Seminar on noncommutative Gröbner bases and minimal resolution

March 27, 2023

The aim of these talks will be to extend the familiar notion of Gröbner basis to the noncommutative case. We are interested specifically in monomial algebras over a field k, particularly path algebras of quivers, quotiented out by an admissible ideal. These are monomial algebras in the sense that they can be seen as

$$k \langle x_e, x_\alpha \mid e \in Q_0, \alpha \in Q_1 \rangle / I,$$

where I is the the ideal with the relations that make the x_e a complete set of orthogonal idempotents, annihilating from the left every x_α with $s(\alpha) \neq e$ and from the right every x_α with $t(\alpha) \neq e$, together with all the relations in the admissible ideal. For such algebras we look at minimal projective resolutions of the algebra as a bimodule over itself and its relevance in computing the Hochschild cohomology.

Talk 1. *In this talk we recollect the notion of a Gröbner basis for ideals in commutative monomial algebras, in order to extend it to the noncommutative case, following [FFG].*

Talk 2. *The second talk will be dedicated to Hochschild cohomology. This topic seems to be to vast to be explored in one talk, but at least some standard definitions will be given in regard to finite dimensional associative algebras following [W] or [L]. Other than that the speaker will be given the freedom motivate to the concept in any way he or she sees fit.*

Talk 3. *After the preliminaries, the second and third section of the main paper of interest [B] shall be discussed. Notation from the introduction should be familiar from the first to talks, but can be recollected. Technical details are not to be feared, contrarily, the computational nature of the procedures described has a lot of emphasis.*

Talk 4. *The last talk is dedicated to sections* 4, 5 *and* 6 *of the aforementioned paper [B], concluding the main goal of this seminar. Time permitting, it would be nice to do some examples from section* 7 *of that same paper, or (not exclusive) shed some light on remark* 8.3 *from the overarching work of Butler and King [BK] on the subject of minimal resolutions.*

References

[B] M.J. Bardzell, The altenating syzegy behaviour of monomial algebras, 1977

[BK] M.C.R. Butler, A.D. King, Minimal resolutions of algebras, 1999

[L] J. Loday, Cyclic homoolgy, 1998

[FFG] D. Farkas, C. Feustel, E. Green, Synergy in the theory of Gröbner basis and path algebras, Canad. J. Math. 1993

[W] C. Weibel, An introduction to homological algebra, chapter 9, 1994