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Time frequency analysis of the commonalities between Bitcoin and major Cryptocurrencies: Portfolio risk management implications

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ABSTRACT

This paper uses wavelet coherence and cross wavelet transform approaches to examine co-movement between Bitcoin and five major cryptocurrencies (Dash, Ethereum, Litecoin, Monero and Ripple) and their portfolio risk implications. The results show evidence of co-movements in time frequency space with leading relationships of Bitcoin with Dash, Monero and Ripple, lagging relationship with Ethereum, and out of phase movements with Litecoin. By considering different portfolios (risk-minimizing portfolio, equally weighted portfolio and hedging portfolio), we show evidence that a mixed portfolio (Bitcoin with other cryptocurrencies) provides better diversification benefits for investors and portfolio managers. Finally, an Ethereum-Bitcoin (Monero-Bitcoin) hedging portfolio offers the highest risk reductions and hedging effectiveness under medium and long term (short term) horizon. The results of downside risk reductions are time horizon dependent.

1. Introduction

The cryptocurrency ‘revolution’ is attracting serious attention by investors, portfolio managers, the media and academics alike. The pure simplicity of these new forms of virtual money is one reason for their popularity. Another, is that digital money is far less costly than mainstream currencies in terms of transaction costs. What is really revolutionary about cryptocurrencies, however, is the fact that they are traded outside the control and regulation of centralized financial institutions. At the time of writing there are approximately one-hundred different types of cryptocurrency; however, Bitcoin, Dash, Ethereum, Litecoin, Monero, and Ripple are currently the leading forms of digital money in terms of their market capitalization and traded volumes. Bitcoin is now considered as legitimate currency and is being used to purchase tangible (and often high value)[1] goods and services in the real world. Bitcoin has also come to be viewed as an alternative asset and

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[1] For example, high cost residential developments in the UAE are now being traded in Bitcoins. See Ryan Browne, ‘Real estate project in Dubai to be the first major development where you can purchase in bitcoin,’ *CNBC*, 5 Sept. 2017, https://www.cnbc.com/2017/09/05/dubai-real-estate-project-first-to-be-priced-in-bitcoin-michelle-mone.html.

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diversification option by investors looking to reduce risk in their portfolios. Despite the major impact of cryptocurrencies, so far, they remain volatile and interdependent (Ciaian, Rajcaniova, & Kancs, 2018). Therefore, statistical analyses of cryptocurrency linkages are essential in determining actual diversification investment opportunities, assessing optimal hedging strategies, and in the prevention of contagion effects.

While the empirical literature on cryptocurrency is rapidly trying to catch up to the technological, financial, economic and political implications of virtual money (see Urquhart, 2016, 2017; Nadarajah & Chu, 2017; Al-Yahyaee, Mensi, & Yoon, 2018; Phillip, Chan, & Peiris, 2018), the academic literature on co-movements amongst cryptocurrencies remains very limited. Ciaian et al. (2018) explored short and long-term relationships between different virtual currency prices, including Bitcoin, sixteen alternative virtual currencies, and two altcoin markets. The authors found evidence of interdependence between Bitcoin and altcoin markets. This evidence was more apparent in the short-run than in the long-run. Perhaps most importantly in that study, macro-financial factors were proven to determine altcoin price formation to a slightly greater degree than Bitcoin did. Whilst these were important revelations, a question yet to be addressed in the literature is: Does Bitcoin co-move with other cryptocurrencies? Does a diversified cryptocurrency portfolio provide hedging effectiveness and downside risk reductions? To the best of our knowledge, this is the first study that analyses the co-movements between Bitcoin and the other major cryptocurrencies. In this analysis, we endeavor to uncover new insights on the linkages amongst these digital monies on a time-frequency basis and its implications in terms of portfolio diversification benefits and downside risk reductions.

In specific terms, this study makes an important contribution to the cryptocurrency literature by examining co-movements between Bitcoin and other major cryptocurrency markets (i.e. Dash, Ethereum, Litecoin, Monero, and Ripple) over time and under high, medium and low frequencies. Further, we build different portfolios (risk-minimizing portfolio, equally weighted portfolio and hedging portfolio) to assess the risk reductions and downside risk reductions by comparing against a benchmark portfolio composed entirely of Bitcoin. Four popular risk metrics including Value at Risk, Expected shortfall, Semivariance, and Regret are considered. The reason we choose Bitcoin as the central factor of the study is that it is the most widely used virtual currency. It is also the most extensive in terms of market value, total virtual currency market capitalization, and total volume compared to the other digital currencies (see Table 1). Hence, Bitcoin has gradually gained the confidence of consumers, retailers, and service providers (Dahlby, Edgar, Li, & Macnaughton, 2014). As the most impactful and important cryptocurrency, it is feasible that the price-formation of Bitcoin affects those of the other digital currencies.

In this paper, we applied an empirical methodological framework based on a wavelet approach. This method is useful for examining co-movements between two series over time and under different frequencies. This method provides a full picture of the relationship across cryptocurrencies, which is clearly important for investors considering different opportunities, and for policy makers looking at the bigger macroeconomic picture. Of more interest, the wavelet approach allows for consideration of the short and long-run co-movements between digital currencies, which is crucial for risk assessment and hedging strategies. This approach also enables collection of accurate descriptive data regarding the local behavior of heterogeneous market participants with different objectives, preferences, and risk tolerances (Mensi, Hkiri, Al-Yahyaee, & Kang, 2018). The wavelet method encapsulates both short-term speculators and long-term investors, where their expectations are heterogeneous and time-frequency dependent. The multi-resolution decomposition of the wavelet transform allows us to identify spillovers, contagion and interdependence (Mensi et al., 2018). Benhmad (2013) stated the importance of this method in determining potential diversification opportunities. The wavelet approach performs an estimation of the spectral characteristics of a financial time series as a function of time, to reveal how the periodic components of the series vary over time (Aguiar-Conraria & Soares, 2011, 2014; Aguiar-Conraria, Soares, & Sousa, 2018; Aguiar-Conraria, Azevedo, & Soares, 2008). Aguiar-Conraria, Martins, and Soares (2018) indicate that financial time series exhibit stylized facts (most of the times noisy, strongly nonstationary, with possible nonlinear relations) and that continuous wavelet transform is suitable to use with this kind of data.

