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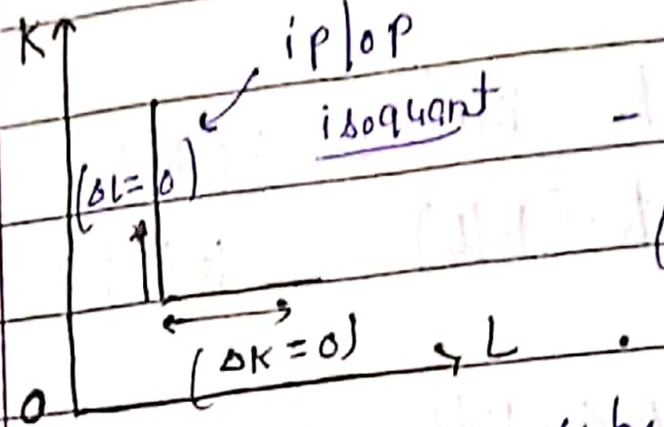
(3) i/p - o/p isoquant -



- Here, Curvature is 'infinite' (very high)
- No substitubility b/w L & K ($\because$ they are perfect Compliments)

$\therefore$ Elasticity of substitution is zero. $\boxed{\sigma = 0}$

$\because$ Curvature of isoquant and elasticity of substitution are inversely or negative related

$\therefore$ mod elasticity will be -

$$\boxed{0 < |\sigma| < +\infty}$$

Illustration of Elasticity of Substitution

Using a specific form of production function, namely, "Cobb Douglas" production function.

It is of the form - $X = b_0 (L^{b_1}) \cdot (K^{b_2})$

where $b_0 > 0$, $b_1 > 0$, $b_2 > 0$, all Constant numbers.

$$MP_L = \frac{\partial X}{\partial L} = b_0 (b_1 L^{b_1 - 1}) \cdot K^{b_2}$$

$$MP_K = \frac{\partial X}{\partial K} = b_0 (L^{b_1}) \cdot (b_2 K^{b_2 - 1})$$

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{b_0 (b_1 L^{b_1 - 1}) \cdot K^{b_2}}{b_0 (L^{b_1}) \cdot (b_2 K^{b_2 - 1})} = \left(\frac{b_1}{b_2}\right) \cdot \left(\frac{K}{L}\right)$$

$$\boxed{MRTS_{L,K} = R \cdot \left(\frac{K}{L}\right)}$$

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$$\% \Delta MRTS_{L,K} = \frac{\Delta MRTS_{L,K}}{MRTS_{L,K}}$$

$$\% \Delta MRTS_{L,K} = \frac{\Delta k \cdot (K/L)}{k \cdot (K/L)} = \frac{\Delta (K/L)}{(K/L)}$$

$$\boxed{\% \Delta MRTS_{L,K} = \% \Delta (K/L)}$$

$$\sigma = \frac{\% \Delta (L/K)}{\% \Delta (MRTS_{L,K})} = \frac{\% \Delta (L/K)}{\% \Delta (K/L)} = -1$$

$\sigma = -1$ Unitary elasticity of substitution

Eg- $b_0 = 100, \; b_1 = 1/2, \; b_2 = 3$

$$X = 100 (L)^{1/2} \cdot (K)^3$$

$$X = 100 \sqrt{L} (K)^3$$

$$MP_L = \frac{\partial X}{\partial L} = b_0 \cdot (b_1 L^{b_1 - 1}) \cdot K^{b_2}$$

$$= 100 \left(\frac{1}{2} L^{1/2 - 1}\right) K^3 = 50 L^{-1/2} K^3$$

$$MP_K = \frac{\partial X}{\partial K} = 100 (L)^{1/2} \left(3 K^{3-1}\right) = 300 L^{1/2} K^2$$

$$\cancel{MRTS = \frac{b_1}{b_2} \left(\frac{K}{L}\right) = \frac{1}{2 \times 3}}$$

$$MRTS = \frac{50 L^{-1/2} K^3}{300 L^{1/2} K^2} = 0.166 L^{-1} K$$

$$MRTS = \frac{0.17 K}{L} \quad 4$$

Cobb Douglas prod^n function eg –

New Output = $X^* = f(kL, kK)$

$$X^* = b_0 (kL)^{b_1} (kK)^{b_2}$$
$$= b_0 k^{b_1} L^{b_1} \cdot k^{b_2} K^{b_2}$$

