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Information volume of Z-number

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ABSTRACT

Given a probability distribution, Shannon entropy can be used to measure its corresponding information volume. However, it is still an open issue to calculate the information volume of Z-number. In this paper, the information volume of the Z-number is presented using the proposed transformation of a Z-number into a mass function. In our proposed method, the mass function degenerates into probability distribution under the circumstance that the uncertainty of the Z-restriction is 0. In this case, the information volume degenerates into Shannon entropy. The information volume of a given Z-number increases approximately linearly with the unreliability of its Z-restriction. In addition, the information volume varies with the value of Z-restriction and fuzzy number A. Some illustrative examples are shown to demonstrate the properties of the proposed information volume of Z-number. A new Weighted Multiple Attribute Decision Making (WMADM) method is also proposed to illustrate the practical advantage of the proposed information volume.

1. Introduction

In the real world, it is necessary to measure the uncertainty of events [20]. Many theories have been proposed, such as probability theory [25], Dempster-Shafer evidence theory [10,35,36], rough sets [24,43], fuzzy set theory [40,34], D-number [30] and Z-number [5,29].

A probability distribution can measure the possibility of a event well. However, it cannot be used to identify the unreliability of the information sources in the process of uncertain information. Dempster and Shafer solved the problem by proposing evidence theory to represent unreliable information [26]. An advantage of the theory is that information from different mass functions can be combined by combination rules of mass functions [37]. In order to deal with the conflicting information, some methods are proposed to calculate the distance between mass functions [9]. D-S Evidence theory have been widely applied in various fields. For example, it can be used for risk evaluation [8], Markov decision [7], reliability analysis [15] and decision making [33].

Zadeh proposed Z-number as another solution to the problem by giving a summary of probability distributions P_x instead of a single probability distribution [41]. A Z-number consists of two ordered fuzzy numbers A and B, where A is a restriction

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on the values that an uncertain variable X can take and B contains the unreliability of the information. The combination of A and B sets a restriction on the distribution of its related uncertain variable X, which can be denoted as $Prob(X \textit{ is } A) \textit{ is } B$.

In the previous decade, Z-number has been widely applied in various areas such as supplier selection [2,39] and clustering [4]. Z-number has the advantage of representing both the uncertainty of the real world and the unreliability of human languages, which facilitates its development in linguistic model. For example, a portfolio manager can express his opinion on the stock price using a linguistic Z-number (*probably rise*, *very likely*), which means he is highly confident that the stock price has a high possibility to rise. Another area that many researchers focused was the operation of Z-numbers. Tian et al. [28] proposed a modified method to generate Z-numbers using Ordered Weighted Averaging (OWA) weights and maximum entropy. Qing et al. [22] proposed a method to derive Z-number from information presented in Dempster-Shafer theory. Liu et al. [21] proposed the negation of discrete Z-numbers to provide an opposite perspective of the information represented in Z-numbers. Aliev et al. [3] first proposed the basic arithmetic rules for discrete Z-numbers to construct a direct calculation model for Z-numbers. Following this, many researches have been done to complete the computing framework of Z-numbers [1]. The concept of Z-Differential Equations was developed to describe uncertain fuzzy systems in the real world. A framework for Z-Differential Equations containing Z-differentiability, Z-integral and Z-Laplace transform was introduced in [23].

Besides the researches on the operations and arithmetic rules of Z-numbers, some works proposed new types of Z-numbers. Zhu et al. proposed a novel concept of rough Z-number and a rough-Z-number-based Decision-Making Trial and Evaluation Laboratory (DEMATEL) was proposed to evaluate sustainability [45]. Some works also extended Z-number by integrating more fuzzy numbers. Allahviranloo and Ezadi [5] proposed the concept of ZA-number, which contains three or more fuzzy numbers. Tian et al. [29] introduced a new extended Z-numbers ZE = ((A, B), E), where the fuzzy number E represents the credibility of the Z-number (A, B). Based on the extended Z-numbers, a new multi-attribute group decision-making method was proposed.

Z-number has been successfully applied in the fields of decision making since it can be used to analyze the uncertain results of possible decisions. It has been applied to construct possibilistic hierachical model to promote sustainable development [16]. Z-number is also applied to assess the results of the coronavirus disease in the paper [19]. Many decision making models have been improved by introducing Z-numbers. Yaakob and Gegov [38] modified Technique for Order Preference by Similarity to an Ideal SOlution (TOPSIS) to facilitate its application in multicriteria decision problems presented in Z-numbers. Z-number was also combined with Bestworst method (BWM) and multi-attribute border approximation area comparison (MABAC) in [17] based on the proposed Z-cloud rough number. Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) was also extended by introducing Z-number to present the reliability of information from experts [42].

Information volume is the basis of the modern information theory as it gives an approach to measure the degree of uncertainty. It is defined as the information that can be obtained when a certain event happens. In probability theory, Shannon entropy can be used to measure the information volume of a probability distribution. Given a mass function in evidence theory, information volume is given based on Deng entropy[11]. The proposed information volume of mass function has been applied in many fields, such as fractal dimension [13] and multi-source information fusion [14]. Xiao also proposed a new method to measure information volume of mass functions, which considers the relative importance of the evidences using a new Belief Jensen-Shannon divergence [32].

From the above literature review of information volume, it was found that few study has been done on the information volume of Z-number. In this paper, a method to calculate information volume of Z-number is proposed by transforming a Z-number into a mass function. The proposed information volume can be applied as an uncertainty measure of Z-number. Greater information volume indicates higher uncertainty of the event. The probability distribution associated with the Z-number is first calculated by the principle of maximum entropy. It assures the information volume of the probability distribution is equivalent to its corresponding Z-restriction. Since there are multiple Z restrictions, a series of possible probability distribution is calculated. Then, they are transformed into mass functions by using the unreliability of Z-restriction as a discounting factor. Finally, all possible mass functions are integrated into the final mass function by weighted average. It is illustrated that the mass function degenerates into probability distribution when the uncertainty of the constraint on the probability distribution is 0. In addition, the information volume increases approximately linearly with the uncertainty of the Z-restriction and it decreases with the value of Z-restriction.

The structure of the rest paper is as follows. In Section 2, some useful theories are briefly introduced. In Section 3, an approach to calculate information volume of Z-number by converting the Z-number into a mass function is proposed. In Section 4, several examples are illustrated to show the efficiency of the algorithm. In addition, a WMADM algorithm is introduced in Section 5, which uses the proposed information volume to calculate weights of multiple attributes. Finally, a conclusion of the paper is given in Section 6.

