Introduction to Physics Approximating Neural Networks (PANN) for Virtual Analog Emulation of Nonlinear Components

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AUDIO ENGINEERING CONVENTION

Preamble & Introduction

- Most, if not all, components inherently exert system insertion behaviour
    - 1st Order: Swapping out a (differently valued/rated) component results in a change in behaviour of system output

    - Higher Order Change: Swapping out a component for an equivalently rated (resistance, capacitance, etc..) results in a change in behaviour of system output that might not be immediately obvious.
        - Transformer materials: Nickel, steel, iron, amorphous metals, and ferrite ceramics
        - Resistor materials: Carbon, Metal film, metalized materials …
        - Capacitor materials: Electrode & Dielectric, (paper-in-oil + tin foil / paper-in-wax + aluminium foil)

- Anecdotally, there have been accounts of people hearing these differences

Preamble & Introduction

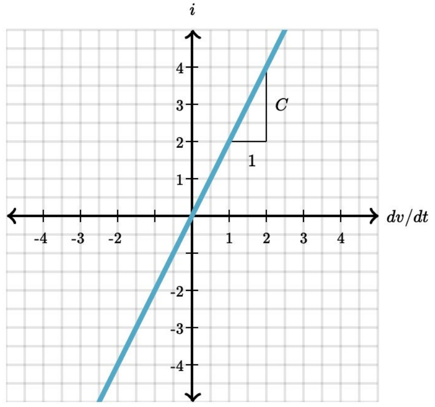
The textbook capacitor is always assumed to be a linear comp

$$i = f\left(\frac{dv}{dt}\right) = \mathrm{C}\,\frac{dv}{dt}$$

Where graphically, if we plot the rate of change of voltage against current,

C = slope

Adheres to scaling (homogeneity)



https://www.khanacademy.org/science/electrical-engineering/ee-circuit-analysis-topic/ee-dc-circuit-analysis/a/ee-linearity

Preamble & Introduction

We have published the results covered in this presentation in:

AES 150th Convention (2021)

- Characterising non-linear behaviour of coupling capacitors through audio feature analysis and machine learning.

ACM SIGGRAPH Research in Adaptive and Convergent Systems

- Computationally efficient physics approximating neural networks for highly nonlinear maps

Audio Developer Conference 2022:

- Pipeline for VA Modelling with Physics-Informed Machine Learning

Clarke, Christopher Johann, Balamurali BT, and Jer-Ming Chen. "Characterising non-linear behaviour of coupling capacitors through audio feature analysis and machine learning." Audio Engineering Society Convention 150. Audio Engineering Society, 2021.

Clarke, Christopher Johann, et al. "Computationally efficient physics approximating neural networks for highly nonlinear maps." *Proceedings of the Conference on Research in Adaptive and Convergent Systems*. 2022.

Clarke, Christopher Johann, Chowdhury, Jatin. "Pipeline for VA Modelling with Physics-Informed Machine Learning" Audio Developer Conference 2022

Preamble & Introduction

THD = Total Harmonic Distortion

SFDR = Spurious Free Dynamic Range

SINAD = Signal-to-Noise-and-Distortion

SNR = Signal-to-Noise

All normalised using Kaiser-Bessel window function.

$$THD = \frac{\sqrt{(P_{F2})^2 + (P_{F3})^2 + (P_{F4})^2 + (P_{F5})^2 + (P_{F6})^2}}{P_{F1}}, \quad (1)$$

where F_1 is the fundamental, P_{F1} is the power of the fundamental, and n in F_n identifies the n^{th} harmonic,

$$SFDR = \frac{\sqrt{(P_{Fx})^2}}{P_{F1}}, \quad (2)$$

where P_{F1} is the power of the fundamental and P_{Fx} is the power of the greatest non-harmonic component,

$$SINAD = 10log_{10}\frac{P_{F1} + P_N + P_D}{P_{F1}}, \quad (3)$$

where P_{F1} is the power of the fundamental, P_N is the power of noise, and P_D is the power of all harmonics of the signal, and

$$SNR = \frac{P_{signal}}{P_N}, \quad (4)$$

where P_{signal} is the power of the signal and P_N is the power of noise.

