \in B$ **do**
  $F_i \leftarrow BroadcastBlock(B_i, F(B_i))$
  $F \leftarrow F \cup F_i$
end for
return F

A step-by-step description of the algorithm $TreeBlock$ follows. All the cut-points U of G are determined in Line 1. The source-node is added to this special set in U_0 (Line 2). A complete weighted graph is built in Lines 3, where the corresponding weights are the distance between every pair of nodes in U_0. A minimum spanning tree T is found using Kruskal algorithm in Line 4. This tree serves as the *skeleton* of the interblock forwarding scheme. The weights in the links represent a *delay* between different cut-points. Observe that there is an additional delay with leaf-blocks, that should be added to reach all nodes in the graph. Therefore, in Lines 5 and 6, the blocks and leaf-blocks from G are found. Recall that a leaf-node has a single cut-point. In the block of Lines 7-11, the eccentricity e_i from the cut-point $z_i \in B_i$ is found for any leaf-block $B_i \in L$. This additional latency is considered by a tree-augmentation, in Line 12. The result is that the new nodes z_i are leaf-nodes in T, and the corresponding links have weights e_i. An inter-block forwarding scheme F is performed using Function $BroadcastTree$ with source-node v_0 in the tree T. Function $BroadcastTree$ returns the optimum forwarding scheme in this weighted tree. Finally, the intra-block forwarding is performed using Function $BroadcastBlock$ (see the last for-loop; Lines 14-17).

Algorithm 2 $F_i = BroadcastBlock(B_i, v'_1, v'_2, \ldots, v'_k)$

1: $T \leftarrow MST(v'_1, B_i)$
2: **for** $i = 1$ **to** k **do**
3: $\quad w_i \leftarrow \{|B_i|/i\}$
4: $\quad w(u_i, v'_i) \leftarrow w_i$
5: $\quad T \leftarrow T \cup w(u_i, v'_i)$
6: **end for**
7: $F_i \leftarrow BroadcastTree(T, v'_1)$
8: **return** F_i

Let us finally consider $BroadcastBlock$. It receives a block B_i and the expected forwarding order $F(B_i)$ of all its k cut-points, to know, $v'_1, v'_2, \ldots, v'_k$. Therefore, node v'_1 is the root-node. In Line 1, Kruskal algorithm is applied into B_i to find the minimum spanning tree, rooted at v'_1. In order to force the correct forwarding order in the cut-points, the weights in their incident links are modified as follows. The incident link to v'_i in T will be assigned a weight $w_i = |B_i|/i$ (Lines 3-4). The resulting tree is updated in Line 5. Finally, Function $BroadcastTree$ is applied into this tree in order to define an intra-block forwarding scheme (Line 7), and the result is returned in Line 8.

4 Proof of Concept

An extensive experimental analysis was carried-out in order to understand the effectivess of $TreeBlock$ heuristic and the capacity of our ILP formulation. Here we highlight a comparison with the latest heuristic we found from the literature [9]. The reader is invited to consult our technical report, with a performance analysis of the heuristic under Harary, Chord Harary, Butterflies, de Bruijn and Small World networks [4].

The broadcast in different grid graphs is successfully found using $TreeBlock$. In a celebrated work from Frank Harary, a family of graphs $H_{n,k}$ with maximum connectivity k is provided for any fixed number of links [7]. The authors from [9] confirm optimality in half of Harary instances under test. We found the optumum broadcast in 10 out of 16 instances, and unit-gap in the others. There are three instances of dense graphs where the exact ILP formulation cannot find the globally optimum broadcast (by a CPU time constraint of 2 hours).

Table 1
Harary. Broadcast scheme [2]

N	k	n	ILP	$TreeBlock$
17	2	17	9	9
17	3	17	5	5
17	5	17	5	5
17	6	17	5	5
17	7	17	5	5
30	2	30	15	15
30	3	30	9	9
30	8	30	5	6
30	9	30	5	6
30	10	30	5	6
50	2	50	25	25
50	3	50	14	14
50	11	50	-	7
50	20	50	-	8
50	21	50	-	7
100	2	100	50	50

5 Conclusions and Trends for Future Work

In this paper, the Minimum Broadcast Time is studied. An efficient ILP formulation and a heuristic is introduced. *TreeBlock* combines the optimal forwarding scheme in trees and an efficient block decomposition of a connected graph. The exact ILP works for small-sized graphs, while *TreeBlock* finds small gaps with respect to the globally optimum solution whenever it is available. Further analysis confirms its wide applicability for large instances as well, with a reasonable CPU time for graphs with thousands of nodes. The heuristic is competitive with respect to previous sub-optimal algorithms from the literature.

As future work, we want to find the broadcast in real-life topologies and have a better understanding of its applications in delay-tolerant systems.

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