CPSC 421: Introduction to Theory of Computing **Winter Term 1 2018-19**

Lecture 21: October 26

Lecturer: Nicholas Harvey *Scribes: Kaitian Xie*

$L = \{x : \exists y, |y| \le |x|^c, V \text{ accepts } x, y\}$

Must show:

1. If G is 3-colourable, then $\exists y$ s.t. V accepts x, y, y should be a valid 3-colouring.
2. If G is <u>not</u> 3-colourable, then $\forall y$, V will not accept x, y.
3. V runs in polynomial. Runtime is $O(|V| + |E|)$ in our example. + decoding time, which is polynomial in input size.

Runtime of M is, let $n = |x|$: $\underbrace{(2^{n^c})}_{\text{if } |\Sigma|=2} \cdot \underbrace{(\text{runtime of } V \text{ on input } x, y)}_{=(|x|+|y|)^k \le (2n^c)^k} = O(2^{n^{c+1}})$. Otherwise, if $|\Sigma| \le d$, it would be $d^{n^c} = 2^{(\log_2 d)n^c} = O(2^{n^{c+1}})$

Definition 21.1 *A function $f : \Sigma^* \to \Sigma^*$ is <u>polytime computable</u> if there exists a TM M that has x as input, runs for time poly$(|x|)$, and halts with $f(x)$ written on the tape.*

Definition 21.2 *f is a polytime reduction from A to B if:*

1. $f(A) \subseteq B$
2. $f(\overline{A}) \subseteq \overline{B}$
3. *f is a polytime computable function.*

Notation: $A \le_P B$