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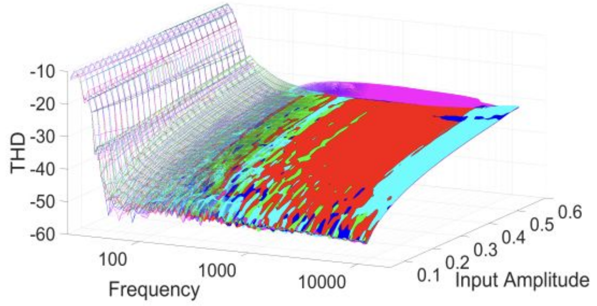
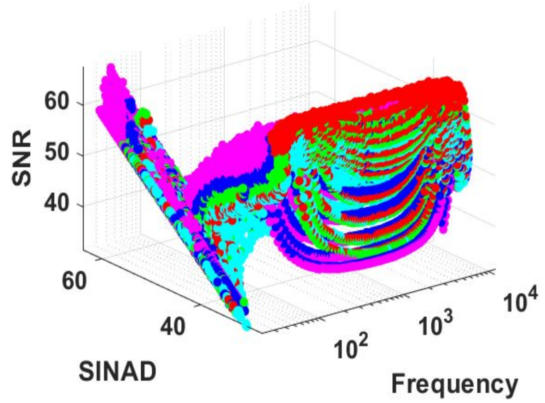
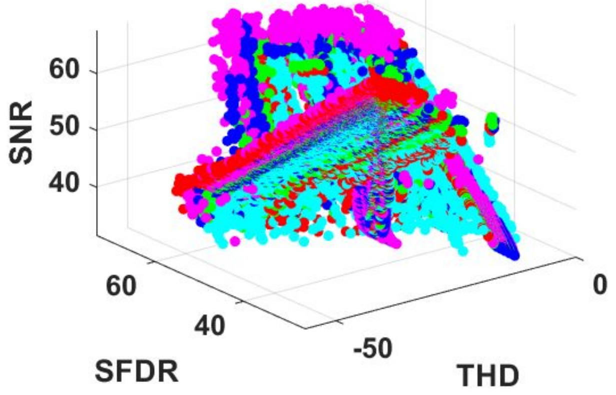
Clarke, Christopher Johann, Balamurali BT, and Jer-Ming Chen. "Characterising non-linear behaviour of coupling capacitors through audio feature analysis and machine learning." Audio Engineering Society Convention 150. Audio Engineering Society, 2021.

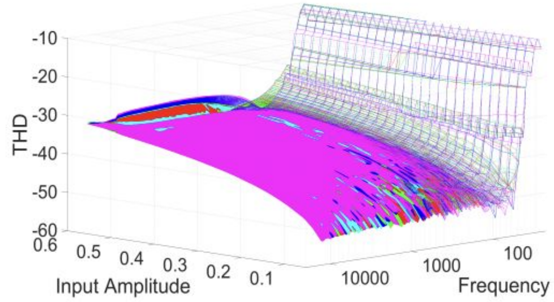
Preamble & Introduction

Characterising non-linear behaviour of coupling capacitors thr
analysis and machine learning. (static response)









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Clarke, Christopher Johann, Balamurali BT, and Jer-Ming Chen. "Characterising non-linear behaviour of coupling capacitors through audio feature analysis and machine learning." Audio Engineering Society Convention 150. Audio Engineering Society, 2021.

Preamble & Introduction

Impasse: If the (ideal) capacitor is indeed linear, but observations of this say otherwise, what gives?

Points to (more questions):

- Different way of modelling the higher order nonlinearities.
- Are the higher order nonlinearities nonlinearly related?
- Are those nonlinear relations also related to each other in a nonlinear manner?
- (etc…)

Preamble & Introduction

Two parts to this workshop:

1. Presentation
2. Code Walkthrough

Link to Github repo: github.com/algoravioli/AES_PANN_WORKSHOP

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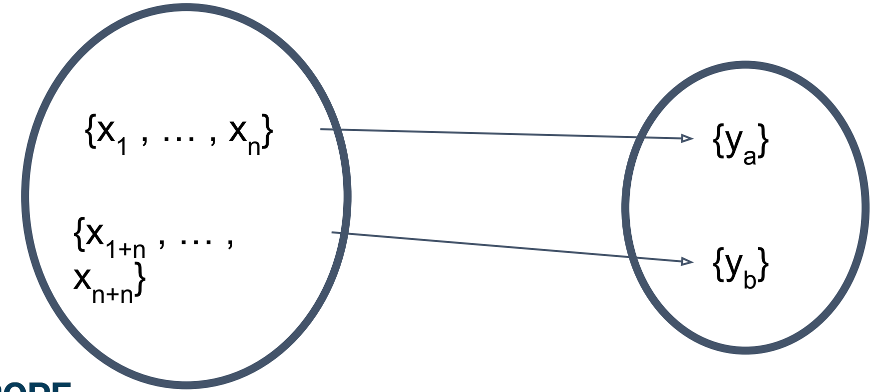
Table of Contents

Pre-motivation

We tend to think of neural networks as function approximators, but really, this is 99% ~~true~~, only if the function is memoizable

- e.g non time-invariant functions not memoizable
- e.g function is trained in one discretization spacing, but inferenced in another



Kovachki, Nikola, et al. "Neural operator: Learning maps between function spaces." *arXiv preprint arXiv:2108.08481* (2021).