2. Preliminaries

Several previous theories are reviewed in this part, including the Z-number, evidence theory, the principle of maximum entropy and information volume of mass function.

2.1. Z-number

Z-number was first proposed by Zadeh as a description of imprecise information [41].

Definition 1. (discrete Z-number)

A discrete Z-number consists of two fuzzy numbers A and B and it can be denoted as an ordered pair $Z = (A, B)$. A Z-number is always associated with an uncertain variable X. The first fuzzy number A depicts the possibility that the variable X is classified as type A. It can be regarded as a fuzzy restriction on values that the uncertain variable X can take and it is denoted as

$$X \text{ is } A \tag{1}$$

Given the values of variable X as $u_1, u_2, \ldots, u_m$, the membership function of A is $\mu_A : [u_1, u_2, \ldots, u_m] \to [0, 1]$. The fuzzy number B is a fuzzy restriction on probability measure of A, whose membership function is noted as $\mu_B : [v_1, v_2, \ldots, v_n] \to [0, 1]$. This fuzzy restriction can be written as

$$Prob(X \text{ is } A) \text{ is } B \tag{2}$$

where $Prob(X \text{ is } A)$ denotes the probability measure of A. In this paper, if there is no special declaration, all Z-numbers are assumed to be discrete Z-numbers.

Definition 2. (Z-restriction)

Given a Z-number associated with an uncertain variable X and let $Z = (A, B)$, then the Z-restriction is [41]

$$R(X) : X \text{ is } Z \tag{3}$$

The restriction posed on X is given as:

$$Prob(X \text{ is } A) \text{ is } B \tag{4}$$

The probability measure $Prob(X \text{ is } A)$ for a discrete Z-number can be noted as:

$$Prob(X \text{ is } A) = \sum_{i=1}^{m} \mu_A(u_i) P_X(u_i) = v \tag{5}$$

where P_X is the probability distribution of the variable X and $\mu_A(u_i)$ is the membership function of A.

Definition 3. ($Z^+ - number$)

A Z^+-number is composed of a fuzzy number and a random number. It can be denoted as Z^+=(A,R), where R represents the probability distribution of an uncertain variable X. A Z-number is always associated with an uncertain variable X and it depicts a summary of possible probability distribution. When the probability distribution in the Z^+-number is compatible with the Z-number, the distribution R is the underlying distribution of the Z-number. The compatibility can be denoted as:

$$\sum_{i=1}^{m} u_i \cdot P_X(u_i) = \frac{\sum_{i=1}^{m} u_i \cdot \mu_A(u_i)}{\sum_{i=1}^{m} \mu_A(u_i)} \tag{6}$$

The compatibility makes sure the membership function and the probability distribution have the same first moment.

2.2. Evidence theory

Evidence theory was first proposed by Dempster and Shafer to deal with multi-source information fusion, which is not considered in probability theory[31].

Definition 4. (Frame of discernment)

Given a variable X, all the possible values it can take constitutes an exhaustive and exclusive set Ω, the set is noted as the frame of discernment. Let the cardinality of set Ω be N, then it is denoted as:

$$\Omega = \{\theta_1, \theta_2, \ldots, \theta_N\} \tag{7}$$

The power set of Ω is denoted as 2^Ω and it contains all the subsets of Ω[27].

Definition 5. (mass function)

A mass function is also known as a basic belief assignment(BPA). Any subset A of the frame of discernment Ω can be mapped to a real number $[0, 1]$, which is defined as:

$$m : 2^\Omega \to [0, 1] \tag{8}$$

The mapping is restricted by the following rules:

$$\begin{cases} \sum_{A \in 2^{\Omega}} m(A) & = 1 \\ m(\emptyset) = 0 \end{cases} \tag{9}$$

2.3. The principle of maximum entropy

The principle of maximum entropy was first proposed by Jaynes in 1947 to link thermodynamic entropy and information theory [18], which points out that a stable system should stay in the state with maximum entropy. In the real world, it is difficult to determine the exact probability distribution of a random variable. In this situation, the principle indicates that restricted to some given conditions, the probability distribution that has the maximum entropy is the best description of the random variable. The probability distribution generated from the principle of maximum entropy contains the same amount of information as that implied in the constraints of the distribution.

Definition 6. (Shannon entropy)

Assume a random variable X that can take values $u_1, u_2, \ldots, u_m$ and its probability distribution is P_X. Its Shannon entropy $H_s(P_X)$ is defined as [25]

$$H_s(P_X) = \sum_{u \in X} P_X(u) log\left(\frac{1}{P_X(u)}\right) \tag{10}$$

where $\sum_{u \in X} P_X(u) = 1$ and $P_X(u) \in [0, 1]$.

If there is no constraint on the probability distribution, the probability distribution satisfies $P(X = u) = \frac{1}{|X|}$, where $|X|$ is the cardinality in X. The corresponding Shannon entropy is

$$H_s(P) = log\left(\frac{1}{P_X(u)}\right) \tag{11}$$

The principle of maximum entropy is very useful in many areas of uncertain information processing. In decision making problem, a novel maximum entropy based Decision-Making Trial and Evaluation Laboratory (DEMATEL) was proposed in [6]. In Evidence theory, entropy was proposed to evaluate the uncertainty and the maximum entropy leads to the maximum uncertainty [44].

2.4. Information volume

Deng defined information volume of mass function using Deng entropy in order to measure the uncertainty of mass function [11].

Definition 7. (Deng entropy)

A mass function defined on the subset A of set Ω is denoted as m(A). Its Deng entropy is calculated as [12]

$$H_{DE}(m) = -\sum_{A \in 2^{\Omega}} m(A) log\left(\frac{m(A)}{2^{|A|} - 1}\right) \tag{12}$$

where H_{DE} represents Deng entropy and $|A|$ is the cardinality of the set A. It reaches the maximum value if and only if

$$m(A) = \frac{2^{|A|} - 1}{\sum_{A \in 2^{\Omega}} \left(2^{|A|} - 1\right)} \tag{13}$$

In this case, the distribution of the mass satisfies the maximum Deng entropy distribution and its corresponding Deng entropy is [12]

$$H_{MDE}(m) = log \sum_{A \in 2^{\Omega}} \left(2^{|A|} - 1\right) \tag{14}$$

Definition 8. (information volume)

For a mass function m(A) defined on the subset A of set Ω, its information volume is defined as the sum of Deng entropy of the mass functions generated by dividing the original mass function based on maximum Deng entropy distribution presented in Eq. 13. The information volume is defined as:

$$H_i = -\sum_{A_i \in \hat{\Omega}} m(A_i) log\left(\frac{m(A_i)}{2^{|A_i|} - 1}\right) \tag{15}$$

where H_i represents the information volume of the mass function $m(A)$ and $\hat{\Omega}$ is the set of all subsets obtained by continuously dividing subsets with a cardinality greater than 1 until the information volume converges.

