Aprendizaje de Máquina

ITAM

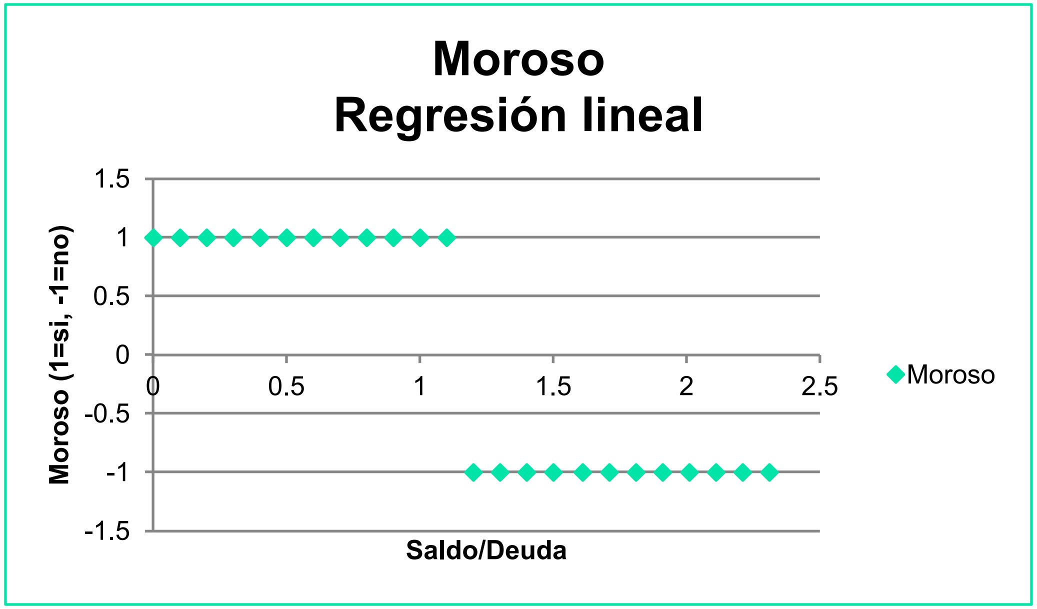
Semestre agosto-diciembre 2017

Menu

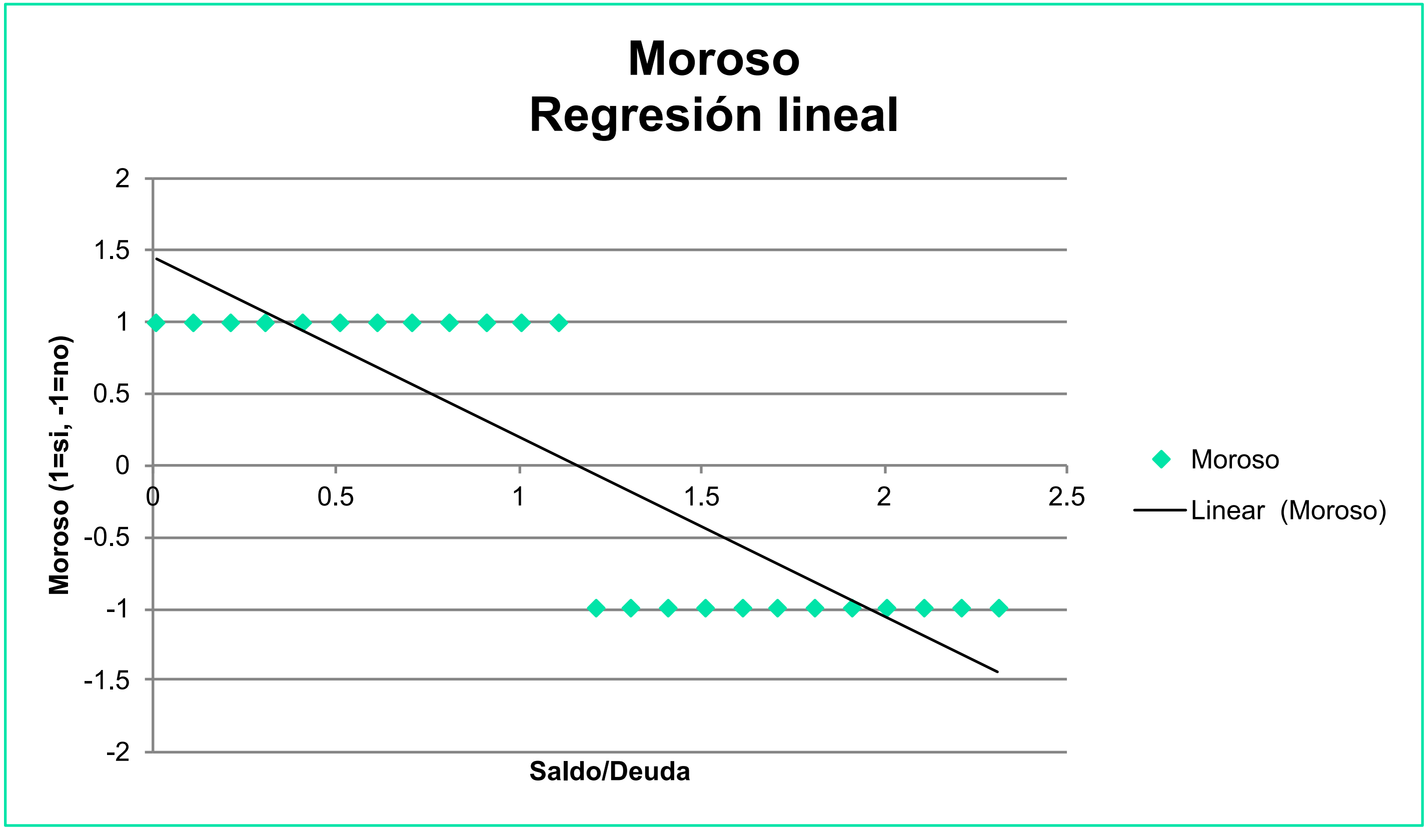
- In this session we will see how to use what we know about doing linear regression to perform classification
  - We are going to see the perceptron model

