CPSC 421: Introduction to Theory of Computing **Winter Term 1 2018-19**

Lecture 20: October 24

Lecturer: Nicholas Harvey *Scribes: Kaitian Xie*

20.1 Time Hierarchy Theorem

Definition 20.1 $EXP = \bigcup_{k \geq 1} TIME(2^{n^k})$ *and* $P = \bigcup_{c>0} TIME(n^c)$

Claim 20.2 $P \subseteq EXP$

Why? Because $n^c = O(2^n)$.

Theorem 20.3 ***Time Hierarchy Theorem****(Sipser Thm 9.10)*
Let $f : \mathbb{N} \to \mathbb{N}$ *be "reasonable", and* $f(n) = \Omega(n \log n)$*. Then* $TIME(f(n)) \subsetneq TIME(f(4n)^4$ *or* $f(n)^2)$

E.g. Let $f(n) = n^c, TIME(n^c) \subsetneq TIME(n^{4c}), \forall c > 1$.

Corollary 20.4 $TIME(n^c) \subsetneq TIME(2^n), \forall c > 1$

Corollary 20.5 $P \subsetneq TIME(2^n) \subseteq EXP$

Claim 20.6 $P \subseteq NP$

Why? Any DTM is trivially an NTM.

Claim 20.7 $3COLORMAP \in NP$

Here's an NTM that decides it: On input x,

1. Reject if $x \notin \langle G \rangle$.
2. Let n be # countries in G.
3. Nondeterministically pick $Z \in \{1, 2, 3\}^n$
4. For all $c, j \in \{1, \cdots, n\}$, if i neighbours j and $Z_i = Z_j$, reject.
5. Accept.

Runtime of NTM is $O(n^2) = O((\text{input length})^2)$.

Claim 20.8 $NP \subseteq EXP$

Open Question 1: Is $P \neq NP$?

Open Question 2: Is $N \neq EXP$?

Theorem 20.9 *Either Question 1 or Question 2 is True.*

Why? Because $P \neq EXP$. It is believed that $P \neq NP \neq EXP$.

Theorem 20.10 *(Sipser 7.20): The following are equivalent:*

1. $L \in NP$
2. *There exists a deterministic, polynomial time TM* V *and a constant* c*, such that* $L = \{x \in \Sigma^* \ \exists y \in \Sigma^*$ *such that* $|y| \leq |x|^c$ *and* V *accepts* $(x, y)\}$

V is called a "verifier". y is called a "certificate".