Preface

- The use of the term “nonlinearity” refers to the property of linearity of a given system.
- When the system is excited with input u_1 and results in an output of z_1
  - and when excited with input u_2 resulting in output z_2– the resulting output of input u_1 + u_2 would be z_1 + z_2 (additivity)

  - The resulting output of input λu_1 would be λz_1 (homogeneity)

Motivation

- Problems that present themselves in this form:

$$\exists F, \forall i \ni F\big(\mathbb{F}(x_i)\big) = y_i \quad \text{OR} \quad F \circ \mathbb{F}(x_i) = y_i$$

- Not all input variables can be collected – Hidden Variables
- Not all interactions between variables can be predicted – Hidden States

$$F(x) : F(x; H_1, \ldots, H_k \mid S_1, , S_m), k \leq m$$

Motivation

- Since each *H* and *S* will cause the function F to alter slightly, the function F can be represented as carrying/containing a family of functions *F*

$$F \circ f_{H \in R^E \mapsto H \in R^F \mid S}(x), E \neq F, S \in R^{\Theta} \quad \text{where } R^{\Theta} \text{ is parameter space.}$$

$$\mathcal{F} = \left\{ \begin{array}{ccc} f_{(H1 \mapsto H2 \mid S_1)} & := & f(x \mid H_1, H_2), \\ f_{(H_{\dots} \mapsto H2 \mid S_1)} & := & f(x \mid H_{\dots}, H_2), \\ f_{(H_n \mapsto H2 \mid S_1)} & := & f(x \mid H_{\dots}, H_2), \\ \dots & \dots & \dots, \\ f_{(H_{\dots} \mapsto H_{\dots} \mid S_1)} & := & f(x \mid H_{\dots}, H_{\dots}), \\ f_{(H_{\dots} \mapsto H_n \mid S_1)} & := & f(x \mid H_{\dots}, H_{\dots}), \\ \dots & \dots & \dots, \\ f_{(H_{\dots} \mapsto H_{\dots} \mid S_{\dots})} & := & f(x \mid H_{\dots}, H_{\dots}), \\ f_{(H_{\dots} \mapsto H_{\dots} \mid S_m)} & := & f(x \mid H_{\dots}, H_{\dots}), \\ f_{(H_{n-1} \mapsto H_n \mid S_m)} & := & f(x \mid H_{n-1}, H_n) \end{array} \right\} \in F : x \mapsto y$$

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Example of Nonlinearity

- Given a system: $$\mathbf{y}_n = \mathbf{f}(\mathbf{x}_n; H; S) = \begin{cases} x + \alpha \mid S_{1\ldots m}, & \text{for } k_1 \leq x \leq 1 \\ x + \beta \mid S_{1\ldots m}, & \text{for } k_2 \leq x \leq k_3 \\ x + \gamma \mid S_{1\ldots m}, & \text{for } -1 \leq x \leq k_4 \end{cases}$$

- The adjustment/perturbation terms (alpha,beta,gamma) are controlled by a set of functions A, B, C

$$\left.\begin{array}{l} \mathbf{A}(\alpha_n, \beta_n, \gamma_n; \alpha_{n-N}, \beta_{n-N}, \gamma_{n-N}; H; S;) \\ \mathbf{B}(\alpha_n, \beta_n, \gamma_n; \alpha_{n-N}, \beta_{n-N}, \gamma_{n-N}; H; S;) \\ \mathbf{C}(\alpha_n, \beta_n, \gamma_n; \alpha_{n-N}, \beta_{n-N}, \gamma_{n-N}; H; S;) \end{array}\right\} = \{\alpha, \beta, \gamma\}$$
$$N = 1, 2, 3, \ldots, N^*$$

- N* denotes the amount of memory a system has.
- The function **f** contains a family $\mathcal{F}$ of functions f that define the relationship of the system's hidden variables and parameters.

Example of Nonlinearity

- For each N in the previous equation, the family $\mathcal{F}$ of functions f alters functions A, B, C at each time step of N:

$$
\begin{aligned}
&N = 1\\
&\left\{\begin{matrix}
f_{(x;H_1,H_{\dots}\,|\,S_{\dots})}\mathbf{:A}_{(\alpha_{n-1}\mapsto\alpha_n)},\mathbf{B}_{(\beta_{n-1}\mapsto\beta_n)},\mathbf{C}_{(\gamma_{n-1}\mapsto\gamma_n)}\\
f_{(x;H_{\dots},H_{\dots}\,|\,S_{\dots})}\mathbf{:A}_{(\alpha_{n-1}\mapsto\alpha_n)},\mathbf{B}_{(\beta_{n-1}\mapsto\beta_n)},\mathbf{C}_{(\gamma_{n-1}\mapsto\gamma_n)}\\
f_{(x;H_{\dots},H_k\,|\,S_{\dots})}\mathbf{:A}_{(\alpha_{n-1}\mapsto\alpha_n)},\mathbf{B}_{(\beta_{n-1}\mapsto\beta_n)},\mathbf{C}_{(\gamma_{n-1}\mapsto\gamma_n)}\\
\dots
\end{matrix}\right\}\\
&N = \dots\\
&\dots\\
&N = N^*\\
&\left\{\begin{matrix}
f_{(x;H_1,H_{\dots}\,|\,S_{\dots})}\mathbf{:A}_{(\alpha_{n-N^*}\mapsto\alpha_n)},\mathbf{B}_{(\beta_{n-N^*}\mapsto\beta_n)},\mathbf{C}_{(\gamma_{n-N^*}\mapsto\gamma_n)}\\
f_{(x;H_{\dots},H_{\dots}\,|\,S_{\dots})}\mathbf{:A}_{(\alpha_{n-N^*}\mapsto\alpha_n)},\mathbf{B}_{(\beta_{n-N^*}\mapsto\beta_n)},\mathbf{C}_{(\gamma_{n-N^*}\mapsto\gamma_n)}\\
f_{(x;H_{\dots},H_k\,|\,S_{\dots})}\mathbf{:A}_{(\alpha_{n-N^*}\mapsto\alpha_n)},\mathbf{B}_{(\beta_{n-N^*}\mapsto\beta_n)},\mathbf{C}_{(\gamma_{n-N^*}\mapsto\gamma_n)}\\
\dots
\end{matrix}\right\}
\end{aligned}
$$

