With the depth to the neutral axis known, the assumption of yielding of the tension steel can be checked. From similar triangles in the linear strain distribution in Fig. 4-18b, the following expression can be derived:

$$\frac{\varepsilon_s}{d - c} = \frac{\varepsilon_{cu}}{c}$$

$$\varepsilon_s = \left(\frac{d - c}{c}\right)\varepsilon_{cu} \tag{4-18}$$

To confirm the assumption that the section is under-reinforced and the steel is yielding, show

$$\varepsilon_s \geq \varepsilon_y = \frac{f_y}{E_s} = \frac{f_y\,(\text{ksi})}{29{,}000\ \text{ksi}} \tag{4-19}$$

Once this assumption is confirmed, the nominal-section moment capacity can be calculated by referring back to the section forces in Fig. 4-18d. The compression force is acting at the middepth of the stress block, and the tension force is acting at a distance d from the extreme compression fiber. Thus, the nominal moment strength can be expressed as either the tension force or the compression force multiplied by the moment arm, $d - a/2$:

$$M_n = T\left(d - \frac{a}{2}\right) = C_c\left(d - \frac{a}{2}\right) \tag{4-20}$$

For singly reinforced sections, it is more common to express the nominal moment strength using the definition of the tension force as

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \tag{4-21}$$

This simple expression can be used for all singly reinforced sections with a rectangular (constant width) compression zone after it has been confirmed that the tension steel is yielding. The same fundamental process as used here to determine M_n for singly reinforced rectangular sections will be applied to other types of beam sections in the following parts of this chapter. However, the reader is urged to concentrate on the process rather than the resulting equations. If the process is understood, it can be applied to any beam section that may be encountered.

Example 4-1 Calculation of M_n for a Singly Reinforced Rectangular Section

For the beam shown in Fig. 4-19a, calculate M_n and confirm that the area of tension steel exceeds the required minimum steel area given by Eq. (4-11). The beam section is made of concrete with a compressive strength, $f'_c = 4000$ psi, and has four No. 8 bars with a yield strength of $f_y = 60$ ksi.

For this beam with a single layer of tension reinforcement, it is reasonable to assume that the effective flexural depth, d, is approximately equal to the total beam depth minus 2.5 in. This accounts for a typical concrete clear cover of 1.5 in., the diameter of the stirrup (typically a No. 3 or No. 4 bar) and half the diameter of the beam longitudinal reinforcement. Depending on the sizes of the stirrup and longitudinal bar, the dimension to the center of the steel layer will vary slightly, but the use of 2.5 in. will be accurate enough for most design work unless adjustments in reinforcement location are required to avoid rebar interference at connections with other members. Small bars are often used in the compression zone to hold the stirrups in position, but these bars normally are ignored unless they were specifically designed to serve as compression-zone reinforcement.

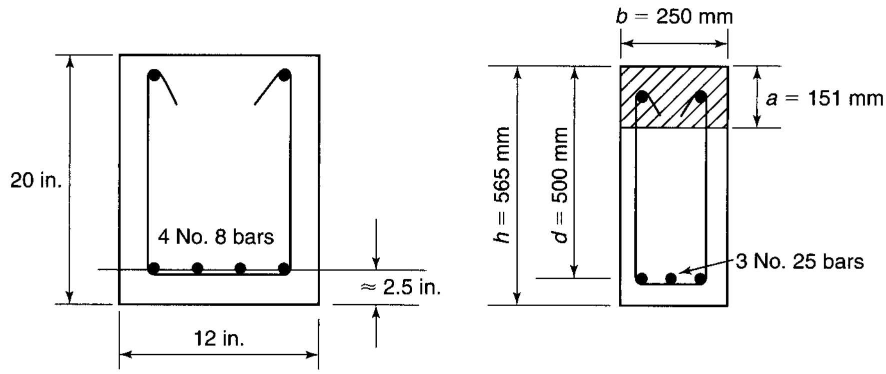


Fig. 4-19
(a) Beam sections for Examples 4-1 and 4-1M.

1. **Following the procedure summarized in Fig. 4-18, assume that the steel strain exceeds the yield strain, and thus, the stress f_s in the tension reinforcement equals the yield strength, f_y. Compute the steel tension force:**

$$A_s = 4 \text{ No. 8 bars} = 4 \times 0.79 \text{ in.}^2 = 3.16 \text{ in.}^2$$

$$T = A_s f_y = 3.16 \text{ in.}^2 \times 60 \text{ ksi} = 190 \text{ kips}$$

The assumption that $\varepsilon_s > \varepsilon_y$ will be checked in step 3. This assumption generally should be true, because the ACI Code requires that the steel area be small enough in beam sections such that the steel will yield before the concrete reaches the maximum useable compression strain.