Our results show evidence of a significant time-frequency relationship between Bitcoin and Dash in most of the frequencies and periods; however, a moderate relationship is exhibited between Bitcoin and Litecoin, Monero and Ripple. Ethereum, on the other hand, shows the least sensitivity to Bitcoin. Lastly, while Bitcoin leads in a cyclical relationship with Ripple, Monero and Dash, it exhibits no cyclical relationship with Litecoin, and lags behind Ethereum in a cyclical relationship. In portfolio risk analysis for raw series, results highlight that among all cryptocurrency pairs, an Ethereun-Bitcoin portfolio pair offers maximum risk reduction benefits. In terms of individual risk reduction strategies, we find that maximum risk reduction in terms of VaR measure is achieved for

Table 1
Cryptocurrency statistics.

Name	Symbol	Market Capitalization	% Market Capitalization	Total Volume	Sample Periods
Bitcoin	BTC	$162.35B	60.00%	42.32%	2013–04-28–2018-01–22
Dash	DASH	$4.22B	1.48%	0.67%	2014–02-14–2018-01–22
Ethereum	ETH	$71.30B	25.03%	10.51%	2015–08-09–2018-01–22
Litecoin	LTC	$10.39B	3.65%	4.20%	2013–04-28–2018-01–22
Monero	XMR	$4.45B	1.56%	0.43%	2014–05-21–2018-01–22
Ripple	XRP	$32.14B	11.28%	2.87%	2013–08-04–2018-01–22
Total		$284.85B	100%		

Note: This table presents the sample period, market capitalization and the total volume for each cryptocurrency. The data is sourced from https://coinmarketcap.com.

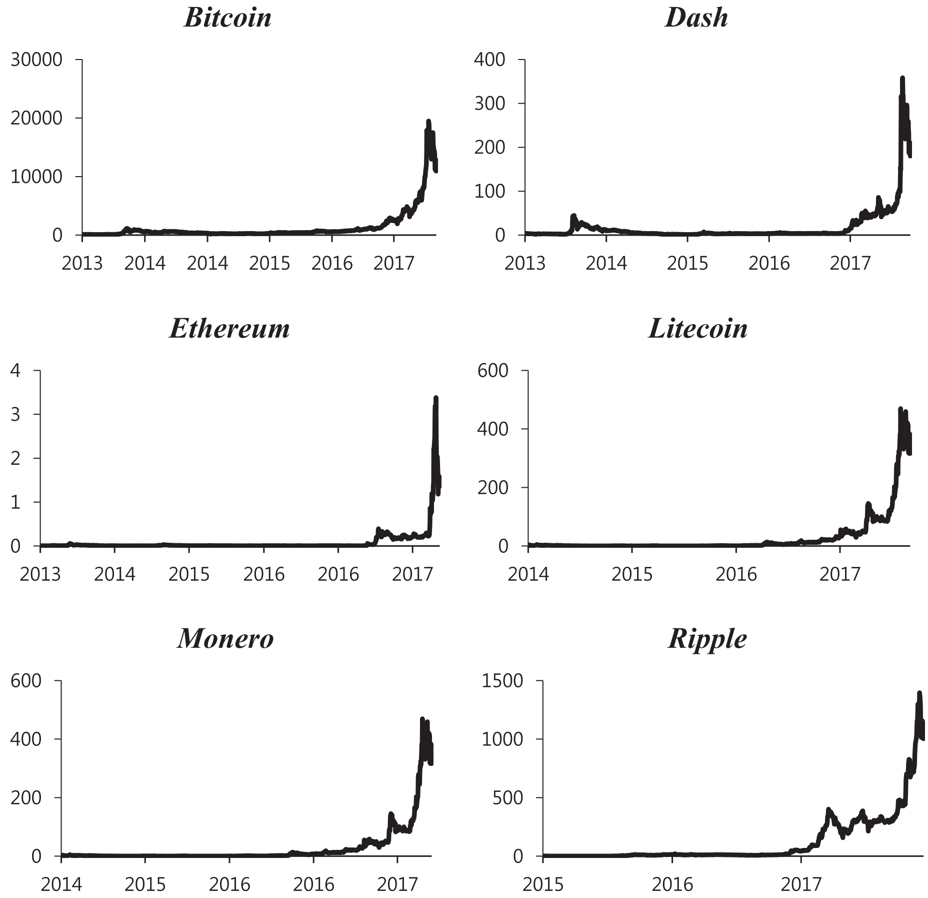


Fig. 1. Time evolution of daily cryptocurrency prices.

an equally weighted portfolio in four out of five cryptocurrencies combinations, with the exception of Litecoin where maximum VaR reduction is achieved for risk-minimizing portfolio. Monero-BTC equally weighted portfolio presents the highest VaR reductions and Regret reductions results are consistent with those of risk reduction. Investors and portfolio managers dealing with Bitcoin should consider other cryptocurrencies particularly Ethereum and to build a risk minimizing portfolio in order to benefit from diversification opportunities. After accounting for the time horizon factor, we find that hedging portfolio (*P-IV)* provides the highest risk reductions. In addition, Ethereun-Bitcoin (Monero-Bitcoin) portfolio offers maximum risk reduction benefits under medium- and long-term (short-term) horizons. As for the downside risk reductions, the results are sensitive to the time horizons.

The remainder of this study is organized as follows. Section 2 discusses the datasets, the methodology and offers a preliminary analysis of results. Section 3 provides an in-depth discussion of the empirical results. Section 4 sums up the findings and appraises the overall conclusions and implications of the study.