Example of Nonlinearity

- The family $\mathcal{F}$ of functions f collapses to return functions A,B,C that produce the adjustment terms.

$$A = \forall H, \forall S, \sum_{N=1}^{N^*} f(\mathbf{A}_{(\alpha_{n-N} \mapsto \alpha_n)})$$

$$B = \forall H, \forall S, \sum_{N=1}^{N^*} f(\mathbf{B}_{(\beta_{n-N} \mapsto \beta_n)})$$

$$C = \forall H, \forall S, \sum_{N=1}^{N^*} f(\mathbf{C}_{(\gamma_{n-N} \mapsto \gamma_n)})$$

- Graphically, this example can be thought of as a linear-piecewise function that changes slightly at each timestep.

Physics Approximating Neural Networks (PANN)

- Constructing a neural network that takes in a positive integer L, and a number of neurons per layer **p** :
- Let H:

$$\mathbb{H}(\mathbf{x}) \triangleq \left[\sigma \mathbf{w}^{\mathrm{T}} \cdot \mathbf{x} + b\right] \ni,$$

$$\mathbb{H}_{(1,\mathbf{p})} \circ \mathbb{H}_{(0,\mathbf{p})}(\mathbf{x}) = \sigma_1 \mathbf{w}_1^{\mathrm{T}} \left[\sigma_0 \mathbf{w}_0^{\mathrm{T}} \cdot \mathbf{x} + b_0\right| + b_1$$

$$\mathbb{H}_L \circ \mathbb{H}_{L-1} \circ \ldots \circ \mathbb{H}_1 \circ \mathbb{H}_0(\mathbf{x}) =$$

$$\sigma_L \mathbf{w}_{\mathrm{L}}^{\mathrm{T}} \sigma_{L-1} \mathbf{w}_{\mathrm{L-1}}^{\mathrm{T}} \ldots \sigma_1 \mathbf{w}_1^{\mathrm{T}} \left[\sigma_2 \mathbf{w}_0^{\mathrm{T}} \cdot \mathbf{x} + b_0\right| + b_1$$

- With shorthand operator:

$$\mathbb{H} \circ \left[\sum_{0}^{(L,\mathbf{p})}\right](\mathbf{x}) = \mathbb{H}_L \circ \mathbb{H}_{L-1} \circ \ldots \circ \mathbb{H}_1 \circ \mathbb{H}_0(\mathbf{x})$$

Physics Approximating Neural Networks

- We want to achieve the mapping M

$$\mathbb{H} \circ \left[\sum_{0}^{(L,\mathbf{p})} \right] (x) = \hat{y} \approx y = F \circ \mathbb{F}(x) \qquad M\left(\mathbb{H} \circ \left[\sum_{0}^{(L,\mathbf{p})} \right] \mapsto F \right)$$

- The Error of the network N approaching 0 as the indicator of the mapping M approaching ground truth

$$Error(\mathbb{N}(L,\mathbf{p})) \to 0 \Leftrightarrow M \to F \circ \mathbb{F}(x)$$

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Physics Approximating Neural Networks

- We can construct an "approximation layer" F*, and embed this layer within the network.

$$\mathbb{H} \circ \left[\sum_{0}^{(l \to L, \mathbf{p})} \right] \circ \mathbb{F}^* \circ \left[\sum_{0}^{(0 \to l, \mathbf{p})} \right] (x) = \hat{y} \approx y = F \circ \mathbb{F}(\mathbf{x})$$

- To give an example:

$$\boxed{G(x) = \frac{T + R\ddot{T}}{2\sqrt{x}} \qquad \gamma(x) = w(x) + b} \longrightarrow \quad G^*(x) = \sigma \frac{\gamma(x_1) + \gamma(x_2)\gamma(x_3)}{2\sqrt{x}}$$

- Variables are replaced with latent network inputs, unless the variable is the input to the model (x)