2. **Compute the area of the compression stress block so that $C_c = T$.** This is done for the equivalent rectangular stress block shown in Fig. 4-16a. The stress block consists of a uniform stress of 0.85 f'_c distributed over a depth $a = \beta_1 c$ which is measured from the extreme compression fiber. For $f'_c = 4000$ psi, Eq. (4-14a) gives $\beta_1 = 0.85$. Using Eq. (4-16), which was developed from section equilibrium,

$$a = \beta_1 c = \frac{A_s f_y}{0.85\, f'_c\, b} = \frac{190 \text{ kips}}{0.85 \times 4 \text{ ksi} \times 12 \text{ in.}} = 4.66 \text{ in.}$$

3. **Check that the tension steel is yielding.** The yield strain is

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29{,}000 \text{ ksi}} = 0.00207$$

From above, $c = a/\beta_1 = 5.48$ in. Now, use strain compatibility, as expressed in Eq. (4-18), to find

$$\varepsilon_s = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{17.5 - 5.48}{5.48}\right)0.003 = 0.00658$$

Clearly, ε_s exceeds ε_y, so the assumption used above to establish section equilibrium is confirmed. Remember that you *must make this check* before proceeding to calculate the section nominal moment strength.

4. **Compute M_n.** Using Eq. (4-21), which was derived for sections with constant width compression zones,

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 190 \text{ kips}\left(17.5 \text{ in.} - \frac{4.66 \text{ in.}}{2}\right)$$

$$M_n = 2880 \text{ k-in.} = 240 \text{ k-ft}$$

5. Confirm that tension steel area exceeds $A_{s,\min}$. For Eq. (4-11), there is a requirement to use the larger of $3\sqrt{f_c'}$ or 200 psi in the numerator. In this case, $3\sqrt{4000 \text{ psi}} = 190$ psi, so use 200 psi. Thus,

$$A_{s,\min} = \frac{200 \text{ psi}}{f_y} b_w d = \frac{200 \text{ psi}}{60{,}000 \text{ psi}} \times 12 \text{ in.} \times 17.5 \text{ in.} = 0.70 \text{ in.}^2$$

A_s exceeds $A_{s,\min}$, so this section satisfies the ACI Code requirement for minimum tension reinforcement. ■

Example 4-1M Analysis of Singly Reinforced Beams: Tension Steel Yielding—SI Units

Compute the nominal moment strength, M_n, of a beam (Fig. 4-19b) with $f_c' = 20$ MPa $(\beta_1 = 0.85)$, $f_y = 420$ MPa, $b = 250$ mm, $d = 500$ mm, and three No. 25 bars (Table A-1M) giving $A_s = 3 \times 510 = 1530 \text{ mm}^2$. Note that the difference between the total section depth, h, and the effect depth, d, is 65 mm, which is a typical value for beam sections designed with metric dimensions.

1. Compute a (assuming the tension steel is yielding).

$$\begin{aligned} a &= \frac{A_s f_y}{0.85 f_c' b} \\ &= \frac{1530 \text{ mm}^2 \times 420 \text{ MPa}}{0.85 \times 20 \text{ MPa} \times 250 \text{ mm}} = 151 \text{ mm} \end{aligned}$$

Therefore, $c = a/\beta_1 = 151/0.85 = 178$ mm.

2. Check whether the tension steel is yielding. The yield strain for the reinforcing steel is

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{420 \text{ MPa}}{200,000 \text{ MPa}} = 0.0021$$

From Eq. (4-18),

$$\varepsilon_s = \left(\frac{500 \text{ mm} - 178 \text{ mm}}{178 \text{ mm}}\right) \times 0.003 = 0.00543$$

Thus, the steel is yielding as assumed in step 1.

3. Compute the nominal moment strength, M_n. From Eq. (4-21), M_n is (where 1 MPa = 1 N/mm^2)

$$\begin{aligned} M_n = A_s f_y\left(d - \frac{a}{2}\right) &= 1530 \text{ mm}^2 \times 420 \text{ N/mm}^2\left(500 - \frac{151}{2}\right) \text{mm} \\ &= 273 \times 10^6 \text{ N-mm} = 273 \text{ kN-m} \end{aligned}$$

Therefore, the design or factored moment strength, ϕM_n, of this beam is $0.9 \times 273 = 246$ kN-m.

