# Package 'HighFreq'

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Type Package

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|--|
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agg\_ohlc

Aggregate a time series of data into a single bar of OHLC data.

#### **Description**

Aggregate a time series of data into a single bar of OHLC data.

#### Usage

```
agg_ohlc(tseries)
```

### **Arguments**

tseries

A time series or a matrix with multiple columns of data.

#### **Details**

The function agg\_ohlc() aggregates a time series of data into a single bar of *OHLC* data. It can accept either a single column of data or four columns of *OHLC* data. It can also accept an additional column containing the trading volume.

The function agg\_ohlc() calculates the *open* value as equal to the *open* value of the first row of tseries. The *high* value as the maximum of the *high* column of tseries. The *low* value as the minimum of the *low* column of tseries. The *close* value as the *close* of the last row of tseries. The *volume* value as the sum of the *volume* column of tseries.

For a single column of data, the open, high, low, and close values are all the same.

# Value

A *matrix* containing a single row, with the *open*, *high*, *low*, and *close* values, and also the total *volume* (if provided as either the second or fifth column of tseries).

```
## Not run:
# Define matrix of OHLC data
ohlc <- coredata(rutils::etfenv$VTI[, 1:5])
# Aggregate to single row matrix
ohlcagg <- HighFreq::agg_ohlc(ohlc)
# Compare with calculation in R
all.equal(drop(ohlcagg),
    c(ohlc[1, 1], max(ohlc[, 2]), min(ohlc[, 3]), ohlc[NROW(ohlc), 4], sum(ohlc[, 5])),
    check.attributes=FALSE)
## End(Not run)</pre>
```

agg\_stats\_r 5

| agg_stats_r | Calculate the aggregation (weighted average) of a statistical estimator over a OHLC time series using R code. |
|-------------|---|
|             |   |

# **Description**

Calculate the aggregation (weighted average) of a statistical estimator over a *OHLC* time series using R code.

#### Usage

```
agg_stats_r(ohlc, calc_bars = "ohlc_variance", weighted = TRUE, ...)
```

#### **Arguments**

|           | additional parameters to the function calc_bars.  |
|-----------|---|
| ohlc      | An OHLC time series of prices and trading volumes, in xts format.   |
| calc_bars | A <i>character</i> string representing a function for calculating statistics for individual <i>OHLC</i> bars. |
| weighted  | Boolean argument: should estimate be weighted by the trading volume? (default                                 |

# Details

The function agg\_stats\_r() calculates a single number representing the volume weighted average of statistics of individual *OHLC* bars. It first calls the function calc\_bars to calculate a vector of statistics for the *OHLC* bars. For example, the statistic may simply be the difference between the *High* minus *Low* prices. In this case the function calc\_bars would calculate a vector of *High* minus *Low* prices. The function agg\_stats\_r() then calculates a trade volume weighted average of the vector of statistics.

The function  $agg\_stats\_r()$  is implemented in R code.

is TRUE)

#### Value

A single *numeric* value equal to the volume weighted average of an estimator over the time series.

```
# Calculate weighted average variance for SPY (single number)
variance <- agg_stats_r(ohlc=HighFreq::SPY, calc_bars="ohlc_variance")
# Calculate time series of daily skew estimates for SPY
skew_daily <- apply.daily(x=HighFreq::SPY, FUN=agg_stats_r, calc_bars="ohlc_skew")</pre>
```

back\_test

| back_test | Simulate (backtest) a rolling portfolio optimization strategy, using RcppArmadillo. |
|-----------|---|
|           |   |

#### **Description**

Simulate (backtest) a rolling portfolio optimization strategy, using RcppArmadillo.

#### Usage

```
back_test(
  excess,
  returns,
  controlv,
  startp,
  endp,
  lambda = 0,
  coeff = 1,
  bid_offer = 0
)
```

# **Arguments**

| returns  | A time series or a matrix of asset returns data.  |  |  |
|----------|---|--|--|
| excess   | A <i>time series</i> or a <i>matrix</i> of excess returns data (the returns in excess of the risk-free rate). |  |  |
| controlv | A list of portfolio optimization model parameters (see Details).  |  |  |
| startp   | An integer vector of start points.  |  |  |
| endp     | An integer vector of end points.  |  |  |
| lambda   | A decay factor which multiplies the past portfolio weights. (The default is $lambda = 0$ - no memory.)        |  |  |
| coeff    | A <i>numeric</i> multiplier of the weights. (The default is 1)  |  |  |

# Details

bid\_offer

The function back\_test() performs a backtest simulation of a rolling portfolio optimization strategy over a *vector* of the end points endp.

A *numeric* bid-offer spread (the default is 0)

It performs a loop over the end points endp, and subsets the *matrix* of the excess asset returns excess along its rows, between the corresponding *start point* and the *end point*.

The function back\_test() passes the subset matrix of excess returns into the function calc\_weights(), which calculates the optimal portfolio weights at each *end point*. It also passes to calc\_weights() the argument controlv, which is the list of portfolio optimization parameters. See the function calc\_weights() for more details. The list of portfolio optimization parameters can be created using the function param\_portf().

The function back\_test() then recursively averages the weights  $w_i$  at the *end point* = i with the weights  $w_{i-1}$  from the previous *end point* = (i-1), using the decay factor lambda =  $\lambda$ :

$$w_i = (1 - \lambda)w_i + \lambda w_{i-1}$$

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The purpose of averaging the weights is to reduce their variance, and improve their out-of-sample performance. It is equivalent to extending the portfolio holding period beyond the time interval between neighboring *end points*.

The function back\_test() then calculates the out-of-sample strategy returns by multiplying the average weights times the future asset returns.

The function back\_test() multiplies the out-of-sample strategy returns by the coefficient coeff (with default equal to 1), which allows simulating either a trending strategy (if coeff = -1), or a reverting strategy (if coeff = -1).

The function back\_test() calculates the transaction costs by multiplying the bid-offer spread bid\_offer times the absolute difference between the current weights minus the weights from the previous period. Then it subtracts the transaction costs from the out-of-sample strategy returns.

The function back\_test() returns a *time series* (column *vector*) of strategy returns, of the same length as the number of rows of returns.

#### Value

A column *vector* of strategy returns, with the same length as the number of rows of returns.

```
## Not run:
# Calculate the ETF daily excess returns
retp <- na.omit(rutils::etfenv$returns[, 1:16])</pre>
# riskf is the daily risk-free rate
riskf <- 0.03/260
retexcess <- retp - riskf
# Define monthly end points without initial warmup period
endp <- rutils::calc_endpoints(retp, interval="months")</pre>
endp <- endp[endp > 0]
nrows <- NROW(endp)</pre>
# Define 12-month look-back interval and start points over sliding window
look back <- 12
startp <- c(rep_len(1, look_back-1), endp[1:(nrows-look_back+1)])</pre>
# Define return shrinkage and dimension reduction
alpha <- 0.5
dimax <- 3
# Create a list of portfolio optimization parameters
controlv <- HighFreq::param_portf(method="maxsharpe", dimax=dimax, alpha=alpha, scalew="sumsq")</pre>
# Simulate a monthly rolling portfolio optimization strategy
pnls <- HighFreq::back_test(retexcess, retp, controlv=controlv, startp=(startp-1), endp=(endp-1))</pre>
pnls <- xts::xts(pnls, index(retp))</pre>
colnames(pnls) <- "strategy"</pre>
# Plot dygraph of strategy
dygraphs::dygraph(cumsum(pnls),
  main="Cumulative Returns of Max Sharpe Portfolio Strategy")
## End(Not run)
```

8 calc\_covar

| calc_covar | Calculate the covariance matrix of the columns of a time series using RcppArmadillo. |
|------------|--|
|            |  |

#### **Description**

Calculate the covariance matrix of the columns of a *time series* using RcppArmadillo.

#### Usage

```
calc_covar(tseries, method = "moment", confl = 0.75)
```

#### **Arguments**

tseries A time series or a matrix of data.

method A character string specifying the type of the covariance model (the default is

method = "moment" - see Details).

confl The confidence level for calculating the quantiles of returns (the default is confl

= 0.75).

#### **Details**

The function calc\_covar() calculates the covariance matrix of the columns of a *time series* or a *matrix* of data using RcppArmadillo C++ code. The covariance is a measure of the codependency of the data.

If method = "moment" (the default) then calc\_covar() calculates the covariance as the second co-moment:

$$\sigma_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

Then calc\_covar() performs the same calculation as the R function stats::cov().

If method = "quantile" then it calculates the covariance as the difference between the quantiles as follows:

$$\mu = q_{\alpha} - q_{1-\alpha}$$

Where  $\alpha$  is the confidence level for calculating the quantiles.

If method = "nonparametric" then it calculates the covariance as the Median Absolute Deviation (MAD):

$$MAD = median(abs(x - median(x)))$$

It also multiplies the  $M\!AD$  by a factor of 1.4826, to make it comparable to the standard deviation.

If method = "nonparametric" then calc\_covar() performs the same calculation as the function stats::mad(), but it's much faster because it uses RcppArmadillo C++ code.

If the number of rows of tseries is less than 3 then it returns zeros.

# Value

A square matrix with the covariance coefficients of the columns of the time series tseries.

calc\_cvar 9

#### **Examples**

```
## Not run:
# Calculate VTI and XLF returns
retp <- na.omit(rutils::etfenv$returns[, c("VTI", "XLF")])</pre>
# Compare HighFreq::calc_covar() with standard var()
all.equal(drop(HighFreq::calc_covar(retp)),
  cov(retp), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with matrixStats and with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::calc_covar(retp),
  Rcode=cov(retp),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Compare HighFreq::calc_covar() with stats::mad()
all.equal(drop(HighFreq::calc_covar(retp, method="nonparametric")),
  sapply(retp, mad), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with stats::mad()
summary(microbenchmark(
  Rcpp=HighFreq::calc_covar(retp, method="nonparametric"),
  Rcode=sapply(retp, mad),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

calc\_cvar

Calculate the Value at Risk (VaR) or the Conditional Value at Risk (CVaR) of an xts time series of returns, using R code.

# **Description**

Calculate the Value at Risk (VaR) or the Conditional Value at Risk (CVaR) of an xts time series of returns, using R code.

# Usage

```
calc_cvar(tseries, method = "var", confi = pnorm(-2))
```

# Arguments

tseries An xts time series of returns with multiple columns.

method A string specifying the type of risk measure (the default is method = "var" - see

Details).

confi The confidence level for calculating the quantile (the default is confi = pnorm(-2)

= 0.02275).

#### **Details**

The function calc\_cvar() calculates the Value at Risk (VaR) or the Conditional Value at Risk (CVaR) of an xts time series of returns, using R

The Value at Risk (VaR) and the Conditional Value at Risk (CVaR) are measures of the tail risk of returns.

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If method = "var" then calc\_cvar() calculates the Value at Risk (VaR) as the quantile of the returns as follows:

$$\alpha = \int_{-\infty}^{\text{VaR}(\alpha)} f(r) \, \mathrm{d}r$$

Where  $\alpha$  is the confidence level for calculating the quantile, and f(r) is the probability density (distribution) of returns.

If method = "cvar" then calc\_cvar() calculates the Value at Risk (VaR) as the Expected Tail Loss (ETL) of the returns as follows:

$$CVaR = \frac{1}{\alpha} \int_0^{\alpha} VaR(p) dp$$

Where  $\alpha$  is the confidence level for calculating the quantile.

#### Value

A vector with the risk measures of the columns of the input time series tseries.

## **Examples**

```
## Not run:
# Calculate VTI and XLF returns
returns <- na.omit(rutils::etfenv$returns[, c("VTI", "XLF")])
# Calculate VaR
all.equal(HighFreq::calc_cvar(returns),
    sapply(returns, quantile, probs=pnorm(-2)), check.attributes=FALSE)
# Calculate CVaR
all.equal(HighFreq::calc_cvar(returns, method="cvar", confi=0.02),
    sapply(returns, function(x) mean(x[x < quantile(x, 0.02)])),
    check.attributes=FALSE)
## End(Not run)</pre>
```

calc\_eigen

 $\label{lem:calculate} \textit{Calculate the eigen decomposition of a square, symmetric matrix using $$ RcppArmadillo. $$$ 

# **Description**

Calculate the eigen decomposition of a square, symmetric matrix using RcppArmadillo.

#### Usage

```
calc_eigen(matrixv, eigenval, eigenvec)
```

# **Arguments**

matrixv A square, symmetric matrix.
eigenval A vector of eigen values.
eigenvec A matrix of eigen vectors.

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#### **Details**

The function calc\_eigen() calculates the eigen decomposition of a square, symmetric matrix using RcppArmadillo. It calls the Armadillo function arma::eig\_sym() to calculate the eigen decomposition.

For small matrices, the function calc\_eigen() is several times faster than the R function eigen(), since calc\_eigen() has no overhead in R code. But for large matrices, they are about the same, since both call C++ code.

#### Value

Void (no return value - passes the eigen values and eigen vectors by reference).

## **Examples**

```
## Not run:
# Create random positive semi-definite matrix
matrixv <- matrix(runif(25), nc=5)</pre>
matrixv <- t(matrixv) %*% matrixv</pre>
# Calculate the eigen decomposition using RcppArmadillo
eigenval <- numeric(5) # Allocate eigen values</pre>
eigenvec <- matrix(numeric(25), nc=5) # Allocate eigen vectors</pre>
HighFreq::calc_eigen(matrixv, eigenval, eigenvec)
# Calculate the eigen decomposition using R
eigenr <- eigen(matrixv)</pre>
# Compare the eigen decompositions
all.equal(eigenr$values, drop(eigenval))
all.equal(abs(eigenr$vectors), abs(eigenvec))
# Compare the speed of Rcpp with R code
summary(microbenchmark(
  Rcpp=HighFreq::calc_eigen(matrixv, eigenval, eigenvec),
  Rcode=eigen(matrixv),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

calc\_eigenp

Calculate the partial eigen decomposition of a dense symmetric matrix using RcppArmadillo.

# Description

Calculate the partial eigen decomposition of a dense symmetric matrix using RcppArmadillo.

# Usage

```
calc_eigenp(matrixv, neigen)
```

#### **Arguments**

matrixv A square matrix.

neigen An *integer* equal to the number of eigenvalues to be calculated.

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#### **Details**

The function calc\_eigenp() calculates the partial eigen decomposition (the lowest order principal components, with the largest eigenvalues) of a dense matrix using RcppArmadillo. It calls the internal Armadillo eigen solver SymEigsSolver in the namespace arma::newarp to calculate the partial eigen decomposition.

The eigen solver SymEigsSolver uses the Implicitly Restarted Lanczos Method (IRLM) which was adapted from the ARPACK library. The eigen solver SymEigsSolver was implemented by Yixuan Oiu.

The function arma::eigs\_sym() also calculates the partial eigen decomposition using the eigen solver SymEigsSolver, but it only works for sparse matrices which are not standard R matrices.

For matrices smaller than 100 rows, the function calc\_eigenp() is slower than the function calc\_eigen() which calculates the full eigen decomposition. But it's faster for very large matrices.

#### Value

A list with two elements: a *vector* of eigenvalues (named "values"), and a *matrix* of eigenvectors (named "vectors").

## **Examples**

```
## Not run:
# Create random positive semi-definite matrix
matrixv <- matrix(runif(100), nc=10)</pre>
matrixv <- t(matrixv) %*% matrixv</pre>
# Calculate the partial eigen decomposition
neigen <- 5
eigenp <- HighFreq::calc_eigenp(matrixv, neigen)</pre>
# Calculate the eigen decomposition using RcppArmadillo
eigenval <- numeric(10) # Allocate eigen values
eigenvec <- matrix(numeric(100), nc=10) # Allocate eigen vectors</pre>
HighFreq::calc_eigen(matrixv, eigenval, eigenvec)
# Compare the eigen decompositions
all.equal(eigenp$values[1:neigen], eigenval[1:neigen])
all.equal(abs(eigenp$vectors), abs(eigenvec[, 1:neigen]))
# Compare the speed of partial versus full decomposition
summary(microbenchmark(
  partial=HighFreq::calc_eigenp(matrixv, neigen),
  full=HighFreq::calc_eigen(matrixv, eigenval, eigenvec),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

calc\_endpoints

Calculate a vector of end points that divides an integer time sequence of time periods into equal time intervals.

## **Description**

Calculate a vector of end points that divides an integer time sequence of time periods into equal time intervals.

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#### Usage

calc\_endpoints(length, step = 1L, stub = 0L, stubs = TRUE)

#### **Arguments**

| length | An <i>integer</i> equal to the length of the time sequence to be divided into equal intervals.        |
|--------|---|
| step   | The number of time periods in each interval between neighboring end points (the default is step = 1). |
| stub   | An <i>integer</i> equal to the first non-zero end point (the default is $stub = 0$ ).                 |
| stubs  | A <i>Boolean</i> specifying whether to include stub intervals (the default is stubs = TRUE).          |

#### **Details**

The end points are a vector of integers which divide the sequence of time periods of length equal to length into equally spaced time intervals. The number of time periods between neighboring end points is equal to the argument step. If a whole number of intervals doesn't fit over the whole sequence, then calc\_endpoints() adds a stub interval at the end. A stub interval is one where the number of periods between neighboring end points is less than the argument step.

If stubs = TRUE (the default) then the first end point is equal to 0 (since indexing in C++ code starts at 0). The first non-zero end point is equal to step or stub (if it's not zero). If stub = 0 (the default) then the first end point is equal to 0 (even if stubs = FALSE). If stubs = TRUE (the default) then the last end point is always equal to length-1. The argument stub should be less than the step: stub < step.

If step = 1 and stub = 0 (the default), then the vector of end points is simply equal to:

$$\{0, 1, 2, ..., length - 1\}$$

If stub = 0 (the default) and stubs = TRUE (the default) then the vector of end points is equal to:

$$\{0, step, 2 * step, ..., length - 1\}$$

If stub = 0 (the default) and stubs = FALSE then the vector of end points is equal to:

$$\{0, step, 2*step, ..., n*step\}$$

If stub > 0 and stubs = TRUE (the default), then the vector of end points is equal to:

$$\{0, stub, stub + step, ..., length - 1\}$$

For example, the end points for length = 20, divided into intervals of step = 5 are equal to: 0, 5, 10, 15, 19.

If stub = 1 then the first non-zero end point is equal to 1 and the end points are equal to: 0, 1, 6, 11, 16, 19. The stub interval at the beginning is equal to 2 (including 0 and 1). The stub interval at the end is equal to 3 = 19 - 16.

The end points for length = 21 divided into intervals of length step = 5, with stub = 0, are equal to: 0, 5, 10, 15, 20. The beginning interval is equal to 5. The end interval is equal to 5 = 20 - 15.

If stub = 1 then the first non-zero end point is equal to 1 and the end points are equal to: 0, 1, 6, 11, 16, 20. The beginning stub interval is equal to 2. The end stub interval is equal to 4 = 20 - 16.

14 calc\_hurst

The function calc\_endpoints() is similar to the function rutils::calc\_endpoints() from package rutils.

But the end points are shifted by -1 compared to R code because indexing starts at 0 in C++ code, while it starts at 1 in R code. So if calc\_endpoints() is used in R code then 1 should be added to it.

#### Value

A vector of equally spaced *integers* representing the end points.

# **Examples**

```
# Calculate the end points without a stub interval
HighFreq::calc_endpoints(length=20, step=5)
# Calculate the end points with a final stub interval
HighFreq::calc_endpoints(length=23, step=5)
# Calculate the end points with initial and final stub intervals
HighFreq::calc_endpoints(length=20, step=5, stub=2)
```

calc\_hurst

Calculate the Hurst exponent from the volatility ratio of aggregated returns.

# **Description**

Calculate the Hurst exponent from the volatility ratio of aggregated returns.

# Usage

```
calc_hurst(tseries, aggv)
```

# Arguments

tseries A *time series* or a *matrix* of log prices. aggv A *vector* of aggregation intervals.

#### **Details**

The function calc\_hurst() calculates the Hurst exponent from the ratios of the volatilities of aggregated returns.

An aggregation interval is equal to the number of time periods between the neighboring aggregation end points.

The aggregated volatility  $\sigma_t$  increases with the length of the aggregation interval  $\Delta t$ . The aggregated volatility increases as the length of the aggregation interval  $\Delta t$  raised to the power of the *Hurst* exponent H:

$$\sigma_t = \sigma \Delta t^H$$

Where  $\sigma$  is the daily return volatility.

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For a single aggregation interval  $\Delta t$ , the *Hurst exponent H* is equal to the logarithm of the ratio of the volatilities divided by the logarithm of the aggregation interval  $\Delta t$ :

$$H = \frac{\log \sigma_t - \log \sigma}{\log \Delta t}$$

For a *vector* of aggregation intervals  $\Delta t_i$ , the *Hurst exponent H* is equal to the regression slope between the logarithms of the aggregated volatilities  $\sigma_i$  versus the logarithms of the aggregation intervals  $\Delta t_i$ :

$$H = \frac{\mathsf{cov}(\log \sigma_i, \log \Delta t_i)}{\mathsf{var}(\log \Delta t_i)}$$

The function calc\_hurst() calls the function calc\_var\_ag() to calculate the variance of aggregated returns  $\sigma_t^2$ .

#### Value

The Hurst exponent calculated from the volatility ratio of aggregated returns. If tseries contains multiple columns, then the function calc\_hurst() returns a single-row matrix of Hurst exponents.

## **Examples**

```
## Not run:
# Calculate the log prices
closep <- na.omit(rutils::etfenv$prices[, c("XLP", "VTI")])
closep <- log(closep)
# Calculate the Hurst exponents for a 21 day aggregation interval
HighFreq::calc_hurst(prices, aggv=21)
# Calculate the Hurst exponents for a vector of aggregation intervals
aggv <- seq.int(from=3, to=35, length.out=9)^2
HighFreq::calc_hurst(prices, aggv=aggv)
## End(Not run)</pre>
```

calc\_hurst\_ohlc

Calculate the Hurst exponent from the volatility ratio of aggregated OHLC prices.

# **Description**

Calculate the Hurst exponent from the volatility ratio of aggregated OHLC prices.

# Usage

```
calc_hurst_ohlc(
  ohlc,
  step,
  method = "yang_zhang",
  closel = 0L,
  scale = TRUE,
  index = 0L
)
```

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#### **Arguments**

ohlc A time series or a matrix of OHLC prices.

step The number of time periods in each interval between neighboring end points.

method A character string representing the price range estimator for calculating the

variance. The estimators include:

• "close" close-to-close estimator,

• "rogers\_satchell" Rogers-Satchell estimator,

• "garman\_klass" Garman-Klass estimator,

• "garman\_klass\_yz" Garman-Klass with account for close-to-open price jumps,

• "yang\_zhang" Yang-Zhang estimator,

(The default is the method = "yang\_zhang".)

closel A vector with the lagged close prices of the OHLC time series. This is an op-

tional argument. (The default is closel = 0).

scale Boolean argument: Should the returns be divided by the time index, the number

of seconds in each period? (The default is scale = TRUE).

index A vector with the time index of the time series. This is an optional argument

(the default is index = 0).

#### **Details**

The function calc\_hurst\_ohlc() calculates the Hurst exponent from the ratios of the variances of aggregated *OHLC* prices.

The aggregated volatility  $\sigma_t$  increases with the length of the aggregation interval  $\Delta t$ . The aggregated volatility increases as the length of the aggregation interval  $\Delta t$  raised to the power of the *Hurst* exponent H:

$$\sigma_t = \sigma \Delta t^H$$

Where  $\sigma$  is the daily return volatility.

The *Hurst exponent H* is equal to the logarithm of the ratio of the volatilities divided by the logarithm of the time interval  $\Delta t$ :

$$H = \frac{\log \sigma_t - \log \sigma}{\log \Delta t}$$

The function calc\_hurst\_ohlc() calls the function calc\_var\_ohlc\_ag() to calculate the aggregated variance  $\sigma_t^2$ .

#### Value

The Hurst exponent calculated from the volatility ratio of aggregated *OHLC* prices.

```
## Not run:
# Calculate the log ohlc prices
ohlc <- log(rutils::etfenv$VTI)
# Calculate the Hurst exponent from 21 day aggregations
calc_hurst_ohlc(ohlc, step=21)
## End(Not run)</pre>
```

calc\_inv 17

| calc_inv | Calculate the regularized inverse of a symmetric matrix of data using eigen decomposition. |
|----------|--|
|          |  |

#### **Description**

Calculate the *regularized inverse* of a symmetric *matrix* of data using eigen decomposition.

# Usage

```
calc_inv(matrixv, dimax = 0L, eigen_thresh = 0)
```

# Arguments

matrixv A symmetric *matrix* of data.

dimax An *integer* equal to the number of *eigen values* used for calculating the *regularized inverse* of the matrix matrixv (the default is dimax = 0 - standard matrix inverse using all the *eigen values*).

eigen\_thresh A *numeric* threshold level for discarding small *eigen values* in order to regularize

gen\_thresh A numeric threshold level for discarding small eigen values in order to regularize the inverse of the matrix matrixv (the default is 0.0).

#### **Details**

The function calc\_inv() calculates the *regularized inverse* of the matrix matrixv using eigen decomposition.

The function  $calc_inv()$  first performs eigen decomposition of the matrix matrixv. The eigen decomposition of a matrix C is defined as the factorization:

$$C = O \Sigma O^T$$

Where O is the matrix of eigen vectors and  $\Sigma$  is a diagonal matrix of eigen values.

The inverse  $C^{-1}$  of the matrix C can be calculated from the eigen decomposition as:

$$C^{-1} = O \Sigma^{-1} O^T$$

The regularized inverse of the matrix C is obtained by removing eigen vectors with very small eigen values:

$$C^{-1} = O_{dimax} \, \Sigma_{dimax}^{-1} \, O_{dimax}^T$$

Where  $O_{dimax}$  is the matrix of eigen vectors that correspond to the largest eigen values  $\Sigma_{dimax}$ .

The function calc\_inv() applies regularization to the matrix inverse using the arguments dimax and eigen\_thresh.

The function calc\_inv() applies regularization by discarding the smallest eigen values  $\Sigma_i$  that are less than the threshold level eigen\_thresh times the sum of all the eigen values:

$$\Sigma_i < eigen\_thresh \cdot (\sum \Sigma_i)$$

It also discards additional *eigen vectors* so that only the highest order *eigen vectors* remain, up to order dimax. It calculates the *regularized inverse* from the eigen decomposition using only the largest *eigen values* up to dimax. For example, if dimax = 3 then it only uses the 3 highest order *eigen vectors*, with the largest *eigen values*. This has the effect of dimension reduction.

If the matrix matrixv has a large number of small eigen values, then the number of remaining eigen values may be less than dimax.

18 calc\_invrec

#### Value

A *matrix* equal to the *regularized inverse* of the matrix matrixv.

#### **Examples**

```
## Not run:
# Calculate ETF returns
retp <- na.omit(rutils::etfenv$returns[, c("VTI", "TLT", "DBC")])</pre>
# Calculate covariance matrix
covmat <- cov(retp)</pre>
# Calculate matrix inverse using RcppArmadillo
invmat <- HighFreq::calc_inv(covmat)</pre>
# Calculate matrix inverse in R
invr <- solve(covmat)</pre>
all.equal(invmat, invr, check.attributes=FALSE)
# Calculate regularized inverse using RcppArmadillo
invmat <- HighFreq::calc_inv(covmat, dimax=3)</pre>
# Calculate regularized inverse using eigen decomposition in R
eigend <- eigen(covmat)</pre>
dimax <- 1:3
invr <- eigend$vectors[, dimax] %*% (t(eigend$vectors[, dimax])/eigend$values[dimax])</pre>
# Compare RcppArmadillo with R
all.equal(invmat, invr)
## End(Not run)
```

calc\_invrec

Calculate the approximate inverse of a square matrix recursively using the Schulz formula (without copying the data in memory).

## **Description**

Calculate the approximate inverse of a square *matrix* recursively using the Schulz formula (without copying the data in memory).

#### Usage

```
calc_invrec(matrixv, invmat, niter = 1L)
```

# **Arguments**

matrixv A *matrix* of data to be inverted.

invmat A *matrix* of data equal to the starting point for the recursion.

niter An *integer* equal to the number of recursion iterations.

# **Details**

The function calc\_invrec() calculates the approximate inverse  $A^{-1}$  of a square matrix A recursively using the Schulz formula:

$$A_{i+1}^{-1} \leftarrow 2A_i^{-1} - A_i^{-1}AA_i^{-1}$$

calc\_invref 19

The Schulz formula is repeated niter times. The Schulz formula is useful for updating the inverse when the matrix A changes only slightly. For example, for updating the inverse of the covariance matrix as it changes slowly over time.

The function calc\_invrec() accepts a *pointer* to the argument invmat (which is the initial value of the inverse matrix for the recursion), and it overwrites the old inverse matrix values with the updated inverse values.

The function calc\_invrec() performs the calculation in place, without copying the matrix in memory, which can significantly increase the computation speed for large matrices.

The function calc\_invrec() doesn't return a value. The function calc\_invrec() performs the calculations using C++ Armadillo code.

#### Value

No return value.

#### **Examples**

```
## Not run:
# Calculate a random matrix
matrixv <- matrix(rnorm(100), nc=10)</pre>
# Define the initial value of the inverse matrix
invmat <- solve(matrixv) + matrix(rnorm(100, sd=0.1), nc=10)</pre>
# Calculate the inverse in place using RcppArmadillo
HighFreq::calc_invrec(matrixv, invmat, 3)
# Multiply the matrix times its inverse
multmat <- matrixv %*% invmat</pre>
round(multmat, 4)
# Calculate the sum of the off-diagonal elements
sum(multmat[upper.tri(multmat)])
# Compare RcppArmadillo with R
all.equal(invmat, solve(matrixv))
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
   rcode=solve(matrixv),
   cppcode=HighFreq::calc_invrec(matrixv, invmat, 3),
   times=10))[, c(1, 4, 5)]
## End(Not run)
```

calc\_invref

Calculate the inverse of a square matrix in place, without copying the data in memory.

# **Description**

Calculate the inverse of a square *matrix* in place, without copying the data in memory.

#### Usage

