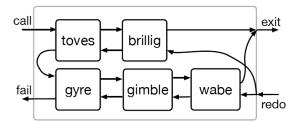


1 Prolog

Sample Box Model

```
1 slithy :- toves, brillig.
2 slithy :- gyre, gimble, wabe.
3 toves :- outgrabe, vorpal.
4 toves :- manxome.
5 brillig :- jubjub.
6 outgrabe.
7 vorpal :- manxome.
8 gyre :- manxome.
9 gyre :- outgrabe.
10 manxome.
11 gimble :- outgrabe.
12 gimble:- brillig.
13 gimble :- vorpal.
14 wabe.
```



Sample Proof Procedure

```
1 wff(A,B) :- terminal(A,Z), binop(Z,Y), wff(Y,B).
2 wff(A,B) :- unop(A,Z), wff(Z,B).
3 wff(A,B) :- terminal(A,B).
4
5 binop([&|T],T).
6 binop([or|T],T).
7 unop([not|T],T).
8
9 terminal([Term|T],T) :- term(Term).
10
11 term(p).
12 term(q).
13 term(r).
```

wff([p,&,not,q,or,r],V).

```
yes(V) :- wff([p,&,not,q,or,r],V).
yes(V) :- terminal([p,&,not,q,or,r],Z1), binop(Z1,Y1),
    wff(Y1,V).
yes(V) :- term(p), binop([&,not,q,or,r],Y1), wff(Y1,V).
yes(V) :- binop([&,not,q,or,r],Y1), wff(Y1,V).
yes(V) :- wff([not,q,or,r],V).
yes(V) :- unop([not,q,or,r],Z2), wff(Z2,V).
yes(V) :- wff([q,or,r],V).
yes(V) :- terminal([q,or,r],Z3), binop(Z3,Y3),
    wff(Y3,V).
yes(V) :- term(q), binop([or,r],Y3), wff(Y3,V).
yes(V) :- binop([or,r],Y3), wff(Y3,V).
yes(V) :- wff([r],V).
yes(V) :- terminal([r],V).
yes([]) :- term(r).
yes([]).
```

Sample Most General Unifier

```
(a) p(W; b f (W)) and p(h(X; Z); X; Y).
(b) ok([fun; course; dream]; A; Y) and ok([a j R; R;
    A]).
(c) bar(A; [o; 1 j B]; B) and bar([p; r j C];C; [o; g])

(a) { W / h(b, Z), X / b, Y / f(h(b, Z))}
(b) does not unify because A would have to be 'fun' and
    [course, dream]
(c) {A / [p, r, o, 1, o, g], B / [o, g], C / [o, 1, o,
    g]}.
```

Delete Exactly 1 Elements

```
1 % del1(E, L, R) is true when R is a list with the same
  elements as
2 % list L (in the same order) but with one instance of E
  removed
3 del1(E,[E|T],T).
4 del1(E,[H|T],[H|R]) :-
5     del1(E,T,R).
```

Delete n Elements

```
1 % deln(N,E,L,R) is true when R is the result of
  removing N instances of E from L.
2 deln(0,_,[],[]).
3 deln(N,E,[E|L],R) :-
4     N>0,
5     N1 is N-1,
6     deln(N1,E,L,R).
```

```
7 deln(N,E,[H|L],[H|R]) :-
8     deln(N,E,L,R).
9 % ?- deln(2,a,[a,v,a,t,a,r],R).
10
11 % Here is another solution
12 % deln_alt(N,E,L,R) is true when R is the result of
  removing N instances of E from L.
13 deln_alt(0,_,R,R).
14 deln_alt(N,E,[E|L],R) :-
15     N>0,
16     N1 is N-1,
17     deln_alt(N1,E,L,R).
18 deln_alt(N,E,[H|L],[H|R]) :-
19     N>0,
20     deln_alt(N,E,L,R).
```

Remove Duplicates (Keep 1st Occurrence)

```
1 myremoveduplicates_helper([], _, []).
2 myremoveduplicates_helper([H|T], Duplicates,
    RecursiveResult) :-
3     member(H, Duplicates),
4     myremoveduplicates_helper(T, Duplicates,
        RecursiveResult).
5 myremoveduplicates_helper([H|T], Duplicates,
    [H|RecursiveResult]) :-
6     \+ member(H, Duplicates),
7     myremoveduplicates_helper(T, [H|Duplicates],
        RecursiveResult).
8
9 myremoveduplicates(Lst, Result) :-
10     myremoveduplicates_helper(Lst, [], Result).
```