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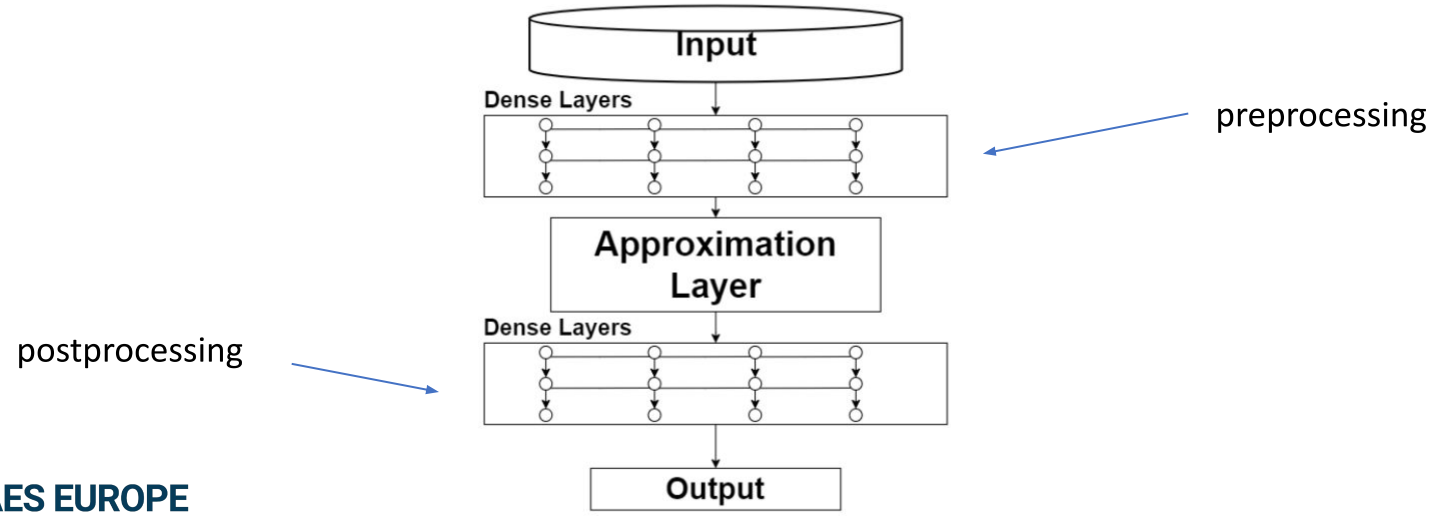
Physics Approximating Neural Networks

- In this paper the preprocessing layer was a 2 x 8 (2 layers by 8 neurons) and a postprocessing layer was 5 x 8 (5 layers by 8 neurons)



Problem Setup and Nonlinear Capacitor Model

- Van der Veen and Van Maanen's model of a nonlinear capacitor

$$V_t^2 = \frac{2x_t^2.(m\ddot{x}+k_1\dot{x}+k_2[x_t-x_0])}{\epsilon_C.A}$$

- Since there is finite stiffness and an acting damping constant, the plates are always in a non-uniform state

- Possibility of superposition of a multitude of constants K_1 and K_2

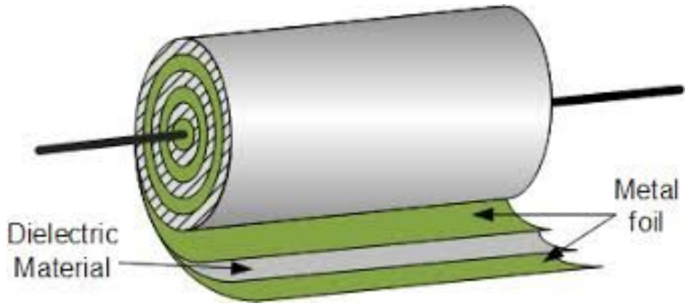
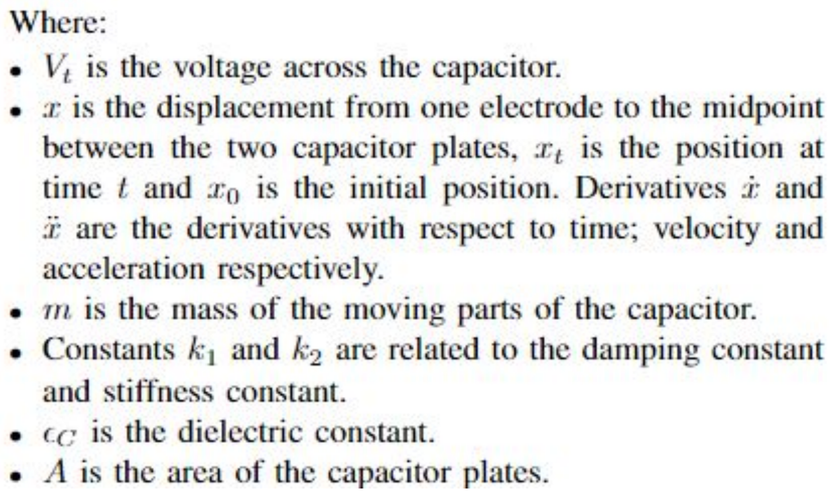


Image: https://www.electronics-tutorials.ws/capacitor/cap_2.html



Where:

- V_t is the voltage across the capacitor.
- x is the displacement from one electrode to the midpoint between the two capacitor plates, x_t is the position at time t and x_0 is the initial position. Derivatives $\dot{x}$ and $\ddot{x}$ are the derivatives with respect to time; velocity and acceleration respectively.
- m is the mass of the moving parts of the capacitor.
- Constants k_1 and k_2 are related to the damping constant and stiffness constant.
- ϵ_C is the dielectric constant.
- A is the area of the capacitor plates.

