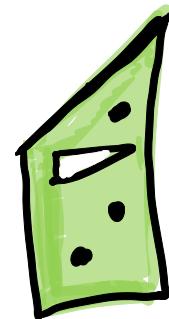


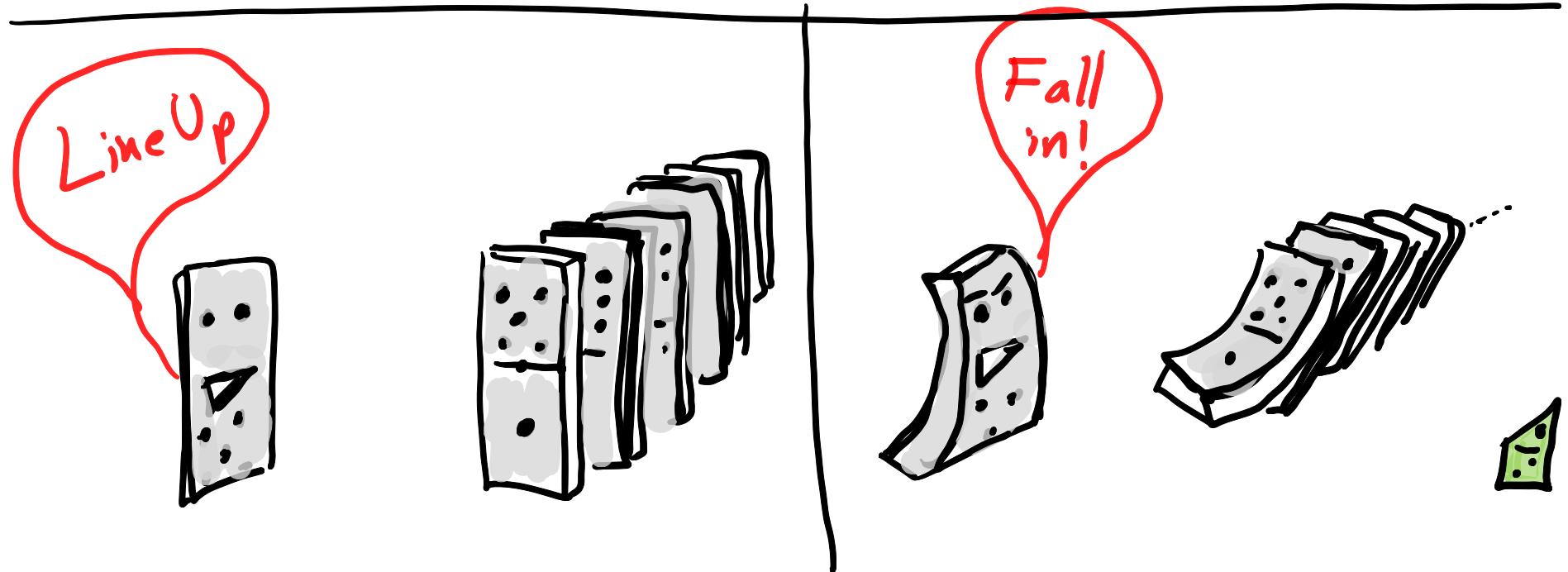
A tale of what
comes next

by James B. Wilson



February 11 2020

In the land of induction there is a well-ordered civilization of dominos.