2. Data and methodology

2.1. Data and descriptive statistics

The dataset for our study was derived from daily closing-price information of six major cryptocurrencies, namely Bitcoin, Dash, Ethereum, Litecoin, Monero and Ripple. All sample data were extracted from the website of the Coindesk Price Index. Table 1 presents the sample period, market capitalization and total volume for each of the monitored cryptocurrencies. As shown, Bitcoin represents 60% of total market capitalization, followed by Ethereum (25%), and Ripple (11.28%). Dash, Monero and Litecoin have low market capitalization. In addition, Bitcoin and Ethereum together represent more than 50% of the total volume.

Fig. 1 plots the daily prices of all cryptocurrencies and reveals similar patterns for all cryptocurrencies. At least two phases are observable. The first phase is characterized by a constant trajectory, whereas the second phase consists of spectacular price rises from 2017. The daily price returns shown in Fig. 2 indicate fat tails (e.g., volatility clustering), which indicate the presence of non-linear behavior and justify our use of the wavelet approach.

Table 2 presents the descriptive statistics of the six cryptocurrencies' return series. The average returns of the six cryptocurrencies

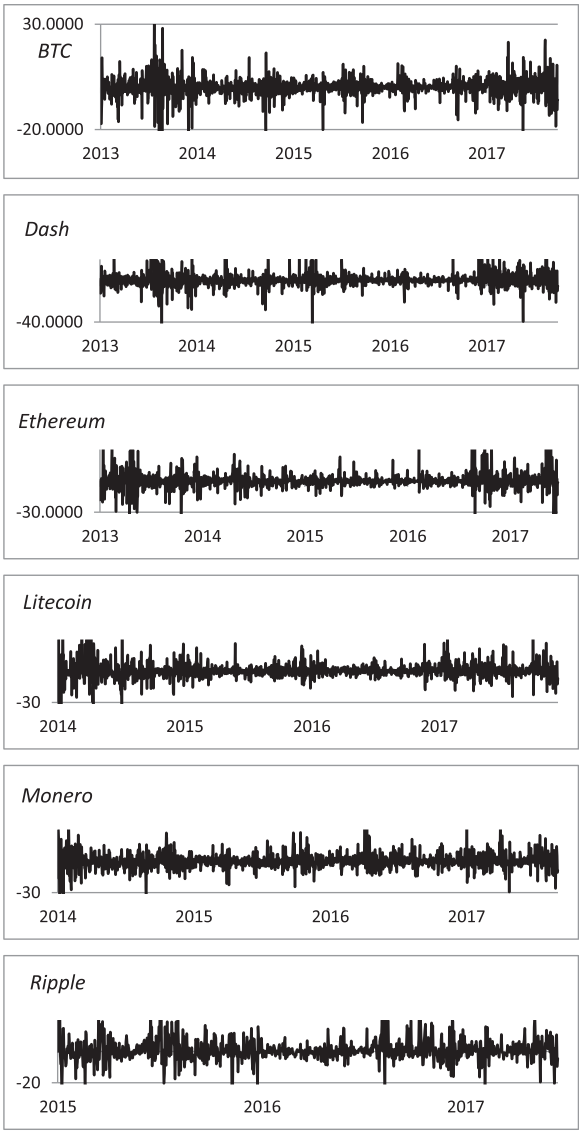


Fig. 2. Dynamics of cryptocurrency price returns.

Table 2
Descriptive Statistics.

Statistics	Bitcoin	Dash	Ethereum	Litecoin	Monero	Ripple
No. of obs.	1731	1731	1633	1439	1343	898
Minimum	−26.61	−51.39	−61.62	−46.75	−37.82	−31.54
Maximum	35.74	82.89	102.73	127.05	58.4	41.23
Mean	0.254	0.215	0.333	0.529	0.393	0.809
Standard deviation	4.448	6.929	8.056	8.593	7.820	7.309
Coefficient of Var.	17.686	27.367	23.950	15.463	20.662	8.990
Kurtosis	8.544	26.214	27.588	40.889	5.946	4.482
Jarque Bera	5243.*	46810.3*	53444.*	10397.01*	2114.6*	789.6*
Skewness	−0.169	1.838	2.075	3.094	0.708	0.551
Correlation	–	0.627*	0.298*	0.316*	0.370*	0.267*

* denotes significance at the 1% level.

are positive, with Ripple exhibiting the highest average returns followed by Litecoin. Bitcoin and Dash present lower level average returns. Dash and Ethereum represent the higher coefficients of variation, while Ripple exhibits the lowest coefficients of variation. Ethereum and Litecoin present the same risk level, with similar results showing for Monero and Ripple. Overall, Bitcoin is the least volatile cryptocurrency among the six observed cryptocurrencies. All returns series are asymmetric and leptokurtic, as revealed by Skewness and Kurtosis. According to the Jarque Bera test, the null of normal distributions applies for all return series. A close inspection on the linear correlation shows that Bitcoin is highly correlated with Dash and Monero, while for the other cryptocurrencies, the correlation level is consistent.

2.2. Wavelet coherence analysis

To analyze the time-frequency relationship of Bitcoin to other cryptocurrencies, we employed the wavelet methodology developed by Hudgins, Friehe, and Mayer (1993) and Torrence and Compo (1998). Phase difference and wavelet coherence (WTC) techniques are useful for analyzing time-frequency dependencies between any two series. Phase difference provides information on delays or synchronization of co-movement between two different time series; whereas cross-wavelet coherency acts as a correlation coefficient in time-frequency space.