Problem Setup and Nonlinear Capacitor Model

- Given at time t, an "interactions tensor" I_t:

$$\text{Forces } \mathbf{F_t} = \begin{matrix} F_{1,1} & F_{1,2} & \dots \\ \vdots & \ddots & \\ F_{n,1} & & F_{n,n} \end{matrix} \qquad \text{Stiffness } \mathbf{K2} = \begin{matrix} k2_{1,1} & k2_{1,2} & \dots \\ \vdots & \ddots & \\ k2_{n,1} & & k2_{n,n} \end{matrix}$$

$$\text{Damping } \mathbf{K1} = \begin{matrix} k1_{1,1} & k1_{1,2} & \dots \\ \vdots & \ddots & \\ k1_{n,1} & & k1_{n,n} \end{matrix} \qquad \text{Mass } \mathbf{m} = \begin{matrix} m2_{1,1} & m2_{1,2} & \dots \\ \vdots & \ddots & \\ m2_{n,1} & & m2_{n,n} \end{matrix}$$

- The capacitor function C collapses I_t to return the voltage across the capacitor

$$C(\mathbf{I_t}) = C(\mathbf{F_t} \otimes \mathbf{K1} \otimes \dots \mathbf{K2} \otimes \mathbf{m}) = V_t$$

... and all other interaction tensors.

- The family $\mathcal{F}$ takes the function C(I_t) as an input and returns the terms required.

$$F(\mathcal{F}(C_n(I_t) \otimes \cdots \otimes C_{n-N^*}(I_t)) \in F, x) = y \qquad \text{where } n = 0, 1, \dots, N^*$$

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Problem Setup and Nonlinear Capacitor Model

- The Physics Approximating Neural Network can then be constructed thusly:

$$\mathbb{H} \circ \left[\sum_{0}^{(l \to L, \mathrm{p})} \right] \circ \mathbb{F}^* \circ \left[\sum_{0}^{(0 \to l, \mathrm{p})} \right] (x) \approx F(\mathcal{F}(.) \in F, x, H, S)$$

- Which has an F* of the form:

$$\mathbb{F}^* = \frac{2(\gamma(x_1))^2(\gamma(x_2)\gamma(x_3)+\gamma(x_4)\gamma(x_5)+\gamma(x_6)(\gamma(x_7)))}{\gamma(x_8)\gamma(x_9)}$$

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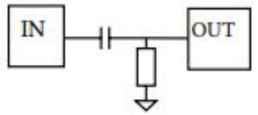
Results

- The dataset used was from the Fraunhofer IDMT-SMT-Guitar Dataset.
- The data was distorted as previous studies show that a distorted signal is associated with or is more correlated to more capacitor nonlinearities.
- A file containing about 2e7 samples was used to train the model with 80:20 split.
- The waveshaper algorithm used was:
- The output of the system was recorded at 44.1 kHz with a Universal Audio Apollo Twin Quad
- The HPF frequency was around 9 Hz, far below the region of interest

