Chapter 62
Integration of Mean–Variance Model and Stochastic Indicator for Portfolio Optimization

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Abstract The mean–variance model proposed by Harry Markowitz is widely used for portfolio optimization. It helps the investors to allocate their capital to a number of assets, however, it does not guide the investors when to buy or sell these assets. In contrast, technical indexes are widely used to help the investors to decide when to buy/sell an asset. In this paper, we integrate the mean–variance model with a commonly used technical index, called stochastic indicator or KD index, to derive three investment strategies. Preliminary performance study is conducted to compare these investment strategies against the mean–variance model during the uptrend, downtrend and correction periods.

Keywords Mean–variance model · KD index · Portfolio optimization

62.1 Introduction

Modern portfolio theory attempts to maximize portfolio expected return for a given level of portfolio risk, or to minimize portfolio risk for a given level of expected return. It was pioneered by Harry Markowitz, who proposed a mathematical formulation of the portfolio optimization problem, and derived a mean–variance model to yield optimized portfolios [1]. These mean–variance optimized portfolios help the investors to allocate their wealth on a number of assets to achieve the goal of diversification.

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A mean–variance optimized portfolio, however, does not provide enough agility to the investors on the timing of buying and selling an asset. In the single-period model, the investors simply buy and sell these assets as suggested by the portfolio at the beginning and the end of a period, respectively [2]. In the multiple-period model, an optimized portfolio is generated for each period, and thus the investors need to adjust his/her holdings at the beginning of each period to match the portfolio of the period through a process called rebalancing [2]. Therefore, it is assumed that there is no buying and selling within each period except at the beginning or the end of a period. In practice, an investor may want to buy/sell an asset anytime with response to the dynamic market conditions. Consequently, using the mean–variance optimized portfolios alone may not be agile enough to meet the investors' need.

Technical analysis is a popular technique to decide the timing of buying and selling an asset. In this paper, we integrate the stochastic indicator (also called KD index) from technical analysis with a mean–variance optimized portfolio to derive three new investment strategies. On one hand, the mean–variance optimized portfolio is responsible of asset allocation. On the other hand, the KD index helps to determine the timing of buying and selling an asset. Overall, the resulting investment strategies provide more agility to the investors than the mean–variance optimized portfolio.

The rest of this paper is organized as follows. Section 62.2 reviews the mean–variance model and the KD index. Section 62.3 describes the proposed investment strategies. Section 62.4 presents the preliminary experimental results. Section 62.5 concludes this paper.

62.2 Preliminaries

62.2.1 Mean–Variance Model

Markowitz's mean–variance model uses the expected return of each asset and the covariance between the returns of any two assets as input. Given the expected returns of n risky assets $\mathbf{r} = (r_1, r_2, \ldots, r_n)$ and their variance–covariance matrix $\boldsymbol{\sigma} = \left(\sigma_{ij}\right)_{n \times n}$, the expected return and the risk of a portfolio $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ are measured as $\mathbf{rw}^T$ and $\mathbf{w\sigma}\,\mathbf{w}^T$, respectively. Then, the mean–variance model is formulated as the following optimization problem.

$$
\begin{aligned}
&\text{Maximize } \ \mathbf{rw}^T \\
&\text{s.t. } \mathbf{w\sigma}\,\mathbf{w}^T \le \mathrm{E}, \\
&\sum_{i=1}^{n} w_i = 1, \ \text{and} \\
&0 \le w_i \le 1, \ \text{for } i = 1 \text{ to } n.
\end{aligned}
$$

Here, E is the given level of risk provided by the investor. Alternatively, this problem can be formulated as maximizing the risk (i.e., $\mathbf{w\sigma}\,\mathbf{w}^T$) for a given level of return. The second constraint enforces that all wealth is invested, and the third constraint prohibits short selling.

62.2.2 KD Index

The stochastic indicator (or the KD index) is a commonly used indicator among future traders. The idea behind the indicator is that asset's price tends to close near the upper/lower end of an uptrend/downtrend period. Thus, the stochastic indicator first calculates the raw stochastic value (RSV) as follows:

$$RSV_n = \frac{(P_n - L_n)}{(H_n - L_n)} \times 100$$

where P_n, L_n, and H_n represent the current closing price, the lowest closing price and the highest closing price during n periods, respectively. Then, the KD index is defined as follows:

$$K_n = RSV_n \times \left(\frac{1}{3}\right) + K_{n-1} \times \left(\frac{2}{3}\right)$$
$$D_n = K_n \times \left(\frac{1}{3}\right) + D_{n-1} \times \left(\frac{2}{3}\right)$$

where $K_1 = D_1 = 50$. In this study, whenever $K_{n-1} \leq D_{n-1}$ and $K_n > D_n$, a buying signal is generated; whenever $K_{n-1} \geq D_{n-1}$ and $K_n < D_n$, a selling signal is generated.

