SOLUTIONS

Queuing Examples (use Excel Template)

Joker Queuing:

⇒ At **Annual Fruit Cake Toss**, ***The Joker*** operates a face painting stand. Time between customer arrivals follows an exponential distribution with an average rate of 40 per hour. ***The Joker*** requires an average of 60 seconds per customer and follows an exponential distribution. Identify/answer the following:

- $\lambda =$ **40 customers/hour**.
- $\mu =$ **1 customer/60 seconds X 60 seconds / 1 minute X 60 minutes / 1 hour = 60 customers / hour**.
- What type of queuing system is this? **M/M/1 (Kendall Notation)**.
  Input above parameters into Queuing Template (Excel)
- On average, what fraction of ***The Joker***'s time will be idle? **P_0 = 0.333; or 1 minus utilization = 1 – 0.667 = 0.333**.
- Average number of customers in the system = **L = 2**.
- Average time a customer spends in the system = **W = 0.050 hours = 3 minutes**.
- Average time a customer spends in the line = **Wq = 0.033 hours = 2 minutes**.
- Average number of customers in the line = **Lq = 1.333**.
- What is $W – W_q$? **0.050 hours – 0.033 hours = 0.017 hours = 1 minute**.
  - What does this represent? **average service time (i.e., 1/μ)**.
- If ***Batman*** decides to help out ***The Joker*** where one line feeds the two of them, what type of queuing system is this? **M/M/2 (Kendall Notation)**.
  - Use **Excel template** to calculate metrics for this system! (Assume Joker and Batman work at same service rate).
    a. L = **0.750 customers**.
    b. L_q = **0.083 customers**.
    c. W = **0.019 hours = 1.125 minutes**.
    d. W_q = **0.002 hours = 0.125 minutes**.
    e. P(if you join line as customer right now, that you wait in line) = **0.1667**.

Notes for Part e:

Implication/Background: This means what is the probability that a customer who is arriving in the next instant has to wait. For this system, there are 2 servers. So, essentially what we are asking, what is the probability that both servers are being used? From our template probability table output, we can look at it and see, what is the probability that there are ***2 or more*** customers currently in the system? (Note, if we have 0 or 1 customers in the system, then at least one of our servers is available.)

Thus, my interpretation:

"Probability of wait?" <u>is the same as</u> "Probability that there are 2 or more customers in the system?"
$P(L \geq 2)=1 – P(L < 2) = 1 – P(L \leq 1) = 1 – (0.8333) = 0.1667 = 16.667\%$
Recall: Cumulative Probability is $P(X \leq \#)$. Be careful with the inequality versus strict inequality; and that we have a discrete situation (i.e., no half people, only integer values of customers in the system).

Now, if we asked "What is the probability that both the Joker and Batman are waiting at the same time for a customer?" Then that implies that the system is empty (i.e., has no customers). And that is 50%.

Factory Queuing:

⇒ On average, 40 jobs per day arrive at a factory. The time between arrivals of jobs is exponentially distributed. The factory can process an average of 42 jobs per day, and the time to process a job is exponentially distributed. Use the Excel template to answer the following questions:

- Using Kendall notation, what type of system is this? **M/M/1**.
- How long does it take before a job is completed (on average, measured from the time the job arrives at the factory)? **W = 0.5 days**.
- On average, how many jobs are waiting to be processed? **Lq = 19.048**.
- On average, what fraction of time is the factory idle? **P_0 = 0.0476**.
- What is the probability that a job will have to wait when it arrives?

  = 1 – P_0 = 1 – 0.0476 = 0.9524.

Franklin Post Office:

At the Franklin Post Office, patrons wait in a single line for the first open window. Patrons enter the post office at an average rate of 100 per hour, and each window can serve an average of 45 patrons per hour. The post office estimates a cost of $10 for each minute that a patron waits in line and believes that it costs $30 per hour to keep a window open. Interarrival times and service times are exponential. To minimize the total expected hourly cost, how many windows should be open?

- Customer arrival ***rate***, λ = **100/hr**.
- Service ***rate***, μ = **45/hr**. *(Note, the minimum number of servers required is 3.)*
- **C_s** = **$30/hr * s**
- **C_w** = **$10/min X 60 min/hr = $600/hr * L**
- **Total Cost** = **Cs + Cw**
- Optimal number of servers? **(6; see below; Use Data Table in Excel)**

s	TC =
3	2372.16
4	1628.18
5	1525.07
6	1523.47
7	1545.68

- At what service rate is the total cost for 5 servers and 6 servers equivalent? **45.46/hr**

Solution//

Arnold Muffler Example: Customers needing new mufflers arrive at the shop on the average of two per hour. Arnold's mechanic is able to perform this service at an average rate of three per hour, or about one every 20 minutes. Arrivals are Poisson and service times are exponentially distributed. The owner, studied queuing models in an MBA program and feels that all the conditions for a single-server queuing model are met. He proceeds to calculate the numeric values of the operating characteristics (i.e., the output measures) of his queuing system.

What are the inputs? $\lambda =$ **2 cust./hr** $\mu =$ **3 cust./hr** s=**1**

Using Excel template:

Find W **= 1 hr = 60 min.** L= **2 cust.** Wq = **0.667 = 40 min.** Lq = **1.333 cust.**

What fraction of time will the mechanic be idle? **= P_0 = 0.333**

What's the likelihood that there are five people in the system? **= P_5 = 0.0439**
(Note that this is exactly 5 people in the system. If we asked for 5 or less, then it would have been 0.9122 {the cumulative probability}.)

What's the likelihood that there are five people in line? **= P_6 = 0.0293**
(Note, it is P_6 since 1 customer's vehicle is with the mechanic.)
(If we had asked for five or less people in line, then it would have been 0.9415 *{the cumulative probability}.)*

Let's add a second server: How busy is each mechanic? **Utilization = 33.3%**

What happened to the average customer's time in the system?
Went down. Now it is W = 0.375 hr = 22.5 minutes.

What's the likelihood that there are five people in the system? **= P_5 = 0.0041**
(Note that this is exactly 5 people in the system. If we asked for 5 or less, then it would have been 0.9979 {the cumulative probability}.)

What's the likelihood that there are five people in line (think harder about this one!)?
= P_7 = 0.0005
(Note, it is P_7 since 2 customers' vehicles are with mechanics.)
(If we had asked for five or less people in line, then it would have been 0.9998 *{the cumulative probability}.)*