How to convert a regressor into a classifier

- Suppose we have the following data



How to convert a regressor into a classifier



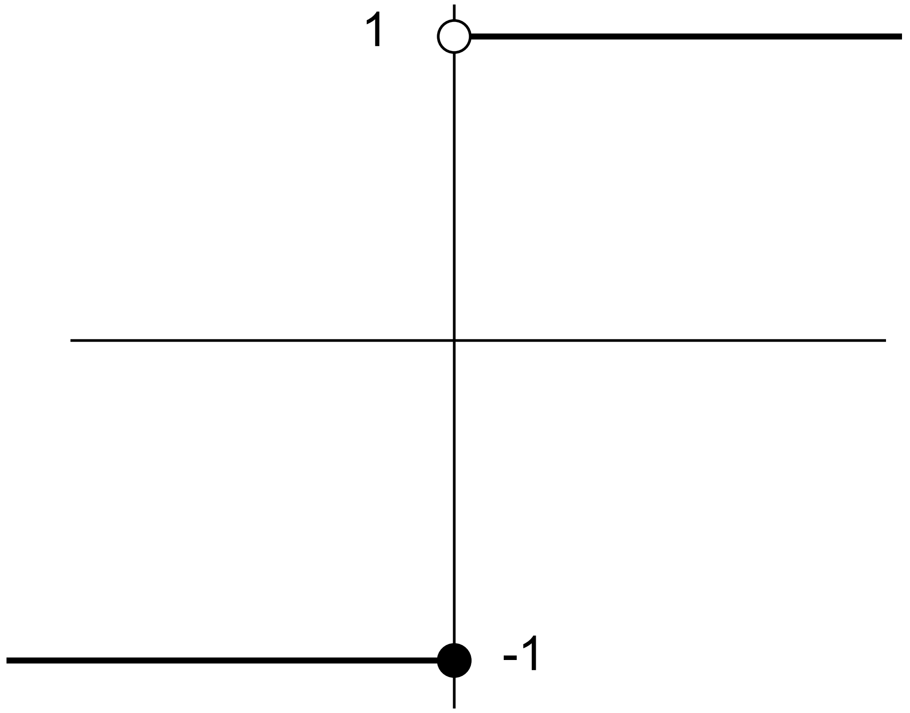
How to convert a regressor into a classifier

- It doesnt make much sense to allow our model to take values higher than 1 and lower than -1. There is no data with such values
- Solution: Limit the possible to this range via a transfer function---a function that takes the output of the regressor and transforms it into someting else

The perceptron: a model neuron

- The function that represents the activation of the perceptron is
  - $g(x_1,x_2,..,x_n)= \begin{cases} 1 \text{ si } w_o+\sum_{i=1,n} w_i x_i >=0 \\ -1 \text{ otherwise} \end{cases}$
  - We can think of w_o as a threshold value since it does not depende on an input variable.
  - We could say that the perceptron fires if there is enough stimulus in the input, if the weighed sum of the inputs is greater than $-w_o$.

