# Example 4-1: Nominal Moment Strength Calculation for a Singly Reinforced Concrete Beam

## Introduction

This document provides a detailed, step-by-step breakdown of the example problem (referenced as Fig. 4-19a). The goal is to calculate the nominal moment strength $M_n$ for the beam and confirm that the area of tension steel exceeds the required minimum steel area as per Equation (4-11) from the relevant design code (specifically ACI 318). All calculations are performed without skipping any micro-steps, including unit conversions and intermediate arithmetic operations.

**Problem statement (paraphrased from the provided excerpt).** The task is to compute $M_n$ for the singly reinforced beam and verify that the provided tension reinforcement area exceeds the minimum required by code.

**Source excerpt (short quote).** "Calculate $M_n$ and confirm that the area of tension steel exceeds the required minimum steel area."

The beam is a rectangular section made of concrete with compressive strength $f'_c = 4000$ psi, reinforced with four No. 8 bars in tension having yield strength $f_y = 60$ ksi. The beam dimensions are width $b = 12$ in. and total height $h = 20$ in. The effective depth $d$ is approximated as $h - 2.5$ in. to account for concrete cover, stirrup diameter, and half the longitudinal bar diameter.

## Given Data

- **Concrete compressive strength:** $f'_c = 4000$ psi
- **Steel yield strength:** $f_y = 60$ ksi $= 60,000$ psi
- **Beam width:** $b = 12$ in.
- **Beam total height:** $h = 20$ in.
- **Effective depth (assumed):** $d = h - 2.5 = 20 - 2.5 = 17.5$ in.

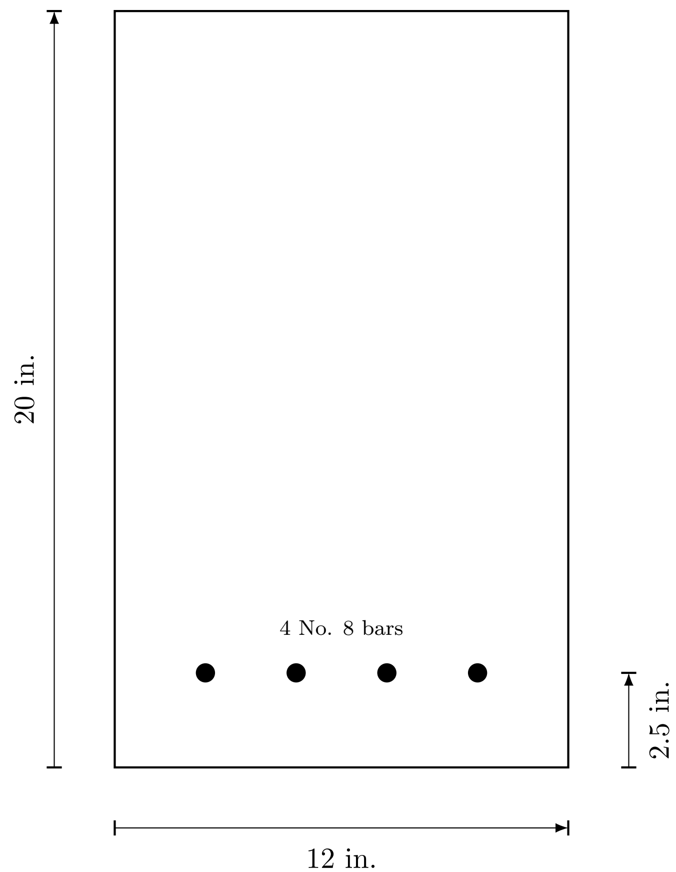


Figure 1: Beam cross-section used in Example 4-1 (redrawn from the provided image).

- **Tension reinforcement:** 4 No. 8 bars
- **Diameter of No. 8 bar:** 1.0 in. (standard ASTM A615/A706 bar size)
- **Area of one No. 8 bar (tabulated):** $A_{bar} = 0.79$ in$^2$.
- **Total tension steel area:**

$$A_s = 4 \times 0.79 = 3.16 \text{ in}^2$$

- **Compression reinforcement:** Ignored (not designed for compression resistance).
- **Rectangular stress block factor:** $\beta_1 = 0.85$ (for $f'_c = 4000$ psi).
- **Minimum steel area rule:** $A_{s,min} = \max\left(\frac{3\sqrt{f'_c}}{f_y}bd, \frac{200}{f_y}bd\right)$

**Note on Effective Depth $d$:** The approximation of 2.5 in. accounts for:

- Clear concrete cover: 1.5 in.
- Stirrup diameter: $\approx 0.5$ in.
- Half the longitudinal bar diameter: 0.5 in.