62.3 Proposed Investment Strategies

This section proposes three investment strategies. The first strategy (called the *conservative strategy*) simply uses mean–variance model to derive the optimized portfolio, but uses KD index to decide when to buy an asset, as shown below.

1. Solve the mean–variance model to yield the optimized portfolio $\mathbf{w} = (w_1, w_2, \ldots, w_n)$;
2. For each asset i;
3. If KD index shows a buying signal, then buy asset i with wealth w_i;
4. Else invest wealth w_i to risk-free asset.

The second strategy (called the *moderate strategy*) relies on both the mean–variance optimized portfolio and the KD index's buying signal to choose assets for investment, but allocate equaled weight to each chosen asset, as follows:

1. Solve the mean–variance model to yield the optimized portfolio $\mathbf{w} = (w_1, w_2, \ldots, w_n)$;
2. B = ϕ;
3. For each asset i satisfying $w_i > 0$;
4. If KD index shows a buying signal, then add i to B;
5. For each asset $i \in$ B;
6. Buy asset i with wealth 1/|B|.

The third strategy (called the *aggressive strategy*) simply relies on the KD index's buying signal to choose assets for investment, but allocate equaled weight to each chosen asset, as follows:

1. B = ϕ;
2. For each asset i;
3. If KD index shows a buying signal, then add i to B;
4. For each asset $i \in$ B;
5. Buy asset i with wealth 1/|B|.

62.4 Experimental Results

The experiment is conducted using a moving window approach. In each window, the last month is used as test data, and the rest as training data. The training data is used both to calculate KD index to generate buying signal, and to construct the mean–variance model to yield the optimized portfolio. The resulting portfolio and the buying signal are then applied to the test data to gather the performance (i.e., return) of each investment strategies.

The experiment uses five assets as recommended in [3]. They are exchange traded funds (ETF): VTI, VEU, BND, VNQ, and DBC. The data spans from May of 2007 to March of 2013. The period from June 2008 to February 2009 is selected for the downtrend period; from February 2009 to December 2009 is for the uptrend period; from January 2010 to March 2013 is for the correction period.

Tables 62.1, 62.2 and 62.3 show the average monthly return during the downtrend, uptrend and correction periods, respectively. During downtrend and correction periods, all three strategies perform better than the mean–variance model. The conservative strategy performs the best and the worst for the downtrend period and the uptrend period, respectively. The aggressive strategy performs the best for the uptrend period, and the moderate strategy performs the best for the correction period.

Table 62.1 Average monthly return during the downtrend period

Strategy	Average monthly return	Standard deviation of monthly return
Conservative	0.0000	0.00018
Moderate	−0.0016	0.00470
Aggressive	−0.0016	0.00470
Mean–variance model	−0.0227	0.07702

Table 62.2 Average monthly return during the uptrend period

Strategy	Average monthly return	Standard deviation of monthly return
Conservative	0.0007	0.00112
Moderate	0.0120	0.03266
Aggressive	0.0284	0.03618
Mean–variance model	0.0022	0.00889

Table 62.3 Average monthly return during the correction period

Strategy	Average monthly return	Standard deviation of monthly return
Conservative	0.0014	0.00362
Moderate	0.0057	0.01306
Aggressive	0.0034	0.01986
Mean–variance model	0.0000	0.01454

62.5 Conclusions

In the literature, mean–variance optimized portfolios have been shown to put excessive weights on assets with large expected returns [4–7]. Instead of simply buying assets according the mean–variance optimized portfolios, this paper proposes the use of KD index to withhold some wealth to risk-free assets (as in the conservative strategy), or to yield an equaled weighted portfolio of selected assets (as in the moderate and aggressive strategies). Compared to the mean–variance model, the three proposed strategies show promising performance in term of monthly return. The mean–variance model only outperforms the conservative strategy during the uptrend period.

Further study with more datasets is needed to strengthen the results of this study. Also, we leave as future study for investigating the possibility of integrating more sophisticate techniques from technical analysis to improve the performance of the proposed strategies.

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References

1. Markowitz, H. (1952). Portfolio selection. *The Journal of Finance, 7*, 77–91.
2. Chapados, N. (2011). *Portfolio choice problems: An introductory survey of single and multiperiod models*. New York: Springer.
3. Faber, M. T., & Richardson, E. W. (2009). *The Ivy Portfolio: How to invest like the top endowments and avoid bear markets*. Hoboken, NJ: Wiley.
4. Huang, X. X. (2010). *Portfolio analysis from probabilistic to credibilistic and uncertain approaches*. Berlin: Springer.
5. Best, M. J., & Grauer, R. R. (1991). Sensitivity analysis for mean-variance portfolio problems. *Management Science, 37*(8), 980–989.
6. Green, R. C., & Burton, H. (1992). When will mean-variance efficient portfolios be well diversified? *The Journal of Finance, 47*(5), 1785–1809.
7. Lin, J.-L. (2013). On the diversity constraints for portfolio optimization. *Entropy, 15*(11), 4607–4621.