```
calc_invref(matrixv)
```

20 calc\_invsvd

#### **Arguments**

matrixv

A *matrix* of data to be inverted. (The argument is interpreted as a *pointer* to a *matrix*, and it is overwritten with the inverse matrix.)

#### **Details**

The function calc\_invref() calculates the inverse of a square *matrix* in place, without copying the data in memory. It accepts a *pointer* to the argument matrixv (which is the matrix to be inverted), and it overwrites the old matrix values with the inverse matrix values. It performs the calculation in place, without copying the data in memory, which can significantly increase the computation speed for large matrices.

The function calc\_invref() doesn't return a value. The function calc\_invref() calls the C++ Armadillo function arma::inv() to calculate the matrix inverse.

#### Value

No return value.

#### **Examples**

```
## Not run:
# Calculate a random matrix
matrixv <- matrix(rnorm(100), nc=10)</pre>
# Copy matrixv to a matrix in a different memory location
invmat <- matrixv + 0
# Calculate the inverse in place using RcppArmadillo
HighFreq::calc_invref(invmat)
\# Multiply the matrix times its inverse
multmat <- matrixv %*% invmat</pre>
round(multmat, 4)
# Calculate the sum of the off-diagonal elements
sum(multmat[upper.tri(multmat)])
# Compare RcppArmadillo with R
all.equal(invmat, solve(matrixv))
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
   rcode=solve(matrixv),
   cppcode=calc_invref(matrixv),
   times=10))[, c(1, 4, 5)]
## End(Not run)
```

calc\_invsvd

Calculate the regularized inverse of a matrix of data using Singular Value Decomposition (SVD).

# **Description**

Calculate the regularized inverse of a matrix of data using Singular Value Decomposition (SVD).

calc\_invsvd 21

## Usage

calc\_invsvd(matrixv, dimax = 0L, eigen\_thresh = 0)

#### Arguments

matrixv A *matrix* of data.

dimax An integer equal to the number of singular values used for calculating the regu-

larized inverse of the matrix matrixv (the default is dimax = 0 - standard matrix

inverse using all the singular values).

eigen\_thresh A numeric threshold level for discarding small singular values in order to regu-

larize the inverse of the matrix matrixv (the default is 0.0).

#### **Details**

The function calc\_invsvd() calculates the *regularized inverse* of the matrix matrixv using Singular Value Decomposition (SVD).

The function calc\_invsvd() first performs Singular Value Decomposition (SVD) of the matrix matrixv. The SVD of a matrix C is defined as the factorization:

$$C = U \Sigma V^T$$

Where U and V are the left and right singular matrices, and  $\Sigma$  is a diagonal matrix of singular values.

The inverse  $C^{-1}$  of the matrix C can be calculated from the SVD matrices as:

$$C^{-1} = V \Sigma^{-1} U^T$$

The regularized inverse of the matrix C is obtained by removing singular vectors with very small singular values:

$$C^{-1} = V_n \, \Sigma_n^{-1} \, U_n^T$$

Where  $U_n$ ,  $V_n$  and  $\Sigma_n$  are the SVD matrices with the rows and columns corresponding to very small singular values removed.

The function calc\_invsvd() applies regularization to the matrix inverse using the arguments dimax and eigen\_thresh.

The function calc\_invsvd() applies regularization by discarding the smallest *singular values*  $\sigma_i$  that are less than the threshold level eigen\_thresh times the sum of all the *singular values*:

$$\sigma_i < eigen\_thresh \cdot (\sum \sigma_i)$$

It also discards additional *singular vectors* so that only the highest order *singular vectors* remain, up to order dimax. It calculates the *regularized inverse* from the *SVD* matrices using only the largest *singular values* up to order dimax. For example, if dimax = 3 then it only uses the 3 highest order *singular vectors*, with the largest *singular values*. This has the effect of dimension reduction.

If the matrix matrixv has a large number of small *singular values*, then the number of remaining *singular values* may be less than dimax.

#### Value

A matrix equal to the regularized inverse of the matrix matrixv.

22 calc\_kurtosis

#### **Examples**

```
## Not run:
# Calculate ETF returns
retp <- na.omit(rutils::etfenv$returns[, c("VTI", "TLT", "DBC")])</pre>
# Calculate covariance matrix
covmat <- cov(retp)</pre>
# Calculate matrix inverse using RcppArmadillo
invmat <- HighFreq::calc_invsvd(covmat)</pre>
# Calculate matrix inverse in R
invr <- solve(covmat)</pre>
all.equal(invmat, invr, check.attributes=FALSE)
# Calculate regularized inverse using RcppArmadillo
invmat <- HighFreq::calc_invsvd(covmat, dimax=3)</pre>
# Calculate regularized inverse from SVD in R
svdec <- svd(covmat)</pre>
dimax <- 1:3
invr <- svdec$v[, dimax] %*% (t(svdec$u[, dimax])/svdec$d[dimax])</pre>
# Compare RcppArmadillo with R
all.equal(invmat, invr)
## End(Not run)
```

calc\_kurtosis

Calculate the kurtosis of the columns of a time series or a matrix using RcppArmadillo.

# **Description**

Calculate the kurtosis of the columns of a time series or a matrix using RcppArmadillo.

# Usage

```
calc_kurtosis(tseries, method = "moment", confl = 0.75)
```

#### **Arguments**

tseries A time series or a matrix of data.

method A character string specifying the type of the kurtosis model (the default is

method = "moment" - see Details).

confl The confidence level for calculating the quantiles of returns (the default is confl

= 0.75).

# **Details**

The function calc\_kurtosis() calculates the kurtosis of the columns of the *matrix* tseries using RcppArmadillo C++ code.

If method = "moment" (the default) then calc\_kurtosis() calculates the fourth moment of the data. But it doesn't de-mean the columns of tseries because that requires copying the matrix tseries in memory, so it's time-consuming.

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If method = "quantile" then it calculates the skewness  $\kappa$  from the differences between the quantiles of the data as follows:

$$\kappa = \frac{q_{\alpha} - q_{1-\alpha}}{q_{0.75} - q_{0.25}}$$

Where  $\alpha$  is the confidence level for calculating the quantiles.

If method = "nonparametric" then it calculates the kurtosis as the difference between the mean of the data minus its median, divided by the standard deviation.

If the number of rows of tseries is less than 3 then it returns zeros.

The code examples below compare the function calc\_kurtosis() with the kurtosis calculated using R code.

#### Value

A single-row matrix with the kurtosis of the columns of tseries.

```
## Not run:
# Define a single-column time series of returns
retp <- na.omit(rutils::etfenv$returns$VTI)</pre>
# Calculate the moment kurtosis
HighFreq::calc_kurtosis(retp)
# Calculate the moment kurtosis in R
calc_kurtr <- function(x) {</pre>
  x \leftarrow (x-mean(x))
  sum(x^4)/var(x)^2/NROW(x)
} # end calc_kurtr
all.equal(HighFreq::calc_kurtosis(retp),
  calc_kurtr(retp), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::calc_kurtosis(retp),
  Rcode=calc_kurtr(retp),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Calculate the quantile kurtosis
HighFreq::calc_kurtosis(retp, method="quantile", confl=0.9)
\# Calculate the quantile kurtosis in R
calc_kurtq <- function(x, a=0.9) {</pre>
   quantiles <- quantile(x, c(1-a, 0.25, 0.75, a), type=5)
   (quantiles[4] - quantiles[1])/(quantiles[3] - quantiles[2])
} # end calc_kurtq
all.equal(drop(HighFreq::calc_kurtosis(retp, method="quantile", confl=0.9)),
  calc_kurtq(retp, a=0.9), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
summary(microbenchmark(
  Rcpp=HighFreq::calc_kurtosis(retp, method="quantile"),
  Rcode=calc_kurtq(retp),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Calculate the nonparametric kurtosis
HighFreq::calc_kurtosis(retp, method="nonparametric")
# Compare HighFreq::calc_kurtosis() with R nonparametric kurtosis
all.equal(drop(HighFreq::calc_kurtosis(retp, method="nonparametric")),
  (mean(retp)-median(retp))/sd(retp),
  check.attributes=FALSE)
```

24 calc\_lm

```
# Compare the speed of RcppArmadillo with R code
summary(microbenchmark(
   Rcpp=HighFreq::calc_kurtosis(retp, method="nonparametric"),
   Rcode=(mean(retp)-median(retp))/sd(retp),
   times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

calc\_lm

Perform multivariate linear regression using least squares and return a named list of regression coefficients, their t-values, and p-values.

## **Description**

Perform multivariate linear regression using least squares and return a named list of regression coefficients, their t-values, and p-values.

#### Usage

```
calc_lm(respv, predv)
```

#### **Arguments**

respv A single-column *time series* or a *vector* of response data.

predv A time series or a matrix of predictor data.

#### **Details**

The function calc\_lm() performs the same calculations as the function lm() from package *stats*. It uses RcppArmadillo C++ code so it's several times faster than lm(). The code was inspired by this article (but it's not identical to it): http://gallery.rcpp.org/articles/fast-linear-model-with-armadillo/

#### Value

A named list with three elements: a *matrix* of coefficients (named "coefficients"), the z-score of the last residual (named "zscore"), and a vector with the R-squared and F-statistic (named "stats"). The numeric matrix of coefficients named "coefficients" contains the alpha and beta coefficients, and their t-values and p-values.

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("XLF", "VTI", "IEF")])
# Response equals XLF returns
respv <- retp[, 1]
# Predictor matrix equals VTI and IEF returns
predv <- retp[, -1]
# Perform multivariate regression using lm()
lmod <- lm(respv ~ predv)
lmodsum <- summary(lmod)
# Perform multivariate regression using calc_lm()</pre>
```

calc\_mean 25

```
regarma <- HighFreq::calc_lm(respv=respv, predv=predv)
# Compare the outputs of both functions
all.equal(regarma$coefficients[, "coeff"], unname(coef(lmod)))
all.equal(unname(regarma$coefficients), unname(lmodsum$coefficients))
all.equal(unname(regarma$stats), c(lmodsum$r.squared, unname(lmodsum$fstatistic[1])))
# Compare the speed of RcppArmadillo with R code
summary(microbenchmark(
    Rcpp=HighFreq::calc_lm(respv=respv, predv=predv),
    Rcode=lm(respv ~ predv),
    times=10))[, c(1, 4, 5)] # end microbenchmark summary</pre>
## End(Not run)
```

calc\_mean

Calculate the mean (location) of the columns of a time series or a matrix using RcppArmadillo.

# **Description**

Calculate the mean (location) of the columns of a time series or a matrix using RcppArmadillo.

#### Usage

```
calc_mean(tseries, method = "moment", confl = 0.75)
```

# **Arguments**

tseries A *time series* or a *matrix* of data.

method A *character string* specifying the type of the mean (location) model (the default is method = "moment" - see Details).

confl The confidence level for calculating the quantiles of returns (the default is confl = 0.75).

# **Details**

The function calc\_mean() calculates the mean (location) values of the columns of the *time series* tseries using C++ RcppArmadillo code.

If method = "moment" (the default) then calc\_mean() calculates the location as the mean - the first moment of the data.

If method = "quantile" then it calculates the location  $\bar{r}$  as the average of the quantiles as follows:

$$\bar{r} = \frac{q_{\alpha} + q_{1-\alpha}}{2}$$

Where  $\alpha$  is the confidence level for calculating the quantiles (argument conf1).

If method = "nonparametric" then it calculates the location as the median.

The code examples below compare the function calc\_mean() with the mean (location) calculated using R code.

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#### Value

A single-row matrix with the mean (location) of the columns of tseries.

#### **Examples**

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("XLP", "VTI")])</pre>
# Calculate the column means in RcppArmadillo
HighFreq::calc_mean(retp)
# Calculate the column means in R
sapply(retp, mean)
# Compare the values
all.equal(drop(HighFreq::calc_mean(retp)),
  sapply(retp, mean), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::calc_mean(retp),
  Rcode=sapply(retp, mean),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Calculate the quantile mean (location)
HighFreq::calc_mean(retp, method="quantile", confl=0.9)
# Calculate the quantile mean (location) in R
colSums(sapply(retp, quantile, c(0.9, 0.1), type=5))
# Compare the values
all.equal(drop(HighFreq::calc_mean(retp, method="quantile", confl=0.9)),
  colSums(sapply(retp, quantile, c(0.9, 0.1), type=5)),
  check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
summary(microbenchmark(
  Rcpp=HighFreq::calc_mean(retp, method="quantile", confl=0.9),
  Rcode=colSums(sapply(retp, quantile, c(0.9, 0.1), type=5)),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Calculate the column medians in RcppArmadillo
HighFreq::calc_mean(retp, method="nonparametric")
# Calculate the column medians in R
sapply(retp, median)
# Compare the values
all.equal(drop(HighFreq::calc_mean(retp, method="nonparametric")),
  sapply(retp, median), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
summary(microbenchmark(
  Rcpp=HighFreq::calc_mean(retp, method="nonparametric"),
  Rcode=sapply(retp, median),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

Calculate the ranks of the elements of a single-column time series, matrix, or a vector using RcppArmadillo.

calc\_ranks 27

#### **Description**

Calculate the ranks of the elements of a single-column *time series*, *matrix*, or a *vector* using RcppArmadillo.

#### Usage

```
calc_ranks(tseries)
```

#### **Arguments**

tseries

A single-column time series, matrix, or a vector.

#### **Details**

The function calc\_ranks() calculates the ranks of the elements of a single-column *time series*, *matrix*, or a *vector*.

The permutation index is an integer vector which sorts a given vector into ascending order. The permutation index of the permutation index is the *reverse* permutation index, because it sorts the vector from ascending order back into its original unsorted order. The ranks of the elements are equal to the *reverse* permutation index. The function calc\_ranks() calculates the *reverse* permutation index.

The ranks produced by calc\_ranks() start at zero, following the C++ convention.

The Armadillo function arma::sort\_index() calculates the permutation index which sorts a given vector into an ascending order. Applying the function arma::sort\_index() twice: arma::sort\_index(arma::sort\_index(arma::sort\_index(arma::sort\_index)) and unsorted order.

The function calc\_ranks() calls the Armadillo function arma::sort\_index() twice to calculate the *reverse* permutation index, to sort the vector from ascending order back into its original unsorted order.

# Value

An *integer vector* with the ranks of the elements of the tseries.

```
## Not run:
# Create a vector of data
datav <- rnorm(1e3)
# Calculate the ranks of the elements using R code and RcppArmadillo
all.equal(rank(datav), drop(HighFreq::calc_ranks(datav))+1)
# Compare the speed of R code with RcppArmadillo
library(microbenchmark)
summary(microbenchmark(
   Rcode=rank(datav),
   Rcpp=calc_ranks(datav),
   times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)</pre>
```

28 calc\_ranks\_stl

| calc_ranks_stl Calculate the ranks of the elements of a single-column time matrix, or a vector using RcppArmadillo. | e series, |
|---|-----------|
|---|-----------|

# **Description**

Calculate the ranks of the elements of a single-column *time series*, *matrix*, or a *vector* using RcppArmadillo.

# Usage

```
calc_ranks_stl(tseries)
```

# **Arguments**

tseries

A single-column time series, matrix, or a vector.

#### **Details**

The function calc\_ranks\_stl() calculates the ranks of the elements of a single-column *time series*, *matrix*, or a *vector*. The function calc\_ranks\_stl() is slightly faster than the function calc\_ranks().

The permutation index is an integer vector which sorts a given vector into ascending order. The permutation index of the permutation index is the *reverse* permutation index, because it sorts the vector from ascending order back into its original unsorted order. The ranks of the elements are equal to the *reverse* permutation index. The function calc\_ranks() calculates the *reverse* permutation index.

The ranks produced by calc\_ranks\_stl() start at zero, following the C++ convention.

The STL C++ function std::sort() sorts a vector into ascending order. It can also be used to calculate the permutation index which sorts the vector into an ascending order.

The function calc\_ranks\_stl() calls the function std::sort() twice: First, it calculates the permutation index which sorts the vector tseries into ascending order. Second, it calculates the permutation index of the permutation index, which are the ranks (the *reverse* permutation index) of the vector tseries.

#### Value

An *integer vector* with the ranks of the elements of tseries.

```
## Not run:
# Create a vector of data
datav <- rnorm(1e3)
# Calculate the ranks of the elements using R code and RcppArmadillo
all.equal(rank(datav), drop(HighFreq::calc_ranks_stl(datav))+1)
# Compare the speed of R code with RcppArmadillo
library(microbenchmark)
summary(microbenchmark(
   Rcode=rank(datav),
   Rcpp=HighFreq::calc_ranks_stl(datav),
   times=10))[, c(1, 4, 5)] # end microbenchmark summary</pre>
```

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| ## | End | (Not | run) |
|----|-----|------|------|
|----|-----|------|------|

| calc_reg | Perform multivariate regression using different methods, and return a vector of regression coefficients, their t-values, and the last residual |
|----------|--|
|          | z-score.   |

#### **Description**

Perform multivariate regression using different methods, and return a vector of regression coefficients, their t-values, and the last residual z-score.

#### Usage

```
calc_reg(respv, predv, controlv)
```

#### **Arguments**

respv A single-column time series or a vector of response data.

predv A *time series* or a *matrix* of predictor data. controlv A *list* of model parameters (see Details).

#### Details

The function calc\_reg() performs multivariate regression using different methods, and returns a vector of regression coefficients, their t-values, and the last residual z-score.

The function calc\_reg() accepts a list of regression model parameters through the argument controlv. The list of model parameters can be created using the function param\_reg(). Below is a description of the model parameters.

If method = "least\_squares" (the default) then it performs the standard least squares regression, the same as the function calc\_lm(), and the function lm() from the R package *stats*. But it uses RcppArmadillo C++ code so it's several times faster than lm().

If method = "regular" then it performs shrinkage regression. It calculates the *regularized inverse* of the predictor matrix from its singular value decomposition. It performs regularization by selecting only the largest *singular values* equal in number to dimax.

If method = "quantile" then it performs quantile regression (not implemented yet).

If intercept = TRUE then an extra intercept column (unit column) is added to the predictor matrix (the default is intercept = TRUE).

The length of the return vector depends on the number of columns of the predictor matrix (including the intercept column, if it's added). The length of the return vector is equal to the number of regression coefficients, plus their t-values, plus the z-score. The number of regression coefficients is equal to the number of columns of the predictor matrix (including the intercept column, if it's added). The number of t-values is equal to the number of coefficients.

For example, if the number of columns of the predictor matrix is equal to n, and if intercept = TRUE (the default), then calc\_reg() returns a vector with 2n+3 elements: n+1 regression coefficients (including the intercept coefficient), n+1 corresponding t-values, and 1 z-score value.

If intercept = FALSE, then calc\_reg() returns a vector with 2n+1 elements: n regression coefficients (without the intercept coefficient), n corresponding t-values, and 1 z-score value.

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#### Value

A single-row matrix with the regression coefficients, their t-values, and the last residual z-score.

#### **Examples**

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("XLF", "VTI", "IEF")])</pre>
# Response equals XLF returns
respv <- retp[, 1]</pre>
# Predictor matrix equals VTI and IEF returns
predv <- retp[, -1]</pre>
# Perform multivariate regression using lm()
lmod <- lm(respv ~ predv)</pre>
lmodsum <- summary(lmod)</pre>
coeff <- lmodsum$coefficients</pre>
# Create a default list of regression parameters
controlv <- HighFreq::param_reg()</pre>
# Perform multivariate regression using calc_reg()
regarma <- drop(HighFreq::calc_reg(respv=respv, predv=predv, controlv=controlv))</pre>
# Compare the outputs of both functions
all.equal(regarma[1:(2*(1+NCOL(predv)))],
  c(coeff[, "Estimate"], coeff[, "t value"]), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::calc_reg(respv=respv, predv=predv, controlv=controlv),
  Rcode=lm(respv ~ predv),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

calc\_scale

Standardize (center and scale) the columns of a time series of data in place, without copying the data in memory, using RcppArmadillo.

## **Description**

Standardize (center and scale) the columns of a *time series* of data in place, without copying the data in memory, using RcppArmadillo.

# Usage

```
calc_scale(tseries, center = TRUE, scale = TRUE, use_median = FALSE)
```

# Arguments

| tseries | A time series or matrix of data.  |
|---------|---|
| center  | A <i>Boolean</i> argument: if TRUE then center the columns so that they have zero mean or median (the default is TRUE).           |
| scale   | A <i>Boolean</i> argument: if TRUE then scale the columns so that they have unit standard deviation or MAD (the default is TRUE). |

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use median

A *Boolean* argument: if TRUE then the centrality (central tendency) is calculated as the *median* and the dispersion is calculated as the *median absolute deviation* (*MAD*) (the default is FALSE). If use\_median = FALSE then the centrality is calculated as the *mean* and the dispersion is calculated as the *standard deviation*.

#### **Details**

The function calc\_scale() standardizes (centers and scales) the columns of a *time series* of data in place, without copying the data in memory, using RcppArmadillo.

If the arguments center and scale are both TRUE and use\_median is FALSE (the defaults), then calc\_scale() performs the same calculation as the standard R function scale(), and it calculates the centrality (central tendency) as the *mean* and the dispersion as the *standard deviation*.

If the arguments center and scale are both TRUE (the defaults), then calc\_scale() standardizes the data. If the argument center is FALSE then calc\_scale() only scales the data (divides it by the standard deviations). If the argument scale is FALSE then calc\_scale() only demeans the data (subtracts the means).

If the argument use\_median is TRUE, then it calculates the centrality as the *median* and the dispersion as the *median absolute deviation (MAD)*.

If the number of rows of tseries is less than 3 then it does nothing and tseries is not scaled.

The function calc\_scale() accepts a *pointer* to the argument tseries, and it overwrites the old data with the standardized data. It performs the calculation in place, without copying the data in memory, which can significantly increase the computation speed for large time series.

The function calc\_scale() uses RcppArmadillo C++ code, so on a typical time series it can be over 10 times faster than the function scale().

#### Value

Void (no return value - modifies the data in place).

```
## Not run:
# Calculate a time series of returns
retp <- zoo::coredata(na.omit(rutils::etfenv$returns[, c("IEF", "VTI")]))</pre>
# Demean the returns
demeaned <- apply(retp, 2, function(x) (x-mean(x)))
HighFreq::calc_scale(retp, scale=FALSE)
all.equal(demeaned, retp, check.attributes=FALSE)
# Calculate a time series of returns
retp <- zoo::coredata(na.omit(rutils::etfenv$returns[, c("IEF", "VTI")]))</pre>
# Standardize the returns
retss <- scale(retp)</pre>
HighFreq::calc_scale(retp)
all.equal(retss, retp, check.attributes=FALSE)
# Compare the speed of Rcpp with R code
library(microbenchmark)
summary(microbenchmark(
  Rcode=scale(retp),
  Rcpp=HighFreq::calc_scale(retp),
  times=100))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

32 calc\_skew

| calc_skew | Calculate the skewness of the columns of a time series or a matrix using RcppArmadillo. |
|-----------|---|
|           |   |

#### **Description**

Calculate the skewness of the columns of a *time series* or a *matrix* using RcppArmadillo.

#### Usage

```
calc_skew(tseries, method = "moment", confl = 0.75)
```

#### **Arguments**

tseries A time series or a matrix of data.

Method A character string specifying the type of the skewness model (the default is method = "moment" - see Details).

Confl The confidence level for calculating the quantiles of returns (the default is confl

= 0.75).

#### **Details**

The function calc\_skew() calculates the skewness of the columns of a *time series* or a *matrix* of data using C++ RcppArmadillo code.

If method = "moment" (the default) then calc\_skew() calculates the skewness as the third moment of the data.

If method = "quantile" then it calculates the skewness  $\varsigma$  from the differences between the quantiles of the data as follows:

$$\varsigma = \frac{q_{\alpha} + q_{1-\alpha} - 2q_{0.5}}{q_{\alpha} - q_{1-\alpha}}$$

Where  $\alpha$  is the confidence level for calculating the quantiles.

If method = "nonparametric" then it calculates the skewness as the difference between the mean of the data minus its median, divided by the standard deviation.

If the number of rows of tseries is less than 3 then it returns zeros.

The code examples below compare the function calc\_skew() with the skewness calculated using R code.

# Value

A single-row matrix with the skewness of the columns of tseries.

```
## Not run:
# Define a single-column time series of returns
retp <- na.omit(rutils::etfenv$returns$VTI)
# Calculate the moment skewness
HighFreq::calc_skew(retp)
# Calculate the moment skewness in R
calc_skewr <- function(x) {</pre>
```

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```
x <- (x-mean(x))
  sum(x^3)/var(x)^1.5/NROW(x)
} # end calc_skewr
all.equal(HighFreq::calc_skew(retp),
  calc_skewr(retp), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::calc_skew(retp),
  Rcode=calc_skewr(retp),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Calculate the quantile skewness
HighFreq::calc_skew(retp, method="quantile", confl=0.9)
# Calculate the quantile skewness in R
calc_skewq \leftarrow function(x, a = 0.75) {
   quantiles <- quantile(x, c(1-a, 0.5, a), type=5)
   (quantiles[3] + quantiles[1] - 2*quantiles[2])/(quantiles[3] - quantiles[1])
} # end calc_skewq
all.equal(drop(HighFreq::calc_skew(retp, method="quantile", confl=0.9)),
  calc_skewq(retp, a=0.9), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
summary(microbenchmark(
  Rcpp=HighFreq::calc_skew(retp, method="quantile"),
  Rcode=calc_skewq(retp),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Calculate the nonparametric skewness
HighFreq::calc_skew(retp, method="nonparametric")
# Compare HighFreq::calc_skew() with R nonparametric skewness
all.equal(drop(HighFreq::calc_skew(retp, method="nonparametric")),
  (mean(retp)-median(retp))/sd(retp),
  check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
summary(microbenchmark(
  Rcpp=HighFreq::calc_skew(retp, method="nonparametric"),
  Rcode=(mean(retp)-median(retp))/sd(retp),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

calc\_startpoints

Calculate a vector of start points by lagging (shifting) a vector of end points.

# **Description**

Calculate a vector of start points by lagging (shifting) a vector of end points.

# Usage

```
calc_startpoints(endp, look_back)
```

34 calc\_var

## **Arguments**

endp An *integer* vector of end points.

look\_back The length of the look-back interval, equal to the lag (shift) applied to the end

points.

#### **Details**

The start points are equal to the values of the vector endp lagged (shifted) by an amount equal to look\_back. In addition, an extra value of 1 is added to them, to avoid data overlaps. The lag operation requires appending a beginning warmup interval containing zeros, so that the vector of start points has the same length as the endp.

For example, consider the end points for a vector of length 25 divided into equal intervals of length 5: 4, 9, 14, 19, 24. (In C++ the vector indexing starts at 0 not 1, so it's shifted by -1.) Then the start points for look\_back = 2 are equal to: 0, 0, 5, 10, 15. The differences between the end points minus the corresponding start points are equal to 9, except for the warmup interval.

#### Value

An *integer* vector with the same number of elements as the vector endp.

#### **Examples**

```
# Calculate end points
endp <- HighFreq::calc_endpoints(length=55, step=5)
# Calculate start points corresponding to the end points
startp <- HighFreq::calc_startpoints(endp, look_back=5)</pre>
```

calc\_var Calculate the dispersion (variance) of the columns of a time series or a matrix using RcppArmadillo.

#### **Description**

Calculate the dispersion (variance) of the columns of a time series or a matrix using RcppArmadillo.

# Usage

```
calc_var(tseries, method = "moment", confl = 0.75)
```

## **Arguments**

tseries A time series or a matrix of data.

method A character string specifying the type of the dispersion model (the default is

method = "moment" - see Details).

confl The confidence level for calculating the quantiles of returns (the default is confl

= 0.75).

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#### **Details**

The function calc\_var() calculates the dispersion of the columns of a *time series* or a *matrix* of data using RcppArmadillo C++ code.

The dispersion is a measure of the variability of the data. Examples of dispersion are the variance and the Median Absolute Deviation (*MAD*).

If method = "moment" (the default) then calc\_var() calculates the dispersion as the second moment of the data (the variance). Then calc\_var() performs the same calculation as the function colVars() from package matrixStats, but it's much faster because it uses RcppArmadillo C++ code.

If method = "quantile" then it calculates the dispersion as the difference between the quantiles as follows:

$$\sigma = q_{\alpha} - q_{1-\alpha}$$

Where  $\alpha$  is the confidence level for calculating the quantiles.

If method = "nonparametric" then it calculates the dispersion as the Median Absolute Deviation (MAD):

$$MAD = median(abs(x - median(x)))$$

It also multiplies the MAD by a factor of 1.4826, to make it comparable to the standard deviation.

If method = "nonparametric" then calc\_var() performs the same calculation as the function stats::mad(), but it's much faster because it uses RcppArmadillo C++ code.

If the number of rows of tseries is less than 3 then it returns zeros.

