

ECON7350 - Tutorial 4 Solutions

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March 30, 2023

Engle-Granger Test

- If x_t and y_t are non-stationary and Order of integration $d=1$, then a linear combination of them must be stationary for some value of β and u_t . In other words:

$$y_t - \beta x_t = u_t$$

where u_t is stationary.

- If we knew β , we could just test it for stationarity with something like a **Dickey–Fuller test** and be done. But because we don't know β , we must estimate this first, generally by using ordinary least squares (**by regressing y_t on x_t and an intercept**) and then run our stationarity test on the estimated u_t series, often denoted \hat{u}_t .
- The null hypothesis of the ADF test is that the residuals have a unit root. Therefore, the Engle-Granger test considers the null hypothesis that there is no cointegration.

Question 1 Summary

- $i3y$: is not empirically distinguishable from $I(1)$
- $i5y$: is not empirically distinguishable from $I(1)$ (ambiguous).
- $i90d$: is empirically distinguishable from $I(1)$
- $i180d$: is not empirically distinguishable from $I(1)$ (ambiguous)

Question 3

$$i5y = \text{constant} + i3y + i90d + i180 + u$$

Assuming first stage regression residuals are mean-independent, there are five possibilities.

- ① $i3y$, $i5y$, $i90d$ and $i180d$ are all $I(0)$.
- ② Any three processes are $I(1)$ and cointegrated while a fourth is $I(0)$; for example, we could have $i3y$, $i5y$ and $i90d$ are cointegrated and $i180d$ is $I(0)$. The same could hold for any other combinations.
- ③ Any two processes are $I(1)$ and cointegrated while the other two are $I(0)$; for example, we could have $i3y$ and $i5y$ cointegrated while $i90d$ and $i180d$ are both $I(0)$.
- ④ Any two processes are $I(1)$ and cointegrated, and the other two processes are also $I(1)$ and cointegrated, but the four processes are not all cointegrated with each other in a single cointegration relation.
- ⑤ All four processes are $I(1)$ and cointegrated in a single cointegration relation.