Alfred Cueva 2020-12738

1) a) $P(a|b,c) = P(b|a,c) \rightarrow P(a|c) = P(b|c)$

$P(A|(B\cap C)) = \dfrac{P((A\cap B)|C)}{P(B|C)}$

$\Rightarrow P(a|b,c) = \dfrac{P((a,b)|c)}{P(b|c)} \wedge P(b|a,c) = \dfrac{P((b,a)|c)}{P(a|c)}$

then $\dfrac{P(\cancel{(a,b)}|c)}{P(b|c)} = \dfrac{P(\cancel{(a,b)}|c)}{P(a|c)} \rightarrow P(a|c) = P(b|c)$ ∎

b) $P(a|b,c) = P(a) \Rightarrow P(b,c) = P(b)$

$P(a|b,c) = P(a) = \dfrac{P(a,b,c)}{P(b,c)} \rightarrow P(b,c) = \dfrac{P(a,b,c)}{P(a)} = \dfrac{P(a)P(b)P(c)}{P(a)} \Leftrightarrow P(a,b) = P(a)P(b)$

$P(b,c) = P(b)P(c)$

$P(a,b,c)$ would mean

using 3D boolean

C is T:

		B: T	B: F
A	T	1	1
	F	2	2

C is F:

	B: T	B: F
A: T	0	0
A: F	1	1

$P(A) = \frac{1}{4}$

$P(B) = \frac{1}{2}$

$P(C) = \frac{3}{4}$

$P(A,B) = \frac{1}{8} \wedge P(A)P(B) = \frac{1}{8}$ (if condition true)

BUT $P(A)P(C) = \frac{3}{16}$ $(\Rightarrow\Leftarrow)$ ∎

c) $P(a|b) = P(a) \longrightarrow P(a|b,c) = P(a|c)$

$a \rightarrow b \rightarrow c$ $\quad c \perp\!\!\!\perp b|a$

then:

$P(a|b,c) = \dfrac{P(a,b,c)}{P(b,c)} = \dfrac{P(a)\,P(\cancel{b|c})\,P(c|a)}{P(\cancel{b|c})\,P(c)} = \dfrac{P(\cancel{a|b})\cdot P(a,c)\cdot \cancel{P(c)}}{\cancel{P(c)}\cdot \cancel{P(a)}}$

$= P(a,c) = P(a|c)\,P(c) \neq P(a|c)$

$(\rightarrow\leftarrow)$ ∎

2) Full joint dist.

a) $P(\text{toothache}) = 0{,}108 + 0{,}012 + 0{,}016 + 0{,}064 = \underline{0{,}2}$

b) $P(\text{catch}) = 0{,}108 + 0{,}016 + 0{,}072 + 0{,}144 = \underline{0{,}34}$

c) $P(\text{cavity}|\text{catch}) = P(\text{cavity}, \text{catch}) / P(\text{catch}) = 0{,}108/0{,}34 = \underline{0{,}318}$

d) $P(\text{cavity} \mid \text{toothach} \vee \text{catch}) = \dfrac{\langle (0{,}108 + 0{,}012 + 0{,}072), (0{,}016 + 0{,}064 + 0{,}144) \rangle}{0{,}108 + 0{,}012 + 0{,}016 + 0{,}064 + 0{,}072 + 0{,}144}$

$= \underline{\langle 0{,}4615; 0{,}5384 \rangle}$

3)

- overall probability that exactly one bit is corrupted: $n\varepsilon(1-\varepsilon)^n$
- Prob of correct message: $(1-\varepsilon)^{n+1} + (n+1)\varepsilon(1-\varepsilon)^n \geq 1-\delta = 1-0{,}01 = 0{,}99$

$x \sim \text{bino}(n+1, 002)$

$P(\text{correct}) = P(x=0) + P(x=1)$

$$= \binom{n+1}{0} p^0 q^{n+1} + \binom{n+1}{1} p^1 q^{n+1-1}$$

Prob. of zero bits corrupted

Prob that one of n+1 bits is corrupted

$n \leq 73{,}7$ using wolfram

$\max(n) = 73$

4)

$$X \to Y \to Z$$

$X \perp\!\!\!\perp Z | Y \Rightarrow P(X,Z|Y) = P(X|Y)\,P(Z|Y)$

a) $$P(z|y) = \frac{P(z,y)}{P(y)} = \frac{\sum_x P(x,y,z)}{\sum_{x,z'} P(x,y,z')}$$

