3.1

G : Gaussian kernel

$\mathcal{E}$: Sobel Edge detector

$\mathcal{M}$: Median filter

Since convolution has Commutative & Associative properties using G & $\mathcal{E}$ in different order gets same answer

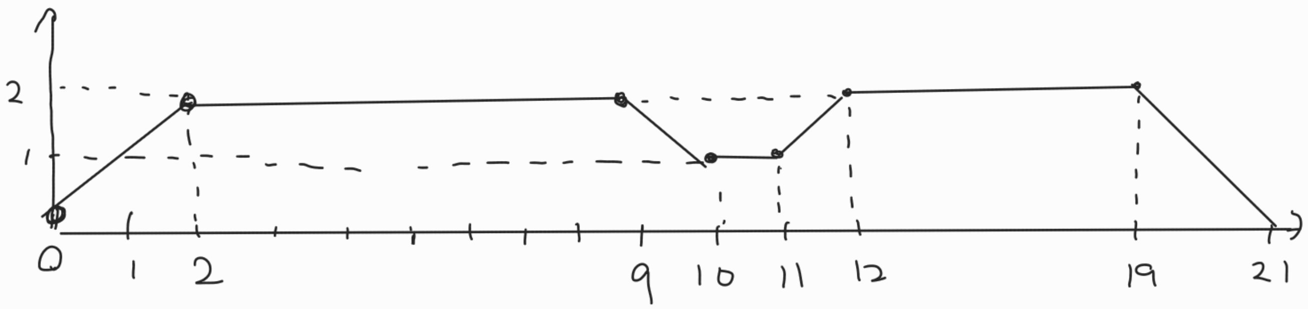
$$G * (\mathcal{E} * A) = (G * \mathcal{E}) * A = (\mathcal{E} * G) * A = \mathcal{E} * (G * A)$$

(Cascade system)

Howabout $\mathcal{M}$ isn't a linear Operation

So obviously, doing $\mathcal{M}$ and $\mathcal{E}$ will produce different output of doing $\mathcal{E}$ and $\mathcal{M}$

3.2



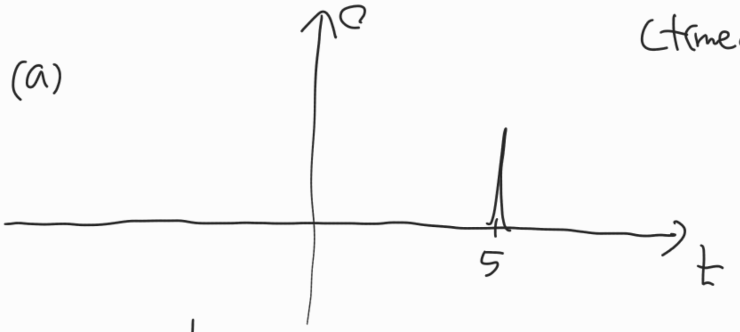
3.3

$$G(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

→ 2D Gaussian filter has $O(n^2)$ complexity when using 2 1D Gaussian filter has $O(2n)$ complexity which makes it much efficient to use $\mathcal{E}_x$ & $\mathcal{E}_y$

3.4 (a)

(time domain)



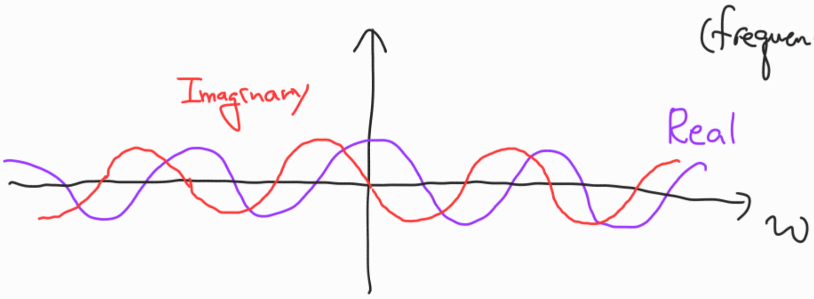
(frequency domain)

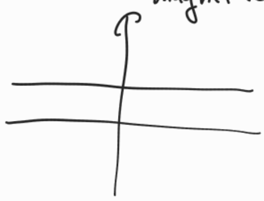
$$h(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx = \int_{-\infty}^{\infty} \delta(x-5) e^{-i\omega x} dx$$

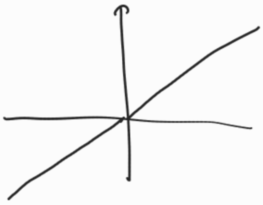
$$= \int_{-\infty}^{\infty} \delta(x-5) e^{-5i\omega} dx = e^{-5i\omega} \rightarrow \text{magnitude}$$

(frequency domain)

$1 = \text{magnitude}$ $\quad 5\omega = \text{phase}$







(b)

$$\underline{X_3(\omega)} = \int_{-\infty}^{\infty} x_3(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} \{a x_1(t) + b x_2(t)\} e^{-i\omega t} dt$$

$$= a\int_{-\infty}^{\infty} x_1(t) e^{-i\omega t} dt + b\int_{-\infty}^{\infty} x_2(t) e^{-i\omega t} dt = \boxed{a X_1(\omega) + b X_2(\omega)}$$