Replace

```
1 replace(_, _, [], []).
2 replace(Old, New, [Old | L], [New | R]) :-
3     replace(Old, New, L, R).
4 replace(Old, New, [Head | L], [Head | R]) :-
5     dif(Old,Head),
6     replace(Old, New, L, R).
7
```

Apply

```
1 % apply(L,S,R) is true if list L becomes R according to
  substitution S
2 % where a substitution is a list of sub(from,to) terms
3 apply([],_,[]).
4 apply([H|L],S,[HR|R]) :-
5     rep(H,S,HR),
6     apply(L,S,R).
7
8 % rep(E,S,R) is true if E gets replaced by R according
  to substitution S
9 rep(H,[],H).
10 rep(H,[sub(H,R)|_],R).
11 rep(H,[sub(H1,_)|S],R) :-
12     dif(H,H1),
13     rep(H,S,R).
```

1 % appla(A,S,R) is true if arithmetic expression A becomes R according to substitution S

```
2 appla(A,S,R) :-
3     atomic(A),
4     rep(A,S,R).
5 appla((A+B),S,(AR+BR)) :-
6     appla(A,S,AR),
7     appla(B,S,BR).
8 appla((A*B),S,(AR*BR)) :-
9     appla(A,S,AR),
10    appla(B,S,BR).
```

Reverse Flatten

```
1 flatr([[A|B]|T],R0,R2) :-
2     flatr([A|B],R1,R2),
3     flatr(T,R0,R1).
```

Shuffle

```
1 % shuffle(L1,L2,L3) is true if L3 is an interleaving of
  the elements of L1 and L2
2 shuffle(L,[],L).
3 shuffle([],L,L) :- dif(L,[]).
4 shuffle([H|T],L,[H|R]) :- shuffle(T,L,R), dif(L,[]).
5 shuffle(T,[H|L],[H|R]) :- shuffle(T,L,R), dif(T,[]).
```

Merge Sort

```
1 split([],[],[]).
2 split([X],[X],[X]).
```

```
3 split([X,Y|R],[X|R1],[Y|R2]) :- split(R,R1,R2).
4
5 merge([],L,L).
6 merge(L,[],L) :- dif(L,[]).
7 merge([A|L1],[B|L2],[A|R]) :-
8     A <= B,
9     merge(L1,[B|L2],R).
10 merge([A|L1],[B|L2],[B|R]) :-
11     A > B,
12     merge([A|L1],L2,R).
13
14 msort([],[]).
15 msort([A],[A]).
16 msort([A,B|L],S) :-
17     split([A,B|L],L1,L2),
18     msort(L1,S1),
19     msort(L2,S2),
20     merge(S1,S2,S).
```

BTree (With Append)

```
1 % Tree to list (with append)
2 tolist(empty,[]).
3 tolist(node(LT,Val,RT),L):-
4     tolist(LT,L0),
5     tolist(RT,L1),
6     append(L0,[Val|L1],L).
```

BTree (Without Append)

```
1 % Tree to list (without append)
2 tolist_d(T,L) :-
3     tolist3(T,L,[]).
4
5 % tolist3(T, L1, L2) is true if L1-L2 is preorder list
  of values in tree T
6 tolist3(empty, L, L).
7 tolist3(node(LT,Val,RT),L0,L2):-
8     tolist3(LT,L0,[Val|L1]),
9     tolist3(RT,L1,L2).
```

Derivative

```
1 % deriv(E,X,DE) is true if DE is the derivative of E
  with respect to X
2 deriv(X,X,1).
3 deriv(C,X,0) :- atomic(C), dif(C,X).
4 deriv(A+B,X,DA+DB) :-
5     deriv(A,X,DA),
6     deriv(B,X,DB).
7 deriv(A*B,X,A*DB+B*DA) :-
8     deriv(A,X,DA),
9     deriv(B,X,DB).
10
11 % Some optional extras
12 deriv(sin(E),X,cos(E)*DE) :-
13     deriv(E,X,DE).
14 deriv(cos(E),X,-sin(E)*DE) :-
15     deriv(E,X,DE).
```