#### Value

A row vector equal to the dispersion of the columns of the matrix tseries.

```
## Not run:
# Calculate VTI and XLF returns
retp <- na.omit(rutils::etfenv$returns[, c("VTI", "XLF")])</pre>
# Compare HighFreq::calc_var() with standard var()
all.equal(drop(HighFreq::calc_var(retp)),
  apply(retp, 2, var), check.attributes=FALSE)
# Compare HighFreq::calc_var() with matrixStats
all.equal(drop(HighFreq::calc_var(retp)),
 matrixStats::colVars(retp), check.attributes=FALSE)
\mbox{\#} Compare the speed of RcppArmadillo with matrixStats and with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::calc_var(retp),
  matrixStats=matrixStats::colVars(retp),
  Rcode=apply(retp, 2, var),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Compare HighFreq::calc_var() with stats::mad()
all.equal(drop(HighFreq::calc_var(retp, method="nonparametric")),
  sapply(retp, mad), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with stats::mad()
summarv(microbenchmark(
  Rcpp=HighFreq::calc_var(retp, method="nonparametric"),
  Rcode=sapply(retp, mad),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
```

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```
## End(Not run)
```

calc\_varvec

 $\label{lem:calculate} \textit{Calculate the variance of a single-column} \ \text{time series} \ \textit{or a} \ \text{vector} \ \textit{using} \\ \text{RcppArmadillo}.$ 

# **Description**

Calculate the variance of a single-column time series or a vector using RcppArmadillo.

#### Usage

```
calc_varvec(tseries)
```

# **Arguments**

tseries

A single-column time series or a vector.

#### **Details**

The function calc\_varvec() calculates the variance of a *vector* using RcppArmadillo C++ code, so it's significantly faster than the R function var().

#### Value

A numeric value equal to the variance of the vector.

```
## Not run:
# Create a vector of random returns
retp <- rnorm(1e6)
# Compare calc_varvec() with standard var()
all.equal(HighFreq::calc_varvec(retp), var(retp))
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
   Rcpp=HighFreq::calc_varvec(retp),
   Rcode=var(retp),
   times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)</pre>
```

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calc\_var\_ag

Calculate the variance of returns aggregated over the end points.

## **Description**

Calculate the variance of returns aggregated over the end points.

## Usage

```
calc_var_ag(tseries, step = 1L)
```

### **Arguments**

tseries A time series or a matrix of log prices.

step The number of time periods in each interval between neighboring end points

(the default is step = 1).

#### **Details**

The function calc\_var\_ag() calculates the variance of returns aggregated over the end points.

It first calculates the end points spaced apart by the number of periods equal to the argument step. Then it calculates the aggregated returns by differencing the prices tseries calculated at the end points. Finally it calculates the variance of the returns.

The choice of the first end point is arbitrary, so calc\_var\_ag() calculates the different end points for all the possible starting points. It then calculates the variance values for all the different end points and averages them.

The aggregated volatility  $\sigma_t$  increases with the length of the aggregation interval  $\Delta t$ . The aggregated volatility increases as the length of the aggregation interval  $\Delta t$  raised to the power of the *Hurst* exponent H:

$$\sigma_t = \sigma \Delta t^H$$

Where  $\sigma$  is the daily return volatility.

The function calc\_var\_ag() can therefore be used to calculate the *Hurst exponent* from the variance ratio.

### Value

The variance of aggregated returns.

```
## Not run:
# Calculate the log prices
closep <- na.omit(rutils::etfenv$prices[, c("XLP", "VTI")])
closep <- log(closep)
# Calculate the daily variance of percentage returns
calc_var_ag(prices, step=1)
# Calculate the daily variance using R
sapply(rutils::diffit(closep), var)
# Calculate the variance of returns aggregated over 21 days
calc_var_ag(prices, step=21)</pre>
```

38 calc\_var\_ohlc

```
# The variance over 21 days is approximately 21 times the daily variance
21*calc_var_ag(prices, step=1)
## End(Not run)
```

calc\_var\_ohlc

Calculate the variance of returns from OHLC prices using different price range estimators.

## **Description**

Calculate the variance of returns from OHLC prices using different price range estimators.

## Usage

```
calc_var_ohlc(
  ohlc,
  method = "yang_zhang",
  closel = 0L,
  scale = TRUE,
  index = 0L
)
```

### **Arguments**

ohlc

A time series or a matrix of OHLC prices.

method

A *character string* representing the price range estimator for calculating the variance. The estimators include:

- "close" close-to-close estimator,
- "rogers\_satchell" Rogers-Satchell estimator,
- "garman\_klass" Garman-Klass estimator,
- "garman\_klass\_yz" Garman-Klass with account for close-to-open price jumps,
- "yang\_zhang" Yang-Zhang estimator,

(The default is the method = "yang\_zhang".)

closel

A *vector* with the lagged *close* prices of the *OHLC time series*. This is an optional argument. (The default is closel = 0).

scale

*Boolean* argument: Should the returns be divided by the time index, the number of seconds in each period? (The default is scale = TRUE).

index

A *vector* with the time index of the *time series*. This is an optional argument (the default is index = 0).

## Details

The function calc\_var\_ohlc() calculates the variance from all the different intra-day and day-over-day returns (defined as the differences of *OHLC* prices), using several different variance estimation methods.

The function calc\_var\_ohlc() does not calculate the logarithm of the prices. So if the argument ohlc contains dollar prices then calc\_var\_ohlc() calculates the dollar variance. If the argument ohlc contains the log prices then calc\_var\_ohlc() calculates the percentage variance.

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The default method is "yang\_zhang", which theoretically has the lowest standard error among unbiased estimators. The methods "close", "garman\_klass\_yz", and "yang\_zhang" do account for close-to-open price jumps, while the methods "garman\_klass" and "rogers\_satchell" do not account for close-to-open price jumps.

If scale is TRUE (the default), then the returns are divided by the differences of the time index (which scales the variance to the units of variance per second squared). This is useful when calculating the variance from minutely bar data, because dividing returns by the number of seconds decreases the effect of overnight price jumps. If the time index is in days, then the variance is equal to the variance per day squared.

If the number of rows of ohlc is less than 3 then it returns zero.

The optional argument index is the time index of the *time series* ohlc. If the time index is in seconds, then the differences of the index are equal to the number of seconds in each time period. If the time index is in days, then the differences are equal to the number of days in each time period.

The optional argument closel are the lagged *close* prices of the *OHLC time series*. Passing in the lagged *close* prices speeds up the calculation, so it's useful for rolling calculations.

The function calc\_var\_ohlc() is implemented in RcppArmadillo C++ code, and it's over 10 times faster than calc\_var\_ohlc\_r(), which is implemented in R code.

#### Value

A single *numeric* value equal to the variance of the *OHLC time series*.

```
## Not run:
# Extract the log OHLC prices of SPY
ohlc <- log(HighFreq::SPY)</pre>
# Extract the time index of SPY prices
indeks <- c(1, diff(xts::.index(ohlc)))</pre>
# Calculate the variance of SPY returns, with scaling of the returns
HighFreq::calc_var_ohlc(ohlc,
method="yang_zhang", scale=TRUE, index=indeks)
# Calculate variance without accounting for overnight jumps
HighFreq::calc_var_ohlc(ohlc,
 method="rogers_satchell", scale=TRUE, index=indeks)
# Calculate the variance without scaling the returns
HighFreq::calc_var_ohlc(ohlc, scale=FALSE)
# Calculate the variance by passing in the lagged close prices
closel <- HighFreq::lagit(ohlc[, 4])</pre>
all.equal(HighFreq::calc_var_ohlc(ohlc),
  HighFreq::calc_var_ohlc(ohlc, closel=closel))
# Compare with HighFreq::calc_var_ohlc_r()
all.equal(HighFreq::calc_var_ohlc(ohlc, index=indeks),
  HighFreq::calc_var_ohlc_r(ohlc))
# Compare the speed of Rcpp with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::calc_var_ohlc(ohlc),
  Rcode=HighFreq::calc_var_ohlc_r(ohlc),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

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| calc_var_ohlc_ag | Calculate the variance of aggregated OHLC prices using different price range estimators. |
|------------------|--|
|------------------|--|

### **Description**

Calculate the variance of aggregated OHLC prices using different price range estimators.

# Usage

```
calc_var_ohlc_ag(
  ohlc,
  step,
  method = "yang_zhang",
  closel = 0L,
  scale = TRUE,
  index = 0L
)
```

## **Arguments**

ohlc A time series or a matrix of OHLC prices.

step The number of time periods in each interval between neighboring end points.

method A character string representing the price range estimator for calculating the

variance. The estimators include:

• "close" close-to-close estimator.

• "rogers\_satchell" Rogers-Satchell estimator,

• "garman\_klass" Garman-Klass estimator,

• "garman\_klass\_yz" Garman-Klass with account for close-to-open price jumps,

• "yang zhang" Yang-Zhang estimator,

(The default is the method = "yang\_zhang".)

closel A vector with the lagged close prices of the OHLC time series. This is an op-

tional argument. (The default is closel = 0).

scale Boolean argument: Should the returns be divided by the time index, the number

of seconds in each period? (The default is scale = TRUE).

index A vector with the time index of the time series. This is an optional argument

(the default is index = 0).

#### **Details**

The function calc\_var\_ohlc\_ag() calculates the variance of *OHLC* prices aggregated over the end points.

It first calculates the end points spaced apart by the number of periods equal to the argument step. Then it aggregates the *OHLC* prices to the end points. Finally it calculates the variance of the aggregated *OHLC* prices.

The choice of the first end point is arbitrary, so calc\_var\_ohlc\_ag() calculates the different end points for all the possible starting points. It then calculates the variance values for all the different end points and averages them.

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The aggregated volatility  $\sigma_t$  increases with the length of the aggregation interval  $\Delta t$ . The aggregated volatility increases as the length of the aggregation interval  $\Delta t$  raised to the power of the *Hurst* exponent H:

$$\sigma_t = \sigma \Delta t^H$$

Where  $\sigma$  is the daily return volatility.

The function calc\_var\_ohlc\_ag() can therefore be used to calculate the *Hurst exponent* from the variance ratio.

#### Value

The variance of aggregated *OHLC* prices.

### **Examples**

```
## Not run:
# Calculate the log ohlc prices
ohlc <- log(rutils::etfenv$VTI)
# Calculate the daily variance of percentage returns
calc_var_ohlc_ag(ohlc, step=1)
# Calculate the variance of returns aggregated over 21 days
calc_var_ohlc_ag(ohlc, step=21)
# The variance over 21 days is approximately 21 times the daily variance
21*calc_var_ohlc_ag(ohlc, step=1)
## End(Not run)</pre>
```

calc\_var\_ohlc\_r

Calculate the variance of an OHLC time series, using different range estimators for variance.

## **Description**

Calculate the variance of an *OHLC* time series, using different range estimators for variance.

# Usage

```
calc_var_ohlc_r(ohlc, method = "yang_zhang", scalit = TRUE)
```

### **Arguments**

ohlc

An OHLC time series of prices in xts format.

method

A *character* string representing the method for estimating variance. The methods include:

- "close" close to close,
- "garman\_klass" Garman-Klass,
- "garman\_klass\_yz" Garman-Klass with account for close-to-open price jumps,
- "rogers\_satchell" Rogers-Satchell,
- "yang\_zhang" Yang-Zhang,

(default is "yang\_zhang")

scalit

*Boolean* argument: should the returns be divided by the number of seconds in each period? (default is TRUE)

42 calc\_weights

#### **Details**

The function calc\_var\_ohlc\_r() calculates the variance from all the different intra-day and day-over-day returns (defined as the differences of *OHLC* prices), using several different variance estimation methods.

The default method is "yang\_zhang", which theoretically has the lowest standard error among unbiased estimators. The methods "close", "garman\_klass\_yz", and "yang\_zhang" do account for close-to-open price jumps, while the methods "garman\_klass" and "rogers\_satchell" do not account for close-to-open price jumps.

If scalit is TRUE (the default), then the returns are divided by the differences of the time index (which scales the variance to the units of variance per second squared.) This is useful when calculating the variance from minutely bar data, because dividing returns by the number of seconds decreases the effect of overnight price jumps. If the time index is in days, then the variance is equal to the variance per day squared.

The function  $calc\_var\_ohlc\_r()$  is implemented in R code.

#### Value

A single *numeric* value equal to the variance.

## **Examples**

```
# Calculate the variance of SPY returns
HighFreq::calc_var_ohlc_r(HighFreq::SPY, method="yang_zhang")
# Calculate variance without accounting for overnight jumps
HighFreq::calc_var_ohlc_r(HighFreq::SPY, method="rogers_satchell")
# Calculate the variance without scaling the returns
HighFreq::calc_var_ohlc_r(HighFreq::SPY, scalit=FALSE)
```

calc\_weights Calculate the optimal portfolio weights using a variety of different objective functions.

# Description

Calculate the optimal portfolio weights using a variety of different objective functions.

### Usage

```
calc_weights(returns, controlv)
```

# **Arguments**

returns A time series or a matrix of returns data (the returns in excess of the risk-free

rate).

controlv A *list* of portfolio optimization model parameters (see Details).

calc\_weights 43

#### **Details**

The function calc\_weights() calculates the optimal portfolio weights using a variety of different objective functions.

The function calc\_weights() accepts a list of portfolio optimization parameters through the argument controlv. The list of portfolio optimization parameters can be created using the function param\_portf(). Below is a description of the parameters.

If method = "maxsharpe" (the default) then calc\_weights() calculates the weights of the maximum Sharpe portfolio, by multiplying the *regularized inverse* of the *covariance matrix*  $C^{-1}$  times the mean column returns  $\bar{r}$ :

$$w = C^{-1}\bar{r}$$

If method = "maxsharpemed" then calc\_weights() uses the medians instead of the means.

If method = "minvarlin" then it calculates the weights of the minimum variance portfolio under linear constraint, by multiplying the *regularized inverse* of the *covariance matrix* times the unit vector:

$$w = C^{-1}1$$

If method = "minvarquad" then it calculates the weights of the minimum variance portfolio under quadratic constraint (which is the highest order principal component).

If method = "sharpem" then it calculates the momentum weights equal to the Sharpe ratios (the returns divided by their standard deviations):

$$w = \frac{\bar{r}}{\sigma}$$

If method = "kellym" then it calculates the momentum weights equal to the Kelly ratios (the returns divided by their variance):

$$w = \frac{\bar{r}}{\sigma^2}$$

calc\_weights() calls the function calc\_inv() to calculate the *regularized inverse* of the *covariance matrix* of returns. It performs regularization by selecting only the largest eigenvalues equal in number to dimax.

In addition, calc\_weights() applies shrinkage to the columns of returns, by shrinking their means to their common mean value:

$$r_i' = (1 - \alpha) \, \bar{r}_i + \alpha \, \mu$$

Where  $\bar{r}_i$  is the mean of column i and  $\mu$  is the average of all the column means. The shrinkage intensity alpha determines the amount of shrinkage that is applied, with alpha = 0 representing no shrinkage (with the column means  $\bar{r}_i$  unchanged), and alpha = 1 representing complete shrinkage (with the column means all equal to the single mean of all the columns:  $\bar{r}_i = \mu$ ).

After the weights are calculated, they are scaled, depending on several arguments.

If rankw = TRUE then the weights are converted into their ranks. The default is rankw = FALSE.

If centerw = TRUE then the weights are centered so that their sum is equal to 0. The default is centerw = FALSE.

If scalew = "voltarget" (the default) then the weights are scaled (multiplied by a factor) so that the weighted portfolio has an in-sample volatility equal to vol\_target.

If scalew = "voleqw" then the weights are scaled so that the weighted portfolio has the same volatility as the equal weight portfolio.

44 decode\_it

If scalew = "sumone" then the weights are scaled so that their sum is equal to 1. If scalew = "sumsq" then the weights are scaled so that their sum of squares is equal to 1. If scalew = "none" then the weights are not scaled.

The function calc\_weights() is written in C++ RcppArmadillo code.

#### Value

A column *vector* of the same length as the number of columns of returns.

#### **Examples**

```
## Not run:
# Calculate covariance matrix and eigen decomposition of ETF returns
retp <- na.omit(rutils::etfenv$returns[, 1:16])</pre>
ncols <- NCOL(retp)</pre>
eigend <- eigen(cov(retp))</pre>
# Calculate regularized inverse of covariance matrix
dimax <- 3
eigenvec <- eigend$vectors[, 1:dimax]</pre>
eigenval <- eigend$values[1:dimax]</pre>
invmat <- eigenvec %*% (t(eigenvec) / eigenval)</pre>
# Define shrinkage intensity and apply shrinkage to the mean returns
alpha <- 0.5
colmeans <- colMeans(retp)</pre>
colmeans <- ((1-alpha)*colmeans + alpha*mean(colmeans))</pre>
# Calculate weights using R
weightr <- drop(invmat %*% colmeans)</pre>
# Apply weights scaling
weightr <- weightr*sd(rowMeans(retp))/sd(retp %*% weightr)</pre>
weightr <- 0.01*weightr/sd(retp %*% weightr)</pre>
weightr <- weightr/sqrt(sum(weightr^2))</pre>
# Create a list of portfolio optimization parameters
controlv <- HighFreq::param_portf(method="maxsharpe", dimax=dimax, alpha=alpha, scalew="sumsq")</pre>
# Calculate weights using RcppArmadillo
weightcpp <- drop(HighFreq::calc_weights(retp, controlv=controlv))</pre>
all.equal(weightcpp, weightr)
## End(Not run)
```

decode\_it

Calculate the vector of data from its run length encoding.

### **Description**

Calculate the *vector* of data from its run length encoding.

## Usage

```
decode_it(encodel)
```

# Arguments

encodel

A *list* with two *vectors*: a *numeric vector* of encoded data and an *integer vector* of data counts (repeats).

diffit 45

#### **Details**

The function decode\_it() the *vector* of data from its run length encoding.

The run length encoding of a *vector* consists of two *vectors*: a *numeric vector* of encoded data (consecutive data values) and of an *integer vector* of the data counts (the number of times the same value repeats in succession).

Run length encoding (RLE) is a data compression algorithm which encodes the data in two *vectors*: the consecutive data values and their counts. If a data value occurs several times in succession then it is recorded only once and its corresponding count is equal to the number of times it occurs. Run-length encoding is different from a contingency table.

#### Value

A numeric vector.

### **Examples**

```
## Not run:
# Create a vector of data
datav <- sample(5, 31, replace=TRUE)
# Calculate the run length encoding of datav
rle <- HighFreq::encode_it(datav)
# Decode the data from its run length encoding
decodev <- HighFreq::decode_it(rle)
all.equal(datav, decodev)
## End(Not run)</pre>
```

diffit

Calculate the row differences of a time series or a matrix using Rcp-pArmadillo.

### **Description**

Calculate the row differences of a time series or a matrix using RcppArmadillo.

# Usage

```
diffit(tseries, lagg = 1L, pad_zeros = TRUE)
```

# Arguments

tseries A time series or a matrix.

lagg An *integer* equal to the number of rows (time periods) to lag when calculating

the differences (the default is lagg = 1).

pad\_zeros Boolean argument: Should the output matrix be padded (extended) with zero

values, in order to return a matrix with the same number of rows as the input?

(the default is pad\_zeros = TRUE)

46 diff\_vec

#### **Details**

The function diffit() calculates the differences between the rows of the input *matrix* tseries and its lagged version.

The argument lagg specifies the number of lags applied to the rows of the lagged version of tseries. For positive lagg values, the lagged version of tseries has its rows shifted *forward* (down) by the number equal to lagg rows. For negative lagg values, the lagged version of tseries has its rows shifted *backward* (up) by the number equal to -lagg rows. For example, if lagg=3 then the lagged version will have its rows shifted down by 3 rows, and the differences will be taken between each row minus the row three time periods before it (in the past). The default is lagg = 1.

The argument pad\_zeros specifies whether the output *matrix* should be padded (extended) with zero values in order to return a *matrix* with the same number of rows as the input tseries. The default is pad\_zeros = TRUE. If pad\_zeros = FALSE then the return *matrix* has a smaller number of rows than the input tseries. The padding operation can be time-consuming, because it requires the copying the data in memory.

The function diffit() is implemented in RcppArmadillo C++ code, which makes it several times faster than R code.

#### Value

A *matrix* containing the differences between the rows of the input *matrix* tseries.

## **Examples**

```
## Not run:
# Create a matrix of random data
datav <- matrix(sample(15), nc=3)</pre>
# Calculate differences with lagged rows
HighFreq::diffit(datav, lagg=2)
# Calculate differences with advanced rows
HighFreq::diffit(datav, lagg=-2)
# Compare HighFreq::diffit() with rutils::diffit()
all.equal(HighFreq::diffit(datav, lagg=2),
  rutils::diffit(datav, lagg=2),
  check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::diffit(datav, lagg=2),
  Rcode=rutils::diffit(datav, lagg=2),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

diff\_vec

Calculate the differences between the neighboring elements of a single-column time series or a vector.

### **Description**

Calculate the differences between the neighboring elements of a single-column *time series* or a *vector*.

diff\_vec 47

## Usage

```
diff_vec(tseries, lagg = 1L, pad_zeros = TRUE)
```

### **Arguments**

tseries A single-column time series or a vector.

lagg An *integer* equal to the number of time periods to lag when calculating the dif-

ferences (the default is lagg = 1).

pad\_zeros Boolean argument: Should the output vector be padded (extended) with ze-

ros, in order to return a vector of the same length as the input? (the default is

pad\_zeros = TRUE)

#### **Details**

The function diff\_vec() calculates the differences between the input *time series* or *vector* and its lagged version.

The argument lagg specifies the number of lags. For example, if lagg=3 then the differences will be taken between each element minus the element three time periods before it (in the past). The default is lagg = 1.

The argument pad\_zeros specifies whether the output *vector* should be padded (extended) with zeros at the front, in order to return a *vector* of the same length as the input. The default is pad\_zeros = TRUE. The padding operation can be time-consuming, because it requires the copying the data in memory.

The function diff\_vec() is implemented in RcppArmadillo C++ code, which makes it several times faster than R code.

## Value

A column *vector* containing the differences between the elements of the input vector.

```
## Not run:
# Create a vector of random returns
retp <- rnorm(1e6)
# Compare diff_vec() with rutils::diffit()
all.equal(drop(HighFreq::diff_vec(retp, lagg=3, pad=TRUE)),
    rutils::diffit(retp, lagg=3))
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
    Rcpp=HighFreq::diff_vec(retp, lagg=3, pad=TRUE),
    Rcode=rutils::diffit(retp, lagg=3),
    times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)</pre>
```

48 encode\_it

| encode_it | Calculate the run length encoding of a single-column time series, matrix, or a vector. |
|-----------|--|
|           |  |

# Description

Calculate the run length encoding of a single-column time series, matrix, or a vector.

### Usage

```
encode_it(tseries)
```

# **Arguments**

tseries

A single-column time series, matrix, or a vector.

## **Details**

The function encode\_it() calculates the run length encoding of a single-column *time series*, *matrix*, or a *vector*.

The run length encoding of a *vector* consists of two *vectors*: a *vector* of encoded data (consecutive data values) and of an *integer vector* of the data counts (the number of times the same value repeats in succession).

Run length encoding (RLE) is a data compression algorithm which encodes the data in two *vectors*: the consecutive data values and their counts. If a data value occurs several times in succession then it is recorded only once and its corresponding count is equal to the number of times it occurs. Run-length encoding is different from a contingency table.

### Value

A list with two vectors: a vector of encoded data and an integer vector of data counts (repeats).

```
## Not run:
# Create a vector of data
datav <- sample(5, 31, replace=TRUE)
# Calculate the run length encoding of datav
HighFreq::encode_it(datav)
## End(Not run)</pre>
```

hf\_data 49

hf\_data

High frequency data sets

# Description

hf\_data.RData is a file containing the datasets:

**SPY** an xts time series containing 1-minute OHLC bar data for the SPY etf, from 2008-01-02 to 2014-05-19. SPY contains 625,425 rows of data, each row contains a single minute bar.

**TLT** an xts time series containing 1-minute OHLC bar data for the TLT etf, up to 2014-05-19.

VXX an xts time series containing 1-minute OHLC bar data for the VXX etf, up to 2014-05-19.

# Usage

```
data(hf_data) # not required - data is lazy load
```

### **Format**

Each xts time series contains OHLC data, with each row containing a single minute bar:

Open Open price in the bar

**High** High price in the bar

Low Low price in the bar

Close Close price in the bar

Volume trading volume in the bar

#### **Source**

```
https://wrds-web.wharton.upenn.edu/wrds/
```

### References

Wharton Research Data Service (WRDS)

```
# data(hf_data) # not required - data is lazy load
head(SPY)
chart_Series(x=SPY["2009"])
```

50 lagit

| RcppArmadillo. |
|----------------|
|----------------|

## **Description**

Apply a lag to the rows of a time series or a matrix using RcppArmadillo.

### Usage

```
lagit(tseries, lagg = 1L, pad_zeros = TRUE)
```

## **Arguments**

tseries A time series or a matrix.

lagg An *integer* equal to the number of periods to lag (the default is lagg = 1).

pad\_zeros Boolean argument: Should the output be padded with zeros? (The default is

pad\_zeros = TRUE.)

#### **Details**

The function lagit() applies a lag to the input *matrix* by shifting its rows by the number equal to the argument lagg. For positive lagg values, the rows are shifted *forward* (down), and for negative lagg values they are shifted *backward* (up).

The output *matrix* is padded with either zeros (the default), or with rows of data from tseries, so that it has the same dimensions as tseries. If the lagg is positive, then the first row is copied and added upfront. If the lagg is negative, then the last row is copied and added to the end.

As a rule, if tseries contains returns data, then the output *matrix* should be padded with zeros, to avoid data snooping. If tseries contains prices, then the output *matrix* should be padded with the prices.

#### Value

A *matrix* with the same dimensions as the input argument tseries.

```
## Not run:
# Create a matrix of random returns
retp <- matrix(rnorm(5e6), nc=5)
# Compare lagit() with rutils::lagit()
all.equal(HighFreq::lagit(retp), rutils::lagit(retp))
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
    Rcpp=HighFreq::lagit(retp),
    Rcode=rutils::lagit(retp),
    times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)</pre>
```

lag\_vec 51

| lag_vec | Apply a lag to a single-column time series or a vector using |
|---------|--|
|         | RcppArmadillo.   |

## **Description**

Apply a lag to a single-column time series or a vector using RcppArmadillo.

## Usage

```
lag_vec(tseries, lagg = 1L, pad_zeros = TRUE)
```

### **Arguments**

tseries A single-column *time series* or a *vector*.

lagg An *integer* equal to the number of periods to lag. (The default is lagg = 1.)

pad\_zeros Boolean argument: Should the output be padded with zeros? (The default is

pad\_zeros = TRUE.)

## **Details**

The function lag\_vec() applies a lag to the input *time series* tseries by shifting its elements by the number equal to the argument lagg. For positive lagg values, the elements are shifted forward in time (down), and for negative lagg values they are shifted backward (up).

The output *vector* is padded with either zeros (the default), or with data from tseries, so that it has the same number of element as tseries. If the lagg is positive, then the first element is copied and added upfront. If the lagg is negative, then the last element is copied and added to the end.

As a rule, if tseries contains returns data, then the output *matrix* should be padded with zeros, to avoid data snooping. If tseries contains prices, then the output *matrix* should be padded with the prices.

## Value

A column *vector* with the same number of elements as the input time series.

```
## Not run:
# Create a vector of random returns
retp <- rnorm(1e6)
# Compare lag_vec() with rutils::lagit()
all.equal(drop(HighFreq::lag_vec(retp)),
    rutils::lagit(retp))
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
    Rcpp=HighFreq::lag_vec(retp),
    Rcode=rutils::lagit(retp),
    times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)</pre>
```

52 lik\_garch

| lik_garch | Calculate the log-likelihood of a time series of returns assuming a GARCH(1,1) process. |
|-----------|---|
|           | GARCH(1,1) process.   |

# Description

Calculate the log-likelihood of a time series of returns assuming a *GARCH*(1,1) process.

## Usage

```
lik_garch(omega, alpha, beta, returns, minval = 1e-06)
```

## **Arguments**

| omega   | Parameter proportional to the long-term average level of variance.                                 |
|---------|--|
| alpha   | The weight associated with recent realized variance updates.                                       |
| beta    | The weight associated with the past variance estimates.  |
| returns | A single-column <i>matrix</i> of returns.  |
| minval  | The floor value applied to the variance, to avoid zero values. (The default is minval = 0.000001.) |

### **Details**

The function  $lik_garch()$  calculates the log-likelihood of a time series of returns assuming a GARCH(1,1) process.

It first estimates the rolling variance of the returns argument using function sim\_garch():

$$\sigma_i^2 = \omega + \alpha r_i^2 + \beta \sigma_{i-1}^2$$

Where  $r_i$  is the time series of returns, and  $\sigma_i^2$  is the estimated rolling variance. And  $\omega$ ,  $\alpha$ , and  $\beta$  are the *GARCH* parameters. It applies the floor value minval to the variance, to avoid zero values. So the minimum value of the variance is equal to minval.

The function lik\_garch() calculates the log-likelihood assuming a normal distribution of returns conditional on the variance  $\sigma_{i-1}^2$  in the previous period, as follows:

$$likelihood = -\sum_{i=1}^{n} \left(\frac{r_i^2}{\sigma_{i-1}^2} + \log(\sigma_{i-1}^2)\right)$$

## Value

The log-likelihood value.

