Comparing Methods of Stellar Luminosity Function Construction Through Interpolation of Stellar Isochrones

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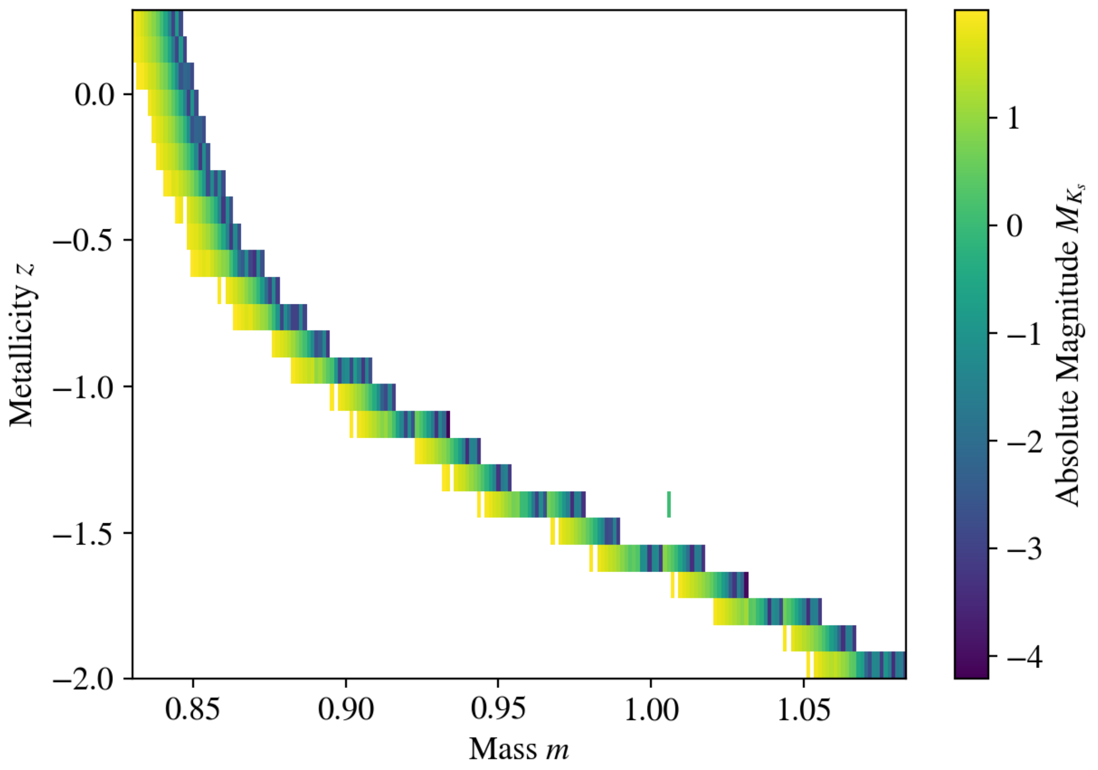
Luminosity Functions & Isochrones

- Investigations into Galactic structure often rely on a luminosity function.
  - Can help to estimate stellar distances using standard candles.
- Luminosity functions can be constructed using isochrones.
  - Sets of values describing a general relationship between several stellar variables (e.g. mass, metallicity, absolute magnitude etc.).
    - Metallicity refers to the abundance of elements present in a star heavier than hydrogen or helium on a logarithmic scale.
    - Absolute magnitude is a measure of a star's luminosity on an inverse logarithmic scale.
- Previous investigations have used a Monte-Carlo method based on repeated sampling for luminosity function construction.
- Paterson et al. (2019) presents an alternative method using a statistical change of variables. Denoted the *semi-analytic* method.
- We compare these two methods in the context of a consistency check of the luminosity function of Paterson et al. (2019).



