**Queuing Formulas (true for any system):**

Single Server: $\rho = \lambda / \mu$ Multiple Servers: $\rho = \lambda / (s\mu)$

**Formula for $L_q$ below assumes single server:*

$L = \lambda W$ $L_q = \lambda W_q$ $L_s = \lambda W_s = \rho$ $L = L_q + L_s$ $L = \dfrac{\rho}{1-\rho} = \dfrac{\lambda}{\mu - \lambda}$

$* \; L_q = \dfrac{\rho}{1-\rho} - \rho = \dfrac{\rho^2}{1-\rho}$

$W = W_q + W_s$ $W = \dfrac{L}{\lambda} = \dfrac{\rho}{\lambda(1-\rho)} = \dfrac{1}{\mu - \lambda}$ $W_q = \dfrac{L_q}{\lambda} = \dfrac{\lambda}{\mu(\mu - \lambda)}$

Formulas (Assuming Two Servers): $L_q = \frac{2\rho^3}{1-\rho^2}$ $L_s = \rho$ $L = \frac{2\rho^3}{1-\rho^2} + \rho$

$W_q = L_q / \lambda$ $W_s = 1 / \mu$ $W = (L_q / \lambda) + (1 / \mu)$

***Formulas for additional servers and other special cases available in various references.***

**M/M/1 Formulas: page 377 of** Managerial Decision Modeling with Spreadsheets, 3rd Edition by Balakrishnan, Render, and Stair.

## Operating Characteristic Equations for an M/M/1 Queuing System

We let

$\lambda$ = average number of arrivals per time period (e.g., per hour)

$\mu$ = average number of people or items served per time period

It is very important to note two issues here. First, both $\lambda$ and $\mu$ must be rates. That is, they must denote the average number of occurrences per a given time interval. Second, both $\lambda$ and $\mu$ must be defined for the *same time interval*. That is, if $\lambda$ denotes the average number of units arriving *per hour*, then $\mu$ must denote the average number of units served *per hour*. As noted earlier, it is necessary for the average service rate to be greater than the average arrival rate (i.e., $\mu > \lambda$). The operating characteristic equations for the M/M/1 queuing system are as follows:

1. Average server utilization in the system:

$$\rho = \lambda/\mu \tag{9-3}$$

2. Average number of customers or units waiting in line for service:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{9-4}$$

3. Average number of customers or units in the system:

$$L = L_q + \lambda/\mu \tag{9-5}$$

4. Average time a customer or unit spends waiting in line for service:

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu - \lambda)} \tag{9-6}$$

5. Average time a customer or unit spends in the system (namely, in the queue or being served):

$$W = W_q + 1/\mu \tag{9-7}$$

6. Probability that there are zero customers or units in the system:

$$P_0 = 1 - \lambda/\mu \tag{9-8}$$

7. Probability that there are $n$ customers or units in the system:

$$P_n = (\lambda/\mu)^n P_0 \tag{9-9}$$

**M/M/s Formulas: page 383 of** Managerial Decision Modeling with Spreadsheets, 3rd Edition by Balakrishnan, Render, and Stair.

## Operating Characteristic Equations for an M/M/s Queuing System

We let

$\lambda$ = average number of arrivals per time (e.g., per hour)

$\mu$ = average number of customers served per time *per server*

$s$ = number of servers

As with the M/M/1 system, with an M/M/s system, it is very important that we define both $\lambda$ and $\mu$ for the *same time interval*. It is also important to note that the average service rate $\mu$ is defined *per server*. That is, if there are two servers and each server is capable of handling an average of three customers per hour, $\mu$ is defined as three per hour, *not* six per hour ($= 2 \times 3$). Finally, as noted earlier, it is necessary for the average total service rate to be greater than the average arrival rate (that is, $s\mu > \lambda$).

The operating characteristic equations for the M/M/s queuing system are as follows:

1. Average server utilization in the system:

$$\rho = \lambda/(s\mu) \quad \textbf{(9-11)}$$

2. Probability that there are zero customers or units in the system:

$$P_0 = \frac{1}{\left[\sum_{k=0}^{s-1} \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k\right] + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{(s\mu - \lambda)}} \quad \textbf{(9-12)}$$

3. Average number of customers or units waiting in line for service:

$$L_q = \frac{(\lambda/\mu)^s \lambda\mu}{(s-1)!(s\mu - \lambda)^2} P_0 \quad \textbf{(9-13)}$$

4. Average number of customers or units in the system:

$$L = L_q + \lambda/\mu \quad \textbf{(9-14)}$$

5. Average time a customer or unit spends waiting in line for service:

$$W_q = L_q/\lambda \quad \textbf{(9-15)}$$

6. Average time a customer or unit spends in the system:

$$W = W_q + 1/\mu \quad \textbf{(9-16)}$$

7. Probability that there are $n$ customers or units in the system:

$$P_n = \frac{(\lambda/\mu)^n}{n!} P_0 \quad \text{for } n \le s \quad \textbf{(9-17)}$$

$$P_n = \frac{(\lambda/\mu)^n}{s!\,s^{(n-s)}} P_0 \quad \text{for } n > s \quad \textbf{(9-18)}$$