The script `specialVoganDiagrams.sage`

`specialVoganDiagrams.sage` is a SageMath script which allows to find special Vogan diagrams, where the definition of special is given below. Let $\mathfrak{g}$ be a real semisimple Lie algebra of rank ℓ with Vogan diagram having indices of painted nodes P and set of positive roots Δ^+. Let $\{\gamma_1, \ldots, \gamma_\ell\}$ be the set of simple roots and $\{\varphi_1, \ldots, \varphi_\ell\}$ be the set of fundamental dominant weights. We assign to each root α a coefficient ε_α which is -1 if α is compact and 1 otherwise. Define the vector $\eta = \sum_{\alpha \in \Delta^+} \varepsilon_\alpha \alpha$ and consider the vector

$$\varphi_P := \eta - 2 \sum_{\alpha \in \mathrm{span}\{\gamma_i | i \in P^c\} \cap \Delta^+} \alpha,$$

which may be expressed in the basis of fundamental dominant weights as $\sum_{i \in P} a_i \varphi_i$, with $a_i \in \mathbb{Z}$. We say that a Vogan diagram is *special* if the a_i's are all negative, zero or positive. In particular, the diagram is said to be *symplectic general type*, *symplectic Calabi-Yau* and *symplectic Fano* respectively. For more details about the theory beyond special Vogan diagrams see [1].

The script may be run as a SageMath script and, once it is executed, the user is asked to insert the Lie type of the Vogan diagram (so one among the letters A, B, C, D, E, F, G) and the rank (so a positive integer number). Then for special Vogan diagrams the following information are printed.

- The vector φ_P expressed in the basis of fundamental dominant weights;

- The set P of indices of non-compact simple roots (which determines the Vogan diagram);

- The type of the Vogan diagram, so sGT, sCY and sF if the diagram is symplectic general type, symplectic Calabi-Yau and symplectic Fano respectively.

References

[1] A. DELLA VEDOVA AND A. GATTI, *Almost Kaehler geometry of adjoint orbits of semisimple Lie groups*, arXiv:1811.06958v2 [math.DG].