$$X^* = k^{(b_1+b_2)} \underbrace{b_0 L^{b_1} K^{b_2}}_{\longrightarrow\ f(L,K) = X_0 \text{ old output}}$$

$$X^* = f(kL, kK) = k^{(b_1+b_2)} \cdot X_0$$

C.D Homogeneous prod^n fun^n

$$X^* = f(kL, kK) = k^{b_1+b_2} X_0$$
$$= k^r X_0$$

when $r \rightarrow$ Constant.

$\therefore$ for CD prod^n fun^n – $r = (b_1 + b_2)$

$\therefore$ (1) b_1, b_2 – Constant $\Rightarrow r = (b_1 + b_2)$

(2) $b_1 > 0, b_2 > 0 \Rightarrow r = (b_1 + b_2) > 0$ Constant

~~Now – if (i) $b_1 + b_2 = 1$ → Constant return to scale~~

~~(ii) $b_1 + b_2$~~ -

Now – (i) $k^r = 1$ → Constant return to scale

(ii) $k^r > 1$ → increase in return to scale

(iii) $k^r < 1$ → decrease in return to scale

$\therefore$ CD Prod^n fun^n is homogeneous of degree $\boxed{r = (b_1 + b_2)}$

Product line of Curves or isoclines

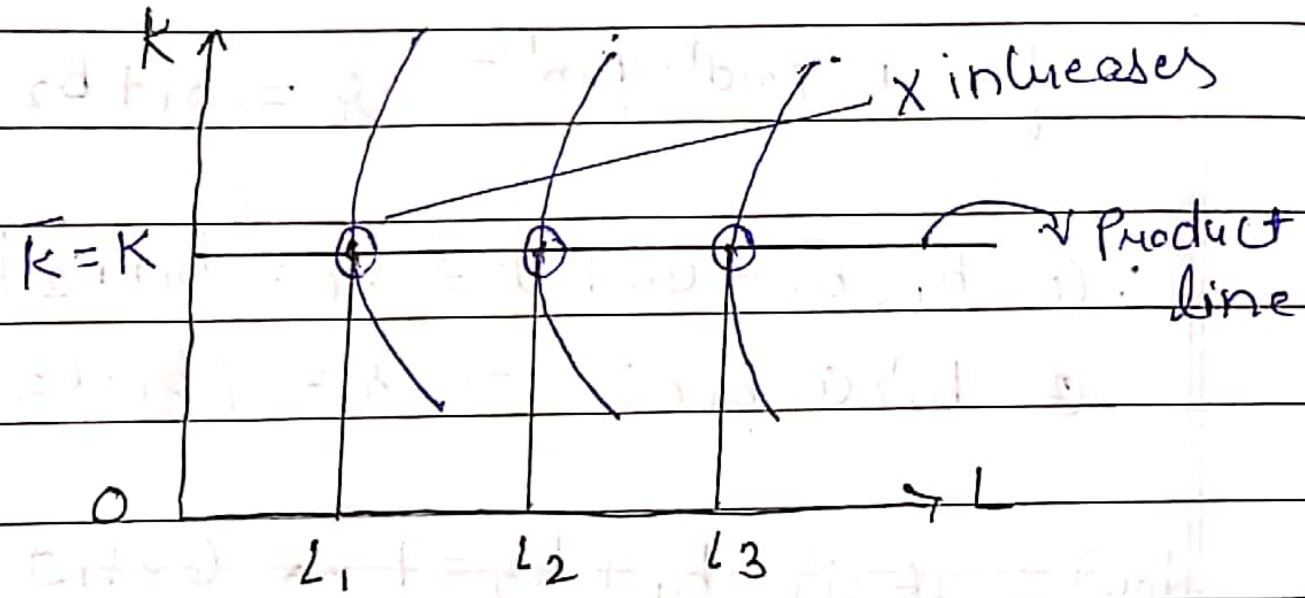
Product line → It shows the expansion of Output (X increases). It shows movement from one isoquant to the other as we change all factor.