An example is shown in Fig. 1 to illustrate the division rule. Given a subset $(\theta_1, \theta_2)^i$ with a cardinality of 2, it can be divided into $\theta_1^{i+1}, \theta_2^{i+1}, (\theta_1, \theta_2)^{i+1}$, where i and i + 1 denotes the iteration of the division. The BPA attributed to the subsets satisfies the distribution of maximum Deng entropy, which is denoted as

$$m\left(\theta_1^{i+1}\right) : m\left(\theta_2^{i+1}\right) : m\left((\theta_1, \theta_2)^{i+1}\right) = 1 : 1 : 3 \tag{16}$$

Let $m\left((\theta_1, \theta_2)^i\right) = r_i$, the results is calculated as:

$$m\left(\theta_1^{i+1}\right) = m\left(\theta_2^{i+1}\right) = \frac{1}{5} \cdot r_i, \; m\left((\theta_1, \theta_2)^{i+1}\right) = \frac{3}{5} \cdot r_i \tag{17}$$

Since the cardinality of $(\theta_1, \theta_2)^{i+1}$ is greater than 1, it should be divided in the next iteration. The division process can fully represent all information contained in the multi-element subsets and as a result it can measure the information volume of the mass function.

3. The proposed method

BPA

According to the previous introduction of Z-number and evidence theory, both of them can depict uncertainty and unreliability. In this part, an approach to convert a discrete Z-number into a mass function is proposed. Information volume of the Z-number can then be calculated using the generated mass function. Assume a Z-number $Z = (A, B)$ associated with an uncertain variable X. The Z-number contains two levels of information, the probability measure $Prob(X \text{ is } A)$ and the reliability of the statement X *is* A. The probability measure can be explained well using a probability distribution. However, the reliability part of the Z-number is not considered in the probability distribution because information source is regarded completely credible in probability theory. As a result, a mass function is introduced to represent information from the reliability part. In the proposed method, the information of the Z-number is first decoupled into a series of Z-restrictions to show the probability measure and a membership function μ_B to represent the reliability of the Z-restrictions. Using the maximum entropy theory, the Z-restrictions can be transformed to a group of probability distributions. The derived distributions are then coupled with the reliability information to generate a mass function. The final mass function is the weighted average of several possible mass functions. The process of the proposed method is shown in Fig. 2.

Formally, assume there is a Z-number $Z = (A, B)$ related with a variable $X = (u_1, u_2, \ldots, u_m)$

$$Z = \left(\sum_{i=1}^{n} \frac{\mu_A(u_i)}{u_i}, \sum_{i=1}^{n} \frac{\mu_B(v_i)}{v_i} \right) \tag{18}$$

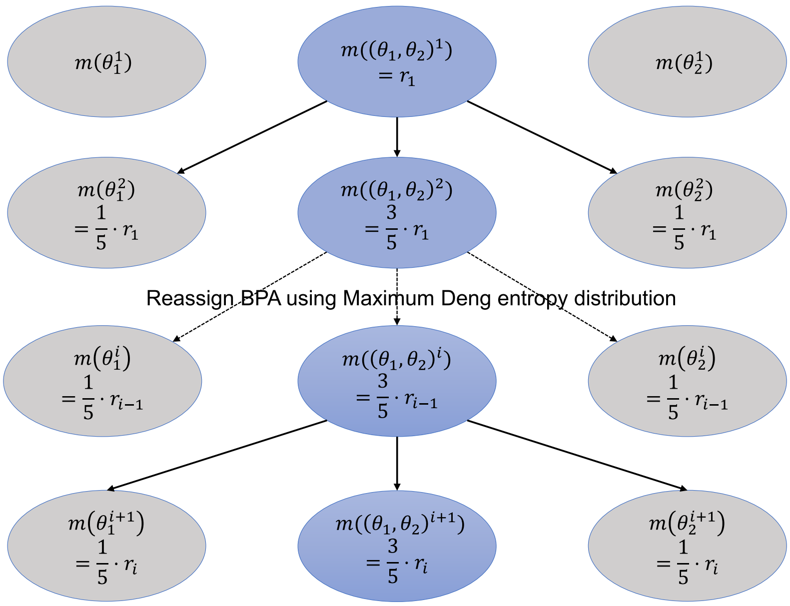

Fig. 1. An example to illustrate the division rule.

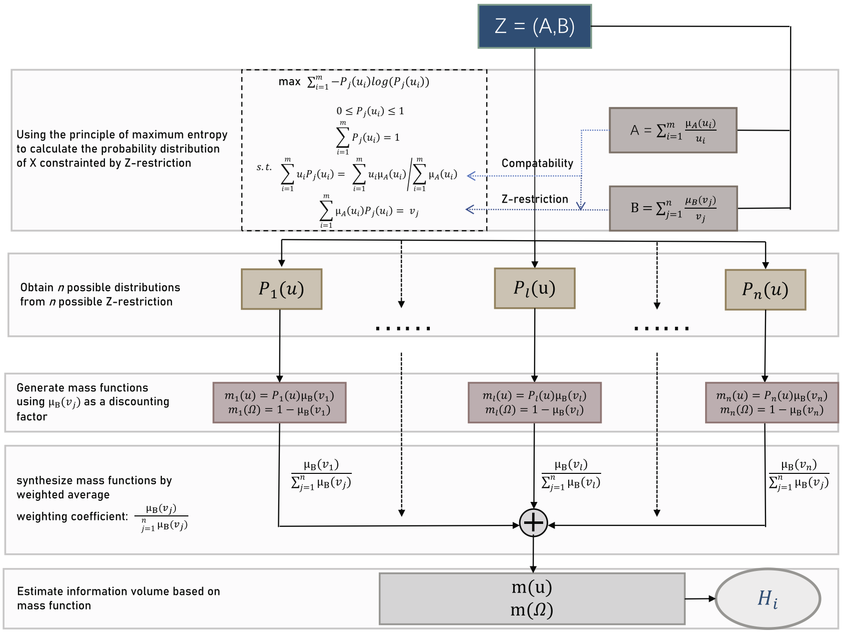

Fig. 2. The process to obtain information volume of a Z-number.

where μ_A is the membership function of A and μ_B represents the membership function of $(X\ is\ A)$. For the Z-number, the Z-restriction of X is given as

$$\begin{cases} \sum_{i=1}^{m}\mu_A(u_i)P_{X,1}(u_i) = v_1 \\ \sum_{i=1}^{m}\mu_A(u_i)P_{X,2}(u_i) = v_2 \\ \cdots \\ \sum_{i=1}^{m}\mu_A(u_i)P_{X,l}(u_i) = v_l \\ \cdots \\ \sum_{i=1}^{m}\mu_A(u_i)P_{X,n}(u_i) = v_n \end{cases} \tag{19}$$

Besides, since the probability distribution generated should be compatible with the fuzzy number A, another compatibility restriction from Eq. 6 should also be considered.