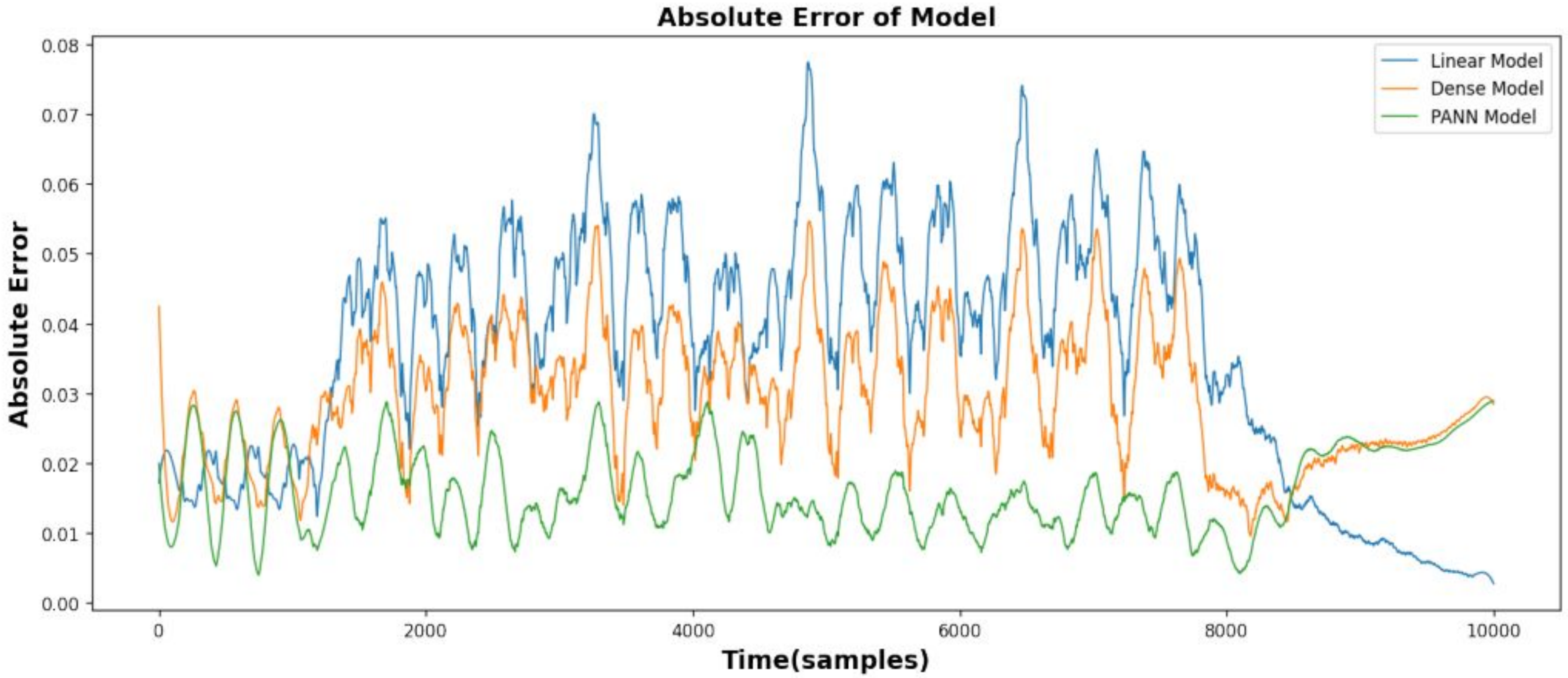
$$x = \begin{cases} 0.4tanh(25x), & \text{if sign}(x) = 1. \\ 0.4tanh(10x), & \text{if sign}(x) = -1. \\ 0, & \text{if sign}(x) = 0. \end{cases}$$



Clarke, Christopher Johann, Balamurali BT, and Jer-Ming Chen. "Characterising non-linear behaviour of coupling capacitors through audio feature analysis and machine learning." Audio Engineering Society Convention 150. Audio Engineering Society, 2021.

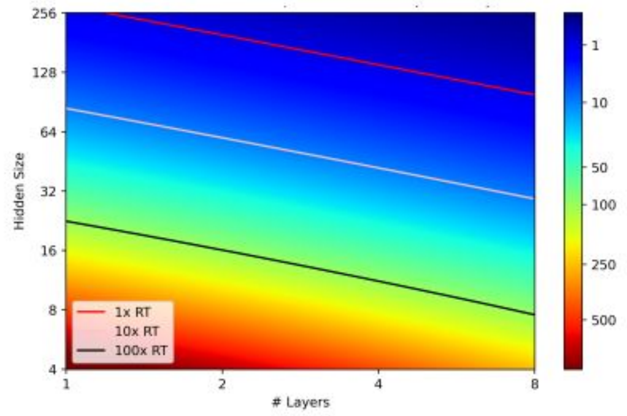
Results

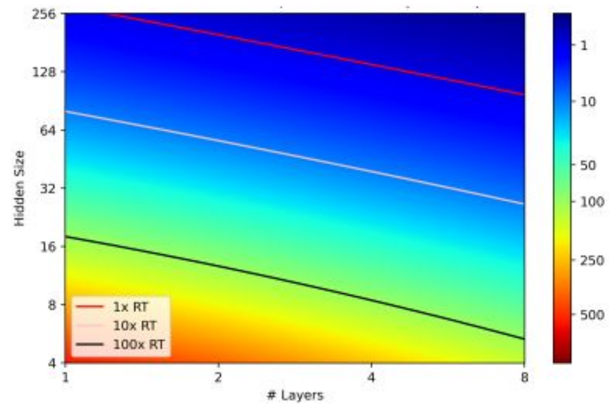
- The PANN model was trained alongside a Dense Model made of only densely-connected layers. This dense model was the same size as the PANN at 8 x 8. A linear model was also used to compare the results.



Conclusion

- The transposability to a realtime deployment is possible with SIMD (possibly with RTNeural)
- The model could be improved through the use of Model Discovery using Genetic Algorithms or Network Architecture Optimizers.
- The methodology displayed shows the possibilities of PANN, ultimately achieving more accuracy with the same computational resources.
- Encourages the use of more informed/empirical models.