Interpolating From the Isochrone

- Need to be able to treat the isochrone as a continuous function.
  - Mass & Metallicity $\rightarrow$ Magnitude
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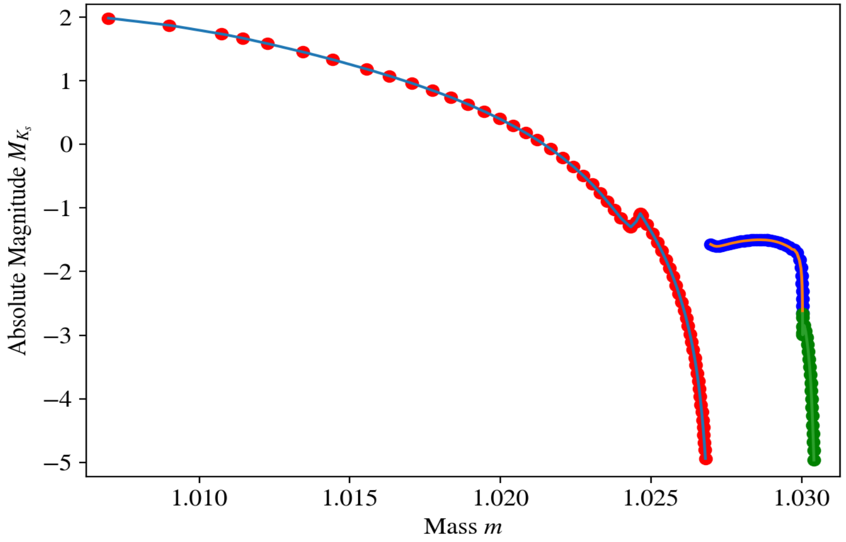
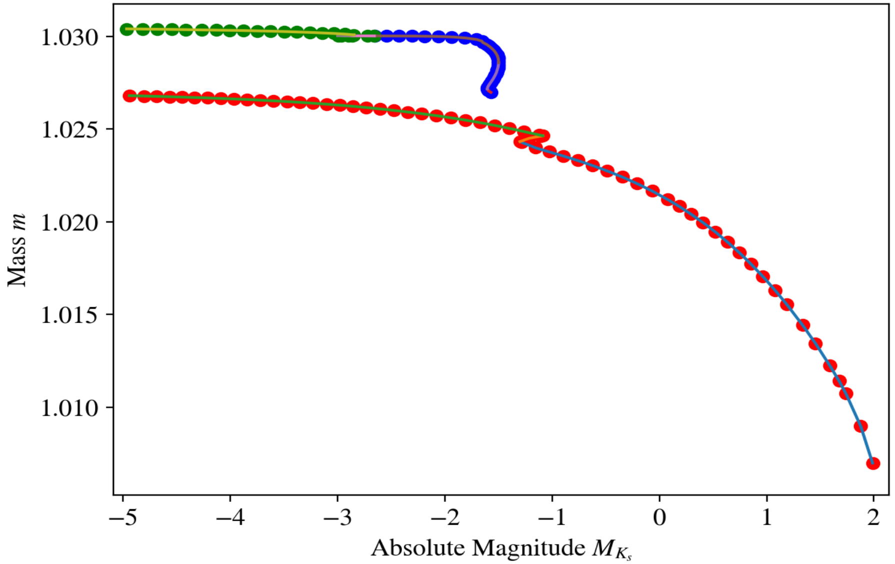
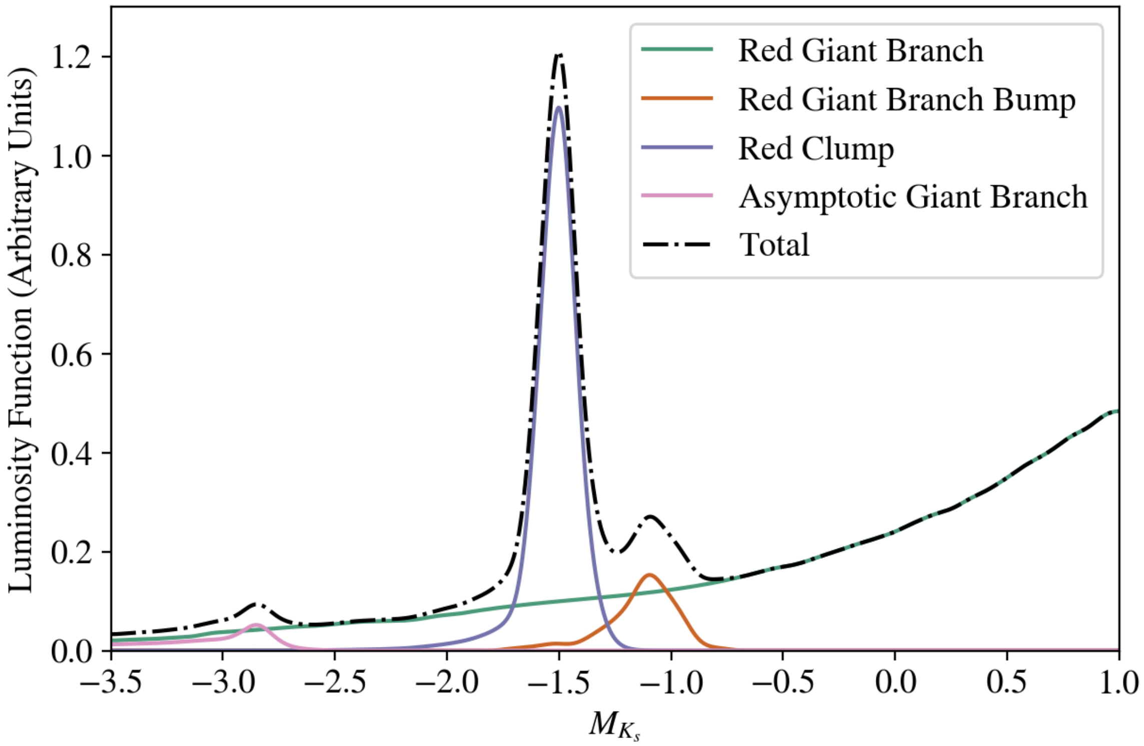
- Use a linear spline to interpolate between values.
- For the inverse splines, break the isochrone into one-to-one sections and spline each section individually.
- When working with splines it is important to clean the data.
  - Outliers can have a far outsized influence on results by extending the spline beyond its plausible range.
- Figures show splines and inverse splines for constant-metallicity cross-section of isochrone data $z = -0.0315$.