According to the principle of maximum entropy, given a restriction on a random variable, its probability distribution should be the one with the maximum entropy. In order to decide the restricted probability that contains the same amount of information volume as the Z-restriction, the probability distribution should satisfy

$$\max_{P_{X,l}(u_i)} -\sum_{i=1}^{m}P_{X,l}(u_i)\cdot log\left(P_{X,l}(u_i)\right)$$
$$s.t.\quad \begin{cases} \sum_{i=1}^{m}P_{X,l}(u_i) = 1 \\ \sum_{i=1}^{m}P_{X,l}(u_i)\cdot\mu_A(u_i) = v_l \\ \sum_{i=1}^{m}P_{X,l}(u_i)\cdot u_i = \dfrac{\sum_{i=1}^{m}\mu_A(u_i)\cdot u_i}{\sum_{i=1}^{m}\mu_A(u_i)} \\ P_{X,l}(u_i)\in\{0,1\} \end{cases} \tag{20}$$

where $P_{X,l}$ represents the probability distribution of variable X in the Z-restriction restricted by v_l.

A group of possible probability distributions can be calculated by solving the above optimization problem. A Z-number can be explained as a summary of restrictions on variable X and $\mu_B(v_l)$ represents the reliability of the restrictions [41]. The uncertainty of the calculated probability is $1 - v_l$. A mass function can then be generated using uncertainty can be used as a discounting factor to generate a mass function.

$$\begin{aligned} m_l(U_i) &= P_{X,l}(u_i) \cdot \mu_B(v_l) \\ m_l(\Omega) &= 1 - \mu_B(v_l) \end{aligned} \tag{21}$$

where U_i is the single-element subsets composed of u_i. The final synthesized mass function is the weighted average of the n possible mass functions and the weight is their $\mu_B(v_i)$.

$$m(U) = \frac{\sum_{l=1}^{n} \mu_B(v_i) \cdot m_l(U)}{\sum_{l=1}^{n} \mu_B(v_l)} \tag{22}$$

where U can take values $(U_1, U_2, \ldots, U_m, \Omega)$. The last step is to calculate the information volume of the mass function through the method introduced in Definition 8.

4. Numerical examples

In this part, 5 examples are given to illustrate the efficiency of the proposed information volume of Z-number. Some properties of the proposed information volume are also demonstrated using control variable method. It was found that information volume of Z-number is primarily determined by three components: unreliability engendered by fuzzy number B, uncertainty from Z-restriction and compatibility between fuzzy number A and probability distribution of X. The fuzziness of fuzzy number A also has a slight contribution to the information volume.

Example 1. Assume a Z number $Z_1 = (A_1, B_1)$ describing the scores of a student. In order to simplify the question, the possible scores are assumed to be discrete, which can take values 100, 200, 300, 400, 500. This means the cardinality of the Frame of Discernment is 5. Let A_1 = "not bad" means that the student has a high possibility to get a score close to $\geqslant$300, and let the corresponding membership function is $\mu_{A_1}(100) = 0.2,\ \mu_{A_1}(200) = 0.5,\ \mu_{A_1}(300) = 1,\ \mu_{A_1}(400) = 0.8,\ \mu_{A_1}(500) = 0.6$, which is equivalent to

$$\begin{aligned} A_1 &= \left(\tfrac{0.2}{100}, \tfrac{0.5}{200}, \tfrac{1}{300}, \tfrac{0.8}{400}, \tfrac{0.6}{500}\right) \\ B_1 &= \left(\tfrac{0.6}{0.5}, \tfrac{0.8}{0.8}, \tfrac{0.4}{0.9}\right) \end{aligned} \tag{23}$$

where the number B_1 represents the reliability of the information. Using three possible numbers in B_1, 3 mass functions can be calculated respectively. When $v = 0.5$, the probability distribution can be calculated by solving

$$\begin{aligned} &\max_{P_{X,0.5}(u_i)} -\sum_{i=1}^{5} P_{X,0.5}(u_i) \cdot log(P_X(u_i)) \\ &s.t. \begin{cases} \sum_{i=1}^{5} P_X(u_i) = 1 \\ \sum_{i=1}^{5} P_{X,0.5}(u_i) \cdot \mu_{A_1}(u_i) = 0.5 \\ \sum_{i=1}^{5} P_{X,0.5}(u_i) \cdot u_i = \frac{\sum_{i=1}^{5} \mu_{A_1}(u_i) \cdot u_i}{\sum_{i=1}^{5} \mu_{A_1}(u_i)} \\ P_{X,0.5}(u_i) \in \{0, 1\} \end{cases} \end{aligned} \tag{24}$$

The Probability distribution is $P(100) = 0.284, P(200) = 0.117, P(300) = 0.017, P(400) = 0.092, P(500) = 0.490$ and using $\mu_B(v_i) = 0.6$ as the discounting factor, the mass function is generated as

$$\begin{aligned} m_{0.5}(\{100\}) &= 0.171\ m_{0.5}(\{200\}) = 0.070\ m_{0.5}(\{300\}) = 0.010 \\ m_{0.5}(\{400\}) &= 0.055\ m_{0.5}(\{500\}) = 0.294\ m_{0.5}(\Omega) = 0.4 \end{aligned} \tag{25}$$

where Ω represents the set $\{100, 200, 300, 400, 500\}$.

Similarly, the rest 2 possible mass functions can be obtained using the rest 2 values of v_i as the restriction in Eq. 24. Finally they can be synthesized by weighted average, shown as

$$\begin{aligned} m(\{100\}) &= 0.058\ m(\{200\}) = 0.056\ m(\{300\}) = 0.350 \\ m(\{400\}) &= 0.171\ m(\{500\}) = 0.152\ m(\Omega) = 0.213 \end{aligned} \tag{26}$$

and the information volume is

$$H_i = 4.925 \tag{27}$$

where the error threshold ϵ is set as 0.001.