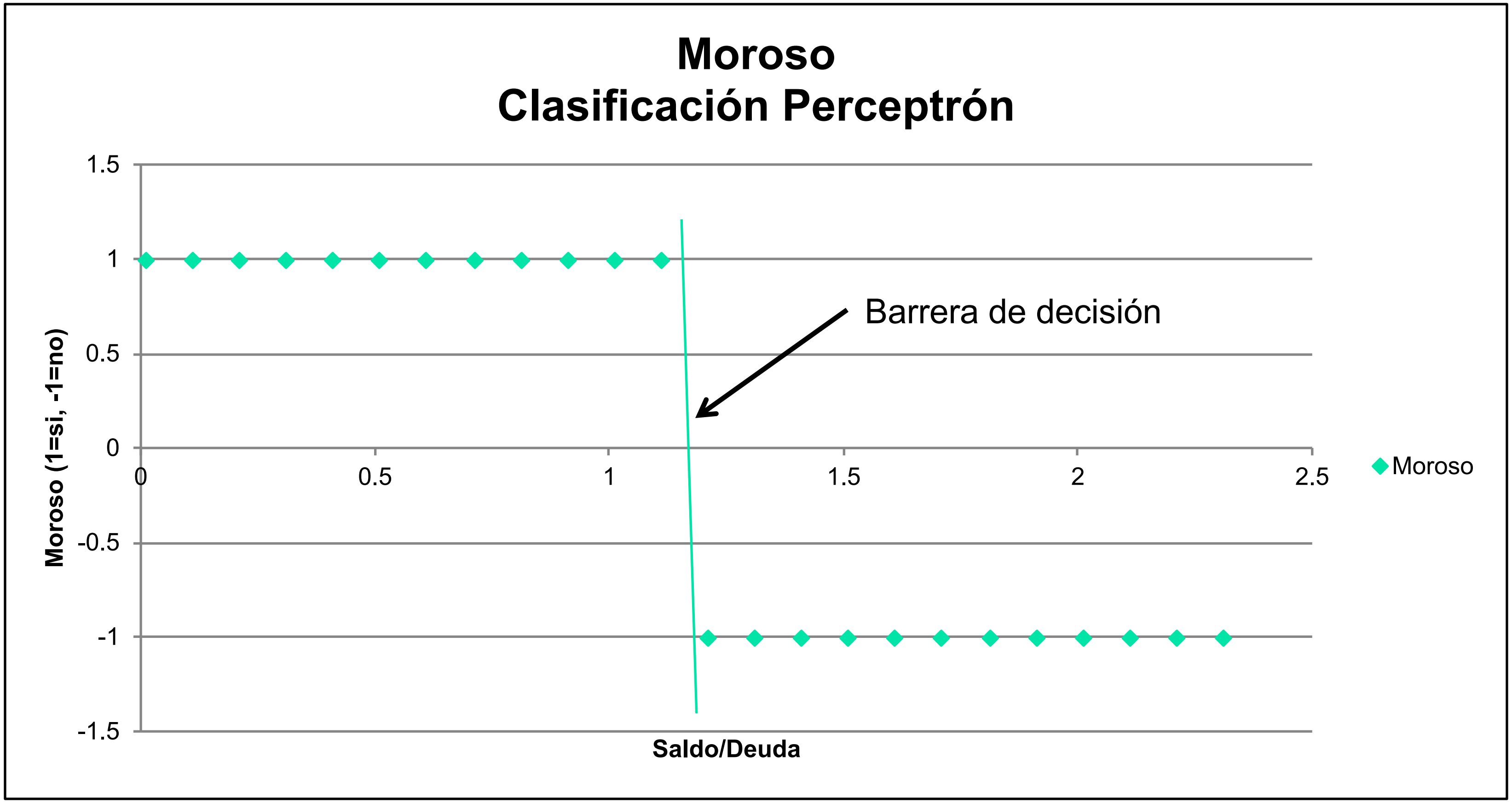
Perceptron
Transfer function: Step function