Simplify

```
1 %simplify(Exp, Exp2) is true if expression Exp
  simplifies to Exp2
2 simplify(X,X) :-
3     atomic(X).
4 simplify((A+B),V) :-
5     simplify(A, VA),
6     simplify(B, VB),
7     simp_vals(VA+VB, V).
8 simplify((A*B),V) :-
9     simplify(A, VA),
10    simplify(B, VB),
11    simp_vals(VA*VB, V).
12
13 %simplify_vals(Exp, Exp2) is true if expression Exp
  simplifies to Exp2,
14 % where the arguments to Exp have already been
  simplified
15 simp_vals(O+V,V).
16 simp_vals(V+0,V).
17 simp_vals(A*B,AB) :-
18     number(A),number(B),
19     AB is A*B.
20 simp_vals(A*B,A*B).
21 simp_vals(0*_,0).
22 simp_vals(_*0,0).
23 simp_vals(V*1,V).
24 simp_vals(1*V,V).
25 simp_vals(A*B,AB) :-
26     number(A),number(B),
27     AB is A*B.
28 simp_vals(A*B,A*B).
```

Arithmetic Operations

```
1 % def(F,A,FA) function F on argument A returns FA
2 def(sq,X,X2) :- X2 is X*X.
3 def(plus,X,plus(X)).
4 def(plus(X),Y,Z) :- Z is X+Y.
5 def(gt,X,gt(X)). % gt(X) is \Y -> X>Y
6 def(gt(X),Y,true) :- X>Y.
7 def(gt(X),Y,false) :- X <= Y.
8
9 % eval(E,V) is true if expression E evaluates to V
10 % this uses square brackets as parentheses, values
  separated by commas
11 eval(N,N) :- number(N).
12 eval[V],V).
13 eval([P,A|R],V) :-
14     eval(A,AV),
15     def(P,AV,R1),
16     eval([R1|R],V).
```

Fibonacci

```
1 fib(1,0).
2 fib(N,F) :- N>1, N1 is N-1, fib2(N1,1,F).
3 fib2(0,_,F,F).
4 fib2(N,F0,F1,F) :- N>0, FS is F0+F1, N1 is N-1,
    fib2(N1,F1,FS,F).
```

Powerset

```
1 % Powerset
2 % Straight Recursion
3 powerset([], [[]]).
4 powerset([H|T],R) :-
5     powerset(T,PT),
6     applytoeach(H,PT,PR),
7     applytoeach(C, [], C).
8     applytoeach(H,[E|R],[[H|E],E|PR]) :-
9         applytoeach(H,R,PR).
10
11 % With an accumulator
12 powset([], [[]]).
13 powset([H|T],R) :-
14     powset(T,TP),
15     add_to_each(H,TP,TP,R).
16 add_to_each(C, [], TP,TP).
17 add_to_each(H,[L|R],TP,[[H|L]|A]) :-
18     add_to_each(H,R,TP,A).
19
20 % With an accumulator, alternative answer
21 powset2([], [[]]).
22 powset2([H|T],R) :-
23     powset2(T,TP),
24     add_to_each2(H,TP,TP,R).
25 add_to_each2(_, [], TP,TP).
26 add_to_each2(H,[L|R],TP,A) :-
27     add_to_each2(H,R,[H|L]|TP),A).
```

Dutch Flag

```
1 % for testing, here is a definition of the colour of
  numbers
2 red(E) :- E < 10.
3 white(10).
4 blue(E) :- E>10.
5
6 % dutch_flag(L,D) is true if D conatins the elements of
  L where,
7 % the red elements are before the white elements, which
  are before the blue elements.
8
9 % dutch flag with difference lists
10 dutch_flag_d1(L,R):-
11     partn_d1(L,R,W,W,B,B,[]).
12
13 %partn_d1(L,R1,R2,W1,W2,B1,B2) is true when
14 % R1-R2 contains elements of L that are red
15 % W1-W2 contains elements of L that are white
16 % B1-B2 contains elements of L that are blue
17 partn_d1([],R,R,W,W,B,B).
18 partn_d1([H|T],[H|R1],[R2,W1,W2,B1,B2] :-
19     red(H),
20     partn_d1(T,R1,R2,W1,W2,B1,B2).
21 partn_d1([H|T],R1,R2,[H|W1],W2,B1,B2) :-
22     white(H),
23     partn_d1(T,R1,R2,W1,W2,B1,B2).
24 partn_d1([H|T],R1,R2,W1,W2,[H|B1],B2) :-
25     blue(H),
26     partn_d1(T,R1,R2,W1,W2,B1,B2).
```

