

Quiz 2:

1.

2.

(a)T (b)F (c)F (d)F (e)F (f)T (g)T (h)F (i)F (j)F (k)T (l)F (m)F

(n)T

3. (a)ABDFH

(b)ABCDEGFH (c)

T:(A,B) (B,C) (C,D) (D,E) (E,G) (G,F) (F,H)

B:(C,A) (D,B) (F,D) (H,G)

F:

C:

(d)

T:(A,B) (B,C) (C,D) (D,E) (E,G) (D,F) (F,H)

B: (C,A)

F: (B,D)

C: (H,G)(F,G)

(e) ABDCFEHG

4.

(a) $X_{i+1} = X_i - \frac{X_i^4 - A}{4X_i^3}$

(b)

As Hint, $B^3 = A^{\frac{3}{4}} - 3A^{\frac{2}{4}}\alpha + 3A^{\frac{1}{4}}\alpha^2 - \alpha^3$, so if A is big enough big $3A^{\frac{2}{4}}\alpha$ may cause the result incorrect.

(c)

$C \leq A^{\frac{3}{4}}$ is logically equal to $C^4 \leq A^3$. We check the latter equation and we get the answer.

(d)

We know if $C = \left\lfloor A^{\frac{3}{4}} \right\rfloor$, there must have $C \leq \left\lfloor A^{\frac{3}{4}} \right\rfloor$ and $C + 1 > \left\lfloor A^{\frac{3}{4}} \right\rfloor$. So we check if $C^4 \leq A^3$ and $(C + 1)^4 > A^3$ is true then we know $C = \left\lfloor A^{\frac{3}{4}} \right\rfloor$ and so on reverse.

(e)

We use Newton's method to compute the $A^{\frac{3}{4}}$, the function we use is $f(x) = A^3 - x^4$, so the formula for the more accurate estimate x_{i+1} is $x_{i+1} = x_i - \frac{A^3}{4x_i^3} + \frac{x_i}{4}$, we then do iteration on x_i and when the x_i doesn't change, we get the correct answer.

5.

We build the G' as follow:

Copy G k times and we get graph $G_1, G_2, G_3 \dots G_k$, then for each graph from G_1 to $G_{k-1}(G_i)$, build directed edge from G_i 's v_i to G_{i+1} 's v_i with weight zero, and we use the whole graph as G' . We note G_1 's v_1 as s, G_k 's v_k as t, input (s, t, G') to the serive and we get the answer.

6.

(a) (s, u_1, u_3, u_2, t) , cost \$20.

(b) $(s, u_1, u_3, u_2, u_1, u_4, t, u_5)$

(c)

In General, we use there layer to represent the state of our oil tank, and a small trick there is

that you can't move from 'NO GAS' node to another 'NO GAS' node except it is t, because if we do so, we will die in 'NO GAS' node. We first remove all edges that from 'NO GAS' to 'NO GAS' in the origin graph, and then get the new graph named L1. We then build L2 as that we copy all vertices from L1, then change every edges in L1 to point to the corresponding vertex in L2, then we copy vertices of 'Gas' type from L2 to L3 and copy edges between L1 and L2 to L2 and L3 to corresponding position, finally every vertices in L2 and L3 can pay the 'Gas Refill' amount to back to the corresponding node in L1. We then run the Dijkstra from s to L1,L2,L3's t and get the answer. We know that build L1,L2 and L3 cost $O(V + E)$, running Dijkstra cost $O(V \log V + E)$, so the total running time is $O(V \log V + E)$.

// Need Be More careful boy! You should