VECM vs. VAR

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Evaluation

VAR Model

- VAR stands for Vector Autoregressive Model.
- It is a multivariate time series model that captures the interdependencies among multiple variables.
- VAR models do not explicitly capture long-term relationships among variables.
- Example equation for a VAR(1) model with three variables (Y_1, Y_2, Y_3):

$$
\begin{aligned}
Y_{1,t} &= a_{11} Y_{1,t-1} + a_{12} Y_{2,t-1} + a_{13} Y_{3,t-1} + u_{1,t} \\
Y_{2,t} &= a_{21} Y_{1,t-1} + a_{22} Y_{2,t-1} + a_{23} Y_{3,t-1} + u_{2,t} \\
Y_{3,t} &= a_{31} Y_{1,t-1} + a_{32} Y_{2,t-1} + a_{33} Y_{3,t-1} + u_{3,t}
\end{aligned}
$$

where $Y_{i,t}$ represents variable i at time t, a_{ij} are coefficient parameters, and $u_{i,t}$ is the error term.

VECM Model

- VECM stands for Vector Error Correction Model.
- It is a special case of VAR models with an added error correction term.
- VECM models are suitable when variables are found to be non-stationary (integrated) of the same order.
- VECM models capture both short-term dynamics and long-term equilibrium relationships.

Rank in VECM

- Rank refers to the number of cointegrating relationships among the variables.
- If the rank is zero, there are no cointegrating relationships, and the variables are purely stationary.
- If the rank is equal to the number of variables, all variables are cointegrated.(VECM is equivalent to VAR)
- If the rank is between zero and the number of variables, there are some cointegrating relationships present.
- The rank determines the number of error correction terms in the VECM model.

Differences between VECM and VAR Models

- VAR models do not explicitly capture long-term relationships, while VECM models do.
- VECM models include an error correction term that captures the adjustment mechanism towards long-run equilibrium.
- VECM models are suitable for integrated variables, whereas VAR models can be used for stationary or integrated variables.
- VECM models can be estimated using the Johansen procedure, which tests for cointegration among variables, while VAR models do not explicitly model cointegration.

Example

The Vector Error Correction Model (VECM) is a generalization of the ECM that we use to analyze cointegrated systems.

Example: $n = 3$, $p = 2$, $r = 2$:

$$\begin{pmatrix} z_{1,t} \\ z_{2,t} \end{pmatrix} = \begin{pmatrix} 1 & \beta_{21} & \beta_{31} \\ \beta_{12} & 1 & \beta_{32} \end{pmatrix} \begin{pmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{pmatrix} = \begin{pmatrix} x_{1,t} + \beta_{21} x_{2,t} + \beta_{31} x_{3,t} \\ \beta_{12} x_{1,t} + x_{2,t} + \beta_{13} x_{3,t} \end{pmatrix} \sim \mathrm{I}(0).$$

Granger Representation Theorem

Let $\mathbf{x}_t$ be $n \times 1$ and $r = \operatorname{rank} \mathbf{A}(1)$. Suppose that for each $x_{i,t}$ in $\mathbf{x}_t$, $i = 1, \ldots, n$, either $x_{i,t} \sim \mathsf{I}(1)$ or $x_{i,t} \sim \mathsf{I}(0)$ holds.

1. If $r = n$, then $\mathbf{x}_t \sim \mathsf{I}(0)$.
2. If $0 < r < n$, then $\mathbf{A}(1) = -\boldsymbol{\alpha}\boldsymbol{\beta}'$ where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n \times r$ full rank matrices, and
   1. one or more variables in $\mathbf{x}_t$ is characterised by an $\mathsf{I}(1)$ process;
   2. if $\mathbf{x}_t \sim \mathsf{I}(1)$, then $\boldsymbol{\beta}'\mathbf{x}_t \sim \mathsf{I}(0)$, with cointegrating vectors given by the columns of $\boldsymbol{\beta}$.
3. If $r = 0$, then $\mathbf{A}(1) = 0$ and $\mathbf{x}_t \sim \mathsf{I}(1)$ but not cointegrated.

The coefficients in $\boldsymbol{\alpha}$ measure how the elements in $\mathbf{x}_t \sim \mathsf{I}(1)$ are adjusted to the r equilibrium errors in each period; that is the speed of adjustment to long-run equilibria.

As before, if the cointegration rank is r then there exist $n - r$ stochastic trends driving the n $\mathsf{I}(1)$ processes in $\{\mathbf{x}_t\}$.