According to Aguiar-Conraria and Soares (2011), cross-wavelet coherency forms a ratio of cross-spectrum to product of each series and can be considered as a local correlation in terms of time and frequency between the two series. The value of wavelet coherency is approximate to unconditional correlation and indicates higher equivalence (no association) for values close to 0 (zero). Variance in the series is highlighted by a power spectrum, with larger variations in the spectrum indicative of more power. Similarly, cross wavelet power spectrums indicate high levels of covariance at different times and frequencies. According to Torrence and Webster (1999) wavelet coherence for two variables is expressed as:

$$R_n^2(S) = \frac{|S(s^{-1}W_n^{xy}(s))|^2}{S(s^{-1}\,|W_n^x(s)|^2).\ S(s^{-1}\,|W_n^y(s)|^2)}, \tag{1}$$

In Eq. (1), $R_n^2(S)$ is local correlation between two series in time-frequency space. The value of squared wavelet coherency ranges between zero and one such that $0 \leq R^2(u, s) \leq 1$. S represents a smoothing operator in time and scale and generate values in-time frequency window. Value close to one suggests strong correlation whereas value close to zero represents weak correlation. Hence, the squared wavelet coherence is capable of measuring local linear correlation between the two time series at each scale. We also highlight wavelet coherence phase difference with clear indication about the delays in oscillations between two sampled series. n denotes localized time index whereas W_n^{xy} represent cross wavelet transformation of series x and y, where $x = \{x_n\}$ and $y = \{y_n\}$. Futhermore, the cross wavelet power is given by $|W_n^{xy}|$. In addition, the above expression also equates to a traditional correlation coefficient, which explains WTC in time-frequency space. For smoothing function where S equal to 1, the time scaling expression can be re-arranged as:

$$S(W) = S_{Scale}(S_{Time}(W_n(s))), \tag{2}$$

For Eq. (2), S_{Scale} represents smoothing along wavelet scale axis whereas S_{Time} represents smoothing in time. The smoothing operator is designed with a similar footprint of the used wavelet. An expression of the smoothing operator for Mortlet wavelet as given by Torrence and Webster is presented below.

$$S_{time}(W)|_s \;=\; (W_n(s) * c_1^{-t^2/2s^2})|_s \tag{3}$$

$$S_scale(W)|_n = (W_n(s) * c_2\Pi(0.6s))|_n \tag{4}$$

where c_1 and c_2 in the above equation are normalized constants whereas Π represents rectangle function. The value of 0.6 is the scale decorrelation length for Mortlet wavelet.[2]

2.3. Cross wavelet transformation and phase angle

We define cross wavelet transformation for two time series x and y as $W^{xy} = W^xW^{y*}$, whereas * represents the complex conjugation. Furthermore, we define the cross wavelet power as $|W^{xy}|$. Torrence and Compo (1998) presented the theoretical distribution of cross wavelet power spectrum of P_k^X and P_k^Y as

$$D\left(\frac{|W_n^x(s)W_n^{y*}(s)|}{\sigma_X\sigma_Y} < p\right) = \frac{Z_v(p)}{v}\sqrt{P_k^X P_k^Y}, \tag{5}$$

$Z_v(p)$ in the above expression represents confidence level associated with probabilities p defined as a square root of the product of two χ^2 distributions.[3]

To calculate the phase difference between the two time series components, mean value and confidence interval of the phase difference needs to be estimated. To do this, we use circular mean of the phase with higher than 5% statistical significance to quantify such relationship.

Regular window and Gaussian functions are used for scale and time convolutions, respectively (see Torrence & Compo, 1998).

[2] For details, see (Torrence & Compo, 1998). Both the convolutions are discretely computed leading towards the determination of normalized coefficients.

[3] The significance level at 5% is calculated using $Z^2(95\%) = 3.999$.

Monte Carlo simulations are then used to analyze the distribution of WTC, whereas phase difference among different components is estimated through mean and confidence intervals of the time series. We interpret the phase differences in the regions of high coherence as recommended by Aguiar-Conraria and Soares (2014).

3. Empirical results and discussion

This section analyzes the short and long-run co-movements between Bitcoin and other cryptocurrencies in different time-frequency domain and discusses the risk assessment and hedging strategies for the portfolio of Bitcoin-other cryptocurrency pairs. First, we reveal the empirical results of both cross-wavelet transformation and wavelet coherence for five pairs of cryptocurrencies (i.e., Bitcoin relative to five other cryptocurrencies). Second, we assess the risk reductions and downside risk reductions by comparing against a benchmark portfolio composed entirely of Bitcoin.

3.1. Cross wavelet transformation results

Fig. 3 plots both cross wavelet transform and wavelet coherence between Bitcoin-other cryptocurrency, respectively. Regarding the Bitcoin-Dash pair, the graphical results reveal a dominant cyclical cross-wavelet transformation relationship with a few high-power regions - highlighted in green for 2012–13 (see Panel A of Fig. 3). This suggests that the relationship between these two digital currencies followed a cyclical pattern with Bitcoin leading Dash during early 2013. Bitcoin also led Dash in almost all frequency bands in 2012–13; however, cyclical patterns are more evident in the 32–256 days' period suggesting the dominance of Bitcoin over Dash at a 5% level of significance.

Moving to the Bitcoin-Ethereum pair in Panel B, the results show an anti-cyclical relationship in most of the frequency bands, which suggests an out of phase relationship; however, a cyclical path is observed at a 5% level of significance for 2012, and again for 2018. It is interesting to note that prior to 2015 Ethereum led Bitcoin, whereas after 2015 the relationship reverses (i.e. Bitcoin leads Ethereum). These strong co-movements between Bitcoin and Ethereum are likely are due to the market share of these two digital currencies. In fact, at the time of writing they were the two largest digital currency markets in terms of market capitalization. We can also observe a high-power spectrum with higher frequency bands (i.e. 64–256 days) during the sample period; however, few significant wavelets with low frequencies are observable in 2018 for the 32–64 days' period. We also find an out of phase relationship between Bitcoin and Ethereum over the 128–256 days' period for 2013, which suggests that there is not a distinct lead-lag relationship between the two currencies. In contrast to Dash and Ethereum, Bitcoin exhibits very few significant relational patterns with Litecoin across the sample period at a high frequency (i.e. over 64–128 days' period in 2017) (see Fig. 3, Panel C). An out-of-phase relationship is observed in most of the periods, with no distinct lead-lag relationship apparent amongst mixed results.