```
## Not run:
# Define the GARCH model parameters
alpha <- 0.79
betav <- 0.2
om_ega <- 1e-4*(1-alpha-betav)
# Calculate historical VTI returns</pre>
```

mult\_mat 53

```
retp <- na.omit(rutils::etfenv$returns$VTI)
# Calculate the log-likelihood of VTI returns assuming GARCH(1,1)
HighFreq::lik_garch(omega=om_ega, alpha=alpha, beta=betav, returns=retp)
## End(Not run)</pre>
```

mult\_mat

Multiply the rows or columns of a matrix times a vector, element-wise.

## **Description**

Multiply the rows or columns of a *matrix* times a *vector*, element-wise.

## Usage

```
mult_mat(vectorv, matrixv, byrow = TRUE)
```

### **Arguments**

vector A numeric vector.

matrix A numeric matrix.

byrow A Boolean argument: if TRUE then multiply the rows of matrix by vector,

otherwise multiply the columns (the default is byrow = TRUE.)

#### **Details**

The function mult\_mat() multiplies the rows or columns of a *matrix* times a *vector*, element-wise.

If byrow = TRUE (the default), then function mult\_mat() multiplies the rows of the argument matrix times the argument vector. Otherwise it multiplies the columns of matrix.

In R, *matrix* multiplication is performed by columns. Performing multiplication by rows is often required, for example when multiplying asset returns by portfolio weights. But performing multiplication by rows requires explicit loops in R, or it requires *matrix* transpose. And both are slow.

The function mult\_mat() uses RcppArmadillo C++ code, so when multiplying large *matrix* columns it's several times faster than vectorized R code, and it's even much faster compared to R when multiplying the *matrix* rows.

The function mult\_mat() performs loops over the *matrix* rows and columns using the Armadillo operators each\_row() and each\_col(), instead of performing explicit for() loops (both methods are equally fast).

## Value

A *matrix* equal to the product of the arguments matrix times vector, with the same dimensions as the argument matrix.

54 mult\_mat\_ref

### **Examples**

```
## Not run:
# Create vector and matrix data
matrixv <- matrix(round(runif(25e4), 2), nc=5e2)</pre>
vectorv <- round(runif(5e2), 2)</pre>
# Multiply the matrix rows using R
matrixr <- t(vectorv*t(matrixv))</pre>
# Multiply the matrix rows using C++
matrixp <- HighFreq::mult_mat(vectorv, matrixv, byrow=TRUE)</pre>
all.equal(matrixr, matrixp)
\# Compare the speed of Rcpp with R code
library(microbenchmark)
summary(microbenchmark(
    Rcpp=HighFreq::mult_mat(vectorv, matrixv, byrow=TRUE),
    Rcode=t(vectorv*t(matrixv)),
    times=10))[, c(1, 4, 5)] # end microbenchmark summary
\# Multiply the matrix columns using R
matrixr <- vectorv*matrixv</pre>
# Multiply the matrix columns using C++
matrixp <- HighFreq::mult_mat(vectorv, matrixv, byrow=FALSE)</pre>
all.equal(matrixr, matrixp)
\# Compare the speed of Rcpp with R code
library(microbenchmark)
summary(microbenchmark(
    Rcpp=HighFreq::mult_mat(vectorv, matrixv, byrow=FALSE),
    Rcode=vectorv*matrixv,
    times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

mult\_mat\_ref

Multiply the rows or columns of a matrix times a vector, element-wise and in place, without copying the data in memory.

# Description

Multiply the rows or columns of a *matrix* times a *vector*, element-wise and in place, without copying the data in memory.

# Usage

```
mult_mat_ref(vectorv, matrixv, byrow = TRUE)
```

# Arguments

vectorv A numeric vector.
matrixv A numeric matrix.

byrow A Boolean argument: if TRUE then multiply the rows of matrixv by vectorv,

otherwise multiply the columns (the default is byrow = TRUE.)

mult\_mat\_ref 55

#### **Details**

The function mult\_mat\_ref() multiplies the rows or columns of a *matrix* times a *vector*, elementwise and in place, without copying the data in memory.

It accepts a *pointer* to the argument matrixv, and it overwrites the old matrix values with the new values. It performs the calculation in place, without copying the *matrix* in memory, which can significantly increase the computation speed for large matrices.

If byrow = TRUE (the default), then function mult\_mat\_ref() multiplies the rows of the argument matrixv times the argument vectorv. Otherwise it multiplies the columns of matrixv.

In R, *matrix* multiplication is performed by columns. Performing multiplication by rows is often required, for example when multiplying asset returns by portfolio weights. But performing multiplication by rows requires explicit loops in R, or it requires *matrix* transpose. And both are slow.

The function mult\_mat\_ref() uses RcppArmadillo C++ code, so when multiplying large *matrix* columns it's several times faster than vectorized R code, and it's even much faster compared to R when multiplying the *matrix* rows.

The function mult\_mat\_ref() performs loops over the *matrix* rows and columns using the Armadillo operators each\_row() and each\_col(), instead of performing explicit for() loops (both methods are equally fast).

#### Value

Void (no return value - modifies the data in place).

```
## Not run:
# Create vector and matrix data
matrixv <- matrix(round(runif(25e4), 2), nc=5e2)</pre>
vectorv <- round(runif(5e2), 2)</pre>
# Multiply the matrix rows using R
matrixr <- t(vectorv*t(matrixv))</pre>
# Multiply the matrix rows using C++
HighFreq::mult_mat_ref(vectorv, matrixv, byrow=TRUE)
all.equal(matrixr, matrixv)
# Compare the speed of Rcpp with R code
library(microbenchmark)
summary(microbenchmark(
    Rcpp=HighFreq::mult_mat_ref(vectorv, matrixv, byrow=TRUE),
    Rcode=t(vectorv*t(matrixv)),
    times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Multiply the matrix columns using R
matrixr <- vectorv*matrixv
# Multiply the matrix columns using C++
HighFreq::mult_mat_ref(vectorv, matrixv, byrow=FALSE)
all.equal(matrixr, matrixv)
# Compare the speed of Rcpp with R code
library(microbenchmark)
summary(microbenchmark(
    Rcpp=HighFreq::mult_mat_ref(vectorv, matrixv, byrow=FALSE),
    Rcode=vectorv*matrixv,
    times=10))[, c(1, 4, 5)] # end microbenchmark summary
```

56 ohlc\_returns

## End(Not run)

| ohlc_returns | Calculate single period percentage returns from either TAQ or OHLC |
|--------------|--|
|              | prices.  |

# Description

Calculate single period percentage returns from either TAQ or OHLC prices.

## Usage

```
ohlc_returns(xtsv, lagg = 1, colnum = 4, scalit = TRUE)
```

## Arguments

| xtsv   | An xts time series of either TAQ or OHLC data.  |
|--------|---|
| lagg   | An integer equal to the number of time periods of lag. (default is 1)   |
| colnum | The column number to extract from the <i>OHLC</i> data. (default is 4, or the <i>Close</i> prices column)         |
| scalit | <i>Boolean</i> argument: should the returns be divided by the number of seconds in each period? (default is TRUE) |

### **Details**

The function ohlc\_returns() calculates the percentage returns for either *TAQ* or *OHLC* data, defined as the difference of log prices. Multi-period returns can be calculated by setting the lag parameter to values greater than 1 (the default).

If scalit is TRUE (the default), then the returns are divided by the differences of the time index (which scales the returns to units of returns per second.)

The time index of the xtsv time series is assumed to be in *POSIXct* format, so that its internal value is equal to the number of seconds that have elapsed since the *epoch*.

If scalit is TRUE (the default), then the returns are expressed in the scale of the time index of the xtsv time series. For example, if the time index is in seconds, then the returns are given in units of returns per second. If the time index is in days, then the returns are equal to the returns per day.

The function ohlc\_returns() identifies the xtsv time series as *TAQ* data when it has six columns, otherwise assumes it's *OHLC* data. By default, for *OHLC* data, it differences the *Close* prices, but can also difference other prices depending on the value of colnum.

#### Value

A single-column xts time series of returns.

ohlc\_sharpe 57

### **Examples**

```
# Calculate secondly returns from TAQ data
returns <- HighFreq::ohlc_returns(xtsv=HighFreq::SPY_TAQ)
# Calculate close to close returns
returns <- HighFreq::ohlc_returns(xtsv=HighFreq::SPY)
# Calculate open to open returns
returns <- HighFreq::ohlc_returns(xtsv=HighFreq::SPY, colnum=1)</pre>
```

ohlc\_sharpe

Calculate time series of point Sharpe-like statistics for each row of a OHLC time series.

## **Description**

Calculate time series of point Sharpe-like statistics for each row of a *OHLC* time series.

### Usage

```
ohlc_sharpe(ohlc, method = "close")
```

## **Arguments**

ohlc An *OHLC* time series of prices in *xts* format.

method A character string representing method for estimating the Sharpe-like exponent.

### **Details**

The function ohlc\_sharpe() calculates Sharpe-like statistics for each row of a *OHLC* time series. The Sharpe-like statistic is defined as the ratio of the difference between *Close* minus *Open* prices divided by the difference between *High* minus *Low* prices. This statistic may also be interpreted as something like a *Hurst exponent* for a single row of data. The motivation for the Sharpe-like statistic is the notion that if prices are trending in the same direction inside a given time bar of data, then this statistic is close to either 1 or -1.

## Value

An xts time series with the same number of rows as the argument ohlc.

```
# Calculate time series of running Sharpe ratios for SPY
sharpe_running <- ohlc_sharpe(HighFreq::SPY)</pre>
```

58 ohlc\_variance

| ohlc_skew | Calculate time series of point skew estimates from a OHLC time series, assuming zero drift. |
|-----------|---|
|           |   |

## **Description**

Calculate time series of point skew estimates from a *OHLC* time series, assuming zero drift.

### Usage

```
ohlc_skew(ohlc, method = "rogers_satchell")
```

## **Arguments**

ohlc An *OHLC* time series of prices in *xts* format.

method A *character* string representing method for estimating skew.

#### **Details**

The function ohlc\_skew() calculates a time series of skew estimates from *OHLC* prices, one for each row of *OHLC* data. The skew estimates are expressed in the time scale of the index of the *OHLC* time series. For example, if the time index is in seconds, then the skew is given in units of skew per second. If the time index is in days, then the skew is equal to the skew per day.

Currently only the "close" skew estimation method is correct (assuming zero drift), while the "rogers\_satchell" method produces a skew-like indicator, proportional to the skew. The default method is "rogers\_satchell".

### Value

A time series of point skew estimates.

### **Examples**

```
# Calculate time series of skew estimates for SPY
skew <- HighFreq::ohlc_skew(HighFreq::SPY)</pre>
```

ohlc\_variance

Calculate a time series of point estimates of variance for an OHLC time series, using different range estimators for variance.

# Description

Calculates the point variance estimates from individual rows of *OHLC* prices (rows of data), using the squared differences of *OHLC* prices at each point in time, without averaging them over time.

### Usage

```
ohlc_variance(ohlc, method = "yang_zhang", scalit = TRUE)
```

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## **Arguments**

ohlc

An OHLC time series of prices in xts format.

method

A *character* string representing the method for estimating variance. The methods include:

- "close" close to close,
- "garman\_klass" Garman-Klass,
- "garman\_klass\_yz" Garman-Klass with account for close-to-open price jumps,
- "rogers satchell" Rogers-Satchell,
- "yang\_zhang" Yang-Zhang,

(default is "yang\_zhang")

scalit

*Boolean* argument: should the returns be divided by the number of seconds in each period? (default is TRUE)

#### **Details**

The function ohlc\_variance() calculates a time series of point variance estimates of percentage returns, from *OHLC* prices, without averaging them over time. For example, the method "close" simply calculates the squares of the differences of the log *Close* prices.

The other methods calculate the squares of other possible differences of the log *OHLC* prices. This way the point variance estimates only depend on the price differences within individual rows of data (and possibly from the neighboring rows.) All the methods are implemented assuming zero drift, since the calculations are performed only for a single row of data, at a single point in time.

The user can choose from several different variance estimation methods. The methods "close", "garman\_klass\_yz", and "yang\_zhang" do account for close-to-open price jumps, while the methods "garman\_klass" and "rogers\_satchell" do not account for close-to-open price jumps. The default method is "yang\_zhang", which theoretically has the lowest standard error among unbiased estimators.

The point variance estimates can be passed into function roll\_vwap() to perform averaging, to calculate rolling variance estimates. This is appropriate only for the methods "garman\_klass" and "rogers\_satchell", since they don't require subtracting the rolling mean from the point variance estimates.

The point variance estimates can also be considered to be technical indicators, and can be used as inputs into trading models.

If scalit is TRUE (the default), then the variance is divided by the squared differences of the time index (which scales the variance to units of variance per second squared.) This is useful for example, when calculating intra-day variance from minutely bar data, because dividing returns by the number of seconds decreases the effect of overnight price jumps.

If scalit is TRUE (the default), then the variance is expressed in the scale of the time index of the *OHLC* time series. For example, if the time index is in seconds, then the variance is given in units of variance per second squared. If the time index is in days, then the variance is equal to the variance per day squared.

The time index of the ohlc time series is assumed to be in *POSIXct* format, so that its internal value is equal to the number of seconds that have elapsed since the *epoch*.

The function ohlc\_variance() performs similar calculations to the function volatility() from package TTR, but it assumes zero drift, and doesn't calculate a running sum using runSum(). It's also a little faster because it performs less data validation.

param\_portf

### Value

An xts time series with a single column and the same number of rows as the argument ohlc.

# **Examples**

```
# Create minutely OHLC time series of random prices
ohlc <- HighFreq::random_ohlc()
# Calculate variance estimates for ohlc
var_running <- HighFreq::ohlc_variance(ohlc)
# Calculate variance estimates for SPY
var_running <- HighFreq::ohlc_variance(HighFreq::SPY, method="yang_zhang")
# Calculate SPY variance without overnight jumps
var_running <- HighFreq::ohlc_variance(HighFreq::SPY, method="rogers_satchell")</pre>
```

param\_portf

Create a named list of model parameters that can be passed into portfolio optimization functions.

# Description

Create a named list of model parameters that can be passed into portfolio optimization functions.

# Usage

```
param_portf(
  method = "sharpem",
  eigen_thresh = 1e-05,
  dimax = 0L,
  confl = 0.1,
  alpha = 0,
  rankw = FALSE,
  centerw = FALSE,
  scalew = "voltarget",
  vol_target = 0.001
)
```

## **Arguments**

| method       | A <i>character string</i> specifying the method for calculating the portfolio weights (the default is method = "sharpem").   |
|--------------|--|
| eigen_thresh | A <i>numeric</i> threshold level for discarding small <i>singular values</i> in order to regularize the inverse of the covariance matrix of returns (the default is 1e-5).   |
| dimax        | An <i>integer</i> equal to the number of <i>singular values</i> used for calculating the <i>regularized inverse</i> of the covariance matrix of returns matrix (the default is dimax = 0 - standard matrix inverse using all the <i>singular values</i> ). |
| confl        | The confidence level for calculating the quantiles of returns (the default is $confl = 0.75$ ).  |
| alpha        | The shrinkage intensity of returns (with values between 0 and 1 - the default is $0$ ).  |

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| rankw      | A <i>Boolean</i> specifying whether the weights should be ranked (the default is rankw = FALSE).               |
|------------|--|
| centerw    | A <i>Boolean</i> specifying whether the weights should be centered (the default is centerw = FALSE).           |
| scalew     | A <i>character string</i> specifying the method for scaling the weights (the default is scalew = "voltarget"). |
| vol_target | A <i>numeric</i> volatility target for scaling the weights (the default is 0.001)                              |

#### **Details**

The function param\_portf() creates a named list of model parameters that can be passed into portfolio optimization functions. For example into the functions calc\_weights() and back\_test(). See the function calc\_weights() for more details.

The function param\_portf() simplifies the creation of portfolio optimization parameter lists. The users can create a parameter list with the default values, or they can specify custom parameter values.

### Value

A named list of model parameters that can be passed into portfolio optimization functions.

## **Examples**

```
## Not run:
# Create a default list of portfolio optimization parameters
controlv <- HighFreq::param_portf()
unlist(controlv)
# Create a custom list of portfolio optimization parameters
controlv <- HighFreq::param_portf(method="regular", dimax=4)
unlist(controlv)
## End(Not run)</pre>
```

param\_reg Create a named list of model parameters that can be passed into regression and machine learning functions.

## **Description**

Create a named list of model parameters that can be passed into regression and machine learning functions.

# Usage

```
param_reg(
  method = "least_squares",
  intercept = TRUE,
  eigen_thresh = 1e-05,
  dimax = 0L,
  confl = 0.1,
  alpha = 0
)
```

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# Arguments

| method       | A <i>character string</i> specifying the type of regression model (the default is method = "least_squares").  |
|--------------|---|
| intercept    | A <i>Boolean</i> specifying whether an intercept term should be added to the predictor (the default is intercept = TRUE).   |
| eigen_thresh | A <i>numeric</i> threshold level for discarding small <i>singular values</i> in order to regularize the inverse of the predictor matrix (the default is 1e-5).  |
| dimax        | An <i>integer</i> equal to the number of <i>singular values</i> used for calculating the <i>regularized inverse</i> of the predictor matrix (the default is $dimax = 0$ - standard matrix inverse using all the <i>singular values</i> ). |
| confl        | The confidence level for calculating the quantiles of returns (the default is $confl = 0.75$ ).   |
| alpha        | The shrinkage intensity of returns (with values between 0 and 1 - the default is 0).  |

#### **Details**

The function param\_reg() creates a named list of model parameters that can be passed into regression and machine learning functions. For example into the functions calc\_reg() and roll\_reg().

The function param\_reg() simplifies the creation of regression parameter lists. The users can create a parameter list with the default values, or they can specify custom parameter values.

#### Value

A named list of model parameters that can be passed into regression and machine learning functions.

## **Examples**

```
## Not run:
# Create a default list of regression parameters
controlv <- HighFreq::param_reg()
unlist(controlv)
# Create a custom list of regression parameters
controlv <- HighFreq::param_reg(intercept=FALSE, method="regular", dimax=4)
unlist(controlv)
## End(Not run)</pre>
```

push\_cov2cor

Calculate the correlation matrix from the covariance matrix.

## **Description**

Calculate the correlation matrix from the covariance matrix.

### Usage

```
push_cov2cor(covmat)
```

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## **Arguments**

covmat A *matrix* of covariances.

#### Details

The function push\_cov2cor() calculates the correlation matrix from the covariance matrix, in place, without copying the data in memory.

The function push\_cov2cor() accepts a *pointer* to the covariance matrix, and it overwrites it with the correlation matrix.

The function push\_cov2cor() is written in RcppArmadillo C++ so it's much faster than R code.

### Value

Void (no return value - modifies the covariance matrix in place).

## **Examples**

```
## Not run:
# Calculate a time series of returns
retp <- na.omit(rutils::etfenv$returns[, c("IEF", "VTI", "DBC")])
# Calculate the covariance matrix of returns
covmat <- cov(retp)
# Calculate the correlation matrix of returns
push_cov2cor(covmat)
all.equal(covmat, cor(retp))
## End(Not run)</pre>
```

push\_covar

Update the trailing covariance matrix of streaming asset returns, with a row of new returns.

### **Description**

Update the trailing covariance matrix of streaming asset returns, with a row of new returns.

### Usage

```
push_covar(retsn, covmat, meanv, lambdacov)
```

## **Arguments**

retsn A *vector* of new asset returns.

covmat A trailing covariance *matrix* of asset returns.

Mean A vector of trailing means of asset returns.

lambdacov A decay factor which multiplies the past covariance.

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#### **Details**

The function push\_covar() updates the trailing covariance matrix of streaming asset returns, with a row of new returns. It updates the covariance matrix in place, without copying the data in memory.

The streaming asset returns  $r_t$  contain multiple columns and the parameter retsn represents a single row of  $r_t$  - the asset returns at time t. The elements of the vectors retsn and mean represent single rows of data with multiple columns.

The function push\_covar() accepts *pointers* to the arguments covmat and meanv, and it overwrites the old values with the new values. It performs the calculation in place, without copying the data in memory, which can significantly increase the computation speed for large matrices.

First, the function push\_covar() updates the trailing means  $\bar{r}_t$  of the streaming asset returns  $r_t$  by recursively weighting present and past values using the decay factor  $\lambda$ :

$$\bar{r}_t = \lambda \bar{r}_{t-1} + (1 - \lambda)r_t$$

This recursive formula is equivalent to the exponentially weighted moving average of the streaming asset returns  $r_t$ .

It then calculates the demeaned returns:

$$\hat{r}_t = r_t - \bar{r}_t$$

Finally, it updates the trailing covariance matrix of the returns:

$$cov_t = \lambda cov_{t-1} + (1 - \lambda)\hat{r}_t^T \hat{r}_t$$

The decay factor  $\lambda$  determines the strength of the updates, with smaller  $\lambda$  values giving more weight to the new data. If the asset returns are not stationary, then applying more weight to the new returns reduces the bias of the trailing covariance matrix, but it also increases its variance. Simulation can be used to find the value of the  $\lambda$  parameter to achieve the best bias-variance tradeoff.

The function push\_covar() is written in RcppArmadillo C++ so it's much faster than R code.

## Value

Void (no return value - modifies the trailing covariance matrix and the return means in place).

```
## Not run:
# Calculate a time series of returns
retp <- na.omit(rutils::etfenv$returns[, c("IEF", "VTI", "DBC")])
# Calculate the returns without last row
nrows <- NROW(retp)
retss <- retp[-nrows]
# Calculate the covariance of returns
meanv <- colMeans(retss)
covmat <- cov(retss)
# Update the covariance of returns
HighFreq::push_covar(retsn=retp[nrows], covmat=covmat, meanv=meanv, lambdacov=0.9)
## End(Not run)</pre>
```

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| • | Ipdate the trailing eigen values and eigen vectors of streaming asset eturn data, with a row of new returns. |
|---|--|
|---|--|

### **Description**

Update the trailing eigen values and eigen vectors of streaming asset return data, with a row of new returns.

### Usage

push\_eigen(retsn, covmat, eigenval, eigenvec, eigenret, meanv, lambdacov)

### **Arguments**

retsn A *vector* of new asset returns.

covmat A trailing covariance *matrix* of asset returns.

eigenval A *vector* of eigen values.
eigenvec A *matrix* of eigen vectors.

eigenret A *vector* of eigen portfolio returns.

meanv A vector of trailing means of asset returns.

lambdacov A decay factor which multiplies the past covariance.

### **Details**

The function push\_eigen() updates the trailing eigen values, eigen vectors, and the eigen portfolio returns of streaming asset returns, with a row of new data. It updates the eigenelements in place, without copying the data in memory.

The streaming asset returns  $r_t$  contain multiple columns and the parameter retsn represents a single row of  $r_t$  - the asset returns at time t. The elements of the vectors retsn, eigenret, and meanv represent single rows of data with multiple columns.

The function push\_eigen() accepts *pointers* to the arguments eigenval, eigenval, eigenvec, meanv, and eigenret, and it overwrites the old values with the new values. It performs the calculation in place, without copying the data in memory, which can significantly increase the computation speed for large matrices.

First, the function push\_eigen() calls the function HighFreq::push\_covar() to update the trailing covariance matrix of streaming asset returns, with a row of new returns. It updates the covariance matrix in place, without copying the data in memory.

It then calls the Armadillo function arma::eig\_sym to calculate the eigen decomposition of the trailing covariance matrix.

The function push\_eigen() calculates the eigen portfolio returns by multiplying the scaled asset returns times the eigen vectors  $v_{t-1}$ :

$$r_t^{eigen} = v_{t-1} \frac{r_t}{\sigma_{t-1}}$$

Where  $v_{t-1}$  is the matrix of previous eigen vectors that are passed by reference through the parameter eigenvec. The eigen returns  $r_t^{eigen}$  are the returns of the eigen portfolios, with weights equal to

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the eigen vectors  $v_{t-1}$ . The eigen weights are applied to the asset returns scaled by their volatilities. The eigen returns  $r_t^{eigen}$  are passed by reference through the parameter eigenret.

The decay factor  $\lambda$  determines the strength of the updates, with smaller  $\lambda$  values giving more weight to the new data. If the asset returns are not stationary, then applying more weight to the new returns reduces the bias of the trailing covariance matrix, but it also increases its variance. Simulation can be used to find the value of the  $\lambda$  parameter to achieve the best bias-variance tradeoff.

The function push\_eigen() is written in RcppArmadillo C++ so it's much faster than R code.

### Value

Void (no return value - modifies the trailing eigen values, eigen vectors, the eigen portfolio returns, and the return means in place).

## **Examples**

push\_sga

Update the trailing eigen values and eigen vectors of streaming asset return data, with a row of new returns, using the SGA algorithm.

### Description

Update the trailing eigen values and eigen vectors of streaming asset return data, with a row of new returns, using the SGA algorithm.

### Usage

```
push_sga(retsn, eigenval, eigenvec, eigenret, meanv, varv, lambda, gamma)
```

## **Arguments**

retsn A vector of new asset returns. eigenval A vector of eigen values. eigenvec A matrix of eigen vectors. push\_sga 67

eigenret A *vector* of eigen portfolio returns.

meanv A *vector* of trailing means of asset returns.

varv A *vector* of the trailing asset variances.

lambda A decay factor which multiplies the past mean and variance.

gamma A *numeric* gain factor which multiplies the past eigenelements.

#### **Details**

The function push\_sga() updates the trailing eigen values, eigen vectors, and the eigen portfolio returns of streaming asset returns, with a row of new data, using the *SGA* algorithm. It updates the eigenelements in place, without copying the data in memory.

The streaming asset returns  $r_t$  contain multiple columns and the parameter retsn represents a single row of  $r_t$  - the asset returns at time t. The elements of the vectors retsn, meany, and vary represent single rows of data with multiple columns.

The function push\_sga() accepts *pointers* to the arguments eigenval, eigenvec, meanv, and varv, and it overwrites the old values with the new values. It performs the calculation in place, without copying the data in memory, which can significantly increase the computation speed for large matrices.

First, the function push\_sga() updates the trailing means  $\bar{r}_t$  and variances  $\sigma_t^2$  of the streaming asset returns  $r_t$  by recursively weighting present and past values using the decay factor  $\lambda$ :

$$\bar{r}_t = \lambda \bar{r}_{t-1} + (1 - \lambda)r_t$$
$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda)(r_t - \bar{r}_t)^2$$

The past values  $\bar{r}_{t-1}$  and  $\sigma_{t-1}^2$  are passed in by reference through the variables means and varv. The updated values are then passed out by reference.

These recursive formulas are equivalent to the exponentially weighted moving averages of the streaming asset returns  $r_t$ .

It then calculates a vector of the eigen portfolio returns:

$$r_t^{eigen} = v_{t-1} \frac{r_t}{\sigma_{t-1}}$$

Where  $v_{t-1}$  is the matrix of previous eigen vectors that are passed by reference through the parameter eigenvec. The eigen returns  $r_t^{eigen}$  are the returns of the eigen portfolios, with weights equal to the eigen vectors  $v_{t-1}$ . The eigen weights are applied to the asset returns scaled by their volatilities. The eigen returns  $r_t^{eigen}$  are passed by reference through the parameter eigenret.

The function push\_sga() then standardizes the columns of the new returns:

$$\hat{r}_t = \frac{r_t - \bar{r}_t}{\sigma_t}$$

Finally, the vector of eigen values  $\Lambda_{j,t}$  and the matrix of eigen vectors  $v_{j,t}$  (j is the column index) are then updated using the SGA algorithm:

$$\Lambda_{j,t} = (1 - \gamma)\Lambda_{j,t-1} + \gamma\phi_{j,t-1}$$

$$v_{j,t} = v_{j,t-1} + \gamma \phi_{j,t-1}(\hat{r}_t - \phi_{j,t-1}v_{j,t-1} - 2\sum_{i=1}^{j-1} \phi_{i,t-1}v_{i,t-1})$$

Where  $\phi_{j,t-1} = \hat{r}_t v_{j,t-1}$  are the matrix products of the new data times the previous eigen vectors.

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The gain factor  $\gamma$  determines the strength of the updates, with larger  $\gamma$  values giving more weight to the new data. If the asset returns are not stationary, then applying more weight to the new returns reduces the bias of the trailing eigen vectors, but it also increases their variance. Simulation can be used to find the value of the  $\gamma$  parameter to achieve the best bias-variance tradeoff.

A description of the SGA algorithm can be found in the package onlinePCA and in the Online PCA paper.

The function push\_sga() is written in RcppArmadillo C++ code and it calls the Armadillo function arma::qr\_econ() to perform the QR decomposition, to calculate the eigen vectors.

### Value

Void (no return value - modifies the trailing eigen values, eigen vectors, the return means, and the return variances in place).

## **Examples**

```
## Not run:
# Calculate a time series of returns
retp <- na.omit(rutils::etfenv$returns[, c("IEF", "VTI", "DBC")])</pre>
# Calculate the covariance of returns without the last row
nrows <- NROW(retp)</pre>
retss <- retp[-nrows]</pre>
HighFreq::calc_scale(retss)
meanv <- colMeans(retss)</pre>
varv <- sapply(retss, var)</pre>
covmat <- cov(retss)</pre>
ncols <- NCOL(retss)</pre>
# Calculate the eigen decomposition using RcppArmadillo
eigenval <- numeric(ncols) # Allocate eigen values</pre>
eigenvec <- matrix(numeric(ncols^2), nc=ncols) # Allocate eigen vectors</pre>
HighFreq::calc_eigen(covmat, eigenval, eigenvec)
# Update the eigen decomposition using SGA
eigenret <- numeric(NCOL(retp))</pre>
HighFreq::push_sga(retsn=retp[nrows],
  eigenval=eigenval, eigenvec=eigenvec,
  eigenret=eigenret, meanv=meanv, varv=varv, lambda=0.9, gamma=0.1)
## End(Not run)
```

random\_ohlc

Calculate a random OHLC time series of prices and trading volumes, in xts format.

# **Description**

Calculate a random *OHLC* time series either by simulating random prices following geometric Brownian motion, or by randomly sampling from an input time series.

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### Usage

```
random_ohlc(
  ohlc = NULL,
  reducit = TRUE,
  volat = 6.5e-05,
  drift = 0,
  indeks = seq(from = as.POSIXct(paste(Sys.Date() - 3, "09:30:00")), to =
    as.POSIXct(paste(Sys.Date() - 1, "16:00:00")), by = "1 sec"),
  ...
)
```

### **Arguments**

| ohlc    | An <i>OHLC</i> time series of prices and trading volumes, in <i>xts</i> format (default is <i>NULL</i> ).            |
|---------|--|
| volat   | The volatility per period of the indeks time index (default is $6.5e-05$ per second, or about $0.01=1.0\%$ per day). |
| drift   | The drift per period of the indeks time index (default is 0.0).  |
| indeks  | The time index for the <i>OHLC</i> time series.  |
| reducit | Boolean argument: should ohlc time series be transformed to reduced form? (default is TRUE)                          |

# **Details**

If the input ohlc time series is *NULL* (the default), then the function random\_ohlc() simulates a minutely *OHLC* time series of random prices following geometric Brownian motion, over the two previous calendar days.

