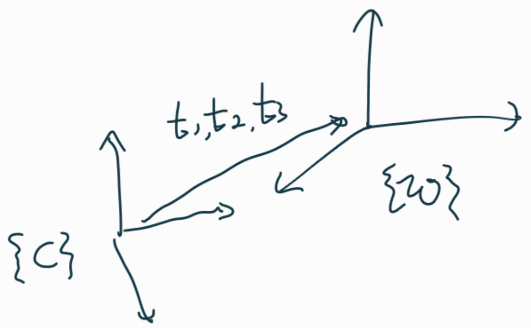
Question 1

(a) $\begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix}\begin{pmatrix} & & & t_1 \\ & R & & t_2 \\ & & & t_3 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix}\begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{matrix} ft_1 \\ ft_2 \\ t_3 \end{matrix}$

$\boxed{\therefore\ u = \dfrac{ft_1}{t_3} \qquad v = \dfrac{ft_2}{t_3}}$

(b) Since Origin of world be t_1, t_2, t_3 at $\{C\}$



O_c at $\{w\}$ be

$-R^{-1}\{t_1, t_2, t_3\}$

$= \boxed{-R^T t}$

(d)

$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K(R \mid t)\begin{pmatrix} dt + X \\ 1 \end{pmatrix}$

$= \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix}\begin{pmatrix} Rdt + RX + t \end{pmatrix}$

$\rightarrow$ let $RX = \begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix} \quad Rd = \begin{matrix} D_1 \\ D_2 \\ D_3 \end{matrix}$

$\rightarrow \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix}\begin{pmatrix} D_1t + X_1 + t_1 \\ D_2t + X_2 + t_2 \\ D_3t + X_3 + t_3 \end{pmatrix}$

$\rightarrow \boxed{\dfrac{f(D_1t + X_1 + t_1)}{D_3t + X_3 + t_3}, \ \dfrac{f(D_2t + X_2 + t_2)}{D_3t + X_3 + t_3}}$

$\rightarrow u, v$ forms linear line to!

(c)

Camera looking $[0, 0, 1]$

this vector be $R^T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \boxed{r_{31}\hat{i} + r_{32}\hat{j} + r_{33}\hat{k}}$

$= \begin{pmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} r_{31} \\ r_{32} \\ r_{33} \end{pmatrix}$

Question 2

$$X_i = [\, X_i \;\; Y_i \;\; Z_i \;\; 1 \,]^T$$

$$A^TA = V\Sigma^T U U^T \Sigma V^T$$

$$= V(\Sigma^T\Sigma)V^T$$

→ for the least solution

$\min \|AP\| \rightarrow \underline{\min P^TA^TAP}$

$= \underline{\min P^TV(\Sigma^T\Sigma)V^TP}$

→ let $V^TP = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

→ $\|AP\|^2$

$= \lambda_1 x_1^2 + \cdots + \lambda_n x_n^2$

& $x_1^2 + \cdots + x_n^2 = 1$

→ min when $x_1^2 = \cdots = x_{n-1}^2 = 0$

$\underline{x_n^2 = 1}$ //

$\therefore V^TP = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \rightarrow P = V\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$

$\therefore$ Last Column of V ($\because$ orthogonal!) $\underline{V^TV = I}$ //

Question 3

(a) $p_1^i = K[R_1 | t] X_i$

$$\begin{bmatrix} u_1^i \\ v_1^i \\ 1 \end{bmatrix} = A\begin{bmatrix} X_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_2^i \\ v_2^i \\ 1 \end{bmatrix} = A\begin{bmatrix} RX_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1^i \\ v_1^i \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} u_2^i \\ v_2^i \\ 1 \end{bmatrix}$$

$A: \begin{bmatrix} u_2^i & v_2^i & 1 & 0 & 0 & 0 & -u_2^iu_1^i & -v_2^iu_1^i & -u_1^i \\ 0 & 0 & 0 & u_2^i & v_2^i & 1 & -u_2^iv_1^i & -v_2^iv_1^i & -v_1^i \end{bmatrix}$

$b: \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Actually h isn't perfectly determined without $\|h\|$

(b) Since $Ah = b = 0$ $\quad \therefore Ah = 0 \rightarrow Ach = 0$ //

$A: 2n \times 9 \quad h: 9 \times 1 \quad b: 2n$

(if) more than ~~4~~ 5 points are given

→ h will 'be determined' or 'non exist' (if $\|h\| = 1$)

(c) To get the H

→ Since $\|Ah\|$ should be mean

→ h be the last column of V of $A = U\Sigma V^T$ //

(it will be exact solution if A contains 0

as a singular value)