Reverse Dutch Flag

```
1 % reversing dutch flag with difference lists
2
3 % rev_dutch_flag(L,D) is true if D conatins the
  elements of L where,
```

```
4 % the red elements are before the white elements, which
  are before the blue elements.
5 % the elements in each colour group are in reverse
  order than they are in L
6 % rev_dutch_flag_d1(L,R):-
7     rev_partn_d1(L,R,W,W,B,B,[]).
8
9 %rev_partn_d1(L,R1,R2,W1,W2,B1,B2) is true when
10 % R1-R2 contains elements of L that are red in reverse
  order
11 % W1-W2 contains elements of L that are white in
  reverse order
12 % B1-B2 contains elements of L that are blue in
  reverse order
13 rev_partn_d1([],R,R,W,W,B,B).
14 rev_partn_d1([H|T],R1,R2,W1,W2,B1,B2) :-
15     red(H),
16     rev_partn_d1(T,R1,[H|R2],W1,W2,B1,B2).
17 rev_partn_d1([H|T],R1,R2,W1,W2,B1,B2) :-
18     white(H),
19     rev_partn_d1(T,R1,R2,W1,[H|W2],B1,B2).
20 rev_partn_d1([H|T],R1,R2,W1,W2,B1,B2) :-
21     blue(H),
22     rev_partn_d1(T,R1,R2,W1,W2,B1,[H|B2]).
```

2 Haskell

Folding

```
foldr ⊕ v[a1,a2,...,an] = a1 ⊕ (a2 ⊕ (... ⊕ (an ⊕ v)))
foldl ⊗ v[a1,a2,...,an] = (((v ⊗ a1) ⊗ a2) ⊗ ... ⊗ an
```

Evaluation

- Different Types of Evaluation
- call-by-value: evaluate argument before applying
 - call-by-name: reduction of function first
 - lazy evaluation (call-by-need): evaluate argument only once, only if needed
 - evaluation from outside in
 - otherwise (if it knows both arguments need to be evaluated) from left to right

Lazy Evaluation vs. Call-By-name
Lazy evaluation evaluates its arguments at most once, whereas call-by-name could evaluate an argument multiple times.

Polymorphic Typing

Definition
Polymorphic typing is when a function can take many types.

IO

```
do v1 <- a1
v2 <- a2
...
vn <- an
return (f v1 v2 ... vn)
```

Tail Recursion

```
1 -- harmonic_tr n evaluates to the nth harmonic number
2 harmonic_tr n = harmonic_tr_help 1 1 n
3 where
4     -- harmonic nc h n evaluates to the nth
        harmonic number given that the nc-th
        harmonic number is h
5     harmonic_tr_help nc v n
6     | nc == n = v
7     | otherwise = harmonic_tr_help (nc+1)
        (v+1/(nc+1)) n
```

Delete Excatly 1 Element (First Occurrence)

```
1 -- del1 e lst -- returns a list with one instance of
  e removed from list lst
2 -- need to make a choice of what happens if e is not in
  list
3 -- it could
4 ---- return lst
5 ---- give a runtime error
6 ---- return Nothing
7 del1 e (h:t)
8 | e==h = t
9 | otherwise = e:del1 e t
```

Delete Excatly 1 Element (All Possibilities)

```
1 deliall :: Eq t => t -> [t] -> [[t]]
2 deliall _ [] = []
3 deliall e (h:t)
4 | e==h = t:[h:1 | 1 <- deliall e t]
5 | otherwise = [h:1 | 1 <- deliall e t]
```

Delete Exactly *n* Elements

```
1 -- delna n e lst returns a list of all of the lists
  ↳ that result from deleting exactly n occurrences
  ↳ of e from lst
2 delna :: Eq t => Int -> t -> [t] -> [[t]]
3 delna 0 _ lst = [lst]
4 delna n e [] = []
5 delna n e (h:t)
6   | e==h = (delna (n-1) e t) ++ cons_to_each h (delna
  ↳         n e t)
7   | otherwise = cons_to_each h (delna n e t)
8   where
9     -- cons_to_each e lst -- conses e to every
  ↳     element of lst
10    cons_to_each _ [] = []
11    cons_to_each e (h:t) = (e:h):cons_to_each e
  ↳     t
```