If the input ohlc time series is not *NULL*, then the rows of ohlc are randomly sampled, to produce a random time series.

If reducit is TRUE (the default), then the ohlc time series is first transformed to reduced form, then randomly sampled, and finally converted to standard form.

Note: randomly sampling from an intraday time series over multiple days will cause the overnight price jumps to be re-arranged into intraday price jumps. This will cause moment estimates to become inflated compared to the original time series.

#### Value

An xts time series with the same dimensions and the same time index as the input ohlc time series.

```
# Create minutely synthetic OHLC time series of random prices
ohlc <- HighFreq::random_ohlc()
# Create random time series from SPY by randomly sampling it
ohlc <- HighFreq::random_ohlc(ohlc=HighFreq::SPY["2012-02-13/2012-02-15"])</pre>
```

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| random_taq | Calculate a random TAQ time series of prices and trading volumes, in xts format. |
|------------|--|
|            |  |

## **Description**

Calculate a TAQ time series of random prices following geometric Brownian motion, combined with random trading volumes.

## Usage

```
random_taq(
  volat = 6.5e-05,
  drift = 0,
  indeks = seq(from = as.POSIXct(paste(Sys.Date() - 3, "09:30:00")), to =
    as.POSIXct(paste(Sys.Date() - 1, "16:00:00")), by = "1 sec"),
  bid_offer = 0.001,
  ...
)
```

### **Arguments**

| bid_offer | The bid-offer spread expressed as a fraction of the prices (default is 0.001=10bps).                            |
|-----------|---|
| volat     | The volatility per period of the indeks time index (default is 6.5e-05 per second, or about 0.01=1.0% per day). |
| drift     | The drift per period of the indeks time index (default is 0.0).   |
| indeks    | The time index for the <i>TAQ</i> time series.  |

### **Details**

The function random\_taq() calculates an *xts* time series with four columns containing random prices following geometric Brownian motion: the bid, ask, and trade prices, combined with random trade volume data. If indeks isn't supplied as an argument, then by default it's equal to the secondly index over the two previous calendar days.

## Value

An *xts* time series, with time index equal to the input indeks time index, and with four columns containing the bid, ask, and trade prices, and the trade volume.

```
# Create secondly TAQ time series of random prices
taq <- HighFreq::random_taq()
# Create random TAQ time series from SPY index
taq <- HighFreq::random_taq(indeks=index(HighFreq::SPY["2012-02-13/2012-02-15"]))</pre>
```

remove\_jumps 71

| remove_jumps | Remove overnight close-to-open price jumps from an OHLC time se- |
|--------------|--|
|              | ries, by adding adjustment terms to its prices.                  |

# Description

Remove overnight close-to-open price jumps from an *OHLC* time series, by adding adjustment terms to its prices.

## Usage

```
remove_jumps(ohlc)
```

## **Arguments**

ohlc

An *OHLC* time series of prices and trading volumes, in xts format.

### **Details**

The function remove\_jumps() removes the overnight close-to-open price jumps from an *OHLC* time series, by adjusting its prices so that the first *Open* price of the day is equal to the last *Close* price of the previous day.

The function remove\_jumps() adds adjustment terms to all the *OHLC* prices, so that intra-day returns and volatilities are not affected.

The function remove\_jumps() identifies overnight periods as those that are greater than 60 seconds. This assumes that intra-day periods between neighboring rows of data are 60 seconds or less.

The time index of the ohlc time series is assumed to be in *POSIXct* format, so that its internal value is equal to the number of seconds that have elapsed since the *epoch*.

### Value

An *OHLC* time series with the same dimensions and the same time index as the input ohlc time series.

```
# Remove overnight close-to-open price jumps from SPY data
ohlc <- remove_jumps(HighFreq::SPY)</pre>
```

72 roll\_apply

| roll_apply | Apply an aggregation function over a rolling look-back interval and the end points of an OHLC time series, using R code. |
|------------|--|
|            |  |

## **Description**

Apply an aggregation function over a rolling look-back interval and the end points of an *OHLC* time series, using R code.

## Usage

```
roll_apply(
   xtsv,
   agg_fun,
   look_back = 2,
   endpoints = seq_along(xtsv),
   by_columns = FALSE,
   out_xts = TRUE,
   ...
)
```

### **Arguments**

|            | additional parameters to the function agg_fun.  |
|------------|---|
| xtsv       | An OHLC time series of prices and trading volumes, in xts format.   |
| agg_fun    | The name of the aggregation function to be applied over a rolling look-back interval.   |
| look_back  | The number of end points in the look-back interval used for applying the aggregation function (including the current row).  |
| by_columns | <i>Boolean</i> argument: should the function agg_fun() be applied column-wise (individually), or should it be applied to all the columns combined? (default is FALSE) |
| out_xts    | Boolean argument: should the output be coerced into an $xts$ series? (default is TRUE)  |
| endpoints  | An integer vector of end points.  |

### **Details**

The function roll\_apply() applies an aggregation function over a rolling look-back interval attached at the end points of an *OHLC* time series.

The function roll\_apply() is implemented in R code.

HighFreq::roll\_apply() performs similar operations to the functions rollapply() and period.apply() from package xts, and also the function apply.rolling() from package PerformanceAnalytics. (The function rollapply() isn't exported from the package xts.)

But HighFreq::roll\_apply() is faster because it performs less type-checking and skips other overhead. Unlike the other functions, roll\_apply() doesn't produce any leading *NA* values.

The function roll\_apply() can be called in two different ways, depending on the argument endpoints. If the argument endpoints isn't explicitly passed to roll\_apply(), then the default value is used, and roll\_apply() performs aggregations over overlapping intervals at each point in time.

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If the argument endpoints is explicitly passed to roll\_apply(), then roll\_apply() performs aggregations over intervals attached at the endpoints. If look\_back=2 then the aggregations are performed over non-overlapping intervals, otherwise they are performed over overlapping intervals.

If the argument out\_xts is TRUE (the default) then the output is coerced into an *xts* series, with the number of rows equal to the length of argument endpoints. Otherwise a list is returned, with the length equal to the length of argument endpoints.

If out\_xts is TRUE and the aggregation function agg\_fun() returns a single value, then roll\_apply() returns an xts time series with a single column. If out\_xts is TRUE and if agg\_fun() returns a vector of values, then roll\_apply() returns an xts time series with multiple columns, equal to the length of the vector returned by the aggregation function agg\_fun().

#### Value

Either an *xts* time series with the number of rows equal to the length of argument endpoints, or a list the length of argument endpoints.

### **Examples**

```
# extract a single day of SPY data
ohlc <- HighFreq::SPY["2012-02-13"]
interval <- 11 # number of data points between end points
look_back <- 4 # number of end points in look-back interval</pre>
# Calculate the rolling sums of ohlc columns over a rolling look-back interval
agg_regations <- roll_apply(ohlc, agg_fun=sum, look_back=look_back, by_columns=TRUE)
# Apply a vector-valued aggregation function over a rolling look-back interval
agg_function <- function(ohlc) c(max(ohlc[, 2]), min(ohlc[, 3]))</pre>
agg_regations <- roll_apply(ohlc, agg_fun=agg_function, look_back=look_back)</pre>
# Define end points at 11-minute intervals (HighFreq::SPY is minutely bars)
endpoints <- rutils::endpoints(ohlc, interval=interval)</pre>
# Calculate the sums of ohlc columns over endpoints using non-overlapping intervals
agg_regations <- roll_apply(ohlc, agg_fun=sum, endpoints=endpoints, by_columns=TRUE)
# Apply a vector-valued aggregation function over the endpoints of ohlc
# using overlapping intervals
agg_regations <- roll_apply(ohlc, agg_fun=agg_function,</pre>
                             look_back=5, endpoints=endpoints)
```

roll\_backtest

Perform a backtest simulation of a trading strategy (model) over a vector of end points along a time series of prices.

# Description

Perform a backtest simulation of a trading strategy (model) over a vector of end points along a time series of prices.

### Usage

```
roll_backtest(
  xtsv,
  train_func,
  trade_func,
```

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```
look_back = look_forward,
look_forward,
endpoints = rutils::calc_endpoints(xtsv, look_forward),
...
)
```

### **Arguments**

|              | additional parameters to the functions train_func() and trade_func().   |
|--------------|---|
| xtsv         | A time series of prices, asset returns, trading volumes, and other data, in <i>xts</i> format.                            |
| train_func   | The name of the function for training (calibrating) a forecasting model, to be applied over a rolling look-back interval. |
| trade_func   | The name of the trading model function, to be applied over a rolling look-forward interval.                               |
| look_back    | The size of the look-back interval, equal to the number of rows of data used for training the forecasting model.          |
| look_forward | The size of the look-forward interval, equal to the number of rows of data used for trading the strategy.                 |
| endpoints    | A vector of end points along the rows of the xtsv time series, given as either integers or dates.                         |

### **Details**

The function roll\_backtest() performs a rolling backtest simulation of a trading strategy over a vector of end points. At each end point, it trains (calibrates) a forecasting model using past data taken from the xtsv time series over the look-back interval, and applies the forecasts to the trade\_func() trading model, using out-of-sample future data from the look-forward interval.

The function trade\_func() should simulate the trading model, and it should return a named list with at least two elements: a named vector of performance statistics, and an xts time series of out-of-sample returns. The list returned by trade\_func() can also have additional elements, like the in-sample calibrated model statistics, etc.

The function roll\_backtest() returns a named list containing the listv returned by function trade\_func(). The list names are equal to the *endpoints* dates. The number of list elements is equal to the number of *endpoints* minus two (because the first and last end points can't be included in the backtest).

### Value

An xts time series with the number of rows equal to the number of end points minus two.

```
## Not run:
# Combine two time series of prices
prices <- cbind(rutils::etfenv$XLU, rutils::etfenv$XLP)
look_back <- 252
look_forward <- 22
# Define end points
endpoints <- rutils::calc_endpoints(prices, look_forward)
# Perform back-test
back_test <- roll_backtest(endpoints=endpoints,</pre>
```

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```
look_forward=look_forward,
look_back=look_back,
train_func = train_model,
trade_func = trade_model,
model_params = model_params,
trading_params = trading_params,
xtsv=prices)
```

roll\_conv

Calculate the rolling convolutions (weighted sums) of a time series with a single-column matrix of weights.

### Description

Calculate the rolling convolutions (weighted sums) of a *time series* with a single-column *matrix* of weights.

### Usage

```
roll_conv(tseries, weightv)
```

### **Arguments**

tseries A *time series* or a *matrix* of data.

weightv A single-column *matrix* of weights.

## **Details**

The function roll\_conv() calculates the convolutions of the *matrix* columns with a single-column *matrix* of weights. It performs a loop over the *matrix* rows and multiplies the past (higher) values by the weights. It calculates the rolling weighted sums of the past data.

The function roll\_conv() uses the Armadillo function arma::conv2(). It performs a similar calculation to the standard R function

filter(x=tseries, filter=weightv, method="convolution", sides=1), but it's over 6 times faster, and it doesn't produce any leading NA values.

#### Value

A *matrix* with the same dimensions as the input argument tseries.

```
## Not run:
# First example
# Calculate a time series of returns
retp <- na.omit(rutils::etfenv$returns[, c("IEF", "VTI")])
# Create simple weights equal to a 1 value plus zeros
weightv <- c(1, rep(0, 10))
# Calculate rolling weighted sums
weighted <- HighFreq::roll_conv(retp, weightv)</pre>
```

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```
# Compare with original
all.equal(coredata(retp), weighted, check.attributes=FALSE)
# Second example
# Calculate exponentially decaying weights
weightv <- exp(-0.2*(1:11))
weightv <- weightv/sum(weightv)
# Calculate rolling weighted sums
weighted <- HighFreq::roll_conv(retp, weightv)
# Calculate rolling weighted sums using filter()
filtered <- filter(x=retp, filter=weightv, method="convolution", sides=1)
# Compare both methods
all.equal(filtered[-(1:11), ], weighted[-(1:11), ], check.attributes=FALSE)
## End(Not run)</pre>
```

roll\_count

Count the number of consecutive TRUE elements in a Boolean vector, and reset the count to zero after every FALSE element.

### **Description**

Count the number of consecutive TRUE elements in a Boolean vector, and reset the count to zero after every FALSE element.

### Usage

```
roll_count(tseries)
```

### **Arguments**

tseries

A Boolean vector of data.

#### **Details**

The function roll\_count() calculates the number of consecutive TRUE elements in a Boolean vector, and it resets the count to zero after every FALSE element.

For example, the Boolean vector FALSE, TRUE, TRUE, TRUE, FALSE, FALSE, TRUE, TRUE,

## Value

An *integer vector* of the same length as the argument tseries.

```
## Not run:
# Calculate the number of consecutive TRUE elements
drop(HighFreq::roll_count(c(FALSE, TRUE, TRUE, FALSE, FALSE, TRUE, TR
```

roll\_hurst 77

| roll_hurst | Calculate a time series of Hurst exponents over a rolling look-back interval. |
|------------|---|
|            |   |

## **Description**

Calculate a time series of *Hurst* exponents over a rolling look-back interval.

### Usage

```
roll_hurst(ohlc, look_back = 11)
```

### **Arguments**

ohlc An *OHLC* time series of prices in *xts* format.

look\_back The size of the look-back interval, equal to the number of rows of data used for

aggregating the OHLC prices.

#### **Details**

The function roll\_hurst() calculates a time series of *Hurst* exponents from *OHLC* prices, over a rolling look-back interval.

The *Hurst* exponent is defined as the logarithm of the ratio of the price range, divided by the standard deviation of returns, and divided by the logarithm of the interval length.

The function roll\_hurst() doesn't use the same definition as the rescaled range definition of the *Hurst* exponent. First, because the price range is calculated using *High* and *Low* prices, which produces bigger range values, and higher *Hurst* exponent estimates. Second, because the *Hurst* exponent is estimated using a single aggregation interval, instead of multiple intervals in the rescaled range definition.

The rationale for using a different definition of the *Hurst* exponent is that it's designed to be a technical indicator for use as input into trading models, rather than an estimator for statistical analysis.

#### Value

An xts time series with a single column and the same number of rows as the argument ohlc.

```
# Calculate rolling Hurst for SPY in March 2009
hurst_rolling <- roll_hurst(ohlc=HighFreq::SPY["2009-03"], look_back=11)
chart_Series(hurst_rolling["2009-03-10/2009-03-12"], name="SPY hurst_rolling")</pre>
```

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| roll_kurtosis | Calculate a matrix of kurtosis estimates over a rolling look-back interval attached at the end points of a time series or a matrix. |
|---------------|---|
|               | var attached at the cha points of a time series of a madrix.  |

### **Description**

Calculate a *matrix* of kurtosis estimates over a rolling look-back interval attached at the end points of a *time series* or a *matrix*.

# Usage

```
roll_kurtosis(
  tseries,
  startp = 0L,
  endp = 0L,
  step = 1L,
  look_back = 1L,
  stub = 0L,
  method = "moment",
  confl = 0.75
)
```

### **Arguments**

| tseries   | A time series or a matrix of data.  |
|-----------|---|
| startp    | An <i>integer</i> vector of start points (the default is $startp = 0$ ).  |
| endp      | An <i>integer</i> vector of end points (the default is endp = $\emptyset$ ).  |
| step      | The number of time periods between the end points (the default is step = 1).  |
| look_back | The number of end points in the look-back interval (the default is look_back = 1).                                      |
| stub      | An <i>integer</i> value equal to the first end point for calculating the end points (the default is $stub = 0$ ).       |
| method    | A <i>character string</i> specifying the type of the kurtosis model (the default is $method = "moment" - see Details).$ |
| confl     | The confidence level for calculating the quantiles of returns (the default is $confl = 0.75$ ).                         |

### **Details**

The function roll\_kurtosis() calculates a *matrix* of kurtosis estimates over rolling look-back intervals attached at the end points of the *time series* tseries.

The function roll\_kurtosis() performs a loop over the end points, and at each end point it subsets the time series over a look-back interval equal to look\_back number of end points.

It passes the subset time series to the function calc\_kurtosis(), which calculates the kurtosis. See the function calc\_kurtosis() for a description of the kurtosis methods.

If the arguments endp and startp are not given then it first calculates a vector of end points separated by step time periods. It calculates the end points along the rows of tseries using the function

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calc\_endpoints(), with the number of time periods between the end points equal to step time periods.

For example, the rolling kurtosis at 25 day end points, with a 75 day look-back, can be calculated using the parameters step = 25 and look\_back = 3.

The function roll\_kurtosis() is implemented in RcppArmadillo C++ code, which makes it several times faster than R code.

#### Value

A *matrix* of kurtosis estimates with the same number of columns as the input time series tseries, and the number of rows equal to the number of end points.

### **Examples**

```
## Not run:
# Define time series of returns using package rutils
retp <- na.omit(rutils::etfenv$returns$VTI)</pre>
# Define end points and start points
endp <- 1 + HighFreq::calc_endpoints(NROW(retp), step=25)</pre>
startp <- HighFreq::calc_startpoints(endp, look_back=3)</pre>
# Calculate the rolling kurtosis at 25 day end points, with a 75 day look-back
kurtosisv <- HighFreq::roll_kurtosis(retp, step=25, look_back=3)</pre>
\# Calculate the rolling kurtosis using R code
kurt_r <- sapply(1:NROW(endp), function(it) {</pre>
 HighFreq::calc_kurtosis(retp[startp[it]:endp[it], ])
}) # end sapply
# Compare the kurtosis estimates
all.equal(drop(kurtosisv), kurt_r, check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::roll_kurtosis(retp, step=25, look_back=3),
  Rcode=sapply(1:NROW(endp), function(it) {
    HighFreq::calc_kurtosis(retp[startp[it]:endp[it], ])
  }),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

roll\_mean

Calculate a matrix of mean (location) estimates over a rolling look-back interval attached at the end points of a time series or a matrix.

# Description

Calculate a *matrix* of mean (location) estimates over a rolling look-back interval attached at the end points of a *time series* or a *matrix*.

## Usage

```
roll_mean(
  tseries,
  look_back = 1L,
```

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```
startp = 0L,
endp = 0L,
step = 1L,
stub = 0L,
method = "moment",
confl = 0.75
)
```

#### **Arguments**

look back The number of end points in the look-back interval (the default is look\_back = 1). A time series or a matrix of data. tseries An *integer* vector of start points (the default is startp = 0). startp An *integer* vector of end points (the default is endp = 0). endp The number of time periods between the end points (the default is step = 1). step An integer value equal to the first end point for calculating the end points (the stub default is stub = 0). method A character string representing the type of mean measure of (the default is method = "moment").

#### **Details**

The function roll\_mean() calculates a *matrix* of mean (location) estimates over rolling look-back intervals attached at the end points of the *time series* tseries.

The function roll\_mean() performs a loop over the end points, and at each end point it subsets the time series tseries over a look-back interval equal to look\_back number of end points.

It passes the subset time series to the function calc\_mean(), which calculates the mean (location). See the function calc\_mean() for a description of the mean methods.

If the arguments endp and startp are not given then it first calculates a vector of end points separated by step time periods. It calculates the end points along the rows of tseries using the function calc\_endpoints(), with the number of time periods between the end points equal to step time periods.

For example, the rolling mean at 25 day end points, with a 75 day look-back, can be calculated using the parameters step = 25 and  $look_back = 3$ .

The function roll\_mean() with the parameter step = 1 performs the same calculation as the function roll\_mean() from package RcppRoll, but it's several times faster because it uses C++ RcppArmadillo code.

The function roll\_mean() is implemented in RcppArmadillo RcppArmadillo C++ code, which makes it several times faster than R code.

If only a simple rolling mean is required (not the median) then other functions like roll\_sum() or roll\_vec() may be even faster.

### Value

A *matrix* of mean (location) estimates with the same number of columns as the input time series tseries, and the number of rows equal to the number of end points.

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#### **Examples**

```
## Not run:
# Define time series of returns using package rutils
retp <- na.omit(rutils::etfenv$returns$VTI)</pre>
# Calculate the rolling means at 25 day end points, with a 75 day look-back
meanv <- HighFreq::roll_mean(retp, look_back=3, step=25)</pre>
# Compare the mean estimates over 11-period look-back intervals
all.equal(HighFreq::roll_mean(retp, look_back=11)[-(1:10), ],
  drop(RcppRoll::roll_mean(retp, n=11)), check.attributes=FALSE)
# Define end points and start points
endp <- HighFreq::calc_endpoints(NROW(retp), step=25)</pre>
startp <- HighFreq::calc_startpoints(endp, look_back=3)</pre>
# Calculate the rolling means using RcppArmadillo
meanv <- HighFreq::roll_mean(retp, startp=startp, endp=endp)</pre>
# Calculate the rolling medians using RcppArmadillo
medianscpp <- HighFreq::roll_mean(retp, startp=startp, endp=endp, method="nonparametric")</pre>
# Calculate the rolling medians using R
medians = sapply(1:NROW(endp), function(i) {
 median(retp[startp[i]:endp[i] + 1])
}) # end sapply
all.equal(medians, drop(medianscpp))
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  \label{lem:condition} {\tt Rcpp=HighFreq::roll\_mean(retp, startp=startp, endp=endp, method="nonparametric"),}
  Rcode=sapply(1:NROW(endp), function(i) {median(retp[startp[i]:endp[i] + 1])}),
  times=10)[, c(1, 4, 5)]
## End(Not run)
```

roll\_moment

Calculate a matrix of moment values over a rolling look-back interval attached at the end points of a time series or a matrix.

### **Description**

Calculate a *matrix* of moment values over a rolling look-back interval attached at the end points of a *time series* or a *matrix*.

### Usage

```
roll_moment(
   tseries,
   funame = "calc_mean",
   method = "moment",
   confl = 0.75,
   startp = 0L,
   endp = 0L,
   step = 1L,
   look_back = 1L,
   stub = 0L
)
```

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#### **Arguments**

tseries A time series or a matrix of data. A character string specifying the moment function (the default is funame = funame "calc\_mean"). method A character string specifying the type of the model for the moment (the default is method = "moment"). conf1 The confidence level for calculating the quantiles of returns (the default is confl = 0.75). An *integer* vector of start points (the default is startp = 0). startp endp An *integer* vector of end points (the default is endp = 0). step The number of time periods between the end points (the default is step = 1). The number of end points in the look-back interval (the default is look\_back = look\_back stub An integer value equal to the first end point for calculating the end points (the default is stub = 0).

#### **Details**

The function roll\_moment() calculates a *matrix* of moment values, over rolling look-back intervals attached at the end points of the *time series* tseries.

The function roll\_moment() performs a loop over the end points, and at each end point it subsets the time series tseries over a look-back interval equal to look\_back number of end points.

It passes the subset time series to the function specified by the argument funame, which calculates the statistic. See the functions calc\_\*() for a description of the different moments. The function name must be one of the following:

- "calc mean" for the estimator of the mean (location),
- "calc\_var" for the estimator of the dispersion (variance),
- "calc\_skew" for the estimator of the skewness,
- "calc\_kurtosis" for the estimator of the kurtosis.

(The default is the funame = "calc\_mean").

If the arguments endp and startp are not given then it first calculates a vector of end points separated by step time periods. It calculates the end points along the rows of tseries using the function calc\_endpoints(), with the number of time periods between the end points equal to step time periods.

For example, the rolling variance at 25 day end points, with a 75 day look-back, can be calculated using the parameters step = 25 and look\_back = 3.

The function roll\_moment() calls the function calc\_momptr() to calculate a pointer to a moment function from the function name funame (string). The function pointer is used internally in the C++ code, but the function calc\_momptr() is not exported to R.

The function roll\_moment() is implemented in RcppArmadillo C++ code, which makes it several times faster than R code.

### Value

A *matrix* with the same number of columns as the input time series tseries, and the number of rows equal to the number of end points.

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#### **Examples**

```
## Not run:
# Define time series of returns using package rutils
retp <- na.omit(rutils::etfenv$returns$VTI)</pre>
# Calculate the rolling variance at 25 day end points, with a 75 day look-back
var_rollfun <- HighFreq::roll_moment(retp, fun="calc_var", step=25, look_back=3)</pre>
# Calculate the rolling variance using roll_var()
var_roll <- HighFreq::roll_var(retp, step=25, look_back=3)</pre>
# Compare the two methods
all.equal(var_rollfun, var_roll, check.attributes=FALSE)
# Define end points and start points
endp <- HighFreq::calc_endpoints(NROW(retp), step=25)</pre>
startp <- HighFreq::calc_startpoints(endp, look_back=3)</pre>
# Calculate the rolling variance using RcppArmadillo
var_rollfun <- HighFreq::roll_moment(retp, fun="calc_var", startp=startp, endp=endp)</pre>
# Calculate the rolling variance using R code
var_roll <- sapply(1:NROW(endp), function(it) {</pre>
 var(retp[startp[it]:endp[it]+1, ])
}) # end sapply
var_roll[1] <- 0</pre>
# Compare the two methods
all.equal(drop(var_rollfun), var_roll, check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::roll_moment(retp, fun="calc_var", startp=startp, endp=endp),
  Rcode=sapply(1:NROW(endp), function(it) {
    var(retp[startp[it]:endp[it]+1, ])
  }),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

roll\_ohlc

Aggregate a time series to an OHLC time series with lower periodicity.

### **Description**

Given a time series of prices at a higher periodicity (say seconds), it calculates the *OHLC* prices at a lower periodicity (say minutes).

### Usage

```
roll_ohlc(tseries, endp)
```

#### **Arguments**

tseries A *time series* or a *matrix* with multiple columns of data.

endp An *integer vector* of end points.

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#### **Details**

The function roll\_ohlc() performs a loop over the end points *endp*, along the rows of the data tseries. At each end point, it selects the past rows of the data tseries, starting at the first bar after the previous end point, and then calls the function agg\_ohlc() on the selected data tseries to calculate the aggregations.

The function roll\_ohlc() can accept either a single column of data or four columns of *OHLC* data. It can also accept an additional column containing the trading volume.

The function roll\_ohlc() performs a similar aggregation as the function to.period() from package xts.

#### Value

A matrix with OHLC data, with the number of rows equal to the number of endp minus one.

# Examples

```
## Not run:
# Define matrix of OHLC data
ohlc <- rutils::etfenv$VTI[, 1:5]
# Define end points at 25 day intervals
endp <- HighFreq::calc_endpoints(NROW(ohlc), step=25)
# Aggregate over endp:
ohlcagg <- HighFreq::roll_ohlc(tseries=ohlc, endp=endp)
# Compare with xts::to.period()
ohlcagg_xts <- .Call("toPeriod", ohlc, as.integer(endp+1), TRUE, NCOL(ohlc), FALSE, FALSE, colnames(ohlc), PAC
all.equal(ohlcagg, coredata(ohlcagg_xts), check.attributes=FALSE)
## End(Not run)</pre>
```

roll\_reg

Calculate a matrix of regression coefficients, their t-values, and z-scores, at the end points of the predictor matrix.

## **Description**

Calculate a *matrix* of regression coefficients, their t-values, and z-scores, at the end points of the predictor matrix.

#### Usage

```
roll_reg(
  respv,
  predv,
  controlv,
  startp = 0L,
  endp = 0L,
  step = 1L,
  look_back = 1L,
  stub = 0L
```

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### **Arguments**

respv A single-column *time series* or a *vector* of response data.

predv A *time series* or a *matrix* of predictor data. controlv A *list* of model parameters (see Details).

startp An *integer* vector of start points (the default is startp =  $\emptyset$ ). endp An *integer* vector of end points (the default is endp =  $\emptyset$ ).

The number of time periods between the end points (the default is step = 1).

look\_back

The number of end points in the look-back interval (the default is look\_back =

1).

stub An *integer* value equal to the first end point for calculating the end points (the

default is stub = 0).

#### **Details**

The function roll\_reg() calculates a *matrix* of regression coefficients, their t-values, and z-scores at the end points of the predictor matrix.

The function roll\_reg() performs a loop over the end points, and at each end point it subsets the time series predv over a look-back interval equal to look\_back number of end points.

If the arguments endp and startp are not given then it first calculates a vector of end points separated by step time periods. It calculates the end points along the rows of predv using the function calc\_endpoints(), with the number of time periods between the end points equal to step time periods.

For example, the rolling regression at 25 day end points, with a 75 day look-back, can be calculated using the parameters step = 25 and  $look_back = 3$ .

It passes the subset time series to the function calc\_reg(), which calculates the regression coefficients, their t-values, and the z-score. The function roll\_reg() accepts a list of model parameters through the argument controlv, and passes it to the function calc\_reg(). The list of model parameters can be created using the function param\_reg(). See the function param\_reg() for a description of the model parameters.

The number of columns of the return matrix depends on the number of columns of the predictor matrix (including the intercept column, if it's added). The number of columns of the return matrix is equal to the number of regression coefficients, plus their t-values, plus the z-score column. The number of regression coefficients is equal to the number of columns of the predictor matrix (including the intercept column, if it's added). The number of t-values is equal to the number of coefficients. For example, if the number of columns of the predictor matrix is equal to n, and if intercept = TRUE (the default), then roll\_reg() returns a matrix with 2n+3 columns: n+1 regression coefficients (including the intercept coefficient), n+1 corresponding t-values, and 1 z-score column.

### Value

A *matrix* with the regression coefficients, their t-values, and z-scores, and with the same number of rows as predv a number of columns equal to 2n+3, where n is the number of columns of predv.