b) $$P(z|y) = \frac{\sum_x \theta(x)\,\theta(y|x)\,\theta(z|y)}{\sum_{x,z'} \theta(x)\,\theta(y|x)\,\theta(z'|y)}$$

c) $$P(z|y) = \frac{\theta(z|y)\sum_x \theta(x)\theta(y|x)}{\sum_x \theta(x)\,\theta(y|x)\;\sum_{z'}(z'|y)} = \frac{\theta(z|y)\sum_x \theta(x)\theta(y|x)}{\sum_x \theta(x)\,\theta(y|x)} = \theta(z|y)$$

d) $\theta(X_i|\text{Parents}(X_i)) = P(X_i|\text{Parents}(X_i))$ Proof:

$\sum_v \prod_i \theta(v_i|\text{Parents}(v_i)) = 1$, and as long as the variables remain to the right $\sum$ can be moved.

joint prob is $P(x_1 \cdots x_n) = \prod_{i=1}^{n} \theta(x_i|\text{Parents}(x_i)) \wedge$ by independence $P(x_1 \cdots x_n) = \prod_{i=1}^{n} P(x_i|\text{Parents}(x_i))$

then $\therefore\ \theta(x_i|\text{Parents}(x_i)) = P(x_i|\text{Parents}(x_i))$

5)

a) $P(B,E) = \sum_{a,j,m} P(B,E,a,j,m) = \sum_{a,j,m} P(B)P(E)P(a|B,E)P(j|a)P(m|a)$

$$= P(B)P(E)\sum_{a,j,m} P(a|B,E)P(j|a)P(m|a) = P(B)P(E)\underbrace{\cancel{\sum_a P(a|B,E)}}_{1}\underbrace{\cancel{\sum_j P(j|a)}}_{1}\underbrace{\cancel{\sum_m P(m|a)}}_{1}$$

$= P(B)P(E) \Leftrightarrow B \perp\!\!\!\perp E$

b) Verify $P(B,E|a) = P(B|a)P(E|a)$

$P(B,E|a) = \alpha P(a|B,E)P(B,E)$

→ then: $B \perp E$

$$\alpha \cdot \begin{cases} 0{,}95 \times 0{,}01 \times 0{,}02 & B=b,\ E=e \\ 0{,}29 \times 0{,}01 \times 0{,}02 & B=\neg b,\ E=e \\ 0{,}94 \times 0{,}01 \times 0{,}02 & B=b,\ E=\neg e \\ 0{,}01 \times 0{,}01 \times 0{,}02 & B=\neg b,\ E=\neg e \end{cases} = \alpha \cdot \begin{cases} 0{,}0008 \\ 0{,}2303 \\ 0{,}3728 \\ 0{,}3962 \end{cases}$$

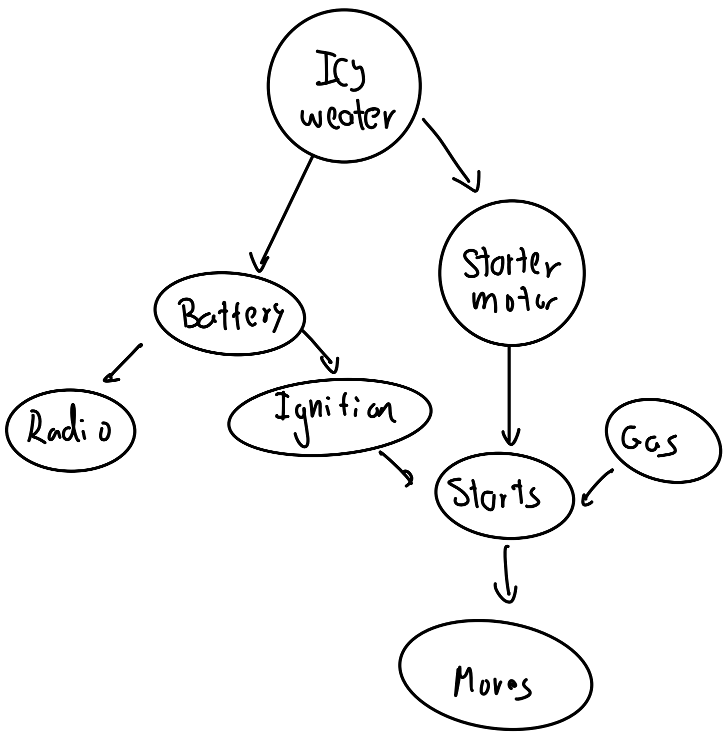
for $\alpha = 1$

$$P(b,e|a) = 0{,}0008 \neq 0{,}0863 = 0{,}3736 \times 0{,}231 = P(b|a)\,P(e|a) = \sum_E P(B,E|a) \cdot \sum_B P(B,E|a)$$