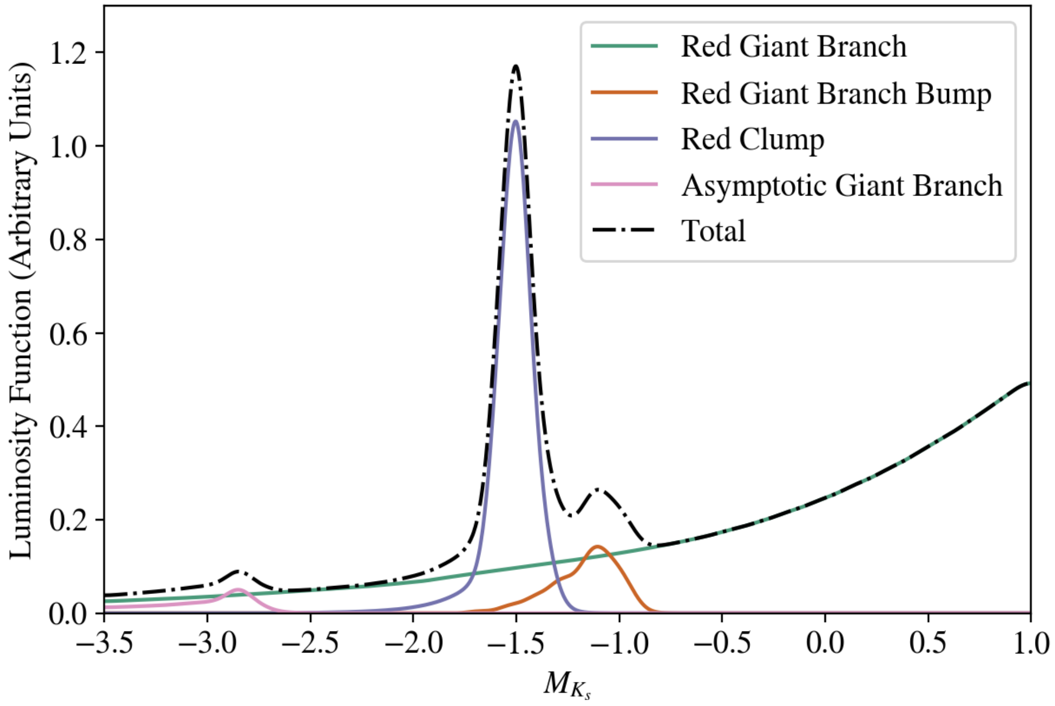
Monte-Carlo Method

- A star's absolute magnitude is largely determined by its initial mass and metallicity.
- Initial mass and metallicity have known distributions. (Denoted ξ and g, respectively)
  - Initial mass was distributed according to a Chabrier (2003) log-normal distribution
  - Metallicity was distributed according to a Zoccali (2008) normal distribution
- The isochrone provides an absolute magnitude for a given initial mass and metallicity.
- We can substitute N samples of the initial distributions into the isochrone to get a number of samples of the magnitude distribution.
  - The histogram formed by these magnitude samples is the luminosity function.