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("XLP", "VTI")])</pre>
```

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```
# Define monthly end points and start points
endp <- xts::endpoints(retp, on="months")[-1]</pre>
look_back <- 12
startp <- c(rep(1, look_back), endp[1:(NROW(endp)-look_back)])</pre>
# Create a default list of regression parameters
controlv <- HighFreq::param_reg()</pre>
# Calculate rolling betas using RcppArmadillo
reg_stats <- HighFreq::roll_reg(respv=retp[, 1], predv=retp[, 2], endp=(endp-1), startp=(startp-1), controlv=</pre>
betas <- reg_stats[, 2]</pre>
# Calculate rolling betas in R
betar <- sapply(1:NROW(endp), FUN=function(ep) {</pre>
  datav <- retp[startp[ep]:endp[ep], ]</pre>
 drop(cov(datav[, 1], datav[, 2])/var(datav[, 2]))
}) # end sapply
# Compare the outputs of both functions
all.equal(betas, betar, check.attributes=FALSE)
## End(Not run)
```

roll\_scale

*Perform a rolling standardization (centering and scaling) of the columns of a* time series *of data using* RcppArmadillo.

## Description

Perform a rolling standardization (centering and scaling) of the columns of a *time series* of data using RcppArmadillo.

### Usage

```
roll_scale(matrix, look_back, center = TRUE, scale = TRUE, use_median = FALSE)
```

### **Arguments**

| tseries    | A time series or matrix of data.  |
|------------|---|
| look_back  | The length of the look-back interval, equal to the number of rows of data used in the scaling.  |
| center     | A $Boolean$ argument: if TRUE then center the columns so that they have zero mean or median (the default is TRUE).  |
| scale      | A <i>Boolean</i> argument: if TRUE then scale the columns so that they have unit standard deviation or MAD (the default is TRUE).   |
| use_median | A <i>Boolean</i> argument: if TRUE then the centrality (central tendency) is calculated as the <i>median</i> and the dispersion is calculated as the <i>median absolute deviation</i> ( <i>MAD</i> ) (the default is FALSE). If use_median = FALSE then the centrality is calculated as the <i>mean</i> and the dispersion is calculated as the <i>standard deviation</i> . |

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#### **Details**

The function roll\_scale() performs a rolling standardization (centering and scaling) of the columns of the tseries argument using RcppArmadillo. The function roll\_scale() performs a loop over the rows of tseries, subsets a number of previous (past) rows equal to look\_back, and standardizes the subset matrix by calling the function calc\_scale(). It assigns the last row of the standardized subset *matrix* to the return matrix.

If the arguments center and scale are both TRUE and use\_median is FALSE (the defaults), then calc\_scale() performs the same calculation as the function roll::roll\_scale().

If the arguments center and scale are both TRUE (the defaults), then calc\_scale() standardizes the data. If the argument center is FALSE then calc\_scale() only scales the data (divides it by the standard deviations). If the argument scale is FALSE then calc\_scale() only demeans the data (subtracts the means).

If the argument use\_median is TRUE, then it calculates the centrality as the *median* and the dispersion as the *median absolute deviation (MAD)*.

#### Value

A *matrix* with the same dimensions as the input argument tseries.

## **Examples**

```
## Not run:
# Calculate a time series of returns
retp <- zoo::coredata(na.omit(rutils::etfenv$returns[, c("IEF", "VTI")]))
look_back <- 11
rolled_scaled <- roll::roll_scale(retp, width=look_back, min_obs=1)
rolled_scaled2 <- HighFreq::roll_scale(retp, look_back=look_back)
all.equal(rolled_scaled[-(1:2), ], rolled_scaled2[-(1:2), ],
    check.attributes=FALSE)
## End(Not run)</pre>
```

roll\_sharpe

Calculate a time series of Sharpe ratios over a rolling look-back interval for an OHLC time series.

### Description

Calculate a time series of Sharpe ratios over a rolling look-back interval for an *OHLC* time series.

# Usage

```
roll_sharpe(ohlc, look_back = 11)
```

# Arguments

ohlc An *OHLC* time series of prices in *xts* format.

look\_back The size of the look-back interval, equal to the number of rows of data used for

aggregating the OHLC prices.

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### **Details**

The function roll\_sharpe() calculates the rolling Sharpe ratio defined as the ratio of percentage returns over the look-back interval, divided by the average volatility of percentage returns.

#### Value

An xts time series with a single column and the same number of rows as the argument ohlc.

### **Examples**

```
# Calculate rolling Sharpe ratio over SPY
sharpe_rolling <- roll_sharpe(ohlc=HighFreq::SPY, look_back=11)</pre>
```

roll\_skew

Calculate a matrix of skewness estimates over a rolling look-back interval attached at the end points of a time series or a matrix.

## **Description**

Calculate a *matrix* of skewness estimates over a rolling look-back interval attached at the end points of a *time series* or a *matrix*.

## Usage

```
roll_skew(
   tseries,
   startp = 0L,
   endp = 0L,
   step = 1L,
   look_back = 1L,
   stub = 0L,
   method = "moment",
   confl = 0.75
)
```

### **Arguments**

| tseries   | A time series or a matrix of data.  |
|-----------|---|
| startp    | An <i>integer</i> vector of start points (the default is $startp = 0$ ).  |
| endp      | An <i>integer</i> vector of end points (the default is endp = $\emptyset$ ).  |
| step      | The number of time periods between the end points (the default is step = 1).  |
| look_back | The number of end points in the look-back interval (the default is look_back = 1).                                    |
| stub      | An $integer$ value equal to the first end point for calculating the end points (the default is stub = 0).             |
| method    | A <i>character string</i> specifying the type of the skewness model (the default is method = "moment" - see Details). |
| confl     | The confidence level for calculating the quantiles of returns (the default is conf1 = $0.75$ ).                       |

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#### **Details**

The function roll\_skew() calculates a *matrix* of skewness estimates over rolling look-back intervals attached at the end points of the *time series* tseries.

The function roll\_skew() performs a loop over the end points, and at each end point it subsets the time series tseries over a look-back interval equal to look\_back number of end points.

It passes the subset time series to the function calc\_skew(), which calculates the skewness. See the function calc\_skew() for a description of the skewness methods.

If the arguments endp and startp are not given then it first calculates a vector of end points separated by step time periods. It calculates the end points along the rows of tseries using the function calc\_endpoints(), with the number of time periods between the end points equal to step time periods.

For example, the rolling skewness at 25 day end points, with a 75 day look-back, can be calculated using the parameters step = 25 and look\_back = 3.

The function roll\_skew() is implemented in RcppArmadillo C++ code, which makes it several times faster than R code.

#### Value

A *matrix* of skewness estimates with the same number of columns as the input time series tseries, and the number of rows equal to the number of end points.

```
## Not run:
# Define time series of returns using package rutils
retp <- na.omit(rutils::etfenv$returns$VTI)</pre>
# Define end points and start points
endp <- 1 + HighFreq::calc_endpoints(NROW(retp), step=25)</pre>
startp <- HighFreq::calc_startpoints(endp, look_back=3)</pre>
# Calculate the rolling skewness at 25 day end points, with a 75 day look-back
skewv <- HighFreq::roll_skew(retp, step=25, look_back=3)</pre>
# Calculate the rolling skewness using R code
skewr <- sapply(1:NROW(endp), function(it) {</pre>
 HighFreq::calc_skew(retp[startp[it]:endp[it], ])
}) # end sapply
# Compare the skewness estimates
all.equal(drop(skewv), skewr, check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::roll_skew(retp, step=25, look_back=3),
  Rcode=sapply(1:NROW(endp), function(it) {
    HighFreq::calc_skew(retp[startp[it]:endp[it], ])
  }),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

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| roll_stats | Calculate a vector of statistics over an OHLC time series, and calculate a rolling mean over the statistics. |
|------------|--|
|            |  |

### **Description**

Calculate a vector of statistics over an *OHLC* time series, and calculate a rolling mean over the statistics.

### Usage

```
roll_stats(
  ohlc,
  calc_stats = "ohlc_variance",
  look_back = 11,
  weighted = TRUE,
  ...
)
```

### **Arguments**

| • • •      | additional parameters to the function calc_stats.   |
|------------|---|
| ohlc       | An OHLC time series of prices and trading volumes, in xts format.   |
| calc_stats | The name of the function for estimating statistics of a single row of <i>OHLC</i> data, such as volatility, skew, and higher moments. |
| look_back  | The size of the look-back interval, equal to the number of rows of data used for calculating the rolling mean.                        |
| weighted   | Boolean argument: should statistic be weighted by trade volume? (default TRUE)  |

#### **Details**

The function roll\_stats() calculates a vector of statistics over an *OHLC* time series, such as volatility, skew, and higher moments. The statistics could also be any other aggregation of a single row of *OHLC* data, for example the *High* price minus the *Low* price squared. The length of the vector of statistics is equal to the number of rows of the argument ohlc. Then it calculates a trade volume weighted rolling mean over the vector of statistics over and calculate statistics.

### Value

An xts time series with a single column and the same number of rows as the argument ohlc.

```
# Calculate time series of rolling variance and skew estimates
var_rolling <- roll_stats(ohlc=HighFreq::SPY, look_back=21)
skew_rolling <- roll_stats(ohlc=HighFreq::SPY, calc_stats="ohlc_skew", look_back=21)
skew_rolling <- skew_rolling/(var_rolling)^(1.5)
skew_rolling[1, ] <- 0
skew_rolling <- rutils::na_locf(skew_rolling)</pre>
```

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roll\_sum

Calculate the rolling sums over a time series or a matrix using Rcpp.

#### **Description**

Calculate the rolling sums over a time series or a matrix using Rcpp.

#### **Usage**

```
roll_sum(tseries, look_back = 1L, weightv = 0L)
```

### **Arguments**

tseries A time series or a matrix.

look\_back The length of the look-back interval, equal to the number of data points included

in calculating the rolling sum (the default is look\_back = 1).

weightv A single-column matrix of weights (the default is weightv = 0).

#### **Details**

The function roll\_sum() calculates the rolling *weighted* sums over the columns of the data tseries. If the argument weightv is equal to zero (the default), then the function roll\_sum() calculates the simple rolling sums of the *time series* data  $p_t$  over the look-back interval  $\Delta$ :

$$\bar{p}_t = \sum_{j=(t-\Delta+1)}^t p_j$$

If the weightv argument has the same number of rows as the argument tseries, then the function roll\_sum() calculates rolling weighted sums of the  $time\ series$  data  $p_t$  in two steps.

It first calculates the rolling sums of the products of the weights  $w_t$  times the *time series* data  $p_t$  over the look-back interval  $\Delta$ :

$$\bar{w}_t = \sum_{j=(t-\Delta+1)}^t w_j$$

$$\bar{p}_t^w = \sum_{j=(t-\Delta+1)}^t w_j p_j$$

It then calculates the rolling *weighted* sums  $\bar{p}_t$  as the ratio of the sum products of the weights and the data, divided by the sums of the weights:

$$\bar{p}_t = \frac{\bar{p}_t^w}{\bar{w}_t}$$

The function roll\_sum() returns a *matrix* with the same dimensions as the input argument tseries. The function roll\_sum() is written in C++ Armadillo code, so it's much faster than equivalent R code.

### Value

A matrix with the same dimensions as the input argument tseries.

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#### **Examples**

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("VTI", "IEF")])</pre>
# Define parameters
look_back <- 11
# Calculate rolling sums and compare with rutils::roll_sum()
sumc <- HighFreq::roll_sum(retp, look_back)</pre>
sumr <- rutils::roll_sum(retp, look_back)</pre>
all.equal(sumc, coredata(sumr), check.attributes=FALSE)
# Calculate rolling sums using R code
sumr <- apply(zoo::coredata(retp), 2, cumsum)</pre>
sumlag <- rbind(matrix(numeric(2*look_back), nc=2), sumr[1:(NROW(sumr) - look_back), ])</pre>
sumr <- (sumr - sumlag)</pre>
all.equal(sumc, sumr, check.attributes=FALSE)
# Calculate weights equal to the trading volumes
weightv <- quantmod::Vo(rutils::etfenv$VTI)</pre>
weightv <- weightv[zoo::index(retp)]</pre>
# Calculate rolling weighted sums
sumc <- HighFreq::roll_sum(retp, look_back, 1/weightv)</pre>
# Plot dygraph of the weighted sums
datav <- cbind(retp$VTI, sumc[, 1])</pre>
colnames(datav) <- c("VTI", "Weighted")</pre>
endp <- rutils::calc_endpoints(datav, interval="weeks")</pre>
dygraphs::dygraph(cumsum(datav)[endp], main=colnames(foo)) %>%
  dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
  dyLegend(width=300)
## End(Not run)
```

roll\_sumep

Calculate the rolling sums at the end points of a time series or a matrix.

## Description

Calculate the rolling sums at the end points of a time series or a matrix.

### Usage

```
roll_sumep(
  tseries,
  startp = 0L,
  endp = 0L,
  step = 1L,
  look_back = 1L,
  stub = 0L
)
```

### **Arguments**

tseries A time series or a matrix. startp An integer vector of start points (the default is startp = 0). roll\_sumw 93

| endp      | An <i>integer</i> vector of end points (the default is endp = $0$ ).                 |
|-----------|--|
| step      | The number of time periods between the end points (the default is $step = 1$ ).      |
| look_back | The number of end points in the look-back interval (the default is look_back = 1).   |
| stub      | An <i>integer</i> value equal to the first end point for calculating the end points. |

### **Details**

The function roll\_sumep() calculates the rolling sums at the end points of the *time series* tseries. The function roll\_sumep() is implemented in RcppArmadillo C++ code, which makes it several times faster than R code.

#### Value

A *matrix* with the same number of columns as the input time series tseries, and the number of rows equal to the number of end points.

## **Examples**

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("VTI", "IEF")])</pre>
# Define end points at 25 day intervals
endp <- HighFreq::calc_endpoints(NROW(retp), step=25)</pre>
# Define start points as 75 day lag of end points
startp <- HighFreq::calc_startpoints(endp, look_back=3)</pre>
# Calculate rolling sums using Rcpp
sumc <- HighFreq::roll_sumep(retp, startp=startp, endp=endp)</pre>
# Calculate rolling sums using R code
sumr <- sapply(1:NROW(endp), function(ep) {</pre>
colSums(retp[(startp[ep]+1):(endp[ep]+1), ])
  }) # end sapply
sumr <- t(sumr)</pre>
all.equal(sumc, sumr, check.attributes=FALSE)
## End(Not run)
```

roll\_sumw Calculate the rolling weighted sums over a time series or a matrix using Rcpp.

## **Description**

Calculate the rolling weighted sums over a time series or a matrix using Rcpp.

# Usage

```
roll_sumw(tseries, endp = NULL, look_back = 1L, stub = NULL, weightv = NULL)
```

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### **Arguments**

tseries A time series or a matrix.

endp An *integer* vector of end points (the default is endp = NULL).

look\_back The length of the look-back interval, equal to the number of data points included

in calculating the rolling sum (the default is look\_back = 1).

stub An *integer* value equal to the first end point for calculating the end points (the

default is stub = NULL).

weightv A single-column *matrix* of weights (the default is weightv = NULL).

#### **Details**

The function roll\_sumw() calculates the rolling weighted sums over the columns of the data tseries.

The function roll\_sumw() calculates the rolling weighted sums as convolutions of the columns of tseries with the *column vector* of weights using the Armadillo function arma::conv2(). It performs a similar calculation to the standard R function

stats::filter(x=retp, filter=weightv, method="convolution", sides=1), but it can be many times faster, and it doesn't produce any leading NA values.

The function roll\_sumw() returns a *matrix* with the same dimensions as the input argument tseries.

The arguments weightv, endp, and stub are optional.

If the argument weightv is not supplied, then simple sums are calculated, not weighted sums.

If either the stub or endp arguments are supplied, then the rolling sums are calculated at the end points.

If only the argument stub is supplied, then the end points are calculated from the stub and look\_back arguments. The first end point is equal to stub and the end points are spaced look\_back periods apart.

If the arguments weighty, endp, and stub are not supplied, then the sums are calculated over a number of data points equal to look\_back.

The function roll\_sumw() is also several times faster than rutils::roll\_sum() which uses vectorized R code.

Technical note: The function roll\_sumw() has arguments with default values equal to NULL, which are implemented in Rcpp code.

### Value

A *matrix* with the same dimensions as the input argument tseries.

```
## Not run:
# First example
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("VTI", "IEF")])
# Define parameters
look_back <- 11
# Calculate rolling sums and compare with rutils::roll_sum()
sumc <- HighFreq::roll_sum(retp, look_back)
sumr <- rutils::roll_sum(retp, look_back)
all.equal(sumc, coredata(sumr), check.attributes=FALSE)</pre>
```

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```
# Calculate rolling sums using R code
sumr <- apply(zoo::coredata(retp), 2, cumsum)</pre>
sumlag <- rbind(matrix(numeric(2*look_back), nc=2), sumr[1:(NROW(sumr) - look_back), ])</pre>
sumr <- (sumr - sumlag)</pre>
all.equal(sumc, sumr, check.attributes=FALSE)
# Calculate rolling sums at end points
stuby <- 21
sumc <- HighFreq::roll_sumw(retp, look_back, stub=stubv)</pre>
endp <- (stubv + look_back*(0:(NROW(retp) %/% look_back)))</pre>
endp <- endp[endp < NROW(retp)]</pre>
sumr <- apply(zoo::coredata(retp), 2, cumsum)</pre>
sumr <- sumr[endp+1, ]</pre>
sumlag <- rbind(numeric(2), sumr[1:(NROW(sumr) - 1), ])</pre>
sumr <- (sumr - sumlag)</pre>
all.equal(sumc, sumr, check.attributes=FALSE)
# Calculate rolling sums at end points - pass in endp
sumc <- HighFreq::roll_sumw(retp, endp=endp)</pre>
all.equal(sumc, sumr, check.attributes=FALSE)
# Create exponentially decaying weights
weightv <- \exp(-0.2*(1:11))
weightv <- matrix(weightv/sum(weightv), nc=1)</pre>
# Calculate rolling weighted sum
sumc <- HighFreq::roll_sumw(retp, weightv=weightv)</pre>
# Calculate rolling weighted sum using filter()
filtered <- filter(x=retp, filter=weightv, method="convolution", sides=1)</pre>
all.equal(sumc[-(1:11), ], filtered[-(1:11), ], check.attributes=FALSE)
# Calculate rolling weighted sums at end points
sumc <- HighFreq::roll_sumw(retp, endp=endp, weightv=weightv)</pre>
all.equal(sumc, filtered[endp+1, ], check.attributes=FALSE)
# Create simple weights equal to a 1 value plus zeros
weightv \leftarrow matrix(c(1, rep(0, 10)), nc=1)
# Calculate rolling weighted sum
weighted <- HighFreq::roll_sumw(retp, weightv=weightv)</pre>
# Compare with original
all.equal(coredata(retp), weighted, check.attributes=FALSE)
## End(Not run)
```

roll\_var

Calculate a matrix of dispersion (variance) estimates over a rolling look-back interval attached at the end points of a time series or a matrix.

### **Description**

Calculate a *matrix* of dispersion (variance) estimates over a rolling look-back interval attached at the end points of a *time series* or a *matrix*.

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#### **Usage**

```
roll_var(
  tseries,
  look_back = 1L,
  startp = 0L,
  endp = 0L,
  step = 1L,
  stub = 0L,
  method = "moment",
  confl = 0.75
)
```

### **Arguments**

tseries A time series or a matrix of data.

look\_back The number of end points in the look-back interval (the default is look\_back = 1).

startp An integer vector of start points (the default is startp = 0).

endp An integer vector of end points (the default is endp = 0).

step The number of time periods between the end points (the default is step = 1).

stub An integer value equal to the first end point for calculating the end points (the default is stub = 0).

A character string representing the type of the measure of dispersion (the default

is method = "moment").

## **Details**

method

The function roll\_var() calculates a *matrix* of dispersion (variance) estimates over rolling look-back intervals attached at the end points of the *time series* tseries.

The function roll\_var() performs a loop over the end points, and at each end point it subsets the time series tseries over a look-back interval equal to look\_back number of end points.

It passes the subset time series to the function calc\_var(), which calculates the dispersion. See the function calc\_var() for a description of the dispersion methods.

If the arguments endp and startp are not given then it first calculates a vector of end points separated by step time periods. It calculates the end points along the rows of tseries using the function calc\_endpoints(), with the number of time periods between the end points equal to step time periods.

For example, the rolling variance at 25 day end points, with a 75 day look-back, can be calculated using the parameters step = 25 and  $look_back = 3$ .

The function roll\_var() with the parameter step = 1 performs the same calculation as the function roll\_var() from package RcppRoll, but it's several times faster because it uses RcppArmadillo C++ code.

The function roll\_var() is implemented in RcppArmadillo RcppArmadillo C++ code, which makes it several times faster than R code.

### Value

A *matrix* dispersion (variance) estimates with the same number of columns as the input time series tseries, and the number of rows equal to the number of end points.

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#### **Examples**

```
## Not run:
# Define time series of returns using package rutils
retp <- na.omit(rutils::etfenv$returns$VTI)</pre>
# Calculate the rolling variance at 25 day end points, with a 75 day look-back
variance <- HighFreq::roll_var(retp, look_back=3, step=25)</pre>
# Compare the variance estimates over 11-period look-back intervals
all.equal(HighFreq::roll_var(retp, look_back=11)[-(1:10), ],
  drop(RcppRoll::roll_var(retp, n=11)), check.attributes=FALSE)
# Compare the speed of HighFreq::roll_var() with RcppRoll::roll_var()
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::roll_var(retp, look_back=11),
  RcppRoll=RcppRoll::roll_var(retp, n=11),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Compare the speed of HighFreq::roll_var() with TTR::runMAD()
summary(microbenchmark(
   Rcpp=HighFreq::roll_var(retp, look_back=11, method="quantile"),
   TTR=TTR::runMAD(retp, n = 11),
   times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

roll\_varvec

Calculate a vector of variance estimates over a rolling look-back interval for a single-column time series or a single-column matrix, using RcppArmadillo.

## Description

Calculate a *vector* of variance estimates over a rolling look-back interval for a single-column *time series* or a single-column *matrix*, using RcppArmadillo.

## Usage

```
roll_varvec(tseries, look_back = 1L)
```

### **Arguments**

tseries A single-column time series or a single-column matrix.

look\_back The length of the look-back interval, equal to the number of *vector* elements

used for calculating a single variance estimate (the default is look\_back = 1).

## **Details**

The function roll\_varvec() calculates a *vector* of variance estimates over a rolling look-back interval for a single-column *time series* or a single-column *matrix*, using RcppArmadillo C++ code.

The function roll\_varvec() uses an expanding look-back interval in the initial warmup period, to calculate the same number of elements as the input argument tseries.

The function roll\_varvec() performs the same calculation as the function roll\_var() from package RcppRoll, but it's several times faster because it uses RcppArmadillo C++ code.

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#### Value

A single-column *matrix* with the same number of elements as the input argument tseries.

### **Examples**

```
## Not run:
# Create a vector of random returns
retp <- rnorm(1e6)
# Compare the variance estimates over 11-period look-back intervals
all.equal(drop(HighFreq::roll_varvec(retp, look_back=11))[-(1:10)],
    RcppRoll::roll_var(retp, n=11))
# Compare the speed of RcppArmadillo with RcppRoll
library(microbenchmark)
summary(microbenchmark(
    Rcpp=HighFreq::roll_varvec(retp, look_back=11),
    RcppRoll=RcppRoll::roll_var(retp, n=11),
    times=10))[, c(1, 4, 5)] # end microbenchmark summary</pre>
## End(Not run)
```

roll\_var\_ohlc

Calculate a vector of variance estimates over a rolling look-back interval attached at the end points of a time series or a matrix with OHLC price data.

## **Description**

Calculate a *vector* of variance estimates over a rolling look-back interval attached at the end points of a *time series* or a *matrix* with *OHLC* price data.

### Usage

```
roll_var_ohlc(
  ohlc,
  startp = 0L,
  endp = 0L,
  step = 1L,
  look_back = 1L,
  stub = 0L,
  method = "yang_zhang",
  scale = TRUE,
  index = 0L
)
```

### **Arguments**

| ohlc   | A time series or a matrix with OHLC price data.                              |
|--------|--|
| startp | An <i>integer</i> vector of start points (the default is $startp = 0$ ).     |
| endp   | An <i>integer</i> vector of end points (the default is endp = $0$ ).         |
| step   | The number of time periods between the end points (the default is step = 1). |

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look\_back The number of end points in the look-back interval (the default is look\_back =

1).

An *integer* value equal to the first end point for calculating the end points (the

default is stub = 0).

method A character string representing the price range estimator for calculating the

variance. The estimators include:

• "close" close-to-close estimator,

• "rogers\_satchell" Rogers-Satchell estimator,

• "garman\_klass" Garman-Klass estimator,

• "garman\_klass\_yz" Garman-Klass with account for close-to-open price jumps,

• "yang\_zhang" Yang-Zhang estimator,

(The default is the "yang\_zhang" estimator.)

scale Boolean argument: Should the returns be divided by the time index, the number

of seconds in each period? (The default is scale = TRUE.)

index A vector with the time index of the time series. This is an optional argument

(the default is index=0).

#### **Details**

The function roll\_var\_ohlc() calculates a *vector* of variance estimates over a rolling look-back interval attached at the end points of the *time series* ohlc.

The input *OHLC time series* ohlc is assumed to contain the log prices.

The function roll\_var\_ohlc() performs a loop over the end points, subsets the previous (past) rows of ohlc, and passes them into the function calc\_var\_ohlc().

At each end point, the variance is calculated over a look-back interval equal to look\_back number of end points. In the initial warmup period, the variance is calculated over an expanding look-back interval.

If the arguments endp and startp are not given then it first calculates a vector of end points separated by step time periods. It calculates the end points along the rows of ohlc using the function calc\_endpoints(), with the number of time periods between the end points equal to step time periods.

For example, the rolling variance at daily end points with an 11 day look-back, can be calculated using the parameters step = 1 and look\_back = 1 (Assuming the ohlc data has daily frequency.)

Similarly, the rolling variance at 25 day end points with a 75 day look-back, can be calculated using the parameters step = 25 and look\_back = 3 (because 3\*25 = 75).

The function roll\_var\_ohlc() calculates the variance from all the different intra-day and day-over-day returns (defined as the differences between *OHLC* prices), using several different variance estimation methods.

The default method is "yang\_zhang", which theoretically has the lowest standard error among unbiased estimators. The methods "close", "garman\_klass\_yz", and "yang\_zhang" do account for close-to-open price jumps, while the methods "garman\_klass" and "rogers\_satchell" do not account for close-to-open price jumps.

If scale is TRUE (the default), then the returns are divided by the differences of the time index (which scales the variance to the units of variance per second squared.) This is useful when calculating the variance from minutely bar data, because dividing returns by the number of seconds decreases the effect of overnight price jumps. If the time index is in days, then the variance is equal to the variance per day squared.

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The optional argument index is the time index of the *time series* ohlc. If the time index is in seconds, then the differences of the index are equal to the number of seconds in each time period. If the time index is in days, then the differences are equal to the number of days in each time period.

The function roll\_var\_ohlc() is implemented in RcppArmadillo C++ code, which makes it several times faster than R code.

#### Value

A column *vector* of variance estimates, with the number of rows equal to the number of end points.

### **Examples**

```
## Not run:
# Extract the log OHLC prices of SPY
ohlc <- log(HighFreq::SPY)</pre>
# Extract the time index of SPY prices
indeks <- c(1, diff(xts::.index(ohlc)))</pre>
# Rolling variance at minutely end points, with a 21 minute look-back
varoll <- HighFreq::roll_var_ohlc(ohlc,</pre>
                                step=1, look_back=21,
                                method="yang_zhang",
                                index=indeks, scale=TRUE)
# Daily OHLC prices
ohlc <- rutils::etfenv$VTI</pre>
indeks <- c(1, diff(xts::.index(ohlc)))</pre>
# Rolling variance at 5 day end points, with a 20 day look-back (20=4*5)
varoll <- HighFreq::roll_var_ohlc(ohlc,</pre>
                                step=5, look_back=4,
                                method="yang_zhang",
                                index=indeks, scale=TRUE)
# Same calculation in R
nrows <- NROW(ohlc)</pre>
closel = HighFreq::lagit(ohlc[, 4])
endp <- drop(HighFreq::calc_endpoints(nrows, 3)) + 1</pre>
startp <- drop(HighFreq::calc_startpoints(endp, 2))</pre>
npts <- NROW(endp)</pre>
varollr <- sapply(2:npts, function(it) {</pre>
  rangev <- startp[it]:endp[it]</pre>
  sub_ohlc = ohlc[rangev, ]
  sub_close = closel[rangev]
  sub_index = indeks[rangev]
 HighFreq::calc_var_ohlc(sub_ohlc, closel=sub_close, scale=TRUE, index=sub_index)
}) # end sapply
varollr <- c(0, varollr)</pre>
all.equal(drop(var_rolling), varollr)
## End(Not run)
```

roll\_vwap

Calculate the volume-weighted average price of an OHLC time series over a rolling look-back interval.

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#### **Description**

Performs the same operation as function VWAP() from package TTR, but using vectorized functions, so it's a little faster.

### Usage

```
roll_vwap(ohlc, close = ohlc[, 4, drop = FALSE], look_back)
```

### **Arguments**

ohlc An OHLC time series of prices in xts format.

close A time series of close prices.

The size of the look-back interval, equal to the number of rows of data used for look\_back

calculating the average price.

#### **Details**

The function roll\_vwap() calculates the volume-weighted average closing price, defined as the sum of the prices multiplied by trading volumes in the look-back interval, divided by the sum of trading volumes in the interval. If the argument close is passed in explicitly, then its volumeweighted average value over time is calculated.

#### Value

An xts time series with a single column and the same number of rows as the argument ohlc.

## **Examples**