Example 2. Given a Z-number $Z = (A_2, B_2) = \left(\frac{\mu_{A_2,1}}{u}, \frac{\mu_{B_2,1}}{v}\right)$, where A_2 represents the membership of a variable X, v shows the restriction and $\mu_B(v)$ depicts the uncertainty of the restriction. When $\mu_B(v) = 1$, it means the restriction is absolutely reliable. In this situation, the probability distribution is consistent with the generated mass function.

Let a Z-number Z_2 describes the flat floor a family lives in and it is denoted as

$$\begin{aligned} A_2 &= \frac{\mu_{A_2,1}}{u} = \frac{0}{1} + \frac{0.3}{2} + \frac{0.5}{3} + \frac{1}{4} + \frac{0.8}{5} + \frac{0.7}{6} \\ B_2 &= \frac{\mu_{B_2,1}}{v} = \frac{1.0}{0.6} \end{aligned} \tag{28}$$

The value of B_2 means that the Z-restriction: Prob(X is A_2) = 0.6 is completely credible. Although the restriction is certain, it is still only restriction on the values that variable X can take. This means the probability distribution of X is not given directly. Using the principle of maximum entropy, the probability distribution that contains the same amount of information volume as the Z-restriction can be obtained as

$$\begin{aligned} &P_X(1) = 0.110\ P_X(2) = 0.104\ P_X(3) = 0.119 \\ &P_X(4) = 0.076\ P_X(5) = 0.192\ P_X(6) = 0.399 \end{aligned} \tag{29}$$

And it is consistent to the mass function calculated by the proposed method

$$\begin{aligned} m(\{1\}) = 0.110\ m(\{2\}) = 0.104\ m(\{3\}) = 0.119 \quad & m(\{4\}) = 0.076\ m(\{5\}) = 0.192\ m(\{6\}) = 0.399 \\ m(\Omega) \quad & = 0 \end{aligned} \tag{30}$$

where Ω is the given set {1,2,3,4,5,6}.

The corresponding information volume of Z_2 is H_i = 2.325. The result illustrates that the equivalent mass function converges to the probability distribution when the uncertainty is 0. As a result, the information volume in the case is the same as Shannon entropy. In addition, it can be seen that the information volume of Z_2 is smaller than that of the Z_1. The reason is that the uncertainty of Z_2 is much less than that of Z_1 .

Example 3. In order to investigate how information volume varies with the unreliability of Z-restriction, a Z-number $Z_3 = (A_3, B_3)$ was assumed to have a settled A_3 and a varying μ_{B_3}

$$\begin{aligned} A_3 &= \left(\frac{0.3}{1}, \frac{0.8}{2}, \frac{0.7}{3}, \frac{0.4}{4}, \frac{0}{5}\right) \\ B_3 &= \left(\frac{C}{0.5}\right) \end{aligned} \tag{31}$$

where $C \in [0, 1]$, the calculated information volume is shown in Table 1, where C is certainty and H_i is information volume. From Fig. 3, the information volume decreases approximately linearly as the certainty of the Z-restriction increases from 0 to 1. It is consistent with the intuition that less uncertainty leads to less information volume.

Example 4. Another important factor to the information volume of a Z-number is the strength of the Z-restriction. Given a Z-number $Z_4 = (A_4, B_4)$ with a varying v_4

$$\begin{aligned} A_4 &= \left(\frac{0.3}{1}, \frac{0.8}{2}, \frac{0.7}{3}, \frac{0.4}{4}, \frac{0}{5}\right) \\ B_4 &= \left(\frac{1}{v_4}\right) \end{aligned} \tag{32}$$

It is noticeable that the value of v_4 is restricted to (0.18,0.75) because of the compatibility of the underlying probability and the fuzzy number A_4. The results are shown in Table 2. The relationship between information volume and v_4 is shown in Fig. 4. It was found that the maximum information volume is obtained when v_4 is around 0.48, which means the strength of Z-restriction is the least at this point. When v_4 is close to 1, the event X observed has a high possibility to be A. On the con-

Table 1
Results of Example 3.

C	Mass function						H_i
	m({1})	m({2})	m({3})	m({4})	m({5})	m(Ω)	
0.000	0.000	0.000	0.000	0.000	0.000	1.000	12.202
0.100	0.027	0.027	0.021	0.015	0.010	0.900	11.676
0.200	0.055	0.053	0.041	0.030	0.021	0.800	10.933
0.300	0.082	0.080	0.062	0.045	0.031	0.700	10.096
0.400	0.109	0.106	0.083	0.060	0.042	0.600	9.189
0.500	0.137	0.133	0.104	0.075	0.052	0.500	8.222
0.600	0.164	0.160	0.124	0.090	0.062	0.400	7.196
0.700	0.191	0.186	0.145	0.074	0.073	0.300	6.110
0.800	0.218	0.213	0.166	0.120	0.083	0.200	4.955
0.900	0.246	0.239	0.186	0.135	0.094	0.100	3.705
1.000	0.273	0.266	0.207	0.150	0.104	0.000	2.240

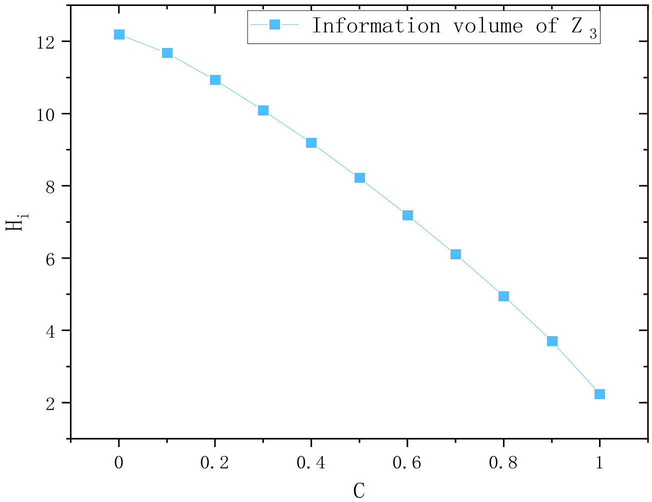

Fig. 3. Information volume of Z_3 in Example 3.

Table 2
Information volume in Example 4.

v_4	H_i	v_4	H_i
0.190	1.076	0.500	2.240
0.200	1.207	0.550	2.184
0.250	1.629	0.600	2.069
0.300	1.895	0.650	1.883
0.350	2.074	0.700	1.594
0.400	2.186	0.710	1.516
0.450	2.240	0.720	1.425
0.470	2.246	0.730	1.315
0.480	2.246	0.740	1.154

trary, when v_4 approaches 0, the event X has a high possibility to be $\overline{A}$, where $\overline{A}$ is the complement of A. The two cases indicate an affirmation or negation on the event X *is* A [21]. As a result. When v_4 is around 0.48, the uncertainty is maximum because it is not known whether X is A or $\overline{A}$. It is also found that the information volume of a Z-number cannot approach 0 in the limited range of v_4 due to the compatibility restriction generated from fuzzy number A_4.

