ECON7350 - VAR Models

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12/05/2023

Summary

We will discuss vector autoregressive (VAR) models.

- VAR models are a type of time series model that can be used to study the relationships between multiple variables.
- **Cholesky decomposition** is a method for decomposing a covariance matrix into a product of lower triangular matrices.
    - **Impulse response functions** are a graphical representation of the effects of a shock to one variable on other variables in a VAR model.
- **Granger causality** is a statistical concept that measures the causal relationship between two time series
- **FEVD** is a tool for decomposing the forecast error variance of a time series into the portions attributable to each of its determinants. determinants.

VAR models

- A VAR model is a linear regression model that regresses each variable in a system on its own lags and the lags of all the other variables in the system. For example, a VAR(1) model with three variables would have the following form:

$$y_t = \beta_{0,1} y_{t-1} + \beta_{1,1} x_{t-1} + \beta_{2,1} z_{t-1} + u_t$$
$$x_t = \beta_{0,2} y_{t-1} + \beta_{1,2} x_{t-1} + \beta_{2,2} z_{t-1} + v_t$$
$$z_t = \beta_{0,3} y_{t-1} + \beta_{1,3} x_{t-1} + \beta_{2,3} z_{t-1} + w_t$$

- Where y_t, x_t, and z_t are the three variables in the system, u_t, v_t, and w_t are error terms, and $\beta_{i,j}$ are the coefficients of the model.

Cholesky decomposition

- The Cholesky decomposition is a method for decomposing a covariance matrix into a product of lower triangular matrices. The Cholesky decomposition of a covariance matrix Σ is written as $\Sigma = LL^T$, where L is a lower triangular matrix.
- The Cholesky decomposition can be used to calculate the impulse response functions of a VAR model.

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- The Cholesky decomposition can be used to calculate the impulse response functions of a VAR model.
- The impulse response function of a VAR model is a graphical representation of the effects of a shock to one variable on other variables in the system.
  - Let Σ be the covariance matrix of a VAR model.
  - Let L be the Cholesky decomposition of Σ.
  - The impulse response function of the VAR model is calculated as $h = L^T$.

Impulse response functions

- The impulse response function of a VAR model is a graphical representation of the effects of a shock to one variable on other variables in the system.
- The impulse response function is calculated by first decomposing the covariance matrix of the VAR model using the Cholesky decomposition. The impulse response function is then calculated by multiplying the Cholesky decomposition by a vector of unit shocks.
- **CAUTION for Cholesky Decomposition:** *IRF is sensitive to orderings of variables.*

Granger Causality

- Granger causality is a statistical concept that measures the causal relationship between two time series.
- It is based on the idea that if a time series X "Granger-causes" another time series Y, **then past values of X should contain information that helps predict current values of Y better than past values of Y alone.**
- In other words, if the inclusion of past values of X in a regression model improves the prediction of Y, then we say that X Granger-causes Y.

Forecast Error Variance Decomposition (FEVD)

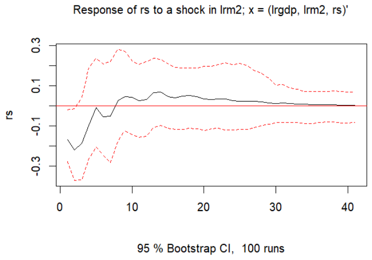
- FEVD is a tool for decomposing the forecast error variance of a time series into the portions attributable to each of its determinants.
- It helps to understand the relative importance of each variable in the forecast.
- FEVD is a useful tool for policy analysis, as it allows policymakers to evaluate the impact of changes in specific variables on the forecast.

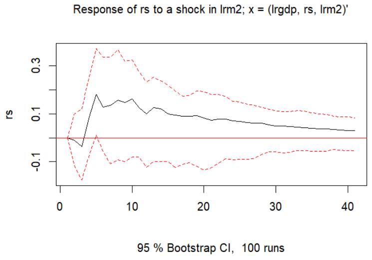
Question 2.a

Results

$(lrgdp_t, lrm2_t, rs_t)$ vs. $(lrgdp_t, rs_t, lrm2_t)$: We observe similar patterns in impulse responses, but note that the response of interest rates to a change in money supply is fairly different within the two orderings. When money supply is ordered prior to interest rates, there is a significant contemporaneous **response of interest rates to a change in money supply**; no significant response is observed in the alternative case (money supply prior to interest rates).