Delete Exactly *n* Elements (List Comprehensions)

```
1 -- Note that this would be easier with list
  ↳ comprehensions:
2 delna2 :: Eq t => Int -> t -> [t] -> [[t]]
3 delna2 0 _ lst = [lst]
4 delna2 n e [] = []
5 delna2 n e (h:t)
6   | e==h = (delna2 (n-1) e t) ++ [h:r | r <- delna2 n
  ↳     e t]
7   | otherwise = [h:r | r <- delna2 n e t]
```

Remove duplicates (Keep 1st Occurrence)

```
1 myremoveduplicatesfirst :: Eq t => [t] -> [t]
2 myremoveduplicatesfirst lst = remdupfirst lst []
3   where
4     -- remdupfirst lst1 lst2 returns the elements
  ↳     in lst1 without duplicates that are not
  ↳     in lst2
5     remdupfirst [] _ = []
6     remdupfirst (h:t) lst2
7       | h `elem` lst2 = remdupfirst t lst2
8       | otherwise    = h : remdupfirst t
  ↳     (h:lst2)
```

Remove Duplicates (Higher-order Functions)

```
1 myremoveduplicates lst = [ head l | l <- tails lst, l
  ↳   <- /- [] , not (head l `elem` tail l)]
2 myremoveduplicates2 lst = [ h | (h:t) <- tails lst,
  ↳   not (h `elem` t)] --using pattern matching
```

Apply

```
1 -- myapply lst sub where sub is a list of (x,y) pairs,
  ↳ replaces each occurrence of x by y in lst.
2 myapply :: Eq t => [t] -> [t] -> [t] -> [t]
3 myapply [] _ = []
4 myapply (h:t) sub = app h sub :myapply t sub
5   where
6     -- app e sub gives the value e is replaced by
  ↳     according to sub
7     app e [] = e
8     app e ((x,y):r)
9       | e==x = y
10      | otherwise = app e r
```

Apply (Higher-Order Functions)

```
1 myapply lst sub = [(\\res -> if res /= [] then head res
  ↳   <- else e) [b | (a,b) <- sub, a==e] | e <- lst]
2 -- myapply "abdcdec" [( 'a','f' ), ( 'c','3' ), ( 'g','7' )]
3 -- myapply "baab" [( 'a','b' ), ( 'b','a' )]
4 -- or even clearer....
5 -- head_with_default lst def = head of list lst,
  ↳ otherwise (if there is no head) evaluates to
  ↳ def
6 head_with_default [] def = def
7 head_with_default (h:t) _ = h
8 myapply1 lst sub = [head_with_default [b | (a,b) <-
  ↳   sub, a==e] e | e <- lst]
9
10
11 -- or even simpler
12 myapply2 lst sub = [head [b | (a,b) <- sub++[(e,e)],
  ↳   a==e] | e <- lst]
```

Append (All Possibilities)

```
1 --- iappend l = [(l1,l2)] where l1 appended to l2 gives
  ↳ l
2 --- returns list of all answers
3 iappend :: [t] -> [[([t],[t])]
4 iappend [] = [[[]],[[]]]
5 iappend (h:t) = ([[],(h:t)) : [(h:l1,l2) | (l1,l2) <-
  ↳   iappend t]
```

Reverse (Pattern Matching)

```
1 -- rev2 l1 l2 = reverse of list l2 followed by l1
2 rev2 l1 [] = l1
3 rev2 l1 (x:xs) = rev2 (x:l1) xs
4 -- rev2 [1,2,4,6] [11,23,45,56]
5
6 rev = rev2 []
```

Reverse (Lambda Calculus)

```
1 re2 = myfoldl (myflip (:)) []
```

Tails

```
1 tails1 :: [t] -> [[t]]
2 tails1 = foldr (\x (h:r) -> (x:h):(h:r) ) [[]]
```