$$\forall k. (P(k) \Rightarrow P(k+1)) , \quad P(1)$$

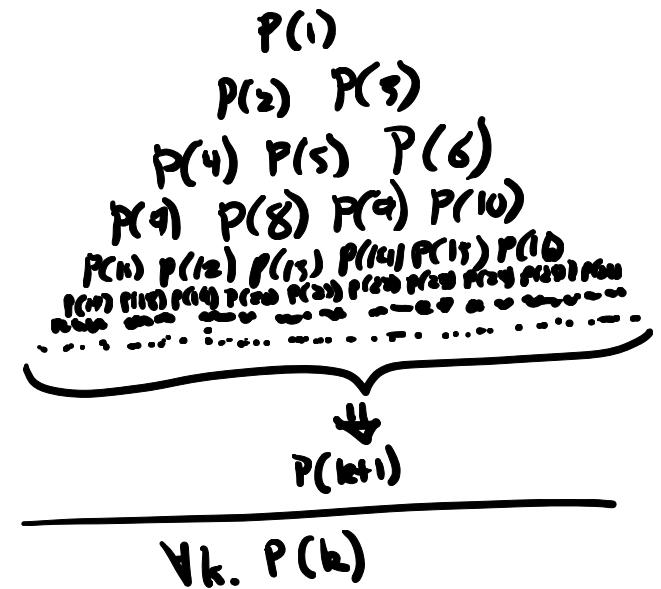
$$\forall k. P(k)$$

The bravest dominos were in the
clan of strong induction

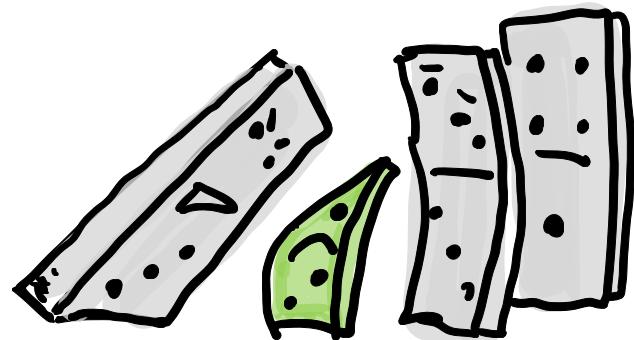
IF WE GO DOWN
THEN WE GO DOWN



TOGETHER

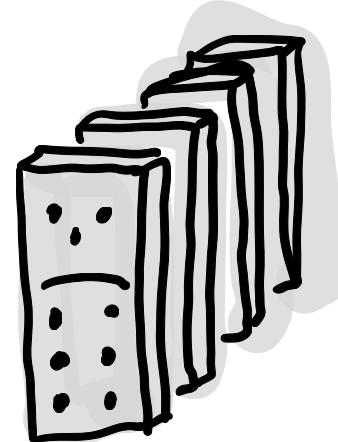


But one domino felt he did not fit



You aren't
 $P(k+1)$!

Sorry taken! $P(1)$ is already



Go to the back of the line
(where ever that is!)

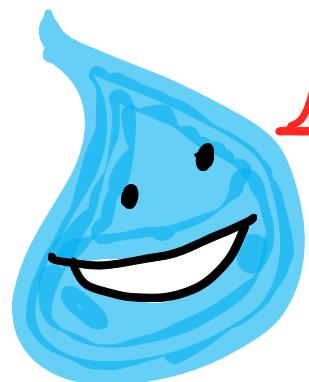
Domie just didn't fit in this order.

... Domie left to find a new pigeon hole... Soon
Domie found a tribe of droplets!



Droplets didn't care about
who was next, so long as someone was!

Domic was stunned and ask their leader Emmy how it was possible...



Our ancestor Noether made
us well-founded.
 $\forall T \subset D, T \neq \emptyset \Rightarrow T$ has a minimum.



Where I am from we had that
too, but only one domino got
to be the minimum

I never got to
be the minimum.



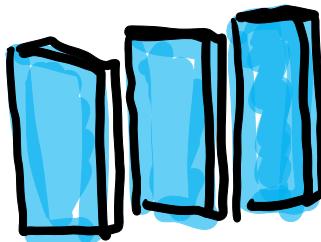
That happened in our society long ago.

Invaders lead by Axiom du Choice
ignored our ways.

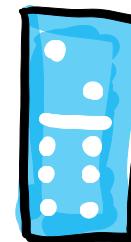
They froze us...



Sliced us...



Gave us meaningless numbers...

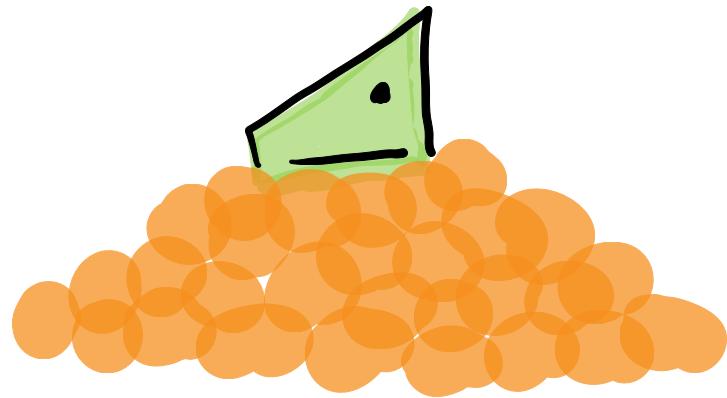


... but it failed.

Sand only gets wet when the
ice melts.



But Domie soon learned he
was no good at getting sand
wet either.



(but he had fun trying.)

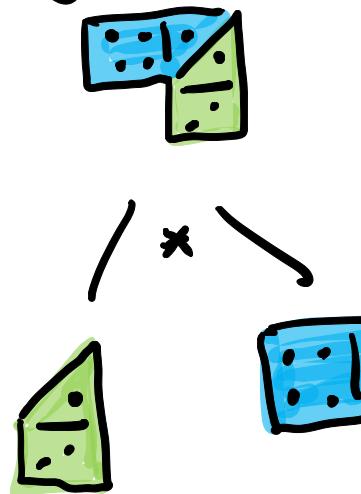
Then one day Domic meet a thinker
who had been thinking about induction