Ex - Labour and Capital both factor or only one factor (ex-labour)

Case-1 Only one factor (Labour) Variable, (K Const)

In this Case, product line will be parallel to x-axis where K/L ratio decreases as L increases because Capital is fixed here.

(ex of Short Run)



Case 2 - All factor Variable (eg L and K Variable)

Both L and K are Variable is a special Case of such product line are Called isoclines.

Isoclines → An isocline is a product line or

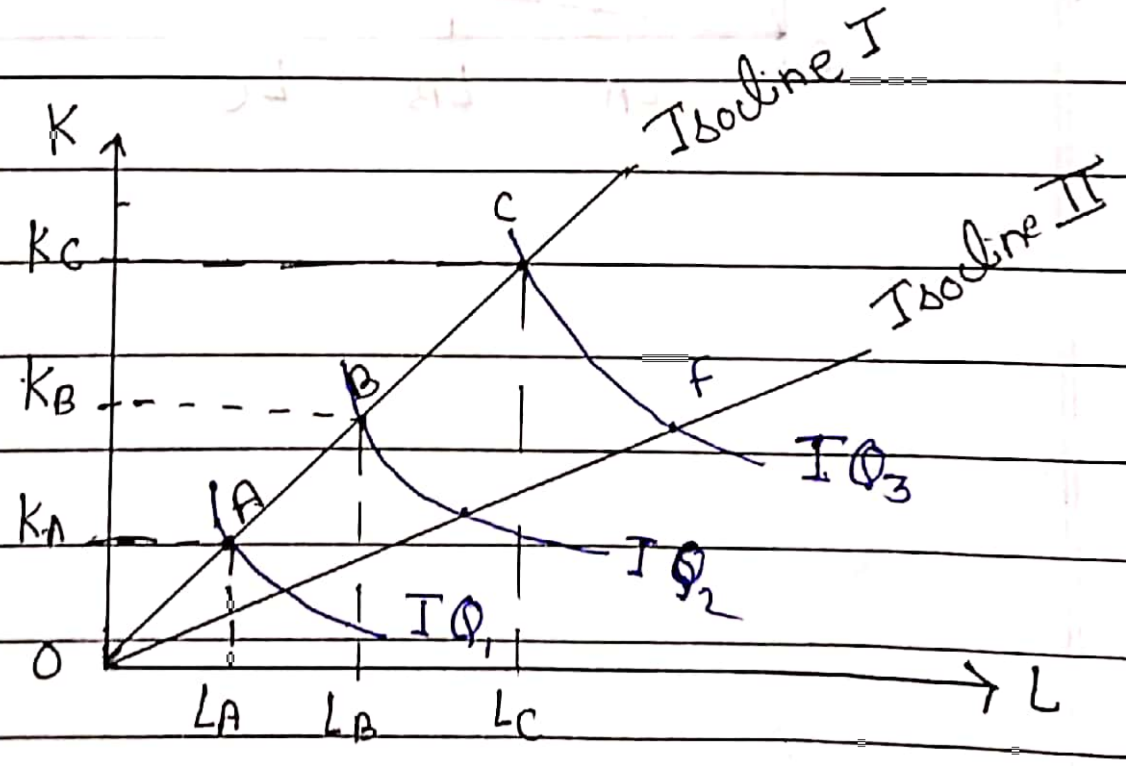
Curve which joins the point on different isoquants at which the MRTS of factors i.e. ($MRTS_{L,K}$) is Constant or unchanged).

* if the production funⁿ is homogeneous ~~then~~ (any degree of r is +ve) then in that case an isoclines will be straight line passing through the origin.

- Along any isoclines K/L ratio stays the same / Constant.