On the other hand, in Panel D Bitcoin exhibits a significant time-frequency relationship with Monero throughout the sample period at a 5% level of significance. Frequent high-power wavelet patches are observable, with cyclical relationships involving Bitcoin leading Monero over the 4–32 days' period in 2012 and again in the 64–128 days' period in 2013. Low frequency wavelets (i.e. 4–32 days') are apparent throughout the sample period, which again demonstrates the leading function of Bitcoin vis-à-vis Monero.

Finally, the Bitcoin-Ripple wavelet transformation is depicted in Panel E of Fig. 3. Here, a cyclical relationship is evident, albeit insignificant, across sample the period prior to 2016. Bitcoin led Ripple in 2016–18 for the 64–128 days' period at a 5% level of significance. Minor significant patches over the 4–8 days' period are also observable for 2017; however, the effects are negligible and only relate to a short period.

3.2. Wavelet coherency test results

Our results regarding cross wavelet transformations highlight the common power of the two processes, without normalizing around a single wavelet spectrum. This technique, however, has some limitations because continuous wavelets in two-time series are usually multiplied. For instance, if one out of two spectrums are local, whereas the other spectrum exhibits high peaks, a cross spectrum may indeed show that high peaks bear no relationship to the two series. This would, therefore, suggest an inability of cross wavelet spectrums to be able to test the significance of any underlying relationship between two series; however, they could be used to estimate the phase spectrum. On the other hand, wavelet coherency has the ability to detect interrelationships between two-time series. We therefore applied a wavelet coherency approach in order to detect both time intervals and frequency bands in which cryptocurrency pairs co-move. Thus, we can report our results of wavelet coherency for the data in the following section.

Starting with the wavelet coherence results for the Bitcoin-Dash currency pair illustrated in Panel A of Fig. 3. It is observable that there is a common time frequency space throughout the sample period. For 2012 to 2015, the two time series co-move and Dash led Bitcoin over the 16–256 days' span with a significance level of 5%. However, after 2015, we can see that Bitcoin took the leading position over a significant 4–32 days' period span. Post-2017, however, co-variation between the two cryptocurrency pairs vanishes.

Among the five currencies, other than Bitcoin, Ethereum demonstrates the least co-variance with Bitcoin over the sample period (Panel B). This lack of covariance is consistent across all frequencies and periods. Although small variations between Bitcoin-Ethereum pairs can be observed, they do not warrant any meaningful interpretation, except to say that more diversification opportunities for cryptocurrencies investors may exist in this area.

Moving to Panel C, the wavelet coherency analysis for the Bitcoin-Litecoin pair displays certain areas of co-variation for the 4–32 days' period, and again in the 64–128 days' span in 2012 and 2013, with respective cyclical patterns present. However, in 2017, Litecoin took the leading position over the 64–128 days' period, with significant covariance extant at the 5% level of significance.

The Bitcoin-Monero pair exhibits significant covariance from 2013 to 2015, and Monero led against Bitcoin in all cases (Panel D). However, significant covariance is observed in the 16–64 days' span with a high-power spectrum. Finally, in Panel E we can observe scattered patches of normalized covariance between the Bitcoin-Ripple pair with significant results observable in the 64–128 days' period in 2016 and thereafter. In 2018, small patches across short spans of 4–16 days' are also evident at the 5% level of significance.

3.3. Implications for portfolio risk management

We serve from our results to analyze its implications in terms of portfolio risk assessment. To do this, we consider and compare the risk of three portfolios having different features against a reference portfolio namely, Portfolio *I* (*P-I*) which is composed only of Bitcoin asset. The other three portfolios are risk-minimizing portfolio (*P-II*), an equally weighted portfolio (*P-III*) and a last portfolio (*P-IV*) whose weights are determined according to a variance minimization hedging strategy. The last portfolio consists of a long position of one USD in the BTC market hedged by a short position of USD in the other cryptocurrency (Ethereum, Ripple, Dash, Monero and Litecoin) market. We first assess the risk reduction effectiveness of each portfolio (i.e. *P-II*, *P-III*, *P-IV*) by comparing the variance reduction of respective portfolio against the benchmark portfolio (*P-I*). We use four downside risk metrics to assess the attractiveness of cryptocurrency portfolio in providing downside risk protection. More precisely, we calculate the Expected Shortfall (ES), the Value-at-Risk (VaR), the Regret (Re), and the Semivariance (SV) for each portfolio (For more details on these measures, see Reboredo (2013) and Mensi, Hammoudeh, and Kang (2015).

Table 3 reports the risk evaluation results for cryptocurrency portfolios where risk-minimizing portfolio (*P-II*) provides the best risk reduction to investors compared to the remaining portfolio (*P-III* and *P-IV*). In addition, adding Ethereun in a risk-minimizing portfolio offers highest risk reduction compared to other cryptocurrencies (Ripple, Dash, Monero and Litecoin). For VaR reduction strategy, we find maximum risk reduction for an equally weighted portfolio (*P-III*) for four-out five cryptocurrencies, however Litecoin offers best VaR reduction in portfolio *P-III*. Among all cryptocurrencies, Monero-BTC portfolio presents highest VaR reduction for portfolio *P-III*. The values of ES are negative for all cases, indicating evidence against the ES reduction strategy however Regret reduction results are in line with those of risk reduction. The risk-minimizing portfolio (*P-II*) provides maximum downside risk reduction for Ethereum compared with the rest of cryptocurrencies. Rational behind such diversification benefit may be due to the fact that Ethereum is the second largest cryptocurrency in terms of trading volume and market capitalization and therefore can act as a hedge when included in the portfolio. Overall, we conclude that the diversified portfolio performs much better compared to the portfolio consisting only of Bitcoin asset.