```
# Calculate and plot rolling volume-weighted average closing prices (VWAP)
prices_rolling <- roll_vwap(ohlc=HighFreq::SPY["2013-11"], look_back=11)</pre>
chart_Series(HighFreq::SPY["2013-11-12"], name="SPY prices")
add_TA(prices_rolling["2013-11-12"], on=1, col="red", lwd=2)
legend("top", legend=c("SPY prices", "VWAP prices"),
bg="white", lty=c(1, 1), lwd=c(2, 2),
col=c("black", "red"), bty="n")
# Calculate running returns
returns_running <- ohlc_returns(xtsv=HighFreq::SPY)</pre>
\ensuremath{\text{\#}} Calculate the rolling volume-weighted average returns
roll_vwap(ohlc=HighFreq::SPY, close=returns_running, look_back=11)
```

roll\_zscores

Calculate a vector of z-scores of the residuals of rolling regressions at the end points of the predictor matrix.

### **Description**

Calculate a vector of z-scores of the residuals of rolling regressions at the end points of the predictor matrix.

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#### Usage

```
roll_zscores(
  respv,
  predv,
  startp = 0L,
  endp = 0L,
  step = 1L,
  look_back = 1L,
  stub = 0L
```

## Arguments

respv A single-column time series or a vector of response data.

predv A time series or a matrix of predictor data.

startp An *integer* vector of start points (the default is start p = 0). endp An *integer* vector of end points (the default is endp = 0).

The number of time periods between the end points (the default is step = 1).

look\_back The number of end points in the look-back interval (the default is look\_back =

1).

stub An integer value equal to the first end point for calculating the end points (the

default is stub = 0).

### **Details**

The function roll\_zscores() calculates a *vector* of z-scores of the residuals of rolling regressions at the end points of the *time series* predv.

The function roll\_zscores() performs a loop over the end points, and at each end point it subsets the time series predv over a look-back interval equal to look\_back number of end points.

It passes the subset time series to the function calc\_lm(), which calculates the regression data.

If the arguments endp and startp are not given then it first calculates a vector of end points separated by step time periods. It calculates the end points along the rows of predv using the function calc\_endpoints(), with the number of time periods between the end points equal to step time periods.

For example, the rolling variance at 25 day end points, with a 75 day look-back, can be calculated using the parameters step = 25 and look\_back = 3.

### Value

A column *vector* of the same length as the number of rows of predv.

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("XLF", "VTI", "IEF")])
# Response equals XLF returns
respv <- retp[, 1]
# Predictor matrix equals VTI and IEF returns
predv <- retp[, -1]</pre>
```

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```
# Calculate Z-scores from rolling time series regression using RcppArmadillo
look_back <- 11
zscores <- HighFreq::roll_zscores(respv=respv, predv=predv, look_back)</pre>
# Calculate z-scores in R from rolling multivariate regression using lm()
zscoresr <- sapply(1:NROW(predv), function(ro_w) {</pre>
  if (ro_w == 1) return(0)
  startpoint <- max(1, ro_w-look_back+1)</pre>
  responsi <- response[startpoint:ro_w]</pre>
  predicti <- predictor[startpoint:ro_w, ]</pre>
  lmod <- lm(responsi ~ predicti)</pre>
  residuals <- lmod$residuals
  residuals[NROW(residuals)]/sd(residuals)
}) # end sapply
# Compare the outputs of both functions
all.equal(zscores[-(1:look_back)], zscoresr[-(1:look_back)],
  check.attributes=FALSE)
## End(Not run)
```

run\_covar

Calculate the trailing covariances of two streaming time series of returns.

### **Description**

Calculate the trailing covariances of two streaming time series of returns.

### Usage

```
run_covar(tseries, lambda)
```

### **Arguments**

tseries A *time series* or a *matrix* with two columns of returns data.

lambda A decay factor which multiplies past estimates.

### **Details**

The function run\_covar() calculates the trailing covariances of two streaming *time series* of returns, by recursively weighting the past covariance estimates  $cov_{t-1}$ , with the products of their returns minus their means, using the decay factor  $\lambda$ :

$$\bar{x}_t = \lambda \bar{x}_{t-1} + (1 - \lambda)x_t$$
$$\bar{y}_t = \lambda \bar{y}_{t-1} + (1 - \lambda)y_t$$
$$cov_t = \lambda cov_{t-1} + (1 - \lambda)(x_t - \bar{x}_t)(y_t - \bar{y}_t)$$

Where  $cov_t$  is the trailing covariance estimate at time t,  $x_t$  and  $y_t$  are the two streaming returns data, and  $\bar{x}_t$  and  $\bar{x}_t$  are the trailing means of the returns.

The above recursive formulas are convenient for processing live streaming data because they don't require maintaining a buffer of past data. The formulas are equivalent to a convolution with exponentially decaying weights, but they're much faster to calculate. Using exponentially decaying

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weights is more natural than using a sliding look-back interval, because it gradually "forgets" about the past data.

The value of the decay factor  $\lambda$  must be in the range between 0 and 1. If  $\lambda$  is close to 1 then the decay is weak and past values have a greater weight, and the trailing covariance values have a stronger dependence on past data. This is equivalent to a long look-back interval. If  $\lambda$  is much less than 1 then the decay is strong and past values have a smaller weight, and the trailing covariance values have a weaker dependence on past data. This is equivalent to a short look-back interval.

The function run\_covar() returns five columns of data: the trailing covariances, the variances, and the mean values of the two columns of the argument tseries. This allows calculating the trailing correlations, betas, and alphas.

#### Value

A *matrix* with five columns of data: the trailing covariances, the variances, and the mean values of the two columns of the argument tseries.

# Examples

```
## Not run:
# Calculate historical returns
retp <- zoo::coredata(na.omit(rutils::etfenv$returns[, c("IEF", "VTI")]))
# Calculate the trailing covariance
lambda <- 0.9
covars <- HighFreq::run_covar(retp, lambda=lambda)
# Calculate trailing covariance using R code
filtered <- (1-lambda)*filter(retp[, 1]*retp[, 2],
    filter=lambda, init=as.numeric(retp[1, 1]*retp[1, 2])/(1-lambda),
    method="recursive")
all.equal(covars[, 1], unclass(filtered), check.attributes=FALSE)
# Calculate the trailing correlation
correl <- covars[, 1]/sqrt(covars[, 2]*covars[, 3])
## End(Not run)</pre>
```

run\_max

Calculate the trailing maximum values of streaming time series data.

## **Description**

Calculate the trailing maximum values of streaming time series data.

#### Usage

```
run_max(tseries, lambda)
```

### **Arguments**

tseries A time series or a matrix.

lambda A decay factor which multiplies past estimates.

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#### **Details**

The function run\_max() calculates the trailing maximum values of streaming *time series* data by recursively weighting present and past values using the decay factor  $\lambda$ .

It calculates the trailing maximum values  $p_t^{max}$  of the streaming data  $p_t$  as follows:

$$p_t^{max} = max(p_t, \lambda p_{t-1}^{max} + (1 - \lambda)p_t)$$

The first term in the sum is the maximum value multiplied by the decay factor  $\lambda$ , so that the past maximum value is gradually "forgotten". The second term pulls the maximum value to the current value  $p_t$ .

The value of the decay factor  $\lambda$  must be in the range between 0 and 1. If  $\lambda$  is close to 1 then the past maximum values persist for longer. This is equivalent to a long look-back interval. If  $\lambda$  is much less than 1 then the past maximum values decay quickly, and the trailing maximum depends on the more recent streaming data. This is equivalent to a short look-back interval.

The above formula can also be expressed as:

$$p_t^{max} = \lambda max(p_t, p_{t-1}^{max}) + (1 - \lambda)p_t$$

The first term is the maximum value multiplied by the decay factor  $\lambda$ , so that the past maximum value is gradually "forgotten". The second term pulls the maximum value to the current value  $p_t$ .

The above recursive formula is convenient for processing live streaming data because it doesn't require maintaining a buffer of past data.

The function run\_max() returns a *matrix* with the same dimensions as the input argument tseries.

### Value

A *matrix* with the same dimensions as the input argument tseries.

```
## Not run:
# Calculate historical prices
closep <- zoo::coredata(quantmod::Cl(rutils::etfenv$VTI))
# Calculate the trailing maximums
lambda <- 0.9
maxv <- HighFreq::run_max(closep, lambda=lambda)
# Plot dygraph of VTI prices and trailing maximums
datav <- cbind(quantmod::Cl(rutils::etfenv$VTI), maxv)
colnames(datav) <- c("prices", "max")
colnamev <- colnames(datav)
dygraphs::dygraph(datav, main="VTI Prices and Trailing Maximums") %>%
    dySeries(name=colnamev[1], label=colnamev[1], strokeWidth=2, col="blue") %>%
    dySeries(name=colnamev[2], label=colnamev[2], strokeWidth=2, col="red")
## End(Not run)
```

106 run\_mean

run\_mean

Calculate the trailing weighted means of streaming time series data.

#### **Description**

Calculate the trailing weighted means of streaming time series data.

#### Usage

run\_mean(tseries, lambda, weightv = 0L)

#### **Arguments**

tseries A time series or a matrix.

lambda A decay factor which multiplies past estimates.

weightv A single-column *matrix* of weights.

#### **Details**

The function run\_mean() calculates the trailing weighted means of the streaming *time series* data  $p_t$  by recursively weighting present and past values using the decay factor  $\lambda$ . If the weightv argument is equal to zero, then the function run\_mean() simply calculates the exponentially weighted moving average value of the streaming *time series* data  $p_t$ :

$$\bar{p}_t = \lambda \bar{p}_{t-1} + (1 - \lambda)p_t = (1 - \lambda)\sum_{j=0}^n \lambda^j p_{t-j}$$

Some applications require applying additional weight factors, like for example the volume-weighted average price indicator (VWAP). Then the streaming prices can be multiplied by the streaming trading volumes.

If the argument weightv has the same number of rows as the argument tseries, then the function run\_mean() calculates the trailing weighted means in two steps.

First it calculates the trailing mean weights  $\bar{w}_t$ :

$$\bar{w}_t = \lambda \bar{w}_{t-1} + (1 - \lambda) w_t$$

Second it calculates the trailing mean products  $\bar{w}p_t$  of the weights  $w_t$  and the data  $p_t$ :

$$\bar{w}p_t = \lambda \bar{w}p_{t-1} + (1 - \lambda)w_t p_t$$

Where  $p_t$  is the streaming data,  $w_t$  are the streaming weights,  $\bar{w}_t$  are the trailing mean weights, and  $\bar{w}p_t$  are the trailing mean products of the data and the weights.

The trailing mean weighted value  $\bar{p}_t$  is equal to the ratio of the data and weights products, divided by the mean weights:

$$\bar{p}_t = \frac{\bar{w}p_t}{\bar{w}_t}$$

The above recursive formulas are convenient for processing live streaming data because they don't require maintaining a buffer of past data. The formulas are equivalent to a convolution with exponentially decaying weights, but they're much faster to calculate. Using exponentially decaying

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weights is more natural than using a sliding look-back interval, because it gradually "forgets" about the past data.

The value of the decay factor  $\lambda$  must be in the range between 0 and 1. If  $\lambda$  is close to 1 then the decay is weak and past values have a greater weight, and the trailing mean values have a stronger dependence on past data. This is equivalent to a long look-back interval. If  $\lambda$  is much less than 1 then the decay is strong and past values have a smaller weight, and the trailing mean values have a weaker dependence on past data. This is equivalent to a short look-back interval.

The function run\_mean() performs the same calculation as the standard R function stats::filter(x=series, filter=lambda, method="recursive"), but it's several times faster.

The function run\_mean() returns a *matrix* with the same dimensions as the input argument tseries.

### Value

A *matrix* with the same dimensions as the input argument tseries.

```
## Not run:
# Calculate historical prices
ohlc <- rutils::etfenv$VTI</pre>
closep <- quantmod::Cl(ohlc)</pre>
# Calculate the trailing means
lambda <- 0.95
meanv <- HighFreq::run_mean(closep, lambda=lambda, weightv = 0)</pre>
# Calculate trailing means using R code
filtered <- (1-lambda)*filter(closep,</pre>
  filter=lambda, init=as.numeric(closep[1, 1])/(1-lambda),
  method="recursive")
all.equal(drop(meanv), unclass(filtered), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::run_mean(closep, lambda=lambda, weightv = 0),
 Rcode=filter(closep, filter=lambda, init=as.numeric(closep[1, 1])/(1-lambda), method="recursive"),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
# Calculate weights equal to the trading volumes
weightv <- quantmod::Vo(ohlc)</pre>
# Calculate the trailing weighted means
meanw <- HighFreq::run_mean(closep, lambda=lambda, weightv=weightv)</pre>
# Plot dygraph of the trailing weighted means
datav <- xts(cbind(meanv, meanw), zoo::index(ohlc))</pre>
colnames(datav) <- c("means trailing", "means weighted")</pre>
dygraphs::dygraph(datav, main="Trailing Means") %>%
  dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
  dyLegend(show="always", width=500)
## End(Not run)
```

108 run\_min

run\_min

Calculate the trailing minimum values of streaming time series data.

### **Description**

Calculate the trailing minimum values of streaming time series data.

#### **Usage**

run\_min(tseries, lambda)

### **Arguments**

tseries A time series or a matrix.

lambda A decay factor which multiplies past estimates.

### **Details**

The function run\_min() calculates the trailing minimum values of streaming *time series* data by recursively weighting present and past values using the decay factor  $\lambda$ .

It calculates the trailing minimum values  $p_t^{min}$  of the streaming data  $p_t$  as follows:

$$p_t^{min} = min(p_t, \lambda p_{t-1}^{min} + (1 - \lambda)p_t)$$

The first term in the sum is the minimum value multiplied by the decay factor  $\lambda$ , so that the past minimum value is gradually "forgotten". The second term pulls the minimum value to the current value  $p_t$ .

The value of the decay factor  $\lambda$  must be in the range between 0 and 1. If  $\lambda$  is close to 1 then the past minimum values persist for longer. This is equivalent to a long look-back interval. If  $\lambda$  is much less than 1 then the past minimum values decay quickly, and the trailing minimum depends on the more recent streaming data. This is equivalent to a short look-back interval.

The above formula can also be expressed as:

$$p_t^{min} = \lambda min(p_t, p_{t-1}^{min}) + (1 - \lambda)p_t$$

The first term is the minimum value multiplied by the decay factor  $\lambda$ , so that the past minimum value is gradually "forgotten". The second term pulls the minimum value to the current value  $p_t$ .

The above recursive formula is convenient for processing live streaming data because it doesn't require maintaining a buffer of past data.

The function run\_min() returns a *matrix* with the same dimensions as the input argument tseries.

### Value

A matrix with the same dimensions as the input argument tseries.

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#### **Examples**

```
## Not run:
# Calculate historical prices
closep <- zoo::coredata(quantmod::Cl(rutils::etfenv$VTI))
# Calculate the trailing minimums
lambda <- 0.9
minv <- HighFreq::run_min(closep, lambda=lambda)
# Plot dygraph of VTI prices and trailing minimums
datav <- cbind(quantmod::Cl(rutils::etfenv$VTI), minv)
colnames(datav) <- c("prices", "min")
colnamev <- colnames(datav)
dygraphs::dygraph(datav, main="VTI Prices and Trailing Minimums") %>%
    dySeries(name=colnamev[1], label=colnamev[1], strokeWidth=2, col="blue") %>%
    dySeries(name=colnamev[2], label=colnamev[2], strokeWidth=2, col="red")
## End(Not run)
```

run\_portf

Simulate a portfolio optimization strategy using online (recursive) updating of the covariance matrix.

## **Description**

Simulate a portfolio optimization strategy using online (recursive) updating of the covariance matrix.

#### **Usage**

```
run_portf(rets, dimax, lambda, lambdacov, lambdaw)
```

# **Arguments**

rets A *time series* or *matrix* of asset returns.

dimax An integer equal to the number of eigen values used for calculating the regular-

ized inverse of the covariance matrix (the default is dimax = 0 - standard matrix

inverse using all the eigen values).

lambda A decay factor which multiplies the past asset returns.

lambdacov A decay factor which multiplies the past covariance.

lambdaw A decay factor which multiplies the past portfolio weights.

### **Details**

The function run\_portf() simulates a portfolio optimization strategy. The strategy calculates the maximum Sharpe portfolio weights *in-sample* at every point in time, and applies them in the *out-of-sample* time interval. It updates the trailing covariance matrix recursively, instead of using past batches of data. The function run\_portf() uses three different decay factors for averaging past values, to reduce the variance of its forecasts.

The function run\_portf() first scales the returns by their trailing volatilities:

$$r_t^s = \frac{r_t}{\sigma_{t-1}}$$

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Returns scaled by their volatility are more stationary so they're easier to model.

Then at every point in time, the function run\_portf() calls the function HighFreq::push\_covar() to update the trailing covariance matrix of the returns:

$$\bar{r}_t = \lambda_c \bar{r}_{t-1} + (1 - \lambda_c) r_t^s$$
$$\hat{r}_t = r_t^s - \bar{r}_t$$
$$cov_t = \lambda_c cov_{t-1} + (1 - \lambda_c) \hat{r}_t^T \hat{r}_t$$

Where  $\lambda_c$  is the decay factor which multiplies the past mean and covariance.

It then calls the function HighFreq::calc\_inv() to calculate the *regularized inverse* of the covariance matrix using its eigen decomposition:

$$C^{-1} = O_{dimax} \, \Sigma_{dimax}^{-1} \, O_{dimax}^T$$

See the function HighFreq::calc\_inv() for details.

It then calculates the *in-sample* weights of the maximum Sharpe portfolio, by multiplying the inverse covariance matrix times the trailing means of the asset returns:

$$\bar{r}_t = \lambda \bar{r}_{t-1} + (1 - \lambda)r_t^s$$
$$w_t = C^{-1}\bar{r}_t$$

Note that the decay factor  $\lambda$  is different from the decay factor  $\lambda_c$  used for updating the trailing covariance matrix.

It then scales the weights so their sum of squares is equal to one:

$$w_t = \frac{w_t}{\sqrt{\sum w_t^2}}$$

It then calculates the trailing mean of the weights:

$$\bar{w}_t = \lambda_w \bar{w}_{t-1} + (1 - \lambda_w) w_t$$

Note that the decay factor  $\lambda_w$  is different from the decay factor  $\lambda$  used for updating the trailing means.

It finally calculates the *out-of-sample* portfolio returns by multiplying the trailing mean weights times the scaled asset returns:

$$r_t^p = \bar{w}_{t-1} r_t^s$$

Applying weights to scaled returns means trading stock amounts with unit dollar volatility. So if the weight is equal to 2 then we should purchase an amount of stock with dollar volatility equal to 2 dollars. Trading stock amounts with unit dollar volatility improves portfolio diversification.

The function run\_portf() uses three different decay factors for averaging past values, to reduce the variance of its forecasts. The value of the decay factor  $\lambda$  must be in the range between 0 and 1. If  $\lambda$  is close to 1 then the decay is weak and past values have a greater weight, so the trailing values have a greater dependence on past data. This is equivalent to a long look-back interval. If  $\lambda$  is much less than 1 then the decay is strong and past values have a smaller weight, so the trailing values have a weaker dependence on past data. This is equivalent to a short look-back interval.

The function run\_portf() returns multiple columns of data, with the same number of rows as the input argument rets. The first column contains the strategy returns and the remaining columns contain the portfolio weights.

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#### Value

A *matrix* of strategy returns and the portfolio weights, with the same number of rows as the argument rets.

### **Examples**

```
## Not run:
# Load ETF returns
retp <- rutils::etfenv$returns[, c("VTI", "TLT", "DBC", "USO", "XLF", "XLK")]</pre>
retp <- na.omit(retp)</pre>
datev <- zoo::index(retp) # dates</pre>
# Simulate a portfolio optimization strategy
dimax <- 6
lambda <- 0.978
lambdacov <- 0.995
lambdaw <- 0.9
pnls <- HighFreq::run_portf(retp, dimax, lambda, lambdacov, lambdaw)</pre>
colnames(pnls) <- c("pnls", "VTI", "TLT", "DBC", "USO", "XLF", "XLK")</pre>
pnls <- xts::xts(pnls, order.by=datev)</pre>
# Plot dygraph of strategy
wealthv <- cbind(retp$VTI, pnls$pnls*sd(retp$VTI)/sd(pnls$pnls))</pre>
colnames(wealthv) <- c("VTI", "Strategy")</pre>
endd <- rutils::calc_endpoints(wealthv, interval="weeks")</pre>
dygraphs::dygraph(cumsum(wealthy)[endd], main="Portfolio Optimization Strategy Returns") %>%
 dyOptions(colors=c("blue", "red"), strokeWidth=2) %>%
 dyLegend(width=300)
# Plot dygraph of weights
symbolv <- "VTI"
stockweights <- cbind(cumsum(get(symbolv, retp)), get(symbolv, pnls))</pre>
colnames(stockweights)[2] <- "Weight"</pre>
colnamev <- colnames(stockweights)</pre>
endd <- rutils::calc_endpoints(pnls, interval="weeks")</pre>
dygraphs::dygraph(stockweights[endd], main="Returns and Weight") %>%
  dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
  dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
  dySeries(name=colnamev[1], axis="y", label=colnamev[1], strokeWidth=2, col="blue") %>%
dySeries(name=colnamev[2], axis="y2", label=colnamev[2], strokeWidth=2, col="red")
## End(Not run)
```

run\_reg

Calculate recursively the trailing regressions of streaming time series of response and predictor data, and calculate the residuals, alphas, and betas.

# **Description**

Calculate recursively the trailing regressions of streaming *time series* of response and predictor data, and calculate the residuals, alphas, and betas.

### Usage

```
run_reg(respv, predv, lambda, method = "none")
```

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### **Arguments**

respv A single-column *time series* or a single-column *matrix* of response data.

predv A *time series* or a *matrix* of predictor data.

lambda A decay factor which multiplies past estimates.

method A character string specifying the method for scaling the residuals (the default is

method = "none" - no scaling).

#### **Details**

The function run\_reg() calculates the vectors of alphas  $\alpha_t$ , betas  $\beta_t$ , and the residuals  $\epsilon_t$  of trailing regressions, by recursively weighting the current estimates with past estimates, using the decay factor  $\lambda$ :

$$\bar{r}_t = \lambda \bar{r}_{t-1} + (1 - \lambda)r_t$$

$$\bar{p}_t = \lambda \bar{p}_{t-1} + (1 - \lambda)p_t$$

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda)(p_t - \bar{p}_t)^T(p_t - \bar{p}_t)$$

$$cov_t = \lambda cov_{t-1} + (1 - \lambda)(r_t - \bar{r}_t)^T(p_t - \bar{p}_t)$$

$$\beta_t = \frac{cov_t}{\sigma_t^2}$$

$$\alpha_t = \bar{r}_t - \beta_t \bar{p}_t$$

$$\epsilon_t = r_t - \alpha_t - \beta_t p_t$$

$$\bar{\epsilon}_t = \lambda \bar{\epsilon}_{t-1} + (1 - \lambda)(\epsilon_t - \bar{\epsilon}_t)^2$$

Where  $r_t$  and  $p_t$  are the response and predictor data,  $cov_t$  is the covariance matrix between the response and the predictor data,  $\sigma_t^2$  is the covariance matrix of the predictors, and  $\varsigma_t^2$  is the residual variance.

The matrices  $\sigma^2$ , cov,  $\alpha$ , and  $\beta$  have the same number of rows as the predictor argument predv.

The function run\_reg() calculates the regressions with an intercept (constant) term. The vector of alphas  $\alpha_t$  is the intercept value.

The above recursive formulas are equivalent to a convolution with exponentially decaying weights, but they're much faster to calculate. The recursive formulas are convenient for processing live streaming data because they don't require maintaining a buffer of past data. Using exponentially decaying weights is more natural than using a sliding look-back interval, because it gradually "forgets" about the past data.

The value of the decay factor  $\lambda$  must be in the range between 0 and 1. If  $\lambda$  is close to 1 then the decay is weak and past values have a greater weight, so the trailing values have a greater dependence on past data. This is equivalent to a long look-back interval. If  $\lambda$  is much less than 1 then the decay is strong and past values have a smaller weight, so the trailing values have a weaker dependence on past data. This is equivalent to a short look-back interval.

The *residuals* may be scaled by their volatilities to obtain the *z-scores*. The default is method = "none" - no scaling. If the argument method = "scale" then the *residuals*  $\epsilon_t$  are divided by their volatilities  $\epsilon_t$  without subtracting their means:

$$\epsilon_t = \frac{\epsilon_t}{\varsigma_t}$$

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If the argument method = "standardize" then the residual means  $\bar{\epsilon}$  are subtracted from the *residuals*, and then they are divided by their volatilities  $\varsigma_t$ :

$$\epsilon_t = \frac{\epsilon_t - \bar{\epsilon}}{\varsigma_t}$$

Which are equal to the z-scores.

The function run\_reg() returns multiple columns of data, with the same number of rows as the input argument predv. If the predictor matrix predv has n columns then run\_reg() returns a matrix with n+2 columns. The first n columns contain the *betas*, the n+1 column the *alphas*, and the last column contains the *residuals*.

#### Value

A *matrix* with the regression betas, alphas, and the residuals (in that order - see details), with the same number of rows as the predictor argument predv.

### **Examples**

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("XLF", "VTI", "IEF")])</pre>
# Response equals XLF returns
respv <- retp[, 1]
# Predictor matrix equals VTI and IEF returns
predv <- retp[, -1]</pre>
# Calculate the trailing regressions
lambda <- 0.9
regs <- HighFreq::run_reg(respv=respv, predv=predv, lambda=lambda)</pre>
# Plot the trailing residuals
datav <- cbind(cumsum(respv), regs[, 5])</pre>
colnames(datav) <- c("XLF", "residuals")</pre>
colnamev <- colnames(datav)</pre>
dygraphs::dygraph(datav["2008/2009"], main="Residuals of XLF Versus VTI and IEF") %>%
  dyAxis("y", label=colnamev[1], independentTicks=TRUE) %>%
  dyAxis("y2", label=colnamev[2], independentTicks=TRUE) %>%
  dySeries(name=colnamev[1], axis="y", label=colnamev[1], strokeWidth=2, col="blue") %>%
  dySeries(name=colnamev[2], axis="y2", label=colnamev[2], strokeWidth=2, col="red")
## End(Not run)
```

run\_scale

Standardize (center and scale) the columns of a time series of data over time and in place, without copying the data in memory, using RcppArmadillo.

### **Description**

Standardize (center and scale) the columns of a *time series* of data over time and in place, without copying the data in memory, using RcppArmadillo.

### Usage

```
run_scale(tseries, lambda, center = TRUE, scale = TRUE)
```

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### **Arguments**

tseries A time series or matrix of data.

lambda A decay factor which multiplies past estimates.

center A Boolean argument: if TRUE then center the columns so that they have zero

mean or median (the default is TRUE).

scale A Boolean argument: if TRUE then scale the columns so that they have unit

standard deviation or MAD (the default is TRUE).

#### **Details**

The function run\_scale() performs a trailing standardization (centering and scaling) of the columns of the tseries argument using RcppArmadillo.

The function run\_scale() accepts a *pointer* to the argument tseries, and it overwrites the old data with the standardized data. It performs the calculation in place, without copying the data in memory, which can significantly increase the computation speed for large time series.

The function run\_scale() performs a loop over the rows of tseries, and standardizes the data using its trailing means and standard deviations.

The function run\_scale() calculates the trailing mean and variance of streaming *time series* data  $r_t$ , by recursively weighting the past estimates with the new data, using the decay factor  $\lambda$ :

$$\bar{r}_t = \lambda \bar{r}_{t-1} + (1 - \lambda)r_t$$

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda)(r_t - \bar{r}_t)^2$$

Where  $\bar{r}_t$  is the trailing mean and  $\sigma_t^2$  is the trailing variance.

It then calculates the standardized data as follows:

$$r_t' = \frac{r_t - \bar{r}_t}{\sigma_t}$$

If the arguments center and scale are both TRUE (the defaults), then calc\_scale() standardizes the data. If the argument center is FALSE then calc\_scale() only scales the data (divides it by the standard deviations). If the argument scale is FALSE then calc\_scale() only demeans the data (subtracts the means).

The value of the decay factor  $\lambda$  must be in the range between 0 and 1. If  $\lambda$  is close to 1 then the decay is weak and past values have a greater weight, and the trailing variance values have a stronger dependence on past data. This is equivalent to a long look-back interval. If  $\lambda$  is much less than 1 then the decay is strong and past values have a smaller weight, and the trailing variance values have a weaker dependence on past data. This is equivalent to a short look-back interval.

The above recursive formulas are convenient for processing live streaming data because they don't require maintaining a buffer of past data. The formulas are equivalent to a convolution with exponentially decaying weights, but they're much faster to calculate. Using exponentially decaying weights is more natural than using a sliding look-back interval, because it gradually "forgets" about the past data.

The function run\_scale() uses RcppArmadillo C++ code, so it can be over 100 times faster than the equivalent R code.

### Value

Void (no return value - modifies the data in place).