Representational ability Perceptron

- To ilustrate the representation power of the perceptron we can plot the equiation $\sum_{i=0,n} w_i x_i = 0$
- Since when $\sum_{i=0,n} w_i x_i$ is greater that or equal to zero it classifies an input as 1 and -1 otherwise
  - $\sum_{i=0,n} w_i x_i = 0$ represents a decision barrier

From the above example



Learning Algorithm Perceptron

- For each training example($\boldsymbol{X}$,y)
  - Calculate g with the current w's
  - For each w_i ,
    - w_i <---$w_i + \eta(y-g(\mathbf{X}))\ x_i$
- Where η is a small constant lower than 1 (learning constant)
- The rule is applied iteratively a fixed number of times or until the error reaches a desired value or if no further decrease in the error is detected
- Note again that the difference with the iterative regression is the function g

$$g(x_1,x_2,..,x_n)= \begin{cases} 1 \text{ si } \sum_{i=0,n} w_i x_i > 0 \\ -1 \text{ de otra forma} \end{cases}$$

Example

	X0	X1	X2	X3	X4	X5	X6
x´s	1	1	1	0	2	0	1
w´s	-1	-0.5	1	0.5	0	1	1
$x_i w_i$	-1	-0.5	1	0	0	0	1

Before without g: y=-1, V^(**X**)=0.5, Error=-1-0.5=-1.5, η =0.1

Now: y=-1, V^(**X**)=g(**X**)=1, **Error=-1-1=-2.0**, η =0.1

w0 = -1 + 0.1(-2.0)1=-1.2

w1 = -0.5 + 0.1(-2.0)1=-0.7

w2 = 1 + 0.1(-2.0)1 = 0.8

w4 = 0 + 0.1(-2.0)2 = -0.4

w6 =1 + 0.1(-2.0)1 = 0.8

Exercise

- Modify the iterative regression algorithm to include the step function as a transfer function
- Generate data for the logical and function

X1	X2	X1 and X2
0	0	0
0	1	0
1	0	0
1	1	1

- Train the perceptron this data set
  - Visualize the data
  - Plot the decision boundary
  - Calculate the classification error
    - Number of misses over number of examples

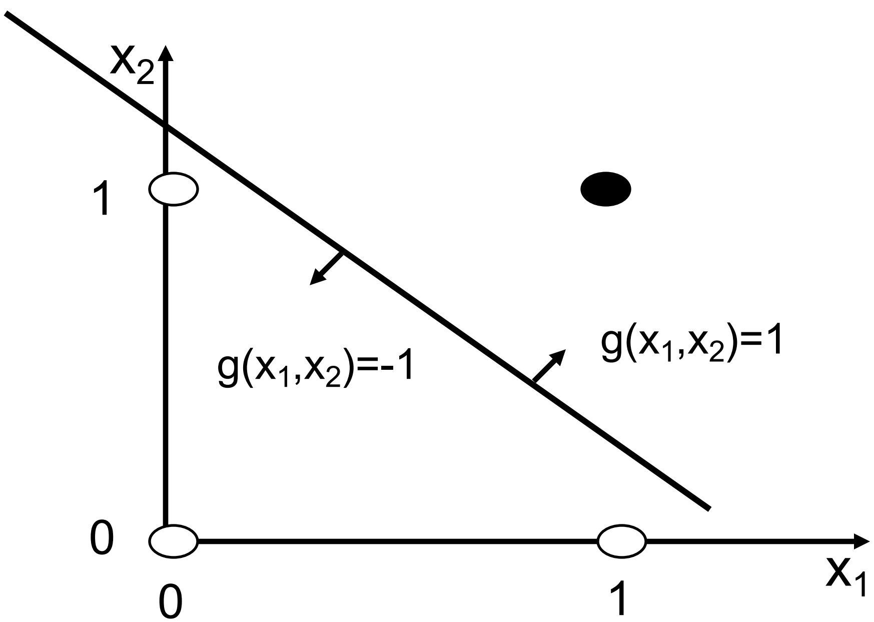
Exercise

- Repeat for the XOR function

X1	X2	X1 xor X2
0	0	0
0	1	1
1	0	1
1	1	0

- For those that finish early
  - Repeat the and exercise
  - How much does the separating hyperplane change from run to run?
    - Using or not using regularization (0.001)

Representation Power Perceptrón



- White and black circles belong to different categories