4. Confirm that the tension steel area exceeds $A_{s,\min}$. For the given concrete strength of 20 MPa, the quantity $0.25\sqrt{f_c'} = 1.12$ MPa, which is less than 1.4 MPa. Therefore, the second part of Eq. (4-11M) governs for $A_{s,\min}$ as

$$A_{s,\min} = \frac{1.4 b_w d}{f_y} = \frac{1.4 \text{ MPa} \times 250 \text{ mm} \times 500 \text{ mm}}{420 \text{ MPa}} = 417 \text{ mm}^2$$

A_s exceeds $A_{s,\min}$, so this section satisfies the ACI Code requirement for minimum tension reinforcement. ■

Example 4-2 Calculation of the Nominal Moment Strength for an Irregular Cross Section

The beam shown in Fig. 4-20 is made of concrete with a compressive strength, $f'_c = 3000$ psi and has three No. 8 bars with a yield strength, $f_y = 60$ ksi. This example is presented to demonstrate the general use of strain compatibility and section equilibrium equations for any type of beam section.

1. Initially, assume that the stress f_s in the tension reinforcement equals the yield strength f_y, and compute the tension force $T = A_s f_y$:

$$A_s = 3 \text{ No. 8 bars} = 3 \times 0.79 \text{ in.}^2 = 2.37 \text{ in.}^2$$

$$T = A_s f_y = 2.37 \text{ in.}^2 \times 60 \text{ ksi} = 142 \text{ kips}$$

The assumption that the tension steel is yielding will be checked in step 3.

2. Compute the area of the compression stress block so that $C_c = T$. As in the prior problem, this is done using the equivalent rectangular stress block shown in Fig. 4-16a. The stress block consists of a uniform stress of $0.85\, f'_c$ distributed over a depth $a = \beta_1 c$, which is measured from the extreme compression fiber. For $f'_c = 3000$ psi, Eq. (4-14a) gives $\beta_1 = 0.85$. The magnitude of the compression force is obtained from equilibrium as

$$C_c = T = 142 \text{ kips} = 142{,}000 \text{ lbs}$$

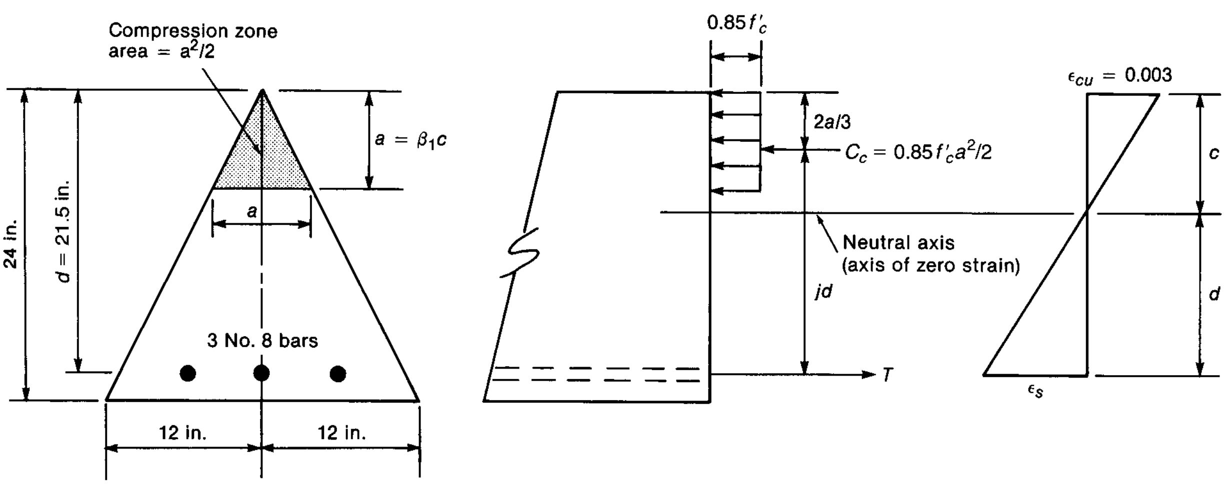
By the geometry of this particular triangular beam, shown in Fig. 4-20, if the depth of the compression zone is a, the width at the bottom of the compression zone is also a, and the area is $a^2/2$. This is, of course, true only for a beam of this particular triangular shape.

Therefore, $C_c = (0.85\, f'_c)(a^2/2)$ and

$$a = \sqrt{\frac{142{,}000 \text{ lb} \times 2}{0.85 \times 3000 \text{ psi}}} = 10.6 \text{ in.}$$

3. Check whether $f_s = f_y$. This is done by using strain compatibility. The strain distribution at ultimate is shown in Fig. 4-20c. As before,

$$c = \frac{a}{\beta_1} = \frac{10.6 \text{ in.}}{0.85} = 12.4 \text{ in.}$$



(a) Cross section. (b) Elevation. (c) Strain distribution.

Fig. 4-20
Analysis of arbitrary cross section—Example 4-2.