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#### **Examples**

```
## Not run:
# Calculate historical returns
retp <- na.omit(rutils::etfenv$returns[, c("XLF", "VTI")])</pre>
# Calculate the trailing standardized returns using R code
lambda <- 0.9
lambda1 <- 1 - lambda
scaled <- zoo::coredata(retp)</pre>
meanm <- scaled[1, ];</pre>
vars <- scaled[1, ]^2;</pre>
for (it in 2:NROW(retp)) {
  meanm <- lambda*meanm + lambda1*scaled[it, ];</pre>
  vars <- lambda*vars + lambda1*(scaled[it, ] - meanm)^2;</pre>
  scaled[it, ] <- (scaled[it, ] - meanm)/sqrt(vars)</pre>
} # end for
# Calculate the trailing standardized returns using C++ code
HighFreq::run_scale(retp, lambda=lambda)
all.equal(zoo::coredata(retp), scaled, check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::run_scale(retp, lambda=lambda),
  Rcode={for (it in 2:NROW(retp)) {
   meanm <- lambda*meanm + lambda1*scaled[it, ];</pre>
   vars <- lambda*vars + lambda1*(scaled[it, ] - meanm)^2;</pre>
   scaled[it, ] <- (scaled[it, ] - meanm)/sqrt(vars)</pre>
  }}, # end for
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

run\_var

Calculate the trailing variance of streaming time series of returns.

### **Description**

Calculate the trailing variance of streaming time series of returns.

# Usage

```
run_var(tseries, lambda)
```

### **Arguments**

tseries A time series or a matrix of returns.

lambda A decay factor which multiplies past estimates.

#### **Details**

The function run\_var() calculates the trailing variance of streaming *time series* of returns, by recursively weighting the past variance estimates  $\sigma_{t-1}^2$ , with the squared differences of the returns minus the trailing means  $(r_t - \bar{r}_t)^2$ , using the decay factor  $\lambda$ :

$$\bar{r}_t = \lambda \bar{r}_{t-1} + (1 - \lambda)r_t$$

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$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda)(r_t - \bar{r}_t)^2$$

Where  $\sigma_t^2$  is the variance estimate at time t, and  $r_t$  are the streaming returns data.

The above recursive formulas are convenient for processing live streaming data because they don't require maintaining a buffer of past data. The formulas are equivalent to a convolution with exponentially decaying weights, but they're much faster to calculate. Using exponentially decaying weights is more natural than using a sliding look-back interval, because it gradually "forgets" about the past data.

The value of the decay factor  $\lambda$  must be in the range between 0 and 1. If  $\lambda$  is close to 1 then the decay is weak and past values have a greater weight, and the trailing variance values have a stronger dependence on past data. This is equivalent to a long look-back interval. If  $\lambda$  is much less than 1 then the decay is strong and past values have a smaller weight, and the trailing variance values have a weaker dependence on past data. This is equivalent to a short look-back interval.

The function run\_var() performs the same calculation as the standard R function stats::filter(x=series, filter=weightv, method="recursive"), but it's several times faster.

The function run\_var() returns a matrix with the same dimensions as the input argument tseries.

### Value

A *matrix* with the same dimensions as the input argument tseries.

## **Examples**

```
## Not run:
# Calculate historical returns
retp <- zoo::coredata(na.omit(rutils::etfenv$returns$VTI))</pre>
# Calculate the trailing variance
lambda <- 0.9
vars <- HighFreq::run_var(retp, lambda=lambda)</pre>
# Calculate centered returns
retc <- (retp - HighFreq::run_mean(retp, lambda=lambda))</pre>
# Calculate trailing variance using R code
filtered <- (1-lambda)*filter(retc^2, filter=lambda,</pre>
  init=as.numeric(retc[1, 1])^2/(1-lambda),
  method="recursive")
all.equal(vars, unclass(filtered), check.attributes=FALSE)
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)
summary(microbenchmark(
  Rcpp=HighFreq::run_var(retp, lambda=lambda),
 Rcode=filter(retc^2, filter=lambda, init=as.numeric(retc[1, 1])^2/(1-lambda), method="recursive"),
  times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

run\_var\_ohlc

Calculate the trailing variance of streaming OHLC price data.

### **Description**

Calculate the trailing variance of streaming *OHLC* price data.

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### Usage

```
run_var_ohlc(ohlc, lambda)
```

#### **Arguments**

ohlc A *time series* or a *matrix* with *OHLC* price data.

lambda A decay factor which multiplies past estimates.

### **Details**

The function run\_var\_ohlc() calculates a single-column *matrix* of variance estimates of streaming *OHLC* price data.

The function run\_var\_ohlc() calculates the variance from the differences between the *Open*, *High*, *Low*, and *Close* prices, using the *Yang-Zhang* range volatility estimator:

$$\sigma_t^2 = (1 - \lambda)((O_t - C_{t-1})^2 + 0.134(C_t - O_t)^2 + 0.866((H_i - O_i)(H_i - C_i) + (L_i - O_i)(L_i - C_i))) + \lambda \sigma_{t-1}^2 + 0.134(C_t - O_t)^2 + 0.13$$

It recursively weighs the current variance estimate with the past estimates  $\sigma_{t-1}^2$ , using the decay factor  $\lambda$ .

The above recursive formula is convenient for processing live streaming data because it doesn't require maintaining a buffer of past data. The formula is equivalent to a convolution with exponentially decaying weights, but it's faster. Using exponentially decaying weights is more natural than using a sliding look-back interval, because it gradually "forgets" about the past data.

The function run\_var\_ohlc() does not calculate the logarithm of the prices. So if the argument ohlc contains dollar prices then run\_var\_ohlc() calculates the dollar variance. If the argument ohlc contains the log prices then run\_var\_ohlc() calculates the percentage variance.

The function run\_var\_ohlc() is implemented in RcppArmadillo C++ code, which makes it several times faster than R code.

#### Value

A single-column *matrix* of variance estimates, with the same number of rows as the input ohlc price data.

```
## Not run:
# Extract the log OHLC prices of VTI
ohlc <- log(rutils::etfenv$VTI)</pre>
# Calculate the trailing variance
vart <- HighFreq::run_var_ohlc(ohlc, lambda=0.8)</pre>
# Calculate the rolling variance
varol1 <- HighFreq::roll_var_ohlc(ohlc, look_back=5, method="yang_zhang", scale=FALSE)</pre>
datav <- cbind(vart, varoll)</pre>
colnames(datav) <- c("trailing", "rolling")</pre>
colnamev <- colnames(datav)</pre>
datav <- xts::xts(datav, index(ohlc))</pre>
# dygraph plot of VTI trailing versus rolling volatility
dygraphs::dygraph(sqrt(datav[-(1:111), ]), main="Trailing and Rolling Volatility of VTI") %>%
  dyOptions(colors=c("red", "blue"), strokeWidth=2) %>%
  dyLegend(show="always", width=500)
# Compare the speed of trailing versus rolling volatility
library(microbenchmark)
```

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```
summary(microbenchmark(
   trailing=HighFreq::run_var_ohlc(ohlc, lambda=0.8),
   rolling=HighFreq::roll_var_ohlc(ohlc, look_back=5, method="yang_zhang", scale=FALSE),
   times=10))[, c(1, 4, 5)]
## End(Not run)
```

save\_rets

Load, scrub, aggregate, and rbind multiple days of TAQ data for a single symbol. Calculate returns and save them to a single '\*.RData' file.

# **Description**

Load, scrub, aggregate, and rbind multiple days of TAQ data for a single symbol. Calculate returns and save them to a single '\*.RData' file.

## Usage

```
save_rets(
  symbol,
  data_dir = "E:/mktdata/sec/",
  output_dir = "E:/output/data/",
  look_back = 51,
  vol_mult = 2,
  period = "minutes",
  tzone = "America/New_York"
)
```

#### **Details**

The function save\_rets loads multiple days of TAQ data, then scrubs, aggregates, and rbinds them into a OHLC time series. It then calculates returns using function ohlc\_returns(), and stores them in a variable named 'symbol.rets', and saves them to a file called 'symbol.rets.RData'. The TAQ data files are assumed to be stored in separate directories for each 'symbol'. Each 'symbol' has its own directory (named 'symbol') in the 'data\_dir' directory. Each 'symbol' directory contains multiple daily '\*.RData' files, each file containing one day of TAQ data.

### Value

A time series of returns and volume in xts format.

```
## Not run:
save_rets("SPY")
## End(Not run)
```

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save\_rets\_ohlc

Load OHLC time series data for a single symbol, calculate its returns, and save them to a single '\*.RData' file, without aggregation.

### **Description**

Load *OHLC* time series data for a single symbol, calculate its returns, and save them to a single '\*.RData' file, without aggregation.

## Usage

```
save_rets_ohlc(
  symbol,
  data_dir = "E:/output/data/",
  output_dir = "E:/output/data/")
```

### **Details**

The function save\_rets\_ohlc() loads *OHLC* time series data from a single file. It then calculates returns using function ohlc\_returns(), and stores them in a variable named 'symbol.rets', and saves them to a file called 'symbol.rets.RData'.

### Value

A time series of returns and volume in xts format.

# **Examples**

```
## Not run:
save_rets_ohlc("SPY")
## End(Not run)
```

save\_scrub\_agg

Load, scrub, aggregate, and rbind multiple days of TAQ data for a single symbol, and save the OHLC time series to a single '\*.RData' file.

# Description

Load, scrub, aggregate, and rbind multiple days of TAQ data for a single symbol, and save the OHLC time series to a single '\*.RData' file.

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#### Usage

```
save_scrub_agg(
   symbol,
   data_dir = "E:/mktdata/sec/",
   output_dir = "E:/output/data/",
   look_back = 51,
   vol_mult = 2,
   period = "minutes",
   tzone = "America/New_York"
)
```

### **Arguments**

symbol A *character* string representing symbol or ticker.

data\_dir A *character* string representing directory containing input '\*.RData' files. output\_dir A *character* string representing directory containing output '\*.RData' files.

### **Details**

The function save\_scrub\_agg() loads multiple days of TAQ data, then scrubs, aggregates, and rbinds them into a OHLC time series, and finally saves it to a single '\*.RData' file. The OHLC time series is stored in a variable named 'symbol', and then it's saved to a file named 'symbol.RData' in the 'output\_dir' directory. The TAQ data files are assumed to be stored in separate directories for each 'symbol'. Each 'symbol' has its own directory (named 'symbol') in the 'data\_dir' directory. Each 'symbol' directory contains multiple daily '\*.RData' files, each file containing one day of TAQ data.

#### Value

An *OHLC* time series in *xts* format.

## **Examples**

```
## Not run:
# set data directories
data_dir <- "C:/Develop/data/hfreq/src/"
output_dir <- "C:/Develop/data/hfreq/scrub/"
symbol <- "SPY"
# Aggregate SPY TAQ data to 15-min OHLC bar data, and save the data to a file
save_scrub_agg(symbol=symbol, data_dir=data_dir, output_dir=output_dir, period="15 min")
## End(Not run)</pre>
```

save\_taq

Load and scrub multiple days of TAQ data for a single symbol, and save it to multiple '\*.RData' files.

### **Description**

Load and scrub multiple days of *TAQ* data for a single symbol, and save it to multiple '\*.RData' files.

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### Usage

```
save_taq(
   symbol,
   data_dir = "E:/mktdata/sec/",
   output_dir = "E:/output/data/",
   look_back = 51,
   vol_mult = 2,
   tzone = "America/New_York"
)
```

### **Details**

The function save\_taq() loads multiple days of TAQ data, scrubs it, and saves the scrubbed TAQ data to individual '\*.RData' files. It uses the same file names for output as the input file names. The TAQ data files are assumed to be stored in separate directories for each 'symbol'. Each 'symbol' has its own directory (named 'symbol') in the 'data\_dir' directory. Each 'symbol' directory contains multiple daily '\*.RData' files, each file containing one day of TAQ data.

### Value

a TAQ time series in xts format.

# **Examples**

```
## Not run:
save_taq("SPY")
## End(Not run)
```

scrub\_agg

Scrub a single day of TAQ data, aggregate it, and convert to OHLC format.

## **Description**

Scrub a single day of TAQ data, aggregate it, and convert to OHLC format.

## Usage

```
scrub_agg(
  taq,
  look_back = 51,
  vol_mult = 2,
  period = "minutes",
  tzone = "America/New_York"
)
```

### **Arguments**

period

The aggregation period.

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#### **Details**

The function scrub\_agg() performs:

- index timezone conversion,
- data subset to trading hours,
- removal of duplicate time stamps,
- scrubbing of quotes with suspect bid-offer spreads,
- scrubbing of quotes with suspect price jumps,
- cbinding of mid prices with volume data,
- aggregation to OHLC using function to.period() from package xts,

Valid 'period' character strings include: "minutes", "3 min", "5 min", "10 min", "15 min", "30 min", and "hours". The time index of the output time series is rounded up to the next integer multiple of 'period'.

#### Value

A OHLC time series in xts format.

#### **Examples**

```
# Create random TAQ prices
taq <- HighFreq::random_taq()
# Aggregate to ten minutes OHLC data
ohlc <- HighFreq::scrub_agg(taq, period="10 min")
chart_Series(ohlc, name="random prices")
# scrub and aggregate a single day of SPY TAQ data to OHLC
ohlc <- HighFreq::scrub_agg(taq=HighFreq::SPY_TAQ)
chart_Series(ohlc, name=symbol)</pre>
```

scrub\_taq

Scrub a single day of TAQ data in xts format, without aggregation.

# Description

Scrub a single day of TAQ data in xts format, without aggregation.

### Usage

```
scrub_taq(taq, look_back = 51, vol_mult = 2, tzone = "America/New_York")
```

# **Arguments**

taq TAQ A time series in xts format.

tzone The timezone to convert.

## **Details**

The function scrub\_taq() performs the same scrubbing operations as scrub\_agg, except it doesn't aggregate, and returns the *TAQ* data in *xts* format.

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#### Value

A TAQ time series in xts format.

#### **Examples**

```
taq <- HighFreq::scrub_taq(taq=HighFreq::SPY_TAQ, look_back=11, vol_mult=1)
# Create random TAQ prices and scrub them
taq <- HighFreq::random_taq()
taq <- HighFreq::scrub_taq(taq=taq)
taq <- HighFreq::scrub_taq(taq=taq, look_back=11, vol_mult=1)</pre>
```

season\_ality

Perform seasonality aggregations over a single-column xts time series.

## **Description**

Perform seasonality aggregations over a single-column xts time series.

## Usage

```
season_ality(xtsv, indeks = format(zoo::index(xtsv), "%H:%M"))
```

### **Arguments**

xtsv A single-column xts time series.

indeks A vector of *character* strings representing points in time, of the same length as

the argument xtsv.

### **Details**

The function season\_ality() calculates the mean of values observed at the same points in time specified by the argument indeks. An example of a daily seasonality aggregation is the average price of a stock between 9:30AM and 10:00AM every day, over many days. The argument indeks is passed into function tapply(), and must be the same length as the argument xtsv.

## Value

An xts time series with mean aggregations over the seasonality interval.

```
# Calculate running variance of each minutely OHLC bar of data
xtsv <- ohlc_variance(HighFreq::SPY)
# Remove overnight variance spikes at "09:31"
indeks <- format(index(xtsv), "%H:%M")
xtsv <- xtsv[!indeks=="09:31", ]
# Calculate daily seasonality of variance
var_seasonal <- season_ality(xtsv=xtsv)
chart_Series(x=var_seasonal, name=paste(colnames(var_seasonal),
    "daily seasonality of variance"))</pre>
```

124 sim\_ar

| sim_ar | Simulate autoregressive returns by recursively filtering a matrix of in- |
|--------|--|
|        | novations through a matrix of autoregressive coefficients.               |

### **Description**

Simulate *autoregressive* returns by recursively filtering a *matrix* of innovations through a *matrix* of *autoregressive* coefficients.

### Usage

```
sim_ar(coeff, innov)
```

### **Arguments**

innov A single-column *matrix* of innovations.

coeff A single-column matrix of autoregressive coefficients.

#### **Details**

The function sim\_ar() recursively filters the *matrix* of innovations innov through the *matrix* of *autoregressive* coefficients coeff, using fast RcppArmadillo C++ code.

The function  $sim_ar()$  simulates an *autoregressive* process AR(n) of order n:

$$r_i = \varphi_1 r_{i-1} + \varphi_2 r_{i-2} + \ldots + \varphi_n r_{i-n} + \xi_i$$

Where  $r_i$  is the simulated output time series,  $\varphi_i$  are the *autoregressive* coefficients, and  $\xi_i$  are the standard normal *innovations*.

The order n of the *autoregressive* process AR(n), is equal to the number of rows of the *autoregressive* coefficients coeff.

The function sim\_ar() performs the same calculation as the standard R function filter(x=innov, filter=coeff, method="recursive"), but it's several times faster.

### Value

A single-column matrix of simulated returns, with the same number of rows as the argument innov.

```
## Not run:
# Define AR coefficients
coeff <- matrix(c(0.1, 0.3, 0.5))
# Calculate matrix of innovations
innov <- matrix(rnorm(1e4, sd=0.01))
# Calculate recursive filter using filter()
filtered <- filter(innov, filter=coeff, method="recursive")
# Calculate recursive filter using RcppArmadillo
retp <- HighFreq::sim_ar(coeff, innov)
# Compare the two methods
all.equal(as.numeric(retp), as.numeric(filtered))
# Compare the speed of RcppArmadillo with R code
library(microbenchmark)</pre>
```

sim\_df

```
summary(microbenchmark(
   Rcpp=HighFreq::sim_ar(coeff, innov),
   Rcode=filter(innov, filter=coeff, method="recursive"),
   times=10))[, c(1, 4, 5)] # end microbenchmark summary
## End(Not run)
```

sim\_df

Simulate a Dickey-Fuller process using Rcpp.

### **Description**

Simulate a *Dickey-Fuller* process using *Rcpp*.

## Usage

```
sim_df(init_price, eq_price, theta, coeff, innov)
```

# Arguments

init\_price The initial price.

eq\_price The equilibrium price.

theta The strength of mean reversion.

coeff A single-column *matrix* of *autoregressive* coefficients.

innov A single-column *matrix* of innovations (random numbers).

#### **Details**

The function sim\_df() simulates the following *Dickey-Fuller* process:

$$r_i = \theta \left( \mu - p_{i-1} \right) + \varphi_1 r_{i-1} + \ldots + \varphi_n r_{i-n} + \xi_i$$
$$p_i = p_{i-1} + r_i$$

Where  $r_i$  and  $p_i$  are the simulated returns and prices,  $\theta$  and  $\mu$  are the *Ornstein-Uhlenbeck* parameters,  $\varphi_i$  are the *autoregressive* coefficients, and  $\xi_i$  are the normal *innovations*. The recursion starts with:  $r_1 = \xi_1$  and  $p_1 = init\_price$ .

The Dickey-Fuller process is a combination of an Ornstein-Uhlenbeck process and an autoregressive process. The order n of the autoregressive process AR(n), is equal to the number of rows of the autoregressive coefficients coeff.

The function sim\_df() simulates the *Dickey-Fuller* process using fast *Rcpp* C++ code.

The function sim\_df() returns a single-column matrix representing the time series of prices.

# Value

A single-column matrix of simulated prices, with the same number of rows as the argument innov.

126 sim\_garch

#### **Examples**

```
## Not run:
# Define the Ornstein-Uhlenbeck model parameters
init_price <- 1.0</pre>
eq_price <- 2.0
thetav <- 0.01
# Define AR coefficients
coeff <- matrix(c(0.1, 0.3, 0.5))
# Calculate matrix of standard normal innovations
innov <- matrix(rnorm(1e3, sd=0.01))</pre>
# Simulate Dickey-Fuller process using Rcpp
prices <- HighFreq::sim_df(init_price=init_price, eq_price=eq_price, theta=thetav, coeff=coeff, innov=innov)</pre>
plot(prices, t="1", main="Simulated Dickey-Fuller Prices")
## End(Not run)
```

Simulate or estimate the rolling variance under a GARCH(1,1) prosim\_garch cess using Rcpp.

### **Description**

Simulate or estimate the rolling variance under a GARCH(1,1) process using Rcpp.

## Usage

```
sim_garch(omega, alpha, beta, innov, is_random = TRUE)
```

# **Arguments**

omega Parameter proportional to the long-term average level of variance. alpha The weight associated with recent realized variance updates. The weight associated with the past variance estimates. beta A single-column *matrix* of innovations. innov Boolean argument: Are the innovations random numbers or historical returns? is\_random

(The default is is\_random = TRUE.)

### **Details**

The function  $sim_garch()$  simulates or estimates the rolling variance under a GARCH(1,1) process using Rcpp.

If is\_random = TRUE (the default) then the innovations innov are treated as random numbers  $\xi_i$  and the GARCH(1,1) process is given by:

$$r_i = \sigma_{i-1} \xi_i$$
 
$$\sigma_i^2 = \omega + \alpha r_i^2 + \beta \sigma_{i-1}^2$$

Where  $r_i$  and  $\sigma_i^2$  are the simulated returns and variance, and  $\omega$ ,  $\alpha$ , and  $\beta$  are the GARCH parameters, and  $\xi_i$  are standard normal *innovations*.

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The long-term equilibrium level of the simulated variance is proportional to the parameter  $\omega$ :

$$\sigma^2 = \frac{\omega}{1 - \alpha - \beta}$$

So the sum of  $\alpha$  plus  $\beta$  should be less than 1, otherwise the volatility becomes explosive.

If is\_random = FALSE then the function  $sim_garch()$  estimates the rolling variance from the historical returns. The innovations innov are equal to the historical returns  $r_i$  and the GARCH(1,1) process is simply:

$$\sigma_i^2 = \omega + \alpha r_i^2 + \beta \sigma_{i-1}^2$$

Where  $\sigma_i^2$  is the rolling variance.

The above should be viewed as a formula for *estimating* the rolling variance from the historical returns, rather than simulating them. It represents exponential smoothing of the squared returns with a decay factor equal to  $\beta$ .

The function sim\_garch() simulates the *GARCH* process using fast *Rcpp* C++ code.

#### Value

A *matrix* with two columns and with the same number of rows as the argument innov. The first column are the simulated returns and the second column is the variance.

### **Examples**

```
## Not run:
# Define the GARCH model parameters
alpha <- 0.79
betav <- 0.2
om_ega <- 1e-4*(1-alpha-betav)
# Calculate matrix of standard normal innovations
innov <- matrix(rnorm(1e3))</pre>
# Simulate the GARCH process using Rcpp
garch_data <- HighFreq::sim_garch(omega=om_ega, alpha=alpha, beta=betav, innov=innov)</pre>
# Plot the GARCH rolling volatility and cumulative returns
plot(sqrt(garch_data[, 2]), t="1", main="Simulated GARCH Volatility", ylab="volatility")
plot(cumsum(garch_data[, 1]), t="1", main="Simulated GARCH Cumulative Returns", ylab="cumulative returns")
# Calculate historical VTI returns
retp <- na.omit(rutils::etfenv$returns$VTI)</pre>
# Estimate the GARCH volatility of VTI returns
garch_data <- HighFreq::sim_garch(omega=om_ega, alpha=alpha, beta=betav,</pre>
  innov=retp, is_random=FALSE)
# Plot dygraph of the estimated GARCH volatility
dygraphs::dygraph(xts::xts(sqrt(garch_data[, 2]), index(retp)),
  main="Estimated GARCH Volatility of VTI")
## End(Not run)
```

sim\_ou

Simulate an Ornstein-Uhlenbeck process using Rcpp.

### **Description**

Simulate an Ornstein-Uhlenbeck process using Rcpp.

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#### Usage

```
sim_ou(init_price, eq_price, theta, innov)
```

#### **Arguments**

init\_price The initial price.

eq\_price The equilibrium price.

theta The strength of mean reversion.

innov A single-column *matrix* of innovations (random numbers).

### **Details**

The function sim\_ou() simulates the following *Ornstein-Uhlenbeck* process:

$$r_i = p_i - p_{i-1} = \theta (\mu - p_{i-1}) + \xi_i$$
  
 $p_i = p_{i-1} + r_i$ 

Where  $r_i$  and  $p_i$  are the simulated returns and prices,  $\theta$ ,  $\mu$ , and  $\sigma$  are the *Ornstein-Uhlenbeck* parameters, and  $\xi_i$  are the standard *innovations*. The recursion starts with:  $r_1 = \xi_1$  and  $p_1 = init\_price$ .

The function  $sim_ou()$  simulates the percentage returns as equal to the difference between the equilibrium price  $\mu$  minus the latest price  $p_{i-1}$ , times the mean reversion parameter  $\theta$ , plus a random normal innovation. The log prices are calculated as the sum of returns (not compounded), so they can become negative.

The function sim\_ou() simulates the *Ornstein-Uhlenbeck* process using fast *Rcpp* C++ code.

The function sim\_ou() returns a single-column *matrix* representing the *time series* of simulated prices.

### Value

A single-column *matrix* of simulated prices, with the same number of rows as the argument innov.

```
## Not run:
# Define the Ornstein-Uhlenbeck model parameters
init_price <- 0.0
eq_price <- 1.0
sigmav <- 0.01
thetav <- 0.01
innov <- matrix(rnorm(1e3))
# Simulate Ornstein-Uhlenbeck process using Rcpp
prices <- HighFreq::sim_ou(init_price=init_price, eq_price=eq_price, volat=sigmav, theta=thetav, innov=innov
plot(prices, t="l", main="Simulated Ornstein-Uhlenbeck Prices", ylab="prices")
## End(Not run)</pre>
```

sim\_schwartz 129

| sim_schwartz | Simulate a Schwartz process using Rcpp. |
|--------------|---|
|--------------|---|

### **Description**

Simulate a Schwartz process using Rcpp.

# Usage

```
sim_schwartz(init_price, eq_price, theta, innov)
```

### **Arguments**

 $\verb|init_price| The initial price|.$ 

eq\_price The equilibrium price.

theta The strength of mean reversion.

innov A single-column *matrix* of innovations (random numbers).

## **Details**

The function sim\_schwartz() simulates a *Schwartz* process using fast *Rcpp* C++ code.

The *Schwartz* process is the exponential of the *Ornstein-Uhlenbeck* process, and similar comments apply to it. The prices are calculated as the exponentially compounded returns, so they are never negative. The log prices can be obtained by taking the logarithm of the prices.

The function sim\_schwartz() simulates the percentage returns as equal to the difference between the equilibrium price  $\mu$  minus the latest price  $p_{i-1}$ , times the mean reversion parameter  $\theta$ , plus a random normal innovation.

The function sim\_schwartz() returns a single-column *matrix* representing the *time series* of simulated prices.

# Value

A single-column matrix of simulated prices, with the same number of rows as the argument innov.

```
## Not run:
# Define the Schwartz model parameters
init_price <- 1.0
eq_price <- 2.0
thetav <- 0.01
innov <- matrix(rnorm(1e3, sd=0.01))
# Simulate Schwartz process using Rcpp
prices <- HighFreq::sim_schwartz(init_price=init_price, eq_price=eq_price, theta=thetav, innov=innov)
plot(prices, t="1", main="Simulated Schwartz Prices", ylab="prices")
## End(Not run)</pre>
```

130 which\_extreme

| which_extreme Calculate a Boolean vector that identifies extreme tail values in a single-column xts time series or vector, over a rolling look-back interval. |  |
|---|--|
|---|--|

#### **Description**

Calculate a *Boolean* vector that identifies extreme tail values in a single-column *xts* time series or vector, over a rolling look-back interval.

### Usage

```
which_extreme(xtsv, look_back = 51, vol_mult = 2)
```

## **Arguments**

xtsv A single-column xts time series, or a numeric or Boolean vector.

look\_back The number of data points in rolling look-back interval for estimating rolling

quantile.

vol\_mult The quantile multiplier.

#### Details

The function which\_extreme() calculates a *Boolean* vector, with TRUE for values that belong to the extreme tails of the distribution of values.

The function which\_extreme() applies a version of the Hampel median filter to identify extreme values, but instead of using the median absolute deviation (MAD), it uses the  $\emptyset.9$  quantile values calculated over a rolling look-back interval.

Extreme values are defined as those that exceed the product of the multiplier times the rolling quantile. Extreme values belong to the fat tails of the recent (trailing) distribution of values, so they are present only when the trailing distribution of values has fat tails. If the trailing distribution of values is closer to normal (without fat tails), then there are no extreme values.

The quantile multiplier vol\_mult controls the threshold at which values are identified as extreme. Smaller quantile multiplier values will cause more values to be identified as extreme.

## Value

A Boolean vector with the same number of rows as the input time series or vector.

```
# Create local copy of SPY TAQ data
taq <- HighFreq::SPY_TAQ
# scrub quotes with suspect bid-offer spreads
bid_offer <- taq[, "Ask.Price"] - taq[, "Bid.Price"]
sus_pect <- which_extreme(bid_offer, look_back=51, vol_mult=3)
# Remove suspect values
taq <- taq[!sus_pect]</pre>
```

which\_jumps 131

| which_jumps | Calculate a Boolean vector that identifies isolated jumps (spikes) in a single-column xts time series or vector, over a rolling interval. |
|-------------|---|
|             |   |

### **Description**

Calculate a *Boolean* vector that identifies isolated jumps (spikes) in a single-column *xts* time series or vector, over a rolling interval.

#### **Usage**

```
which_jumps(xtsv, look_back = 51, vol_mult = 2)
```

#### **Details**

The function which\_jumps() calculates a *Boolean* vector, with TRUE for values that are isolated jumps (spikes).

The function which\_jumps() applies a version of the Hampel median filter to identify jumps, but instead of using the median absolute deviation (MAD), it uses the 0.9 quantile of returns calculated over a rolling interval. This is in contrast to function which\_extreme(), which applies a Hampel filter to the values themselves, instead of the returns. Returns are defined as simple differences between neighboring values.

Jumps (or spikes), are defined as isolated values that are very different from the neighboring values, either before or after. Jumps create pairs of large neighboring returns of opposite sign.

Jumps (spikes) must satisfy two conditions:

- 1. Neighboring returns both exceed a multiple of the rolling quantile,
- 2. The sum of neighboring returns doesn't exceed that multiple.

The quantile multiplier vol\_mult controls the threshold at which values are identified as jumps. Smaller quantile multiplier values will cause more values to be identified as jumps.

# Value

A *Boolean* vector with the same number of rows as the input time series or vector.

```
# Create local copy of SPY TAQ data
taq <- SPY_TAQ
# Calculate mid prices
mid_prices <- 0.5 * (taq[, "Bid.Price"] + taq[, "Ask.Price"])
# Replace whole rows containing suspect price jumps with NA, and perform locf()
taq[which_jumps(mid_prices, look_back=31, vol_mult=1.0), ] <- NA
taq <- xts:::na.locf.xts(taq)</pre>
```