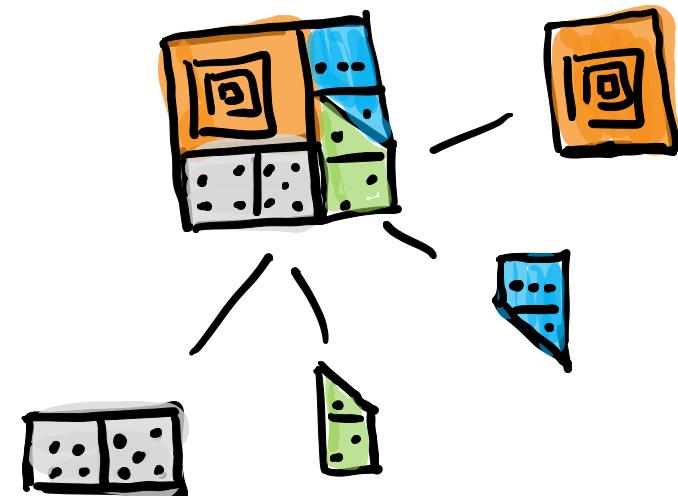
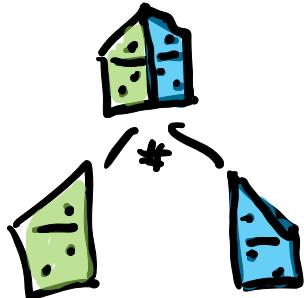
Domic. Why don't you ignore everyone
else's orders. All you need to
multiply your effect is multiplication!

... oh and call me Per, Per Martin-Löf

So he did!



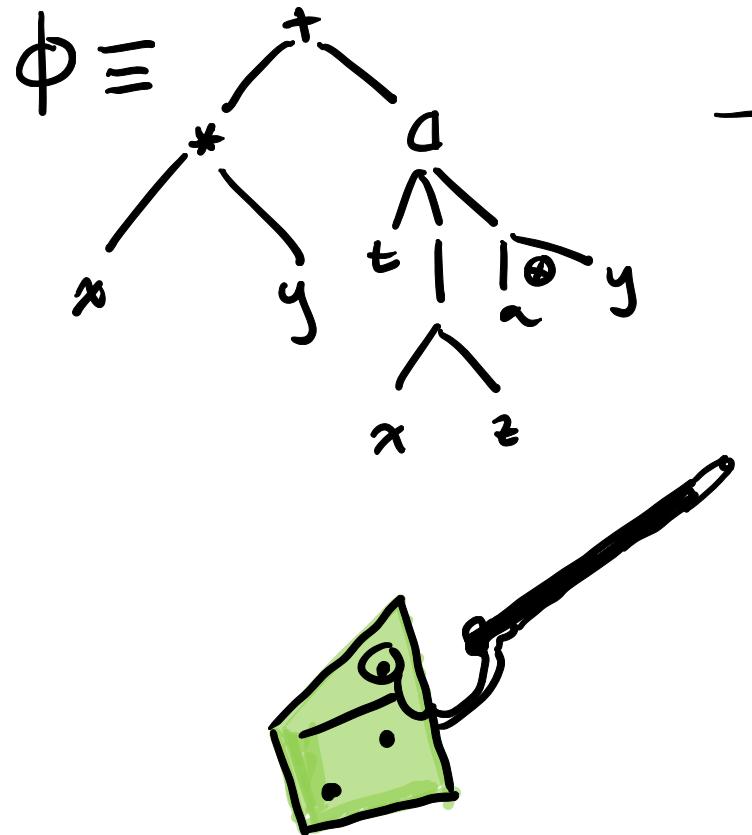
He soon found lots of ways to
multiply



... and old
and new friends
joined

So to Domie, induction became simply
a homomorphism on a free algebra.

Free $X, +, *, \otimes, \mathbb{I}, \neg$



Thm $(\forall \phi. \text{Free}[\dots]). P(\phi)$

$* : \text{Free} \rightarrow (\text{Free} \rightarrow \text{Free})$
 $\phi \mapsto (\tau \mapsto \phi * \tau)$

$P(\phi) \Rightarrow (P(\tau) \Rightarrow P(\phi * \tau))$

In time Domie got even more bold
and began using non-free algebras
like his new pals Schulman & Licata

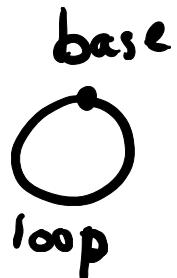
If the free algebra
has relations, say

$$\begin{array}{c} + \\ / \quad \backslash \\ + \quad + \end{array} = \begin{array}{c} + \\ / \quad \backslash \\ + \quad + \end{array}$$

then we call it
"higher induction"

because it not only
affects our path,
it changes our $=$.

Soon the land of induction was filled
with life and invention



$$S^1 := \text{base}$$
$$\mid \text{loop} : \text{base} = \text{base}$$



$$S^2 := \text{base}$$
$$\mid \text{surface} : \text{refl}_{\text{base}} = \text{refl}_{\text{base}} \text{ in base} = \text{base type}$$

And finally it made sense why homology
measures so little about a sphere.
There simply wasn't that much information
in a sphere to begin with.