Total: $1.5 + 0.5 + 0.5 = 2.5$ in.

# Step 1: Confirm Tension Steel Area Exceeds Minimum Required

## Step 1.1: Calculate $\rho_{min}$ Using Equation (4-11)

The minimum reinforcement ratio $\rho_{min}$ is the maximum of two values:

1. $\frac{3\sqrt{f'_c}}{f_y}$
2. $\frac{200}{f_y}$ (in psi units)

**Micro-Calculation for First Term:** $\frac{3\sqrt{f'_c}}{f_y}$

- Calculate $\sqrt{f'_c} = \sqrt{4000} \approx 63.2456$.
- $3 \times 63.2456 = 189.7368$.
- $\frac{189.7368}{60{,}000} = 0.00316228$.

**Micro-Calculation for Second Term:** $\frac{200}{f_y}$

$$\frac{200}{60,000} = 0.00333333$$

**Select** $\rho_{min}$

$$\rho_{min} = \max(0.00316228, 0.00333333) = 0.00333333$$

## Step 1.2: Calculate Minimum Steel Area $A_{s,min}$

$$A_{s,min} = \rho_{min} \times b \times d$$

- $b \times d = 12 \times 17.5 = 210$ in$^2$
- $A_{s,min} = 0.00333333 \times 210 = 0.6999993 \approx 0.70$ in$^2$

## Step 1.3: Compare Actual $A_s$ with $A_{s,min}$

- Actual $A_s = 3.16$ in$^2$.
- Since $3.16 > 0.70$, the tension steel area exceeds the minimum required.
- Reinforcement ratio $\rho = \frac{A_s}{bd} = \frac{3.16}{210} = 0.015048$, which is greater than $\rho_{min} = 0.003333$.

# Step 2: Calculate Nominal Moment Strength $M_n$

For a singly reinforced beam, utilizing the rectangular stress block assumption (Whitney block):

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

where $a$ is the depth of the equivalent rectangular stress block:

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

## Step 2.1: Calculate Depth of Stress Block $a$

**Micro-Calculation for Numerator:** $A_s f_y$

$$A_s f_y = 3.16 \times 60,000 = 189,600 \text{ lb}$$

**Micro-Calculation for Denominator:** $0.85 f'_c b$

$$0.85 \times 4000 \times 12 = 3,400 \times 12 = 40,800 \text{ lb/in}$$

**Calculate** $a$

$$= \frac{189,600}{40,800} \approx 4.6471 \text{ in.}$$

## Step 2.2: Calculate Lever Arm $d - \frac{a}{2}$

- $\frac{a}{2} = \frac{4.6471}{2} = 2.3236$ in.
- $d - \frac{a}{2} = 17.5 - 2.3236 = 15.1764$ in.

## Step 2.3: Calculate $M_n$ in in.-lb

$$M_n = 189,600 \times 15.1764 = 2,877,445.44 \text{ in.-lb}$$

## Step 2.4: Convert $M_n$ to ft-kip

- Convert to ft-lb (divide by 12): $2,877,445.44/12 = 239,787.12$ ft-lb.
- Convert to ft-kip (divide by 1000): $239,787.12/1000 = 239.78712$ ft-kip.
- Rounded: **240 ft-kip**.

# Step 3: Verify Assumptions and Additional Notes

- **Strain compatibility (qualitative check):** The section is expected to be under-reinforced because the provided steel ratio ($\rho \approx 0.015$) is modest for the given section; thus tension steel yielding is the likely controlling behavior for this example.
- **Compression Zone Bars:** Ignored as per problem statement.
- **Accuracy of $d$:** The 2.5 in. assumption is sufficient. Using a No. 3 stirrup would result in $d \approx 17.625$ in., slightly increasing capacity, but 2.5 is conservative/standard for this example.
- **Units:** All consistency checks passed.