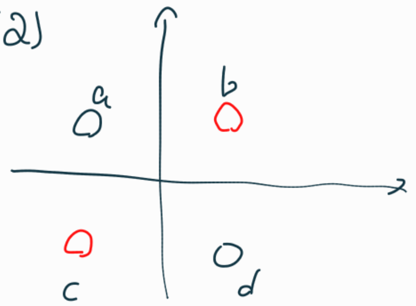
THEORY QUESTION

1.1 SVM 2-D space

$X_+ = \{(1,1), (-1,-1)\}$ $X_- = \{(1,-1), (-1,1)\}$

(a)



→ impossible since

a&d is in the same plane,

it must contain b or c at the

same plane ∴ impossible

(b) $\varphi(X_+) = \begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}$ & $\varphi(X_-) = \begin{pmatrix}1\\-1\\1\\-1\end{pmatrix}, \begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}$

(c) $w : \begin{pmatrix}a\\b\\c\\d\end{pmatrix}$

for $(1,1) \rightarrow$: $a+b+c+d$ $(+1)$

$(-1,-1) \rightarrow$ 〃 $a-b-c+d$ $(+1)$

$(1,-1)$ 〃 $a+b-c-d$ (-1)

$(-1,1)$ 〃 $a-b+c-d$ (-1)

→ to make for $y_i(w^T x_i) \geq 1$

→ $a+b+c+d \geq 1$ --- ①

$a-b-c+d \geq 1$ --- ②

$-a-b+c+d \geq 1$ --- ③

$-a+b-c+d \geq 1$ --- ④

$①^2 + ②^2 + ③^2 + ④^2 \geq 4$

↳ $a^2+b^2+c^2+d^2 \geq 1$

$\rightarrow \max\|w\| = 1$ when $a = b = c = 0$ & $d = 1$

$\therefore w = (0, 0, 0, 1)^T$

1.2 K-means (20 points)

(a)

$$d(x, c) = \sqrt{\sum_{i=1} \frac{(x_i - c_i)^2}{s_i^2}}$$

Since $(x_i - c_i)$ only correlated to $(x_i - c_i)$

$\rightarrow$ Covariance matrix Σ is diagonal matrix

It is so called scaled Euclidean distance

when thinking of x_i axis divided by s_i

for $\forall i \rightarrow$ it's Euclidean distance is same

as Mahalanobis distance

(b) $\mathcal{X} = \{0, 2, 4, 6, 18, 20\}$

when $c_1 = 3$, $c_2 = 4$

$$\begin{array}{cc|cccc} 0 & 2 & 4 & 6 & 18 & 20 \\ \multicolumn{2}{c|}{c_1} & \multicolumn{4}{c}{c_2} \end{array} \rightarrow \underline{c_1' = 1}, \quad \underline{c_2' = 12}$$

6.5

after changing

$0, 2, 4, 6 \rightarrow c_1'$

$18, 20 \rightarrow c_2'$

$C_1'' = 3$ / $C_2' = 19$

(c) $C_1 = 4 \rightarrow D(x) = \|x - 4\|^2$

$\rightarrow$ max at $x = 20$ $\therefore C_2 = 20$

$C_3 = \text{argmax}\left\{ D(x) = \min\left(\|x - 4\|^2, \|x - 20\|^2\right) \right\}$

$\rightarrow C_3 = 0$

$\therefore 4, 20, 0$