Split

```
1 -- split ('elem' ' ','?!") "What? is this thing? called
  ↳ Love." =>
  ↳ ["What","is","this","thing","called","Love"]
2 split sep [] = []
3 split sep (h1:h2:t)
4   | sep h1 = split sep (h2:t)
5   | sep h2 = [h1]:split sep (h2:t)
6   | otherwise = ((h1:w1):rest)
7   where w1:rest = split sep (h2:t)
8 split sep [h] -- if previous patterns do not match
  ↳ the argument must be a single element list
9   | sep h = []
10  | otherwise = [[h]]
```

Subsequence

```
1 subsequence l1 l2 = starts l1 l2 || l2 /= [] &&
  ↳ subsequence l1 (tail l2)
2   where
3     starts [] _ = True
4     starts _ [] = False
5     starts (h1:t1) (h2:t2) = h1==h2 && starts t1 t2
```

Shuffle

```
1 shuffle :: [t] -> [t] -> [[t]] -- list of all shuffles
2 shuffle l1 [] = [l1]
3 shuffle [] l2 = [l2]
4 shuffle (h1:t1) (h2:t2) =
5   [h1:e | e <- shuffle t1 (h2:t2)] ++ [h2:e | e <-
  ↳   shuffle (h1:t1) t2]
```

Merge Sort

```
1 split [] = ([],[])
2 split [x] = ([x],[])
3 split (x:y:r)
4   | let (r1,r2) = split r
5   | in (x:r1, y:r2)
6
7 merge [] l = l
8 merge l [] = l
9 merge (a:l1) (b:l2)
10  | a <= b = a: merge l1 (b:l2)
11  | otherwise = b: merge (a:l1) l2
12
13 msort [] = []
14 msort [a] = [a]
15 msort l =
16   let (l1,l2) = split l
17   in merge (msort l1) (msort l2)
```

BTree

```
1 -- a binary tree where the nodes are labelled with
  ↳ integers
2 data BTree = Empty
3           | Node BTree Int BTree
```

BTree tolist (With Append)

```
1 -- tolist lst returns the list giving the inorder
  ↳ traversal of lst
2 tolist :: BTree -> [Int]
3 tolist Empty = []
4 tolist (Node lt val rt) =
5   tolist lt ++ (val : tolist rt)
```

BTree tolist (Without Append)

```
1 -- tolist without append (++), using accumulators
2 tolista :: BTree -> [Int]
3 tolista lst =
4   tolist2 lst []
5
6 -- tolist2 tree lst returns the the list of elements
  ↳ of tree followed by the elements of lst
7 tolist2 :: BTree -> [Int] -> [Int]
8 tolist2 Empty acc = acc
9 tolist2 (Node lt val rt) acc =
10  tolist2 lt (val : tolist2 rt acc)
```

BSTree

```
1 -- a binary search tree
2 data BSTree k v = Empty
3               | Node k v (BSTree k v) (BSTree k v)
4               deriving (Eq, Show, Read)
```

BSTree tolist (With Append)

```
1 -- tolist lst returns the list giving the inorder
  ↳ traversal of lst
2 tolist :: BSTree k v -> [(k,v)]
3 tolist Empty = []
4 tolist (Node key val lt rt) =
5   tolist lt ++ ((key,val) : tolist rt)
```

BSTree tolist (Without Append)

```
1 tolista :: BSTree k v -> [(k,v)]
2 tolista lst =
3   tolist2 lst []
4   where
5     -- tolist2 tree lst returns the the list of
  ↳ elements of tree followed by the
  ↳ elements of lst
6     -- tolist2 :: BSTree k v -> [(k,v)] -> [(k,v)]
7     tolist2 Empty acc = acc
8     tolist2 (Node key val lt rt) acc =
9       tolist2 lt ((key,val) : tolist2 rt acc)
```

BSTree insert

```
1 -- insert key val tree returns the tree that results
  ↳ from inserting key with value into tree
2 insert :: Ord k => k -> v -> BSTree k v -> BSTree k v
3 insert key val Empty = Node key val Empty Empty
4 insert key val (Node k1 v1 lt rt)
5   | key == k1 = Node key val lt rt
6   | key < k1 = Node k1 v1 (insert key val lt) rt
7   | key > k1 = Node k1 v1 lt (insert key val rt)
```

