

What is algebra

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What's left, $=$? How can that vary?...oh maybe its mod 12.

Max variability allows for wide ranging commonality.

Conic sections

$$x^2 - y^2 = 0$$

Wave propagation

$$\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2} = 0$$

Factoring by quadratic sieve

$$m^2 - n^2 \equiv 0 \pmod{65398}$$

The power of algebra is that every symbol in an equation is a variable, especially the equals sign.