CPSC 421: Introduction to Theory of Computing **Winter Term 1 2018-19**

Lecture 3: September 10

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3.1 Regular Language (Continued)

Definition 3.1 *A regular language is any language L s.t. some finite automaton accepts L.*

We will study operations on the class of regular languages.

For strings x, y their concatenation is denoted $x \circ y$ or just xy.

Definition 3.2 *For languages L_1 and L_2*

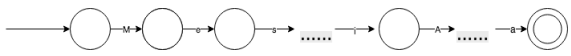
$$L_1 \circ L_2 = \{x \circ y : x \in L_1 and y \in L_2\}$$

If L_1 and L_2 are regular, is $L_1 \circ L_2$ also?

$L_1 = \{Messi\}$

$L_2 = \{Alba\}$

$L_1 \circ L_2 = \{MessiAlba\}$



M_3 accepts $L_1 \circ L_2 \Rightarrow L_1 \circ L_2$ is regular.

3.2 Non-determinism

Definition 3.3 *A non-deterministic finite automaton is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$ s.t.*

- *Q, Σ, q_0, F are the same*
- *$\delta : Q \times (\Sigma \cup \{\epsilon\}) \to 2^Q$*
  - *$\delta(q, s)$ is a subset of Q*

For a set S, 2^S is called the power set of S. It contains all subsets of S.

$2^{\{a,b\}} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Definition 3.4 *The NFA M accepts the string $w = w_1 w_2 \cdots$ if there exists a string $y = y_1 y_2 \cdots y_m \in (\Sigma \cup \{\epsilon\})^*$ and a sequence $r0, r_1, \cdots r_m \in Q$ such that:*

- $w = y_1 \circ y_2 \circ \cdots \circ y_m$
- $r_0 = q_0$
- $r_i \in \delta(r_{i-1}, y_i)$ *for* $i = 1, \cdots, m$
- $r_m \in F$

Input string: $w = 00$

$y = \epsilon 00$, $r = q_0 q_1 q_2 q_1$

$\delta(q_0, \epsilon) = \{q_1, q_3\}$