Example 5. The fuzzy number A can also influence the information volume as it indicates the fuzziness. To illustrate it, a Z-number $Z_5 = (A_5, B_5)$ is assumed to be

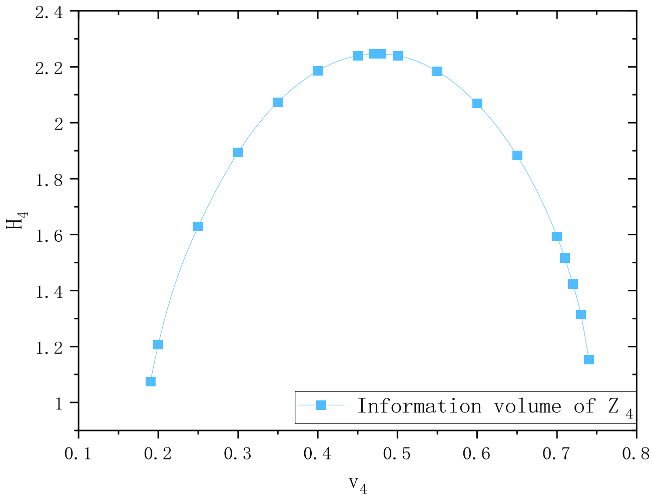

Fig. 4. Information volume of Z_4 in Example 4.

$$\begin{aligned} A_5 &= \left(\tfrac{0}{0}, \tfrac{a}{1}, \tfrac{1-a}{2}, \tfrac{1}{3}, \tfrac{1-a}{4}, \tfrac{a}{5}, \tfrac{0}{6}\right) \\ B_5 &= \left(\tfrac{0.6}{0.5}, \tfrac{0.8}{0.8}, \tfrac{0.4}{0.9}\right) \end{aligned} \tag{33}$$

where a varies from 0 to 0.5. With the special design of fuzzy number, the compatibility restriction can be denoted as:

$$\sum_{i=1}^{5} P_{X,0.5}(u_i) \cdot u_i = 3 \tag{34}$$

which will not vary with the variable a. In the situation, the results of the experiment can show the information volume caused by fuzziness containing in the fuzzy number.

The relationship between the information volume and the variable a is shown in Fig. 5 and the results of calculation are listed in Table 3. The mass function becomes more concentrated as variable a increases. As a result, the information volume decreases. Compared with the variation of information volume shown in Example 3 and Example 4, information volume varies slightly with the value of a. The results show that the fuzziness of the fuzzy number A only plays a minor role in determining information volume of Z-number.

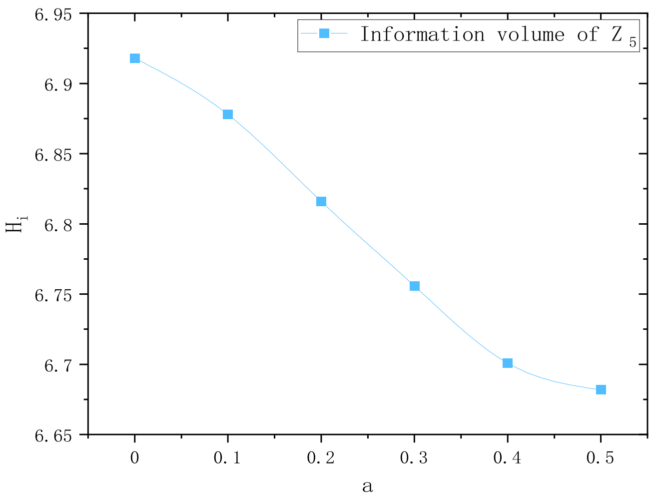

Fig. 5. Information volume of Z_5 in Example 5.

Table 3
Information volume of Z_5 in Example 5.

a	Mass function								H_i
	m({0})	m({1})	m({2})	m({3})	m({4})	m({5})	m({6})	m(Ω)	
0.0	0.045	0.045	0.155	0.155	0.155	0.045	0.045	0.355	6.918
0.1	0.038	0.042	0.150	0.186	0.150	0.042	0.038	0.355	6.878
0.2	0.031	0.041	0.133	0.235	0.133	0.041	0.031	0.355	6.816
0.3	0.029	0.043	0.107	0.288	0.107	0.043	0.029	0.355	6.756
0.4	0.026	0.051	0.084	0.323	0.084	0.051	0.026	0.355	6.701
0.5	0.025	0.065	0.065	0.335	0.065	0.065	0.025	0.355	6.682

5. Application in WMADM

Information volume of Z-number can facilitate the processing of information presented in Z-number. In Z-number based Multiple Attributes Decision Making (MADM), multiple attributes of a set of alternatives are evaluated using linguistic variables given by an expert, which can be translated into Z-numbers for ranking. The expert can be more confident on their opinions on some attributes. For example, when a portfolio manager try to decide the stocks to buy, a series of indexes can be used. Those indexes that he is more professional in can be more convincible in his decision making process. In this section, a new WMADM is proposed to deal with the varying confidence of the expert on different attributes in decision making process.

5.1. Weighted multiple attribute decision making

Under a Z-numbers environment, opinions from experts are represented using a Z-numbers matrix. In WMADM, m alternatives are available and n attributes of the alternatives are considered for ranking. The alternatives are denoted as $K_1, K_2, \ldots, K_m$ and the attributes are denoted as $A_1, A_2, \ldots, A_n$. The decision information can be written as a Z-numbers matrix $M_Z = (Z_{ij})_{m \times n}$, which is denoted as:

$$M_Z = \begin{pmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mn} \end{pmatrix} \tag{35}$$

where Z_{ij} is the value of the attribute A_j of the alternative K_i. Since the expert can be more confident on some of the attributes, more weights should be assigned to them. The confidence of the attributes is denoted as $W_1, W_2, \ldots, W_n$ and it satisfies $\sum_{j=1}^{n} W_j = 1$. The proposed information volume of Z-numbers can be applied to indicate the confidence of the multiple attributes. Given a Z-numbers matrix 36, the process is stated in detail as follows.