The investor behavior is not the same. Some investors are considered as short-term investors (speculators) and they are interested by the short-term co-movements whereas other long-term investors (institutional investors and banker) are concerned by the long term co-movements. Thus considering the heterogeneous market hypothesis is crucial to better analyze the risk evaluation of portfolios and the behavior of each kind of investors. To do this we have decomposed the raw series into lower, intermediate and higher time-scales in order to account for different time investment horizons. More specifically, we have analyzed the risk evaluation for different portfolios under different time horizons by accounting for short-term horizon (0–32 days), medium term horizon (32–256 days) and long horizons (above 256 days). The estimate results are reported in Tables 4–6. We show that the mixed portfolio provides higher risk reductions and downside risk reductions in relative to the benchmark BTC portfolio regardless the time horizon. More importantly, the portfolio *P-IV* offers the highest risk reductions compared to the remaining portfolios (*P-II* and *P-III*). More precisely, ETH-BTC (Monero-BTC) portfolio *P-II* (*P-IV*) offers the highest risk reductions in both medium- and long-term (short-term) investment horizon.

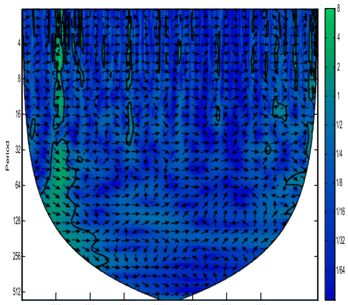
Regarding the downside risk reduction measures, we find that the portfolio *P-IV* generally offers the better downside risk reductions for the short-term horizon (except for Monero and Dash). For medium horizon, the high VaR reductions is obtained with Portfolio *P-III* (except for Dash and Ripple). The Ethereum-Bitcoin portfolio provides the highest VaR reductions whereas the Ripple-Bitcoin portfolio offers the lowest VaR reductions. Regarding the semivariance and regret risk measures, we find that portfolio *P-IV* provides the high downside risk reduction for the majority of cases. Regarding the long-term horizon, the results is similar to the medium term. In fact, the portfolio *P-IV* still provides the highest hedging with some exceptions. For example, for long-term horizon, the highest VaR reduction is reached with Portfolio *P-II* for the case of Dash, Monero and Ripple. This result indicates the importance of time horizon factor when accounting for hedging strategies. Investors, portfolio managers and institutional investors should be caution when constructing their portfolio. In addition, they should change the portfolio structure according to the time horizons.

4. Conclusion

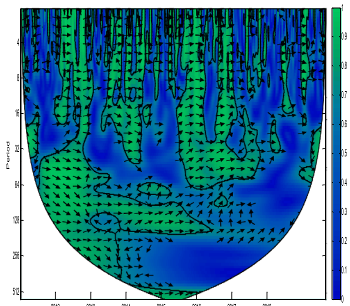
This study investigated co-movements, within a time-frequency space, between Bitcoin and five other major cryptocurrencies (Dash, Ripple, Litecoin, Monero and Ethereum). To achieve this, we used both cross wavelet transforms and wavelet coherence methods. Our results from the cross-wavelet method reveal a significant cyclical relationship between Bitcoin, Dash and Ethereum in 2012–13 within a 32–256 days' period. While Litecoin and Ripple show insensitivity to Bitcoin, Monero exhibits a strongly cyclical lagging relationship with Bitcoin throughout the sample period. To overcome the limitations of the cross-wavelet transform method (i.e. its inability to normalize two series to a single wavelet spectrum and to provide robustness to our results), we determined to apply wavelet coherence methods.

Our results support evidence that Dash has significant covariance throughout the sample period with Bitcoin, whereas Litecoin, Monero and Ripple show sensitivity mostly over the high frequency and 2014–2015 periods. Conversely, Ethereum shows no

