ECON7350 - Tutorial 8

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Breusch-Pagan Test: Hypotheses

The Breusch-Pagan test is used to test for heteroscedasticity in a regression model.

The null hypothesis is that the error variance is constant across all observations:

$$H_0 : \sigma_i^2 = \sigma^2 \quad \text{for all } i$$

The alternative hypothesis is that the error variance is not constant across all observations:

$$H_1 : \sigma_i^2 \neq \sigma^2 \quad \text{for some } i$$

Breusch-Pagan Test: Background

The Breusch-Pagan test is based on regressing the squared residuals from the original regression on the independent variables:

$$e_i^2 = \alpha + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + u_i$$

The null hypothesis of constant variance is equivalent to the null hypothesis that the coefficients on the independent variables are all zero:

$$H_0 : \beta_1 = \cdots = \beta_k = 0$$

The test statistic is the sum of squared residuals from this regression, which has a chi-squared distribution under the null hypothesis.

Breusch-Pagan Test: Steps

The Breusch-Pagan test is easy to implement in practice, and can be carried out using the following steps:

1. Estimate the original regression model.
2. Calculate the squared residuals and regress them on the independent variables.
3. Calculate the sum of squared residuals from the regression in step 2.
4. Test the null hypothesis using the chi-squared distribution with degrees of freedom equal to the number of independent variables.

If the p-value of the test is less than the chosen significance level, then we reject the null hypothesis of constant variance and conclude that there is evidence of heteroscedasticity in the model.

Breusch-Pagan Test: LM Multiplier and Chi-Square Distribution

The **LM multiplier** is defined as the sum of the squared coefficients from this regression:

$$LM = nR^2$$

where n is the number of observations and R^2 is the coefficient of determination.

Under the null hypothesis of constant variance, the **LM multiplier** has an asymptotic **chi-square** distribution with degrees of freedom equal to the number of independent variables.

ARCH/GARCH MODELS

Breusch-Pagan Test and ARCH/GARCH Models

The Breusch-Pagan test is a commonly used test for heteroscedasticity in a regression model.

If the test detects heteroscedasticity, then it may be appropriate to use a model that accounts for the heteroscedasticity, such as an ARCH/GARCH model.

ARCH/GARCH models are time series models that allow for the variance of the errors to vary over time. They are commonly used in finance and economics to model asset returns.

ARCH Models

ARCH stands for Autoregressive Conditional Heteroscedasticity. In an ARCH model, the variance of the error term at time t depends on the past squared errors:

$$y_t = \mu + \epsilon_t$$

$$\epsilon_t = \sigma_t e_t, \quad e_t \sim \mathrm{IID}(0, 1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2$$

where $\omega > 0$, $\alpha_i \geq 0$ for $i = 1, \ldots, p$, and p is the order of the model.

GARCH Models

GARCH stands for Generalized Autoregressive Conditional Heteroscedasticity. In a GARCH model, the variance of the error term at time t depends on both the past squared errors and the past variances:

$$y_t = \mu + \epsilon_t$$

$$\epsilon_t = \sigma_t e_t, \quad e_t \sim \text{IID}(0, 1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

where $\omega > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ for $i = 1, \ldots, p$ and $j = 1, \ldots, q$, and p and q are the orders of the model.