Step 1: The information volume of the Z-numbers in the matrix is calculated as the degree of confidence, which can be represented as a matrix H_i

$$M_{Hi} = \begin{pmatrix} Hi_{11} & Hi_{12} & \cdots & Hi_{1n} \\ Hi_{21} & Hi_{22} & \cdots & Hi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Hi_{m1} & Hi_{m2} & \cdots & Hi_{mn} \end{pmatrix} \tag{36}$$

where Hi_{ij} represents the information volume of the Z-number Z_{ij}.

Step 2: The average confidence of the expert on the attribute j is calculated as:

$$W_j = \frac{\frac{1}{\sum_{i=1}^{m} Hi_{ij}}}{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{m} Hi_{ij}}} \tag{37}$$

Step 3: In this step, the Z-numbers will be translated to crisp numbers for ranking. Z-numbers are first normalized because the scale of different attributes varies. Given a Z-numbers matrix $Z_{ij_{m\times n}} = (A_{ij}, B_{ij})_{m\times n}$, where A_{ij} is a triangular fuzzy number and $A_{ij} = \left(u^l_{ij}, u^m_{ij}, u^r_{ij}\right)$. A_{ij} can be normalized as:

$$A^N_{ij} = \begin{cases} \left(\frac{u^l_{ij}}{\max\limits_i u^r_{ij}}, \frac{u^m_{ij}}{\max\limits_i u^r_{ij}}, \frac{u^r_{ij}}{\max\limits_i u^r_{ij}}\right) \text{ if attribute is beneficial} \\ \left(\frac{\min\limits_i u^l_{ij}}{u^l_{ij}}, \frac{\min\limits_i u^l_{ij}}{u^m_{ij}}, \frac{\min\limits_i u^l_{ij}}{u^r_{ij}}\right) \text{ if attribute is negative} \end{cases} \tag{38}$$

B_{ij} is transformed into crisp value by calculating the mean of the reliability:

$$\alpha_{ij} = \frac{\sum_v v \cdot \mu_{B_{ij}}(v)}{\sum_v \mu_{B_{ij}}(v)} \tag{39}$$

Then the Z-number can be transformed into an equivalent normalized fuzzy number:

$$Z^\alpha_{ij} = \sum_{u^\alpha} \frac{\mu^\alpha_{ij}(u^\alpha)}{u^\alpha} = \sum_u \frac{\mu_{A^N_{ij}}\left(\frac{u}{\sqrt{\alpha}}\right)}{u} \tag{40}$$

Finally, the generated fuzzy number Z^α_{ij} is converted into the corresponding crisp number:

$$r_{ij} = \frac{\sum\limits_{u^\alpha} u^\alpha \cdot \mu^\alpha_{ij}(u^\alpha)}{\sum\limits_{u^\alpha} \mu^\alpha_{ij}(u^\alpha)} \tag{41}$$

Step 4: The alternatives are ranked based on the score obtained from the confidence weight W_j and defuzzified values shown as follows:

$$Score(K_i) = \sum_{j=1}^n W_j \cdot r_{ij} \tag{42}$$

where $Score(K_i)$ presents the score of the alternative K_i and a higher $Score(K_i)$ indicates a higher preference for the alternative K_i.

Step 5: Rank the preference for the alternatives based on the calculated $Score(K_i)$.

5.2. Illustrative example of WMADM

In order to present the process of WMADM clearly, a numerical example is given in this section. In the example, a decision making problem of selecting the best way to travel is solved using the proposed WMADM. Three possible alternatives are given, including K_1 ("Car"), K_2 ("Taxi") and K_3 ("Train"). They are evaluated from three aspects, namely A_1 ("Price"), A_2 ("Travel time") and A_3 ("Comfort Zone"). The opinions presented in Z-numbers given by the expert are shown in Table 4. The first fuzzy number is represented in a triangular fuzzy number, which is translated into membership function in Table 5. The second fuzzy number in the given Z-numbers indicates the reliability of the expert's opinion. The translation of the linguistic evaluation to the fuzzy number is shown in Table 6.

Given the Z-numbers matrix shown in Table 4, WMADM is applied to select the best transport among the three choices.

Step 1: The information volume of the Z-numbers in the Z-numbers matrix is calculated first as the degree of confidence. The calculated confidence matrix is shown in Eq. 43:

$$M_{Hi} = \begin{pmatrix} 3.70 & 6.28 & 5.57 \\ 4.56 & 5.538 & 5.98 \\ 0.67 & 2.822 & 7.19 \end{pmatrix} \tag{43}$$

Step 2: The average confidence of three attributes is computed as their weights:

Table 4
Expert's opinion in Z-numbers matrix.

Alternatives	A_1(pound)	A_2(min)	A_3
K_1(car)	(8,10,12),VH	(70,100,120),L	(3,5,7),M
K_2(taxi)	(20,24,25),VL	(60,70,100),M	(6,8,11),H
K_3(train)	(15,15,15),VH	(70,80,90),H	(4,7,10),M

Table 5
Membership function of the first triangular fuzzy number.

Triangular fuzzy number	Membership function
F_{11} (8,10,12)	$\frac{0}{8}+\frac{0.5}{9}+\frac{1}{10}+\frac{0.5}{11}+\frac{0}{12}$
F_{21} (20,24,25)	$\frac{0}{20}+\frac{0.25}{21}+\frac{0.5}{22}+\frac{0.75}{23}+\frac{1}{24}+\frac{0}{25}$
F_{31} (15,15,15)	$\frac{1}{15}$
F_{12} (70,100,120)	$\frac{0}{70}+\frac{0.33}{80}+\frac{0.66}{90}+\frac{1}{100}+\frac{0.5}{110}+\frac{0}{120}$
F_{22} (60,70,100)	$\frac{0}{60}+\frac{1}{70}+\frac{0.66}{80}+\frac{0.33}{90}+\frac{0}{100}$
F_{32} (70,80,90)	$\frac{0}{70}+\frac{1}{80}+\frac{0}{90}$
F_{13} (3,5,7)	$\frac{0}{3}+\frac{0.5}{4}+\frac{1}{5}+\frac{0.5}{6}+\frac{0}{7}$
F_{23} (6,8,11)	$\frac{0}{6}+\frac{0.5}{7}+\frac{1}{8}+\frac{0.66}{9}+\frac{0.33}{10}+\frac{0}{11}$
F_{33} (4,7,10)	$\frac{0}{4}+\frac{0.33}{5}+\frac{0.66}{6}+\frac{1}{7}+\frac{0.66}{8}+\frac{0.33}{9}+\frac{0}{10}$

Table 6
Transformation of linguistic unreliability to fuzzy number.

