

# Supplementary Material

## Surface Analysis for ST-ESC Optimization

Manuscript: *Data-driven extremum seeking control applied to a biohydrogen production process*

**Authors:** Algemirol Gil-Fernández, Fernando López-Caamal, Juan Gabriel Avina-Cervantes, Ixbalank Torres-Zúñiga

This supplementary document provides additional graphical evidence supporting the concavity, unimodality, and local optimality assumptions used for the ST-ESC design. The results correspond to three different experimental surface-generation strategies evaluated at their respective optimal operating points.

### Test Type I: Small Set Case (Luck)

This test corresponds to the initial surface construction using sparse experimental data. The following figures show the numerical gradient and Hessian evaluations.



Figure 1: Surface analysis for 5 points.

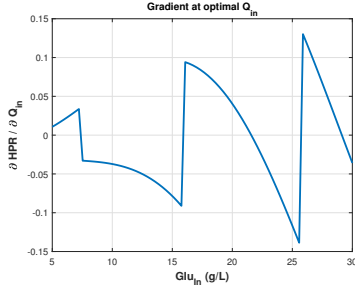
### 1 Test Type II: Incremental New Points

In this test, additional experimental points are incorporated iteratively to refine the steady-state surface. The gradient magnitude and Hessian structure are evaluated at the updated optimal operating point.

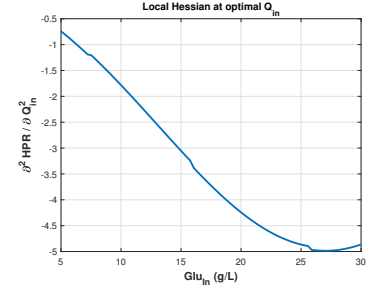
### Test Type III: Cumulative Additional Points

This test represents the final surface used for ST-ESC tuning. The results verify unimodality and local stability at the optimal operating region.

**Note:** All gradients and Hessians were computed numerically using finite-difference approximations at the identified optimal operating points.

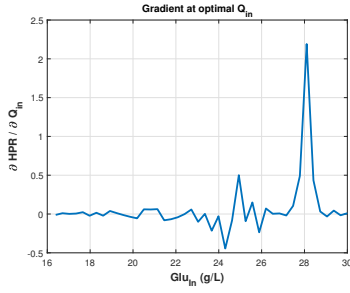


(a) Gradient.

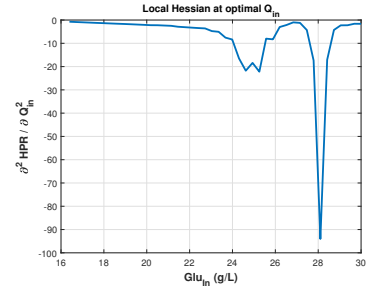


(b) Hessian.

Figure 2: Surface analysis for 5 points.

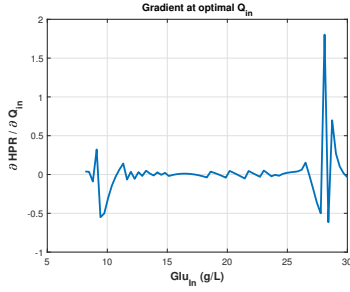


(a) Gradient.

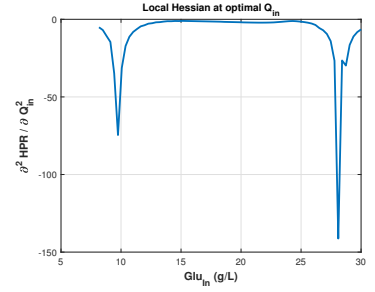


(b) Hessian.

Figure 3: Surface analysis for 10 points.

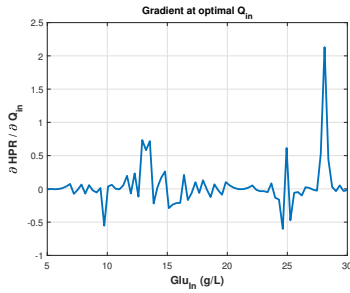


(a) Gradient.

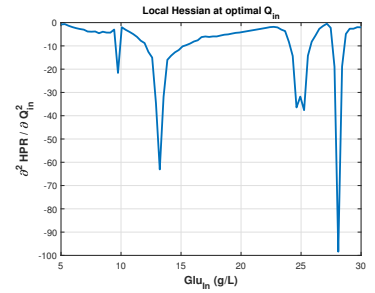


(b) Hessian.

Figure 4: Surface analysis for 15 points.

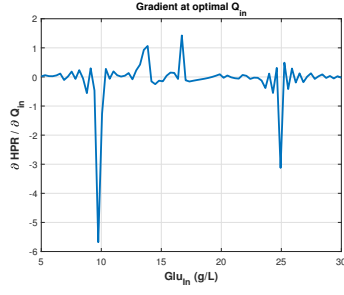


(a) Gradient.

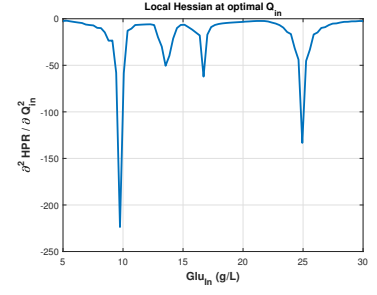


(b) Hessian.

Figure 5: Surface analysis for 20 points.

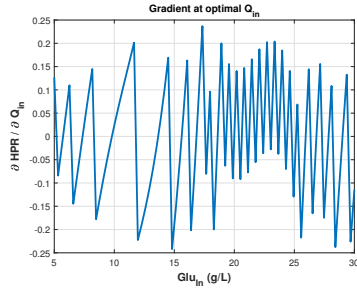


(a) Gradient.

