CPSC 421: Introduction to Theory of Computing **Winter Term 1 2018-19**

Lecture 18: October 19

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18.1 More Reductions

$E_{TM} = \{\langle N \rangle : N \text{ is a TM s.t. } L(N) = \emptyset\}$

Theorem 18.1 *(Sipser 5.1): E_{TM} is undecidable.*

Proof: We know A_{TM} is undecidable. We want to prove $A_{TM} \leq_T E_{TM}$. Then we can conclude that E_{TM} is undecidable. How to do reduction? Assume that R is a (hypothetical) TM solving $E_T M$. We must somehow construct a TM S that decides A_{TM}. $A_{TM} = \{\langle M, w \rangle : M \text{ accepts } w\}$.

Design of S:

On input X:

1. If x is not of form $\langle M, w \rangle$, reject.
2. Construct description of N like this:

   Let N be TM: On input y, (ignore input), simulate M on input w:

   (a) If M accepts N accepts.

   (b) If M rejects N rejects.

S is a decider since R is.

What is $L(N)$:

Case 1: M accepts w. Then $L(N) = \Sigma^*$.

Case 2: M does not accept w:

$2a$: M rejects w. Then N rejects all inputs, so $L(N) = \emptyset$.

$2b$: M runs forever on input w. Then N runs forever on all inputs, so $L(N) = \emptyset$.

Analysis:

In Case 1, then R rejects $\langle N \rangle$, so S accepts (M accepts w). Otherwise, $L(N) = \emptyset$, so R accepts $\langle N \rangle$, so S rejects. ■

$REGULAR_{TM} = \{\langle N \rangle : N \text{ is a TM s.t. } L(N) \text{ is regular}\}$

Proof: We know A_{TM} is undecidable. We want to prove $A_{TM} \leq_T E_{TM}$. Then we can conclude that $REGULAR_{TM}$ is undecidable.

What is $L(N)$:

Case 1: M accepts w. Then $L(N) = \{0^n 1^n : n \in \mathbb{N}\}$ (NOT REGULAR!).

Case 2: M does not accept w:

$2a$: M rejects w. $L(N) = \emptyset$ is regular.

$2b$: M runs forever on input w. $L(N) = \emptyset$ is regular.

Analysis:

If M accepts w, then $L(N)$ is not regular, so R rejects, so S accepts. Otherwise, $L(N) = \emptyset$, so R accepts $\langle N \rangle$, so S rejects. ■