Linguistic term	Membership function
Very high (VH)	$\frac{0}{0.7}+\frac{0.3}{0.8}+\frac{0.7}{0.9}+\frac{1}{1}$
High (H)	$\frac{0}{0.5}+\frac{0.3}{0.6}+\frac{1}{0.7}+\frac{0.7}{0.8}+\frac{0}{1}$
Medium (M)	$\frac{0}{0.3}+\frac{0.5}{0.4}+\frac{1}{0.5}+\frac{0.5}{0.6}+\frac{0}{0.7}$
Low (L)	$\frac{0}{0.1}+\frac{0.7}{0.2}+\frac{1}{0.3}+\frac{0.3}{0.4}+\frac{0}{0.5}$
Very low (VL)	$\frac{1}{0}+\frac{0.7}{0.1}+\frac{0.3}{0.2}+\frac{0}{0.3}$

$$
\begin{aligned}
W_1 &= \frac{\frac{1}{8.93}}{\frac{1}{8.93}+\frac{1}{14.64}+\frac{1}{18.74}} = 0.479 \\
W_2 &= \frac{\frac{1}{14.64}}{\frac{1}{8.93}+\frac{1}{14.64}+\frac{1}{18.74}} = 0.292 \\
W_3 &= \frac{\frac{1}{18.74}}{\frac{1}{8.93}+\frac{1}{14.64}+\frac{1}{18.74}} = 0.228
\end{aligned}
\tag{44}
$$

where W_1 is the weight of price, W_2 is the weight of time and W_3 is that of comfort zone.

Step 3: The Z-numbers should be transformed into crisp numbers for computation in the next step. Before that, the Z-numbers matrix should first be normalized so that the different attributes can be compared on the same scale. The normalized membership function is presented in Table 7. The normalized Z-numbers are then transformed into their corresponding crisp number shown as:

$$
M_{Hi} = \begin{pmatrix} 0.773 & 0.286 & 0.321 \\ 0.170 & 0.555 & 0.659 \\ 0.513 & 0.636 & 0.386 \end{pmatrix} \tag{45}
$$

Step 4: The final score of each alternative is computed as a weighted sum of its three attributes using Eq. 42:

$$
\begin{aligned}
Score(K_1) &= 0.773 \times 0.479 + 0.286 \times 0.292 + 0.321 \times 0.228 = 0.527 \\
Score(K_2) &= 0.170 \times 0.479 + 0.555 \times 0.292 + 0.659 \times 0.228 = 0.394 \\
Score(K_3) &= 0.513 \times 0.479 + 0.636 \times 0.292 + 0.386 \times 0.228 = 0.519
\end{aligned}
\tag{46}
$$

Step 5: From the calculated scores of the three alternatives K_1, K_2 and K_3, it can be found that K_1 is higher than the others. As a result, K_1 is the best choice among the three transports. If the passenger is more concerned with the time spent than money spent, the alternative K_3 can also be applied as its score is close to K_1.

Table 7
Normalized membership function of the first triangular fuzzy number.

Triangular fuzzy number	Membership function
F_{11} (8,10,12)	$\frac{0}{0.6667}+\frac{0.5}{0.7273}+\frac{1}{0.8}+\frac{0.5}{0.8889}+\frac{0}{1.0}$
F_{21} (20,24,25)	$\frac{0}{0.3200}+\frac{1}{0.3333}+\frac{0.75}{0.3478}+\frac{0.5}{0.3636}+\frac{0.25}{0.3810}+\frac{0}{0.4000}$
F_{31} (15,15,15)	$\frac{1}{0.5333}$
F_{12} (70,100,120)	$\frac{0}{0.4992}+\frac{0.5}{0.5454}+\frac{1}{0.6}+\frac{0.66}{0.6667}+\frac{0.33}{0.7500}+\frac{0}{0.8572}$
F_{22} (60,70,100)	$\frac{0}{0.6}+\frac{0.33}{0.6667}+\frac{0.66}{0.7500}+\frac{1}{0.8571}+\frac{0}{1.0}$
F_{32} (70,80,90)	$\frac{0}{0.6667}+\frac{1}{0.7500}+\frac{0}{0.8571}$
F_{13} (3,5,7)	$\frac{0}{0.2727}+\frac{0.5}{3636}+\frac{1}{0.4545}+\frac{0.5}{0.5454}+\frac{0}{0.6363}$
F_{23} (6,8,11)	$\frac{0}{0.5454}+\frac{0.5}{0.6363}+\frac{1}{0.7272}+\frac{0.66}{0.8181}+\frac{0.33}{0.9090}+\frac{0}{1.0}$
F_{33} (4,7,10)	$\frac{0}{0.3636}+\frac{0.33}{0.4545}+\frac{0.66}{0.5454}+\frac{1}{0.6363}+\frac{0.66}{0.7273}+\frac{0.33}{0.8182}+\frac{0}{0.9091}$

WMADM has fully considered the unreliability of the expert's opinion in the transports selection problem. The unreliability contained in the given Z-numbers exists in two forms, including the cardinality of the first fuzzy number and the value of the second fuzzy number. It can be found in Table 4 that the cardinality of attribute A_1 is low while its value of the second fuzzy number is high. As a result, the expert's opinion on attribute A_1 should be the most convincible one. In comparison, the attribute A_2 and A_3 have either a larger cardinality of the first fuzzy number or a lower value of the second fuzzy number, which causes them less credible. It can been seen that the calculated weights $(0.479, 0.292, 0.228)$ is consistent with the analysis.

6. Conclusion

In this paper, a new transformation of Z-number to mass function is presented. Based on the new transformation, the information volume of a Z-number is proposed. In addition, several numerical examples are demonstrated to show the properties of the proposed method to calculate information volume. Some remarkable results are shown as follows.

1) The mass function generated from a Z-number degenerates into probability distribution when the uncertainty of information source is 0.
2) Given a Z-number,greater unreliability of the Z-restriction leads to greater information volume and the relationship is approximately linear.
3) Given a Z-number, a less powerful Z-restriction leads to greater information volume. It was shown that the least powerful Z-restriction happens when the value v of B is around 0.48.
4) Given a Z-number, its first fuzzy number A contributes to the information volume by its compatibility restriction and fuzziness.

The paper also introduced a WMADM to solve the problem that the confidence of expert's opinion varies with attributes. In the future, we will extend the proposed method to calculate information volume of continuous Z-numbers and apply the proposed information volume to more decision making problems.

CRediT authorship contribution statement

Ben Xu: Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review & editing. **Yong Deng:** Validation, Resources, Writing – review & editing, Supervision, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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