CPSC 421: Introduction to Theory of Computing **Winter Term 1 2018-19**

Lecture 1: September 5

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1.1 What is a computational problem?

Exercises:

1. Sorting a list of names
2. Given a polynomial, find its roots
3. Given an integer, find its prime factors

Representation issues: Encode input & output

Generic representation:

Definition 1.1 *An* alphabet *is a finite non-empty set. Typically denoted* Σ *and* Γ *(e.g. ASCII, Unicode:* $\Sigma = \{0, 1\}$*).*

Definition 1.2 *A* string *is a finite sequence of zeros or more symbols from* Σ *(e.g. text file, binary file).*

Definition 1.3 Σ^* *is a set of all strings over alphabet* Σ *(so* Σ^* *is infinitely big).*

A problem is a mapping of strings to strings
e.g. for Ex. 3

```
f("b") = "2, 3"
f("30") = "2, 3, 5"
f("28mT") = "error"
```

Notice: It must be a function.

1.2 What is a decision problem?

Definition 1.4 *A* decision problem *is a problem whose input is yes/no (accept/reject).*
e.g.

1. *Is this list sorted?*
2. *Given integers* (x, y)*. does* x *has a prime factor less than* y*?*

```
f("35, 4") = "reject"
```

Important concept: Decision problem ≡ set of strings for which the function outputs "accept"

Definition 1.5 *A set of strings is called* language, *so any set $S \in \Sigma^*$ is a language. So decision problems ≡ languages*

$$L = \{S : s \text{ is a string of the form } s = \text{“p”} \text{ where p is a prime integer}$$
$$\equiv$$
$$f(s) = \begin{cases} \text{“accept”} & \text{if s = “p” and p is a prime integer} \\ \text{“reject”} & \textit{otherwise} \end{cases}$$