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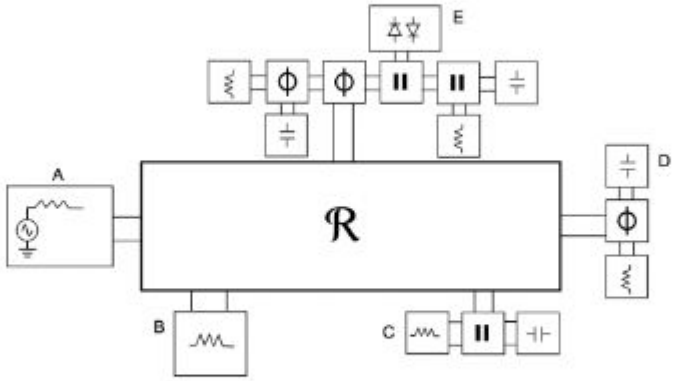
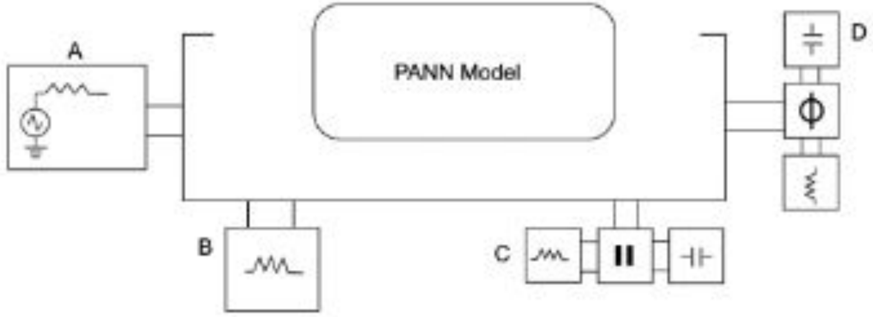
Code Walkthrough

Some experimental work

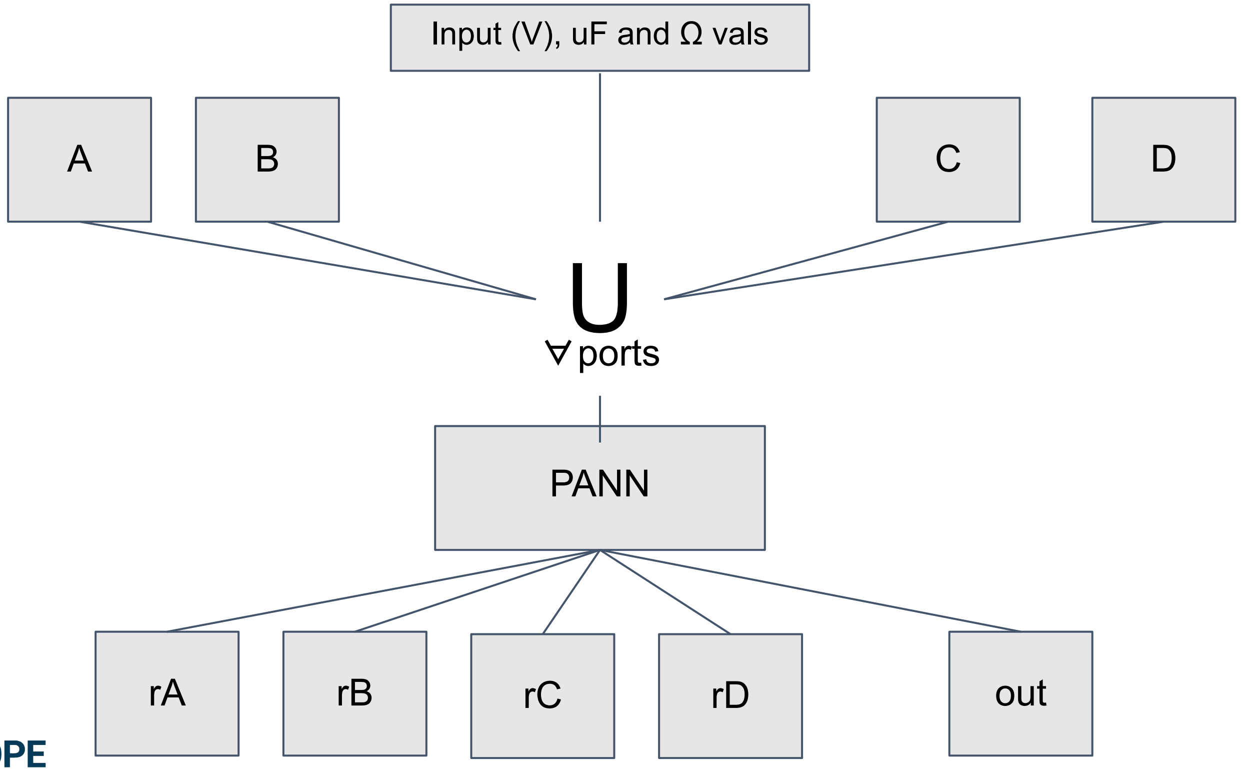


WDF graph of Boss DS-1



R-adaptor replaced with PANN

Some experimental work - MIMO-PANN



Thank You

Please contact me at christopher_clarke@mymail.sutd.edu.sg
for any further questions.