Results

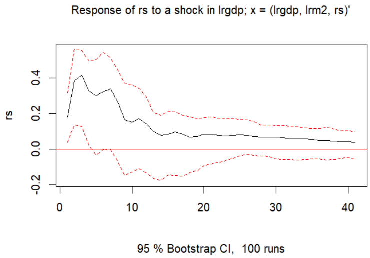

Response of rs to a shock in lrm2; x = (lrgdp, lrm2, rs)'
rs
0.3
0.1
-0.1
-0.3
0
10
20
30
40
95 % Bootstrap CI, 100 runs


Response of rs to a shock in lrm2; x = (lrgdp, rs, lrm2)'
rs
0.3
0.1
-0.1
0
10
20
30
40
95 % Bootstrap CI, 100 runs

Results

$(lrgdp_t, lrm2_t, rs_t)$ vs. $(lrm2_t, rs_t, lrgdp_t)$: These two orderings exhibit patterns that are very similar to IRFs in Part (i). However, **the response of interest rates to a change in GDP** is larger when GDP is ordered prior to interest rates, than the other way around.

Results


Response of rs to a shock in lrgdp; x = (lrgdp, lrm2, rs)'
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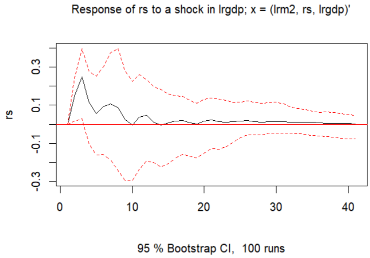
0.2
0.0
-0.2
0
10
20
30
40
95 % Bootstrap CI, 100 runs


Response of rs to a shock in lrgdp; x = (lrm2, rs, lrgdp)'
rs
0.3
0.1
-0.1
-0.3
0
10
20
30
40
95 % Bootstrap CI, 100 runs

Results

$(rs_t, lrgdp_t, lrm2_t)$ vs. $(rs_t, lrm2_t, lrgdp_t)$: The impulse response functions (IRFs) are similar for both orderings and do not exhibit significant sensitivity to the order of GDP and money supply when interest rates are ordered first.

Which Ordering?

- Data is not informative on which ordering is most suitable, so we need to draw on economic theory if we are to focus on one particular ordering.
- A conventional approach used in analysing dynamic responses to monetary policy shocks (e.g., interest rates) separates all non-policy variables into *fast-moving* and *slow-moving* variables.
- Then, all *slow-moving* variables are ordered prior to the interest rate variable and all fast-moving variables are placed after.

Which Order?

- Typically, fast-moving variables are taken to be financial indicators and asset prices, whereas slow-moving variables are those related real economic activity.
- In the literature, it has been shown that under reasonable conditions the particular ordering within groups of slow-moving and fast-moving variables is not important for the purpose of drawing inference on the response of economic variables to a change in interest rates.

So?

- In our setting, one might argue that GDP is certainly a slow-moving variable, so it should definitely be ordered before interest rates.
- It is also reasonable to assume that GDP does not respond to money supply within one quarter. It may be less clear on how to classify money supply.
- Fortunately, the comparison of IRFs carried out in Part (i) suggests that it may not matter much on whether money supply is ordered prior to interest rates or vice-versa.

So?

- If we focus on two orderings that put rs last, i.e., (lrgdp, lrm2, rs) and (lrm2, lrgdp, rs), we observe that (as theory suggests) all responses to interest rates are indistinguishable, and the IRFs for the other impulses / responses are also "qualitatively" the same.
- Therefore, inference drawn based on these IRFs may be considered to be robust to changes in the ordering of GDP and money supply (which is reassuring in case our intuition that GDP does not respond to money supply within one quarter fails).

Caution

- It is tempting to look at the IRFs and choose an ordering (or more generally set of identifying restrictions) based on what yields the **most reasonable** results.
- This, however, is circular reasoning—by undertaking such an approach we are simply finding **the right method** that confirms what we hypothesised before seeing the data. Such an approach has been shown to lead to very dangerous conclusions!