  Eg - At upper graph, we can see along 6 isoclines I_1,

$$\left(\frac{K_A}{L_A}\right) = \left(\frac{K_B}{L_B}\right) = \left(\frac{K_C}{L_C}\right) = \text{slope of line } I_1 \text{ (+ve)}$$



$$\left(\frac{K_A}{L_A}\right) = \left(\frac{K_B}{L_B}\right) = \left(\frac{K_C}{L_C}\right)$$

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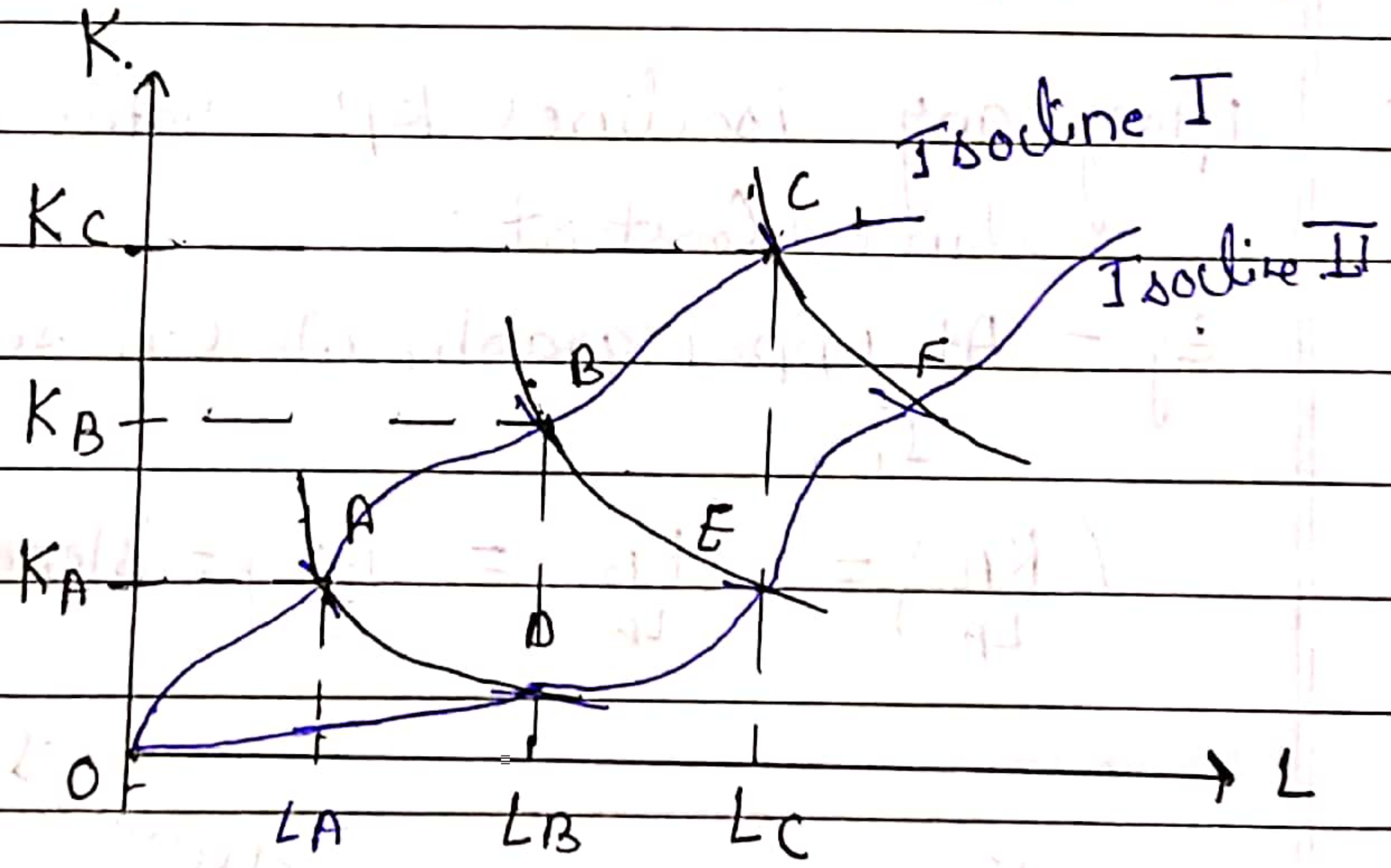
- If Prod^n fun^n is non-homogeneous then an isoclines will not be a straight line instead it will be a Curve.

Here, K/L ratio Can change along an isocline (for example in above isocline I_1) -

$$\text{slope} = \left(\frac{K_A}{L_A}\right) \neq \left(\frac{K_B}{L_B}\right) \neq \left(\frac{K_C}{L_C}\right)$$



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