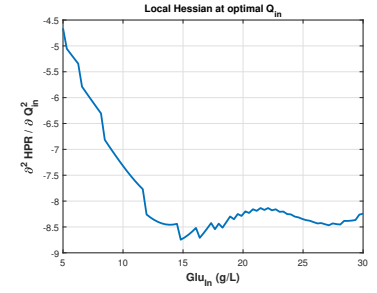


(b) Hessian.

Figure 6: Surface analysis for 25 points.

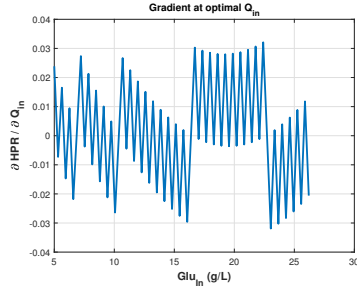


(a) Gradient.

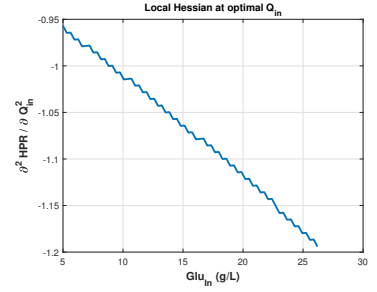


(b) Hessian.

Figure 7: Surface analysis for 30 points.

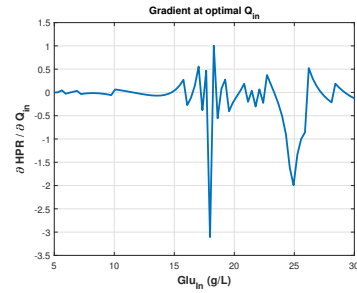


(a) Gradient.

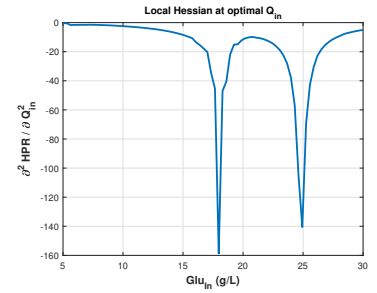


(b) Hessian.

Figure 8: Surface analysis for 5 points.

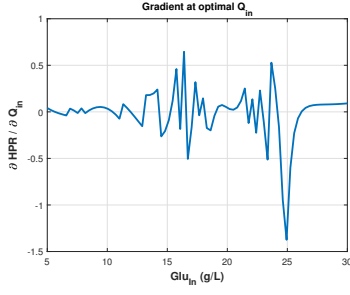


(a) Gradient.

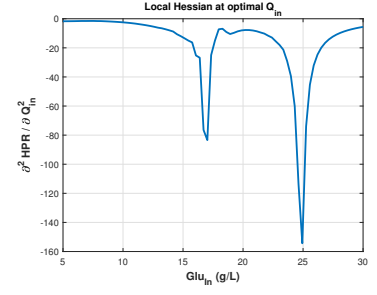


(b) Hessian.

Figure 9: Surface analysis for 10 points.

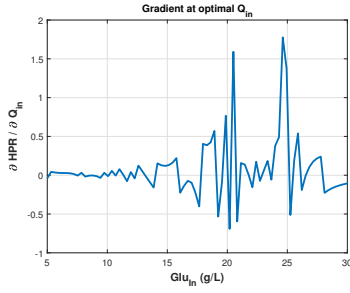


(a) Gradient.

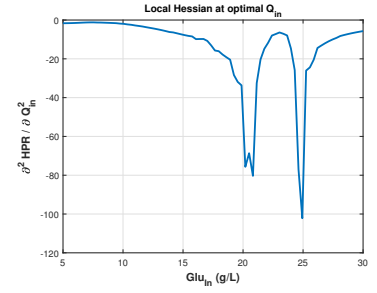


(b) Hessian.

Figure 10: Surface analysis for 15 points.

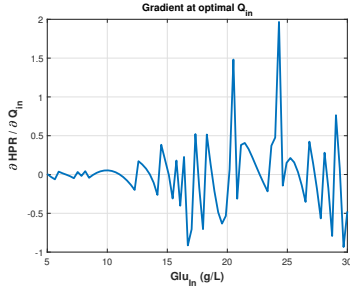


(a) Gradient.

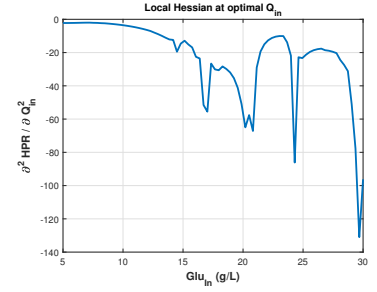


(b) Hessian.

Figure 11: Surface analysis for 20 points.

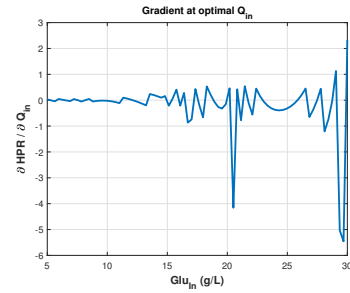


(a) Gradient.

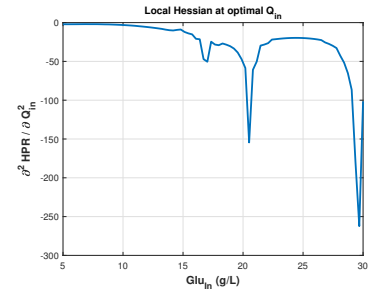


(b) Hessian.

Figure 12: Surface analysis for 25 points.



(a) Gradient.



(b) Hessian.

Figure 13: Surface analysis for 30 points.