Panel A: Bitcoin-Dash

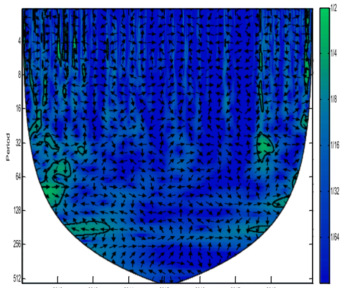


Cross wavelet transform (XWT): BTC-DASH

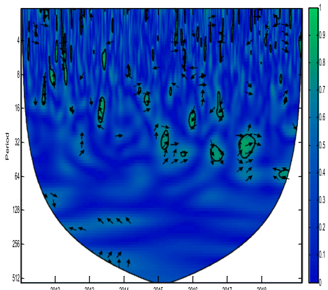


Wavelet coherence (WTC): BTC-DASH

Panel B: Bitcoin-Ethereum

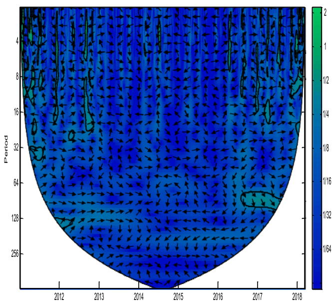


Cross wavelet transform (XWT): BTC-ETH

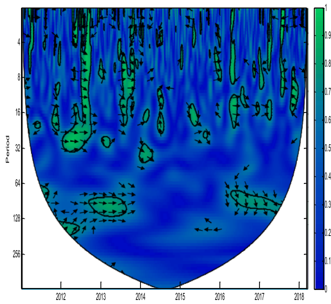


Wavelet coherence (WTC): BTC-ETH

Panel C: Bitcoin-Litecoin

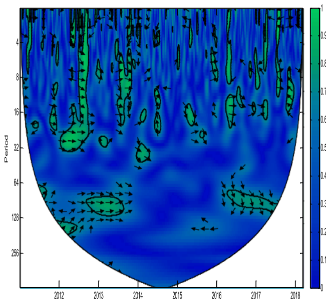


Cross wavelet transform (XWT): BTC-LTC

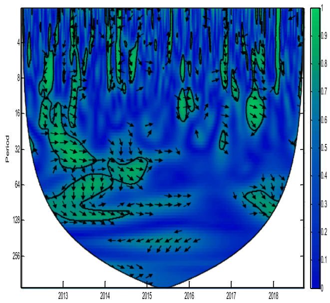


Wavelet coherence (WTC): BTC-LTC

Panel D: Bitcoin-Monero



Cross wavelet transform (XWT): BTC-XMR



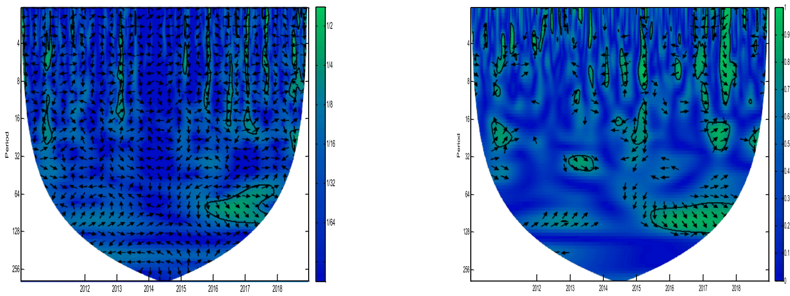
Wavelet coherence (WTC): BTC-XMR

(caption on next page)

Fig. 3. Wavelet transform and coherence test of Bitcoin with other cryptocurrencies. *Notes:* The dense black outlines surrounding green patches indicate five % level of significance. The area outside the cone of influence (at the bottom of every image) highlights the edge effect. We highlight color code at the right side of each image in which blue color indicates low power and green color indicates high-power. More the density of the colors, higher is the power of the wavelet. X-axis presents timeline for each cryptocurrency pair whereas **Y** axis measure scale or frequency. Phase difference between the two series is indicated by arrows direction and detailed interpretation is as follows. (→) = Both the variables are in phase (cyclical effect on each other); (↗) = Bitcoin is leading; (↘) = Bitcoin is lagging; (←) = variables are out of phase (anti-cyclical effect); (↖) = Bitcoin is lagging;(↙); Bitcoin is leading. A value of zero suggests that both the variables co-move with stated frequency. First series lags second series if $\phi_{x,y} \in [0, \pi/2]$. On the other hand "x" leads when $\phi_{x,y} \in [-\pi/2, 0]$. When there is a negative association between the two series (an anti-phase relationship) i.e. phase difference of $\pi\,(or - \pi)$; meaning $\phi_{x,y} \in [-\pi/2, \pi]\mathrm{U}[-\pi, \pi/2]$. If $\phi_{x,y} \in [\pi/2, \pi]$ then "x" leads, and "y" leads if $\phi_{x,y} \in [-\pi/2, -\pi]$.

sensitivity to Bitcoin throughout the sample period. These results hold important implications for investors in terms of portfolio diversification, but also in terms of understanding time frequencies between similar sets of cryptocurrencies. Our results highlight that a pair of cryptocurrencies might indicate covariance over a short-term, but may indeed prove insensitive to one another in the long run. Furthermore, the role of Bitcoin as a leading currency vis-à-vis other cryptocurrencies holds important implications for

Panel E: Bitcoin-Ripple



Cross wavelet transform (XWT): BTC-XRP   Wavelet coherence (WTC): BTC-XRP

Fig. 3. (*continued*)

Table 3
Risk evaluation for cryptocurrency portfolios.

		P-II	P-III	P-IV
Dash	Risk Red.	**0.98857**	0.00144	0.92373
	VaR Red.	0.02816	**0.04654**	0.04225
	ES Red.	−1.3066	−0.10262	−0.39752
	SV Red.	**0.08550**	0.00071	0.00933
	Re Red.	**0.30582**	0.02782	0.09663
Ethereum	Risk Red.	**0.99591**	2.836E-05	0.99082
	VaR Red.	0.04470	**0.04592**	0.02143
	ES Red.	−2.1454	−0.10266	−1.2789
	SV Red.	**0.12315**	0.00073	0.03353
	Re Red.	**0.47258**	0.02705	0.19387
Litecoin	Risk Red.	**0.99507**	0.02449	0.99130
	VaR Red.	**0.04378**	0.04378	0.02849
	ES Red.	−1.5325	−0.09451	−1.1718
	SV Red.	**0.07759**	0.00059	0.02813
	Re Red.	**0.41988**	0.02456	0.18388
Monero	Risk Red.	**0.99585**	0.004829	0.98924
	VaRRed.	0.02680	**0.04616**	0.02755
	ES Red.	−1.55551	−0.08801	−1.2638
	SV Red.	**0.03753**	0.00056	0.02148
	ReRed.	**0.26585**	0.02456	0.18471
Ripple	RiskRed.	0.99212	0.02761	**0.99309**
	VaR Red.	0.02449	**0.04343**	0.03006
	ES Red.	−1.70921	−0.08997	−1.4485
	SV Red.	**0.03939**	0.00056	0.01943
	Re Red.	**0.24715**	0.02544	0.24206

Note: The bold values denote the portfolio that has the best risk reduction and downside reduction among the three portfolios. The shaded values indicate the best cryptocurrency markets' assets among the three portfolios.

Table 4
Risk evaluation for cryptocurrency portfolios over the short-term horizon (0–32 days).

