CPSC 421: Introduction to Theory of Computing **Winter Term 1 2018-19**

Lecture 8: September 21

Lecturer: Nicholas Harvey *Scribes: Kaitian Xie*

8.1 Example of CFG

$$\text{Rules} \begin{cases} S \to Sa \\ S \to Sb \\ S \to \epsilon \end{cases}$$

or

$$S \to Sa|Sb|\epsilon$$

- Variables: S
- Terminals: a, b
- Start variable: S

LHS of a rule is a single variable. RHS of a rule is any string of variables and terminals (and ϵ).

8.2 Deriving Strings from a Grammar

Formally, suppose u, v, w are strings of variables and terminals. Suppose there is a rule $A \to w$. From the string uAv, we can obtain uwv. We unite $uAv \xrightarrow[\text{"yields"}]{} uwv$. If $u_1 \to u_2 \to u_3 \to \cdots \to u_k$, then $u_1 \xrightarrow[\text{derives}]{*} u_k$.

Given a grammar G, the language derived by the grammar is: $L(G) = \{w \in \Sigma^* : \text{ start variable} \to w\}$.

In example, $S \to Sa \to Saa \to baa$, so $baa \in L(G)$ i.e. $S \xrightarrow{*} baa$.

Ex 2: $L = \{0^n 1^n : n \geq 0\}$

- $S \to 0S1|\epsilon$
- $S \to \epsilon$
- $S \to 0S1 \to 01$
- $S \to 0S1 \to 00S11 \to 0011$

A context-free language is a language can be derived from a CFG.

$$
\begin{aligned}
S &\to NP\ VP \\
&\to \textit{Alaska}\ VP \\
&\to \textit{Alaska}\ VerbNP \\
&\to \textit{Alaska want}\ NP \\
&\to \textit{Alaska want you}
\end{aligned}
$$

Can I derive 0101? No.

$$
\begin{aligned}
E &\to E + T \\
&\to T + T \\
&\to F + T \\
&\to 1 + T \\
&\to 1 + T \times F \\
&\to 1 + F \times F \\
&\to \cdots \\
&\to 1 + 2 \times 2
\end{aligned}
$$