Semi-Analytic Method

- The method presented in Paterson et al. (2019)
- We want to find the distribution of a variable M_{K_s} given the distribution of m and z and knowing $M_{K_s} = \theta(m, z)$.
  - Where θ refers to the isochrone.
- Use a statistical change of variables to find the distribution of magnitudes as a function of both magnitude and metallicity.
  - $\phi\left(M_{K_s}, z\right) = \sum_i \xi(\theta^{-1}(M_{K_s}, z)) \left|\frac{d\theta^{-1}(M_{K_s}, z)}{dM_{K_s}}\right|$
- Then integrate to find the expected value with respect to metallicity. i.e. the luminosity function.
  - $\Phi\left(M_{K_s}\right) = \int_{-\infty}^{\infty} \phi\left(M_{K_s}, z\right) g(z) dz$



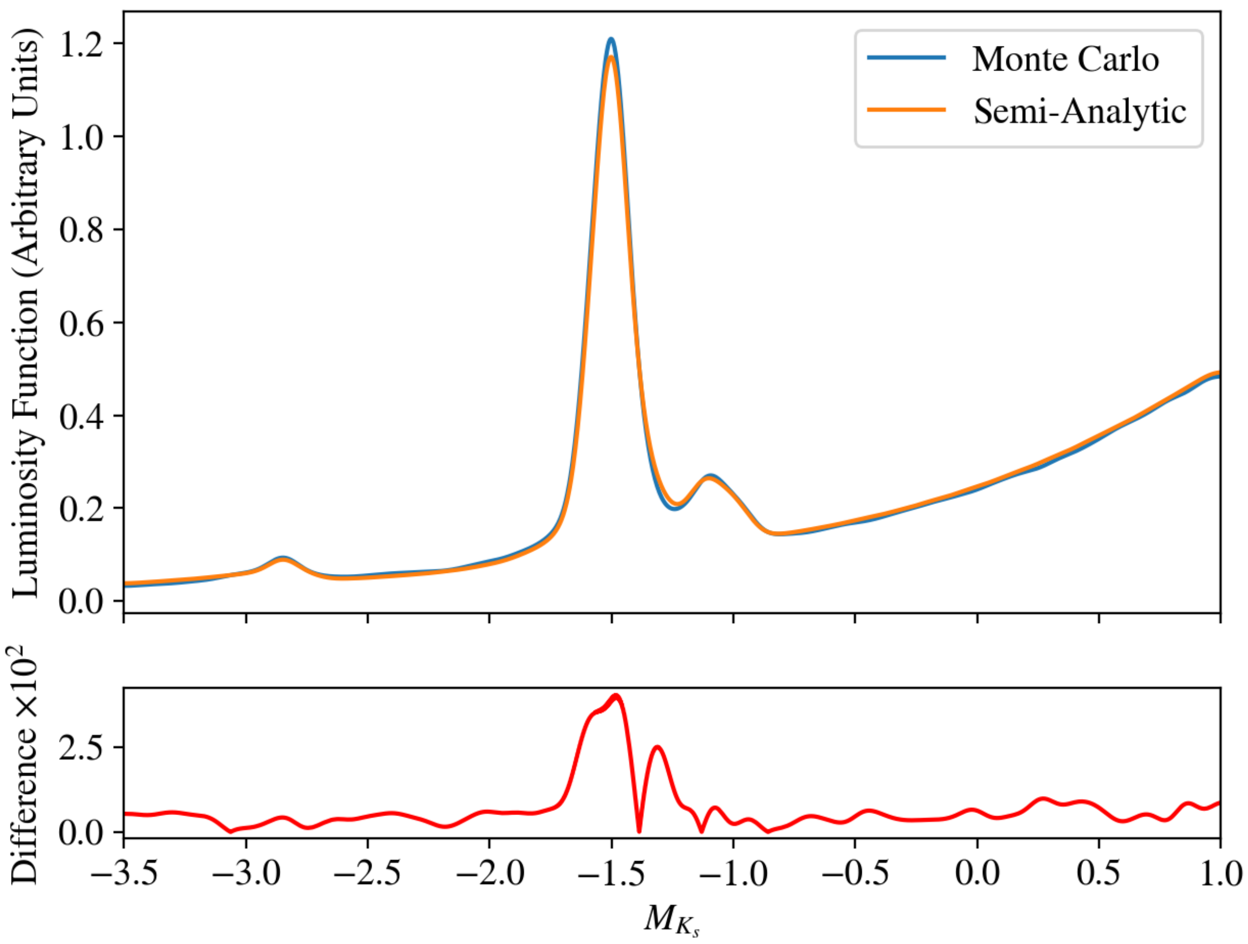
Comparison of Methods

Monte-Carlo

- Inefficient due to high sampling.
- Most samples are thrown away as they don't correspond to any magnitude in the isochrone.
- Very simple implementation

Semi-Analytic

- Much more computationally feasible.
- No sampling error
- More involved implementation



Conclusions

- Described two methods of stellar luminosity function construction from stellar isochrones.
  - Monte-Carlo method involving repeated sampling of initial distributions.
  - *Semi-analytic* method using a statistical change of variables.
- Conclude that the preferred method should be a *semi-analytic* method with a lower-resolution confidence check using a Monte-Carlo method.