		P-II	P-III	P-IV
Dash	Risk Red.	0.57167	0.14287	**0.63901**
	VaR Red.	**0.04854**	0.04352	0.03627
	ES Red.	−6.0926	−4.6949	−6.5647
	SV Red.	1.68642	0.84022	**1.71359**
	Re Red.	**2.66613**	1.99212	2.26376
Ethereum	Risk Red.	0.61353	0.36192	**0.90294**
	VaR Red.	0.02734	0.02622	**0.02846**
	ES Red.	−6.0417	−5.1056	−9.3354
	SV Red.	1.91480	1.20357	**6.40573**
	Re Red.	2.39072	1.98180	**3.51911**
Litecoin	Risk Red.	0.36158	0.29701	**0.82485**
	VaR Red.	0.04073	0.03794	**0.04185**
	ES Red.	−5.1480	−3.6435	−10.121
	SV Red.	1.11648	0.50111	**4.16936**
	Re Red.	2.30639	1.61473	**4.11738**
Monero	Risk Red.	**0.95486**	0.04833	0.66765
	VaRRed.	0.01227	**0.04464**	0.03571
	ES Red.	−41.962	−4.2977	−10.971
	SV Red.	**1.15240**	0.067644	0.21843
	ReRed.	**4.51581**	1.89514	2.68792
Ripple	RiskRed.	0.48637	0.13948	**0.88391**
	VaR Red.	0.03627	**0.04185**	0.03348
	ES Red.	−10.147	−5.8841	−22.911
	SV Red.	1.52736	0.61681	**6.59961**
	Re Red.	2.59005	1.75487	**4.78669**

Note: See notes of Table 3.

Table 5
Risk evaluation for cryptocurrency portfolios over the medium-tem horizon (32–128 days).

		P-II	P-III	P-IV
Dash	Risk Red.	0.56466	0.17907	**0.63901**
	VaR Red.	0.04296	0.04352	**0.04502**
	ES Red.	−4.9667	−3.8807	−2.3954
	SV Red.	0.53494	0.52400	**0.59024**
	Re Red.	2.05347	1.54176	**5.97721**
Ethereum	Risk Red.	0.63326	0.07274	**0.94554**
	VaR Red.	0.04129	**0.05078**	0.04129
	ES Red.	−4.8396	−3.5049	−9.3611
	SV Red.	0.11320	0.47155	**8.66356**
	Re Red.	2.11712	1.43519	**3.28593**
Litecoin	Risk Red.	0.36277	0.17371	**0.77160**
	VaR Red.	0.03906	**0.04743**	0.04129
	ES Red.	−4.4824	−3.2067	−7.2497
	SV Red.	0.64288	0.34745	**1.72273**
	Re Red.	1.79417	1.33698	**2.46808**
Monero	Risk Red.	0.64788	0.21044	**0.97497**
	VaRRed.	0.04352	**0.04966**	0.03683
	ES Red.	−5.3393	−3.7086	−12.433
	SV Red.	1.16883	0.53921	**9.90251**
	ReRed.	2.32251	1.64937	**3.59926**
Ripple	RiskRed.	0.48933	0.03553	**0.87383**
	VaR Red.	**0.03850**	0.03459	0.03013
	ES Red.	−7.3628	−5.2725	−15.023
	SV Red.	0.90509	0.44686	**2.83171**
	Re Red.	2.13515	−1.48865	**3.30956**

Note: See notes of Table 3.

investors. Awareness of these directional relationships can proffer better understandings of the diversification opportunities extant among cryptocurrencies, but furthermore it can also assist in balancing effective combinations of cryptocurrencies and traditional assets. Among all other cryptocurrencies, Ethereum appears less sensitive to Bitcoin, which thereby highlights the opportunities for

Table 6
Risk evaluation for cryptocurrency portfolios over the long-term horizon (218–256 days).

		P-II	P-III	P-IV
Dash	Risk Red.	0.38063	0.00082	**0.91711**
	VaR Red.	**0.04296**	0.04185	0.02901
	ES Red.	−1.4149	−1.2041	−6.3907
	SV Red.	0.12884	0.20177	**2.45068**
	Re Red.	0.78021	1.01891	**2.30654**
Ethereum	Risk Red.	0.42866	0.23873	**0.99461**
	VaR Red.	0.04017	**0.04799**	0.03571
	ES Red.	−1.2041	−1.2141	−17.322
	SV Red.	0.12689	0.23584	**2.46684**
	Re Red.	0.69139	1.02468	**3.87684**
Litecoin	Risk Red.	0.38550	0.16761	**0.82827**
	VaR Red.	0.04241	**0.04687**	0.03738
	ES Red.	−1.1358	−1.1434	−4.7926
	SV Red.	0.11338	0.16231	**1.07870**
	Re Red.	0.73032	0.90773	**2.09494**
Monero	Risk Red.	0.22717	0.10627	**0.96271**
	VaRRed.	**0.05078**	0.04687	0.03404
	ES Red.	−1.4455	−1.4307	−6.8922
	SV Red.	0.15738	0.20717	**2.79887**
	ReRed.	0.84997	1.11448	**2.82635**
Ripple	RiskRed.	0.55301	0.29645	**0.98877**
	VaR Red.	**0.03627**	0.02790	0.01618
	ES Red.	−2.8650	−4.5473	−36.967
	SV Red.	0.13207	0.13091	**1.05442**
	Re Red.	0.77824	1.12836	**4.53177**

Note: See notes of Table 3.

investing in Bitcoin-Ethereum pairs. For raw (original) series, the results of portfolio risk analysis conclude that the risk-minimizing portfolio provides the best risk reductions. In addition, an Ethereun-Bitcoin risk-minimizing portfolio offers maximum risk reduction benefits compared to the other cryptocurrency pairs. The largest VaR reduction is achieved for an equally weighted portfolio in four-out five cryptocurrency pairs, with the exception of Litecoin where a risk-minimizing portfolio offers maximum VaR reduction. Monero-BTC pair for an equally weighted portfolio offers highest VaR reductions. We also report Regret reductions results being consistent with those of other risk reductions. A mixed portfolio provides diversification benefits, hedging effectiveness and downside risk reductions. By accounting for time horizons, we find that an Ethereum-Bitcoin (Monero-Bitcoin) hedging portfolio offers the highest risk reductions and hedging effectiveness under both medium and long term (short term) horizon. On the other hand, the results of downside risk reductions are time horizon dependent. These findings are vital in light of recent sharp escalations in the prices and market capitalizations of cryptocurrencies, which possibly present a 'virtual revolution' in the entire investment market for international investors, both now and into the future.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.najef.2019.02.013.

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