$0{,}0008 + 0{,}3728$

$0{,}2303 + 0{,}0008$

6) a)



Icy weather can't be influenced by car-related nodes.

b) Cond. Prob. table for all nodes

$P(\text{Icy weather}) = 0{,}06$

$P(\text{Battery} | \text{Icy weater}) = 0{,}94$ $\quad$ $P(\text{Battery} | \neg \text{Icy weath.}) = 0{,}996$

$P(\text{Starter} | \text{Icy weath.}) = 0{,}98$ $\quad$ $P(\text{Starter} | \sim \text{Icy weater}) = 0{,}999$

$P(\text{radio} | \text{battery}) = 0{,}9999$ $\quad$ $P(\text{radio} | \sim \text{battery}) = 0{,}06$ $\quad$ $P(\text{Gas}) = 0{,}995$

$P(\text{Ignition} | \text{battery}) = 0{,}998$ $\quad$ $P(\text{ignition} | \sim \text{battery}) = 0{,}01$

$P(\text{Starts} | \text{Ignition, Starter Motor, Gas}) = 0{,}9994$ else 0 $\quad$ $P(\text{moves} | \text{starts}) = 0{,}998$

c) no ind relations $2^n - 1$ → 2^5 are independent

because sum adds up to 1.

d) $1+2+2+2+2+1+8+2=20$ 2 per parent, 1 if node is only a parent

e) For start to happen all conditions should be true (other entries close to none)

$A \wedge B = \sim(\sim A \vee \sim B)$ Morgan Law, opposite of Noisy-OR

Noisy-AND:

$$P(\text{parents}(x_i) \mid x_1 \cdots x_k) = \prod_{\{i : x_i = \text{false}\}} r_i$$

the ith parent being false fails to make the child false.

Noisy-OR:

$$P(\text{Parents}(x_i) \mid x_1 \cdots x_k) = 1 - \prod_{\{i : x_i = \text{true}\}} q_i$$

: when presence of ith Parents makes the child be TRUE.

7)

8) a) & $P(S,R) = \sum_{C,W} P(S,R,C,W) = \sum_{C,W} P(C)\, P(S|C)\, P(R|C)\, P(W|S,R)$

$$= \sum_{C} P(C) P(S|C)\, P(R|C) \cancel{\sum_{W} P(W|S,R)}^{\,1}$$

$$= \sum_{C} \cancel{P(C)}\, P(C|S) \cdot \frac{P(S)}{\cancel{P(C)}}\, P(C|R)\, \frac{P(R)}{P(C)} = P(S)\, P(R) \sum_{C} \frac{1}{P(C)} P(C|S)\, P(C|R) \neq P(S) \cdot P(R)$$

not independant

& $P(S,R|C) = \dfrac{P(S,R,C)}{P(C)}$ $\sum_{W} P(S,R,C,W) = \sum_{W} P(C) P(S|C)\, P(R|C)\, P(W|S,R)$

$$= P(C) \cdot P(S|C) \cdot P(R|C)$$

$$= \frac{\cancel{P(C)} \cdot P(S|C)\, P(R|C)}{\cancel{P(C)}} = P(S|C) \cdot P(R|C) \text{ then } S|C \perp\!\!\!\perp R|C$$

b) $P(S,R|W) = \dfrac{P(S,R,W)}{P(W)} = \dfrac{P(S|C)\, P(R|C)\, P(W|S,R)}{P(W)}$ $S \not\perp\!\!\!\perp R|W$

$\therefore P(S,R|W) = \dfrac{0{,}0891}{0{,}7261} = 0{,}1227$ $P(S,R|\sim W) = \dfrac{0{,}0009}{1-0{,}7261} = 0{,}0033$

$P(s, \sim r | w) = 0{,}1639$ $P(\sim s, r | w) = 0{,}6941$ $P(s, \sim r | \sim w) = 0{,}0767$

$P(\sim s, r | w) = 0{,}46$ $P(\sim s, \sim r | \sim w) = 0{,}4600$ $P(\sim s, \sim r | \sim w) = 0{,}4600$

$P(w) = P(s, r, w) + P(s, \sim r, w) + P(\sim s, r, w) + P(\sim s, \sim r, w) = 0{,}7261$