

# The script `specialVoganDiagrams.sage`

`specialVoganDiagrams.sage` is a SageMath script which allows to find special Vogan diagrams, where the definition of special is given below. Let  $\mathfrak{g}$  be a real semisimple Lie algebra of rank  $\ell$  with Vogan diagram having indices of painted nodes  $P$  and set of positive roots  $\Delta^+$ . Let  $\{\gamma_1, \dots, \gamma_\ell\}$  be the set of simple roots and  $\{\varphi_1, \dots, \varphi_\ell\}$  be the set of fundamental dominant weights. We assign to each positive root  $\alpha$  a coefficient  $\varepsilon_\alpha$  which is  $-1$  if  $\alpha$  is compact and  $1$  otherwise. Define  $\eta = \sum_{\alpha \in \Delta^+} \varepsilon_\alpha \alpha$  and consider the vector

$$\varphi_P := \eta - 2 \sum_{\alpha \in \text{span}\{\gamma_i | i \in P^c\} \cap \Delta^+} \alpha,$$

which may be expressed in the basis of fundamental dominant weights as  $\sum_{i \in P} a_i \varphi_i$ , with  $a_i \in \mathbb{Z}$ . We say that a Vogan diagram is *special* if the  $a_i$ 's are all negative, zero or positive. In particular, the diagram is said to be *symplectic general type*, *symplectic Calabi-Yau* and *symplectic Fano* respectively. For more details about the theory beyond special Vogan diagrams see [1].

The script may be run as a SageMath script and, once it is executed, the user is asked to insert the Lie type of the Vogan diagram (so one among the capital letters  $A, B, C, D, E, F, G$ ) and the rank (so a positive integer number). Then for special Vogan diagrams the following information are printed.

- The vector  $\varphi_P$  expressed in the basis of fundamental dominant weights;
- The set  $P$  of indices of non-compact simple roots (which determines the Vogan diagram);
- The type of the special Vogan diagram.

## References

- [1] A. DELLA VEDOVA AND A. GATTI, *Almost Kaehler geometry of adjoint orbits of semisimple Lie groups*, arXiv:1811.06958v2 [math.DG].