Tutorial 9 - ECON7350

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TGARCH Model

- Definition: Time series model that allows for the volatility to be influenced by past shocks and current value of the series.
- Notation:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \lambda d_{t-1} \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

- Parameters: α_0, α_1, λ, β_1.

TGARCH Model (cont'd)

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \lambda d_{t-1} \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

- Explanation: Volatility affected by negative and positive shocks. Asymmetric response to negative and positive shocks.
- Indicator function: d_{t-1} takes value of 1 if ε_{t-1} is negative, 0 otherwise.
- Negative shock impact: $\lambda d_{t-1} \varepsilon_{t-1}^2$ added to conditional variance equation.
- Positive shock impact: None, as ε_{t-1}^2 already included.

TGARCH Model (cont'd)

- Application: Financial time series analysis, volatility clustering, asymmetric responses to shocks.
- Limitations: Assumes stationary conditional variance, may not capture all complexity and nonlinearity of underlying time series.
- **Leverage Effects**

Leverage Effects

- Definition: Asymmetric response of volatility to positive and negative shocks.
- In the TGARCH model:
  - Negative shocks increase conditional variance through $\lambda d_{t-1}\varepsilon_{t-1}^2$ and $\alpha_1\varepsilon_{t-1}^2$.
  - Positive shocks only increase conditional variance through $\alpha_1\varepsilon_{t-1}^2$.
- Interpretation: Negative shocks have a larger impact on future volatility than positive shocks of equal magnitude.

(G)ARCH-in-Mean

GARCH-M Model

- **Definition**: An extension of the GARCH model that allows for modeling asymmetric volatility, where negative shocks have a larger impact on future volatility than positive shocks of equal magnitude.

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t \\
\mu_t &= \beta + \delta\sqrt{h_t} \\
h_t &= \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{k=1}^{p} w_k r_{t-k}^2
\end{aligned}
$$

- δ: This parameter represents the strength of the relationship between the conditional mean and the conditional variance. A larger value of δ indicates a stronger relationship between the mean and variance, which can capture the time-varying risk premium effect.

GARCH-M Model (Cont'd)

- Allows for time-varying volatility
- Accounts for the mean structure
- Flexible enough to accommodate various types of conditional distributions

Example

Engle, *et al.* (1987) ARCH-M Estimates

The ARCH-M estimates are:

$$\begin{aligned}
\widehat{\mu}_t &= \underset{(-1.29)}{-0.0241} + \underset{(5.15)}{0.687}\sqrt{\widehat{h}_t}, \\
\widehat{h}_t &= \underset{(1.08)}{0.0023} + \underset{(6.30)}{1.64}\left(0.4\varepsilon_{t-1}^2 + 0.3\varepsilon_{t-2}^2 + 0.2\varepsilon_{t-3}^2 + 0.1\varepsilon_{t-4}^2\right).
\end{aligned}$$

Results:

- Estimate of 1.64 implies the unconditional variance is infinite (although conditional variance is finite).
- Risk premium is time-varying.
- During volatile periods, the risk premium rises as risk-averse agents seek assets that are conditionally less risky.