BSTree insertv

```
1 -- insertv key val tree returns the previous value
  ↳ and the resulting tree
2 insertv :: Ord k => k -> v -> BSTree k v -> (Maybe v,
  ↳ BSTree k v)
3 insertv key val Empty = (Nothing, Node key val Empty
  ↳ Empty)
4 insertv key val (Node k1 v1 lt rt)
5   | key == k1 = (Just v1, Node key val lt rt)
6   | key < k1 = let (res,nt) = insertv key val lt
  ↳ in (res, Node k1 v1 nt rt)
7   | key > k1 = let (res,nt) = insertv key val rt
  ↳ in (res, Node k1 v1 lt nt)
```

BSTree btlookup

```
1 -- lookup key tree returns the value of key in tree
2 btlookup :: Ord k => k -> BSTree k v -> Maybe v
3 btlookup key (Node k1 v1 lt rt)
4   | key == k1 = Just v1
5   | key < k1 = btlookup key lt
6   | otherwise = btlookup key rt
7 btlookup key Empty = Nothing
```

BSTree value_in_tree

```
1 value_in_tree BSEmpty _ = False
2 value_in_tree (BSNode key val lt rt) v =
3   val==v || value_in_tree lt v || value_in_tree rt v
```

BSTree lefttree

```
1 lefttree :: BSTree k v -> Maybe (BSTree k v)
2 lefttree Empty = Nothing
3 lefttree (Node _ _ lt _) = Just lt
```

Average (One Pass)

```
1 -- summ lst = (sum lst, lenght lst)
2 summ :: [Int] -> (Int,Int) -- (sum,lenght)
3 summ [e] = (e,1)
4 summ (h:t) =
5   let (s,n) = summ t
6   in (s+h, n+1)
7 ave lst =
8   let (s,n) = summ lst
9   in s `div` n
```

Powerset

```
1 powerset [] = [[]]
2 powerset (h:t) =
3   let pset = powerset t
4   in [h:s | s <- pset] ++ pset
```

Covert to Upper Case

```
1 toUpper :: Char -> Char
2 toUpper x = toEnum( fromEnum x - fromEnum 'a' +
  ↳ fromEnum 'A')
```

3 Misc

Purpose of Uniform Resource Identifier (URI)
A URI is a constant that has a well-defined and standard meaning. It is useful because everyone who uses the constant means the same thing. (It does not imply that there is a unique URI for each thing (the unique names assumption); there can be multiple URIs for the same thing.)
Prolog vs. Haskell
Prolog can do something like `del1(val(x,V),val(y:3),val(x:7),val(x:2),R).` to return the value of x as well as the rest of the list. Haskell can work with higher-order function, e.g., generalizing this `delete1` to act on other functions of the elements (not just equality).
Main Advantage of Strong Typing
Strong typing help programmers eliminate many bugs before the program is put into production.
Advantage of Interactive Development
Programmers can experiment and test their code interactively, and to see results immediately.
Haskell Function Arguments
Q: In Haskell every function takes a single argument. What does this mean for functions that take 2 arguments, such as (+)?
A: (+) applied to a single argument returns a function which takes one argument and returns a value.
Difference List (Prolog vs. Haskell)
Prolog

```
1 wff (A,B) :- terminal (A,Z), binop (Z,Y), wff (Y,B).
2 wff (A,B) :- unop (A,Z), wff (Z,B).
3 wff (A,B) :- terminal (A,B).
4
5 binop (&|T|,T).
6 binop (|or|T|,T).
7 unop (not|T|,T).
8
9 terminal ([Term|T|,T) :- term (Term).
10
11 term (p).
12 term (q).
13 term (r).
```

Haskell

```
1 -- this uses the same naming conventions for variables
  ↳ as in the prolog (except, lower case, of
  ↳ course)
2 wff :: [[Char]] -> [[[Char]]]
3 wff a =
4   [ b | z <- terminal a, y <- binop z, b <- wff y]
5   ++ [b | z <- unop a, b <- wff z]
6   ++ terminal a
7
8 binop ("&":t) = [t]
9 binop ("or":t) = [t]
10 binop ( _:_) = []
11 binop [] = []
12
13 terminal (term:t)
14   | term `elem` ["p","q","r"] = [t]
15   | otherwise = []
16 terminal [] = []
17
18 unop ("not":t) = [t]
19 unop ( _:_) = []
20 unop [] = []
```